

glucat

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# Chapter 1

## Namespace Index

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pade::pade_log_numer< Scalar_T > . . . . .	213
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<a href="#">pade::pade_log_denom&lt; float &gt;</a>	212
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<a href="#">pade::pade_log_numer&lt; qd_real &gt;</a>	213
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<a href="#">glucat::numeric_traits&lt; Scalar_T &gt;::promoted</a>	
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## Chapter 5

# Namespace Documentation

### 5.1 cga3 Namespace Reference

Definitions for 3D Conformal Geometric Algebra [DL].

#### Functions

- `template<typename Multivector_T>`  
`Multivector_T cga3 (const Multivector_T &x)`  
*Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].*
- `template<typename Multivector_T>`  
`Multivector_T cga3std (const Multivector_T &X)`  
*Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].*
- `template<typename Multivector_T>`  
`Multivector_T agc3 (const Multivector_T &X)`  
*Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].*

#### 5.1.1 Detailed Description

Definitions for 3D Conformal Geometric Algebra [DL].

#### 5.1.2 Function Documentation

##### 5.1.2.1 agc3()

```
template<typename Multivector_T>
Multivector_T cga3::agc3 (
    const Multivector_T & X) [inline]
```

Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].

Definition at line 126 of file [PyClical.h](#).

References [cga3std\(\)](#).

### 5.1.2.2 cga3()

```
template<typename Multivector_T>
Multivector_T cga3::cga3 (
    const Multivector_T & x) [inline]
```

Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].

Definition at line 103 of file [PyClical.h](#).

### 5.1.2.3 cga3std()

```
template<typename Multivector_T>
Multivector_T cga3::cga3std (
    const Multivector_T & X) [inline]
```

Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].

Definition at line 114 of file [PyClical.h](#).

Referenced by [agc3\(\)](#).

## 5.2 glucat Namespace Reference

### Namespaces

- namespace [gen](#)
- namespace [matrix](#)
- namespace [timing](#)

### Classes

- class [basis\\_table](#)  
*Table of basis elements used as a cache by basis\_element()*
- class [bool\\_to\\_type](#)  
*Bool to type.*
- class [clifford\\_algebra](#)  
*clifford\_algebra<> declares the operations of a [Clifford algebra](#)*
- class [compare\\_types](#)  
*Type comparison.*
- class [compare\\_types< T, T >](#)
- class [control\\_t](#)  
*Parameters to control tests.*
- struct [CTAssertion](#)  
*Compile time assertion.*
- struct [CTAssertion< true >](#)
- class [error](#)  
*Specific exception class.*
- class [framed\\_multi](#)  
*A framed\_multi<Scalar\_T,LO,HI,Tune\_P> is a framed approximation to a multivector.*

- class [glucat\\_error](#)  
*Abstract exception class.*
- class [index\\_set](#)  
*Index set class based on `std::bitset<>` in Gnu standard C++ library.*
- class [index\\_set\\_hash](#)
- class [matrix\\_multi](#)  
*A `matrix_multi<Scalar_T,LO,HI,Tune_P>` is a matrix approximation to a multivector.*
- class [numeric\\_traits](#)  
*Extra traits which extend numeric limits.*
- class [random\\_generator](#)  
*Random number generator with single instance per `Scalar_T`.*
- class [sorted\\_range](#)  
*Sorted range for use with output.*
- class [sorted\\_range<Sorted\\_Map\\_T,Sorted\\_Map\\_T>](#)

## Typedefs

- using [index\\_t](#) = int  
*Size of `index_t` should be enough to represent LO, HI.*
- using [set\\_value\\_t](#) = unsigned long  
*Size of `set_value_t` