Algebraic Number Theory

(PARI-GP version 2.17.3)

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$	`Qfb(a,b,c)` or `Qfb([a,b,c])`
reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$)	`qfbred(x,{flag},{D},{l},{s})`
return $[y, g]$, $g \in \mathrm{SL}_2(\mathbf{Z})$, $y = g \cdot x$ reduced	`qfbredsl2(x)`
composition of forms	`x*y` or `qfbnucomp(x,y,l)`
n-th power of form	`x^n` or `qfbnupow(x,n)`
composition	`qfbcomp(x,y)`
... without reduction	`qfbcompraw(x,y)`
n-th power	`qfbpow(x,n)`
... without reduction	`qfbpowraw(x,n)`
prime form of disc. x above prime p	`qfbprimeform(x,p)`
class number of disc. x	`qfbclassno(x)`
Hurwitz class number of disc. x	`qfbhclassno(x)`
solve $Q(x, y) = n$ in integers	`qfbsolve(Q,n)`
solve $x^2 + Dy^2 = p$, p prime	`qfbcornacchia(D,p)`
... $x^2 + Dy^2 = 4p$, p prime	`qfbcornacchia(D,4*p)`

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$	`quadgen(x)`
minimal polynomial of ω	`quadpoly(x)`
discriminant of $\mathbf{Q}(\sqrt{x})$	`quaddisc(x)`
regulator of real quadratic field	`quadregulator(x)`
fundamental unit in O_D, $D > 0$	`quadunit(D,{'w})`
norm of fundamental unit in O_D	`quadunitnorm(D)`
index of $O_{Df^2}^\times$ in $O_D^\times$	`quadunitindex(D,f)`
class group of $\mathbf{Q}(\sqrt{D})$	`quadclassunit(D,{flag},{t})`
Hilbert class field of $\mathbf{Q}(\sqrt{D})$	`quadhilbert(D,{flag})`
... using specific class invariant ($D < 0$)	`polclass(D,{inv})`
test if T is `polclass(D)`; if so return D	`polisclass(T)`
ray class field modulo f of $\mathbf{Q}(\sqrt{D})$	`quadray(D,f,{flag})`

General Number Fields: Initializations

The number field $K = \mathbf{Q}[X]/(f)$ is given by irreducible $f \in \mathbf{Q}[X]$. We denote $\theta = \bar{X}$ the canonical root of f in K. A *nf* structure contains a maximal order and allows operations on elements and ideals. A *bnf* adds class group and units. A *bnr* is attached to ray class groups and class field theory. A *rnf* is attached to relative extensions L/K.

init number field structure *nf*	`nfinit(f,{flag})`
known integer basis B	`nfinit([f,B])`
order maximal at $vp = [p_1, \ldots, p_k]$	`nfinit([f,vp])`
order maximal at all $p \le P$	`nfinit([f,P])`
certify maximal order	`nfcertify(nf)`

nf members:

a monic $F \in \mathbf{Z}[X]$ defining K	`nf.pol`
number of real/complex places	`nf.r1/r2/sign`
discriminant of *nf*	`nf.disc`
primes ramified in *nf*	`nf.p`
T_2 matrix	`nf.t2`
complex roots of F	`nf.roots`
integral basis of $\mathbf{Z}_K$ as powers of θ	`nf.zk`
different/codifferent	`nf.diff`, `nf.codiff`
index $[\mathbf{Z}_K : \mathbf{Z}[X]/(F)]$	`nf.index`
recompute *nf* using current precision	`nfnewprec(nf)`
init relative *rnf* $L = K[Y]/(g)$	`rnfinit(nf,g)`
init *bnf* structure	`bnfinit(f,1)`

bnf members: same as *nf*, plus

underlying *nf*	`bnf.nf`
class group, regulator	`bnf.clgp`, `bnf.reg`
fundamental/torsion units	`bnf.fu`, `bnf.tu`
add S-class group and units, yield *bnfS*	`bnfsunit(bnf,S)`
init class field structure *bnr*	`bnrinit(bnf,m,{flag})`

bnr members: same as *bnf*, plus

underlying *bnf*	`bnr.bnf`
big ideal structure	`bnr.bid`
modulus m	`bnr.mod`
structure of $(\mathbf{Z}_K/m)^*$	`bnr.zkst`

Fields, subfields, embeddings

Defining polynomials, embeddings

(some) number fields with Galois group G	`nflist(G)`
... and $\lvert\mathrm{disc}(K)\rvert = N$ and s complex places	`nflist(G,N,{s})`
... and $a \le \lvert\mathrm{disc}(K)\rvert \le b$	`nflist(G,[a,b],{s})`
smallest poly defining $f = 0$ (slow)	`polredabs(f,{flag})`
small poly defining $f = 0$ (fast)	`polredbest(f,{flag})`
monic integral $g = Cf(x/L)$	`poltomonic(f,{&L})`
random Tschirnhausen transform of f	`poltschirnhaus(f)`
$\mathbf{Q}[t]/(f) \subset \mathbf{Q}[t]/(g)$? Isomorphic?	`nfisincl(f,g)`, `nfisisom`
reverse polmod $a = A(t) \bmod T(t)$	`modreverse(a)`
compositum of $\mathbf{Q}[t]/(f)$, $\mathbf{Q}[t]/(g)$	`polcompositum(f,g,{flag})`
compositum of $K[t]/(f)$, $K[t]/(g)$	`nfcompositum(nf,f,g,{flag})`
splitting field of K (degree divides d)	`nfsplitting(nf,{d})`
signs of real embeddings of x	`nfeltsign(nf,x,{pl})`
complex embeddings of x	`nfeltembed(nf,x,{pl})`
$T \in K[t]$, # of real roots of $\sigma(T) \in R[t]$	`nfpolsturm(nf,T,{pl})`
absolute Weil height	`nfweilheight(nf,v)`

Subfields, polynomial factorization

subfields (of degree d) of *nf*	`nfsubfields(nf,{d})`
maximal subfields of *nf*	`nfsubfieldsmax(nf)`
maximal CM subfield of *nf*	`nfsubfieldscm(nf)`
$K_d \subset \mathbf{Q}(\zeta_n)$, using Gaussian periods	`polsubcyclo(n,d,{v})`
... using class field theory	`polsubcyclofast(n,d)`
roots of unity in *nf*	`nfrootsof1(nf)`
roots of g belonging to *nf*	`nfroots(nf,g)`
factor g in *nf*	`nffactor(nf,g)`

Linear and algebraic relations

poly of degree $\le k$ with root $x \in \mathbf{C}$ or $\mathbf{Q}_p$	`algdep(x,k)`
alg. dep. with pol. coeffs for series s	`seralgdep(s,x,y)`
diff. dep. with pol. coeffs for series s	`serdiffdep(s,x,y)`
small linear rel. on coords of vector x	`lindep(x)`

Basic Number Field Arithmetic (nf)

Number field elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis `nf.zk`).

Basic operations

$x + y$	`nfeltadd(nf,x,y)`
$x \times y$	`nfeltmul(nf,x,y)`
x^n, $n \in \mathbf{Z}$	`nfeltpow(nf,x,n)`
x/y	`nfeltdiv(nf,x,y)`
$q = x\backslash/y :=$ `round(x/y)`	`nfeltdiveuc(nf,x,y)`
$r = x\%y := x - (x\backslash/y)y$	`nfeltmod(nf,x,y)`
... $[q, r]$ as above	`nfeltdivrem(nf,x,y)`
reduce x modulo ideal A	`nfeltreduce(nf,x,A)`
absolute trace $\mathrm{Tr}_{K/\mathbf{Q}}(x)$	`nfelttrace(nf,x)`
absolute norm $\mathrm{N}_{K/\mathbf{Q}}(x)$	`nfeltnorm(nf,x)`
is x a square?	`nfeltissquare(nf,x,{&y})`
... an n-th power?	`nfeltispower(nf,x,n,{&y})`

Multiplicative structure of K^*; $K^*/(K^*)^n$

valuation $v_{\mathfrak{p}}(x)$	`nfeltval(nf,x,pr)`
... write $x = \pi^{v_{\mathfrak{p}}(x)} y$	`nfeltval(nf,x,pr,&y)`
quadratic Hilbert symbol (at $\mathfrak{p}$)	`nfhilbert(nf,a,b,{pr})`
b such that $xb^n = v$ is small	`idealredmodpower(nf,x,n)`

Maximal order and discriminant

integral basis of field $\mathbf{Q}[x]/(f)$	`nfbasis(f)`
field discriminant of $\mathbf{Q}[x]/(f)$	`nfdisc(f)`
... and factorization	`nfdiscfactors(f)`
express x on integer basis	`nfalgtobasis(nf,x)`
express element x as a polmod	`nfbasistoalg(nf,x)`

Hecke Grossencharacters

Let K be a number field and m a modulus. A gchar structure describes the group of Hecke Grossencharacters of K of modulus m and allows computations with these characters. A character χ is described by its components modulo `gc.cyc`.

init gchar structure *gc* for modulus m	`gcharinit(bnf,m,{cm})`

gc members:

underlying *bnf*	`gc.bnf`
modulus	`gc.mod`
elementary divisors (including 0s)	`gc.cyc`
recompute *gc* using current precision	`gcharnewprec(gc)`
evaluate Hecke character *chi* at ideal *id*	`gchareval(gc,chi,id)`
exponent column of *id* in $\mathbf{R}^n$	`gcharideallog(gc,id)`
log representation of ideal *id*	`gcharlog(gc,id)`
... of character χ	`gchardualog(gc,chi)`
exponent vector of χ in $\mathbf{R}^n$	`gcharparameters(gc,chi)`
conductor of χ	`gcharconductor(gc,chi)`
L-function of χ	`lfuncreate([gc,chi])`
local component χ_v of χ	`gcharlocal(gc,chi,v)`
χ s.t. $\chi_v \approx Lchiv[i]$ for $v = Lv[i]$	`gcharidentify(gc,Lv,Lchiv)`
basis of group of algebraic characters	`gcharalgebraic(gc)`
algebraic character of given infinity type	`gcharalgebraic(gc,type)`
is χ algebraic?	`gcharisalgebraic(gc,chi)`

Dedekind Zeta Function ζ_K, Hecke L series

$R = [c, w, h]$ in initialization means we restrict $s \in \mathbf{C}$ to domain $\lvert\Re(s) - c\rvert < w$, $\lvert\Im(s)\rvert < h$; $R = [w, h]$ encodes $[1/2, w, h]$ and $[h]$ encodes $R = [1/2, 0, h]$ (critical line up to height h).

ζ_K as Dirichlet series, $N(I) \le b$	`dirzetak(nf,b)`
init $\zeta_K^{(k)}(s)$ for $k \le n$	`L = lfuninit(bnf,R,{n=0})`
compute $\zeta_K(s)$ (n-th derivative)	`lfun(L,s,{n=0})`
compute $\Lambda_K(s)$ (n-th derivative)	`lfunlambda(L,s,{n=0})`
init $L_K^{(k)}(s, \chi)$ for $k \le n$	`L = lfuninit([bnr,chi],R,{n=0})`
compute $L_K(s, \chi)$ (n-th derivative)	`lfun(L,s,{n})`
Artin root number of K	`bnrrootnumber(bnr,chi,{flag})`
$L(1, \chi)$, for all χ trivial on H	`bnrL1(bnr,{H},{flag})`

Class Groups & Units (bnf, bnr)

Class field theory data $a_1, \{a_2\}$ is usually *bnr* (ray class field), *bnr*, H (congruence subgroup) or *bnr*, χ (character on `bnr.clgp`). Any of these define a unique abelian extension of K.

units / S-units	`bnfunits(bnf,{S})`
remove GRH assumption from *bnf*	`bnfcertify(bnf)`

Algebraic Number Theory

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expo. of ideal x on class gp	`bnfisprincipal(bnf, x, {flag})`
...on ray class gp	`bnrisprincipal(bnr, x, {flag})`
expo. of x on fund. units	`bnfisunit(bnf, x)`
...on S-units, U is `bnfunits(bnf, S)`	`bnfisunit(bnfs, x, U)`
signs of real embeddings of `bnf.fu`	`bnfsignunit(bnf)`
narrow class group	`bnfnarrow(bnf)`

Class Field Theory

ray class number for modulus m	`bnrclassno(bnf, m)`
discriminant of class field	`bnrdisc(a1, {a2})`
ray class numbers, l list of moduli	`bnrclassnolist(bnf, l)`
discriminants of class fields	`bnrdisclist(bnf, l, {arch}, {flag})`
decode output from `bnrdisclist`	`bnfdecodemodule(nf, fa)`
is modulus the conductor?	`bnrisconductor(a1, {a2})`
is class field (bnr, H) Galois over K^G	`bnrisgalois(bnr, G, H)`
action of automorphism on `bnr.gen`	`bnrgaloismatrix(bnr, aut)`
apply `bnrgaloismatrix` M to H	`bnrgaloisapply(bnr, M, H)`
characters on `bnr.clgp` s.t. $\chi(g_i) = e(v_i)$	`bnrchar(bnr, g, {v})`
conductor of character χ	`bnrconductor(bnr, chi)`
conductor of extension	`bnrconductor(a1, {a2}, {flag})`
conductor of extension $K[Y]/(g)$	`rnfconductor(bnf, g)`
canonical projection $\mathrm{Cl}_F \to \mathrm{Cl}_f$, $f \mid F$	`bnrmap`
Artin group of extension $K[Y]/(g)$	`rnfnormgroup(bnr, g)`
subgroups of bnr, index $<= b$	`subgrouplist(bnr, b, {flag})`
compositum as `[bnr,H]`	`bnrcompositum([bnr1, H1], [bnr2, H2])`
class field defined by $H < \mathrm{Cl}_f$	`bnrclassfield(bnr, H)`
...low level equivalent, prime degree	`rnfkummer(bnr, H)`
same, using Stark units (real field)	`bnrstark(bnr, {sub}, {flag})`
Stark unit	`bnrstarkunit(bnr, {sub})`
is a an n-th power in K_v ?	`nfislocalpower(nf, v, a, n)`
cyclic L/K satisf. local conditions	`nfgrunwaldwang(nf, P, D, pl)`

Cyclotomic and Abelian fields theory

An Abelian field F given by a subgroup $H \subset (Z/fZ)^*$ is described by an argument F, e.g. f (for $H = 1$, i.e. $Q(\zeta_f)$) or $[G, H]$, where G is `idealstar(f, 1)`, or a minimal polynomial.

minus class number $h^-(F)$	`subcyclohminus(F)`
...p-part	`subcyclohminus(F, p)`
minus part of Iwasawa polynomials	`subcycloiwasawa(F, p)`
p-Sylow of $\mathrm{Cl}(F)$	`subcyclopclgp(F, p)`

Logarithmic class group

logarithmic ℓ-class group	`bnflog(bnf, ℓ)`
$[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$	`bnflogef(bnf, pr)`
$\exp\deg_F(A)$	`bnflogdegree(bnf, A, ℓ)`
is ℓ-extension L/K locally cyclotomic	`rnfislocalcyclo(rnf)`

Ideals: elements, primes, or matrix of generators in HNF

is id an ideal in nf ?	`nfisideal(nf, id)`
is x principal in bnf ?	`bnfisprincipal(bnf, x)`
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$	`idealtwoelt(nf, x, {a})`
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form	`idealhnf(nf, a, {b})`
norm of ideal x	`idealnorm(nf, x)`
minimum of ideal x (direction v)	`idealmin(nf, x, v)`
LLL-reduce the ideal x (direction v)	`idealred(nf, x, {v})`

Ideal Operations

add ideals x and y	`idealadd(nf, x, y)`
multiply ideals x and y	`idealmul(nf, x, y, {flag})`
intersection of ideal x with Q	`idealdown(nf, x)`
intersection of ideals x and y	`idealintersect(nf, x, y, {flag})`
n-th power of ideal x	`idealpow(nf, x, n, {flag})`
inverse of ideal x	`idealinv(nf, x)`
divide ideal x by y	`idealdiv(nf, x, y, {flag})`
Find $(a, b) \in x \times y$, $a + b = 1$	`idealaddtoone(nf, x, {y})`
coprime integral A, B such that $x = A/B$	`idealnumden(nf, x)`

Primes and Multiplicative Structure

check whether x is a maximal ideal	`idealismaximal(nf, x)`
factor ideal x in $\mathbf{Z}_K$	`idealfactor(nf, x)`
expand ideal factorization in K	`idealfactorback(nf, f, {e})`
is ideal A an n-th power ?	`idealispower(nf, A, n)`
expand elt factorization in K	`nffactorback(nf, f, {e})`
decomposition of prime p in $\mathbf{Z}_K$	`idealprimedec(nf, p)`
valuation of x at prime ideal pr	`idealval(nf, x, pr)`
weak approximation theorem in nf	`idealchinese(nf, x, y)`
$a \in K$, s.t. $v_{\mathfrak{p}}(a) = v_{\mathfrak{p}}(x)$ if $v_{\mathfrak{p}}(x) \neq 0$	`idealappr(nf, x)`
$a \in K$ such that $(a \cdot x, y) = 1$	`idealcoprime(nf, x, y)`
give bid =structure of $(\mathbf{Z}_K/id)^*$	`idealstar(nf, id, {flag})`
structure of $(1 + \mathfrak{p})/(1 + \mathfrak{p}^k)$	`idealprincipalunits(nf, pr, k)`
discrete log of x in $(\mathbf{Z}_K/bid)^*$	`ideallog(nf, x, bid)`
`idealstar` of all ideals of norm $\leq b$	`ideallist(nf, b, {flag})`
add Archimedean places	`ideallistarch(nf, b, {ar}, {flag})`
init `modpr` structure	`nfmodprinit(nf, pr, {v})`
project t to $\mathbf{Z}_K/pr$	`nfmodpr(nf, t, modpr)`
lift from $\mathbf{Z}_K/pr$	`nfmodprlift(nf, t, modpr)`

Galois theory over $\mathbf{Q}$

conjugates of a root θ of nf	`nfgaloisconj(nf, {flag})`
apply Galois automorphism s to x	`nfgaloisapply(nf, s, x)`
Galois group of field $\mathbf{Q}[x]/(f)$	`polgalois(f)`
resolvent field of $\mathbf{Q}[x]/(f)$	`nfresolvent(f)`
initializes a Galois group structure G	`galoisinit(pol, {den})`
...for the splitting field of pol	`galoissplittinginit(pol, {d})`
character table of G	`galoischartable(G)`
conjugacy classes of G	`galoisconjclasses(G)`
$\det(1 - \rho(g)T)$, χ character of ρ	`galoischarpoly(G, χ, {o})`
$\det(\rho(g))$, χ character of ρ	`galoischardet(G, χ, {o})`
action of p in nfgaloisconj form	`galoispermtopol(G, {p})`
identify as abstract group	`galoisidentify(G)`
export a group for GAP/MAGMA	`galoisexport(G, {flag})`
subgroups of the Galois group G	`galoissubgroups(G)`
is subgroup H normal?	`galoisisnormal(G, H)`
subfields from subgroups	`galoissubfields(G, {flag}, {v})`
fixed field	`galoisfixedfield(G, perm, {flag}, {v})`
Frobenius at maximal ideal P	`idealfrobenius(nf, G, P)`
ramification groups at P	`idealramgroups(nf, G, P)`
is G abelian?	`galoisisabelian(G, {flag})`
abelian number fields/$\mathbf{Q}$	`galoissubcyclo(N,H,{flag},{v})`

The `galpol` package

query the package: polynomial	`galoisgetpol(a,b,{s})`
...: permutation group	`galoisgetgroup(a,b)`
...: group description	`galoisgetname(a,b)`

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.

absolute equation of L	`rnfequation(nf, T, {flag})`
is L/K abelian?	`rnfisabelian(nf, T)`
relative `nfalgtobasis`	`rnfalgtobasis(rnf, x)`
relative `nfbasistoalg`	`rnfbasistoalg(rnf, x)`
relative `idealhnf`	`rnfidealhnf(rnf, x)`
relative `idealmul`	`rnfidealmul(rnf, x, y)`
relative `idealtwoelt`	`rnfidealtwoelt(rnf, x)`

Lifts and Push-downs

absolute $\to$ relative representation for x	`rnfeltabstorel(rnf, x)`
relative $\to$ absolute representation for x	`rnfeltreltoabs(rnf, x)`
lift x to the relative field	`rnfeltup(rnf, x)`
push x down to the base field	`rnfeltdown(rnf, x)`

idem for x ideal: `(rnfideal)reltoabs`, `abstorel`, `up`, `down`

Norms and Trace

relative norm of element $x \in L$	`rnfeltnorm(rnf, x)`
relative trace of element $x \in L$	`rnfelttrace(rnf, x)`
absolute norm of ideal x	`rnfidealnormabs(rnf, x)`
relative norm of ideal x	`rnfidealnormrel(rnf, x)`
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$	`bnfisintnorm(bnf, x)`
is $x \in \mathbf{Q}$ a norm from K?	`bnfisnorm(bnf, x, {flag})`
initialize T for norm eq. solver	`rnfisnorminit(K, pol, {flag})`
is $a \in K$ a norm from L?	`rnfisnorm(T, a, {flag})`
initialize t for Thue equation solver	`thueinit(f)`
solve Thue equation $f(x, y) = a$	`thue(t, a, {sol})`
characteristic poly. of a mod T	`rnfcharpoly(nf, T, a, {v})`

Factorization

factor ideal x in L	`rnfidealfactor(rnf, x)`
$[S, T]$: $T_{i,j} \mid S_i$; S primes of K above p	`rnfidealprimedec(rnf, p)`

Maximal order $\mathbf{Z}_L$ as a $\mathbf{Z}_K$-module

relative `polredbest`	`rnfpolredbest(nf, T)`
relative `polredabs`	`rnfpolredabs(nf, T)`
relative Dedekind criterion, prime pr	`rnfdedekind(nf, T, pr)`
discriminant of relative extension	`rnfdisc(nf, T)`
pseudo-basis of $\mathbf{Z}_L$	`rnfpseudobasis(nf, T)`

General $\mathbf{Z}_K$-modules: $M = [\text{matrix}, \text{vec. of ideals}] \subset L$

relative HNF / SNF	`nfhnf(nf, M)`, `nfsnf`
multiple of det M	`nfdetint(nf, M)`
HNF of M where $d = nfdetint(M)$	`nfhnfmod(x, d)`
reduced basis for M	`rnflllgram(nf, T, M)`
determinant of pseudo-matrix M	`rnfdet(nf, M)`
Steinitz class of M	`rnfsteinitz(nf, M)`
$\mathbf{Z}_K$-basis of M if $\mathbf{Z}_K$-free, or 0	`rnfhnfbasis(bnf, M)`
n-basis of M, or $(n + 1)$-generating set	`rnfbasis(bnf, M)`
is M a free $\mathbf{Z}_K$-module?	`rnfisfree(bnf, M)`

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Associative Algebras

A is a general associative algebra given by a multiplication table mt (over $\mathbf{Q}$ or $\mathbf{F}_p$); represented by al from `algtableinit`.

create al from mt (over $\mathbf{F}_p$)	$\texttt{algtableinit}(mt, \{p=0\})$
group algebra $\mathbf{Q}[G]$ (or $\mathbf{F}_p[G]$)	$\texttt{alggroup}(G, \{p=0\})$
center of group algebra	$\texttt{alggroupcenter}(G, \{p=0\})$

Properties

is (mt, p) OK for algtableinit?	$\texttt{algisassociative}(mt, \{p=0\})$
multiplication table mt	$\texttt{algmultable}(al)$
dimension of A over prime subfield	$\texttt{algdim}(al)$
characteristic of A	$\texttt{algchar}(al)$
is A commutative?	$\texttt{algiscommutative}(al)$
is A simple?	$\texttt{algissimple}(al)$
is A semi-simple?	$\texttt{algissemisimple}(al)$
center of A	$\texttt{algcenter}(al)$
Jacobson radical of A	$\texttt{algradical}(al)$
radical J and simple factors of A/J	$\texttt{algsimpledec}(al)$

Operations on algebras

create A/I, I two-sided ideal	$\texttt{algquotient}(al, I)$
create $A_1 \otimes A_2$	$\texttt{algtensor}(al1, al2)$
create subalgebra from basis B	$\texttt{algsubalg}(al, B)$
quotients by ortho. central idempotents e	$\texttt{algcentralproj}(al, e)$
isomorphic alg. with integral mult. table	$\texttt{algmakeintegral}(mt)$
prime subalgebra of semi-simple A over $\mathbf{F}_p$	$\texttt{algprimesubalg}(al)$
find isomorphism $A \cong M_d(\mathbf{F}_q)$	$\texttt{algsplit}(al)$

Operations on lattices in algebras

lattice generated by cols. of M	$\texttt{alglathnf}(al, M)$
...by the products xy, $x \in lat1$, $y \in lat2$	$\texttt{alglatmul}(al, lat1, lat2)$
sum $lat1 + lat2$ of the lattices	$\texttt{alglatadd}(al, lat1, lat2)$
intersection $lat1 \cap lat2$	$\texttt{alglatinter}(al, lat1, lat2)$
test $lat1 \subset lat2$	$\texttt{alglatsubset}(al, lat1, lat2)$
generalized index $(lat2 : lat1)$	$\texttt{alglatindex}(al, lat1, lat2)$
$\{x \in al \mid x \cdot lat1 \subset lat2\}$	$\texttt{alglatlefttransporter}(al, lat1, lat2)$
$\{x \in al \mid lat1 \cdot x \subset lat2\}$	$\texttt{alglatrighttransporter}(al, lat1, lat2)$
test $x \in lat$ (set $c =$ coord. of x)	$\texttt{alglatcontains}(al, lat, x, \{\&c\})$
element of lat with coordinates c	$\texttt{alglatelement}(al, lat, c)$

Operations on elements

$a+b$, $a-b$, $-a$	$\texttt{algadd}(al, a, b)$, `algsub`, `algneg`
$a \times b$, a^2	$\texttt{algmul}(al, a, b)$, `algsqr`
a^n, a^{-1}	$\texttt{algpow}(al, a, n)$, `alginv`
is x invertible ? (then set $z = x^{-1}$)	$\texttt{algisinv}(al, x, \{\&z\})$
find z such that $x \times z = y$	$\texttt{algdivl}(al, x, y)$
find z such that $z \times x = y$	$\texttt{algdivr}(al, x, y)$
does z s.t. $x \times z = y$ exist? (set it)	$\texttt{algisdivl}(al, x, y, \{\&z\})$
matrix of $v \mapsto x \cdot v$	$\texttt{algtomatrix}(al, x)$
absolute norm	$\texttt{algnorm}(al, x)$
absolute trace	$\texttt{algtrace}(al, x)$
absolute char. polynomial	$\texttt{algcharpoly}(al, x)$
given $a \in A$ and polynomial T, return $T(a)$	$\texttt{algpoleval}(al, T, a)$
random element in a box	$\texttt{algrandom}(al, b)$

Central Simple Algebras

A is a central simple algebra over a number field K; represented by al from `alginit`; K is given by a nf structure.

create CSA from data	$\texttt{alginit}(B, C, \{v\}, \{maxord = 1\})$
multiplication table over K	$B = K$, $C = mt$
cyclic algebra $(L/K, \sigma, b)$	$B = rnf$, $C = [sigma, b]$
quaternion algebra $(a, b)_K$	$B = K$, $C = [a, b]$
matrix algebra $M_d(K)$	$B = K$, $C = d$
local Hasse invariants over K	$B = K$, $C = [d, [PR, HF], HI]$

Properties

type of al (mt, CSA)	$\texttt{algtype}(al)$
dimension of A over $\mathbf{Q}$	$\texttt{algdim}(al, 1)$
dimension of al over its center K	$\texttt{algdim}(al)$
degree of A $(= \sqrt{\dim_K A})$	$\texttt{algdegree}(al)$
al a cyclic algebra $(L/K, \sigma, b)$; return σ	$\texttt{algaut}(al)$
...return b	$\texttt{algb}(al)$
...return L/K, as an rnf	$\texttt{algsplittingfield}(al)$
split A over an extension of K	$\texttt{algsplittingdata}(al)$
splitting field of A as an rnf over center	$\texttt{algsplittingfield}(al)$
multiplication table over center	$\texttt{algrelmultable}(al)$
places of K at which A ramifies	$\texttt{algramifiedplaces}(al)$
Hasse invariants at finite places of K	$\texttt{alghassef}(al)$
Hasse invariants at infinite places of K	$\texttt{alghassei}(al)$
Hasse invariant at place v	$\texttt{alghasse}(al, v)$
index of A over K (at place v)	$\texttt{algindex}(al, \{v\})$
is al a division algebra? (at place v)	$\texttt{algisdivision}(al, \{v\})$
is A ramified? (at place v)	$\texttt{algisramified}(al, \{v\})$
is A split? (at place v)	$\texttt{algissplit}(al, \{v\})$

Operations on elements

reduced norm	$\texttt{algnorm}(al, x)$
reduced trace	$\texttt{algtrace}(al, x)$
reduced char. polynomial	$\texttt{algcharpoly}(al, x)$
express x on integral basis	$\texttt{algalgtobasis}(al, x)$
convert x to algebraic form	$\texttt{algbasistoalg}(al, x)$
map $x \in A$ to $M_d(L)$, L split. field	$\texttt{algtomatrix}(al, x)$

Orders

$\mathbf{Z}$-basis of order $\mathcal{O}_0$	$\texttt{algbasis}(al)$
discriminant of order $\mathcal{O}_0$	$\texttt{algdisc}(al)$
$\mathbf{Z}$-basis of natural order in terms $\mathcal{O}_0$'s basis	$\texttt{alginvbasis}(al)$

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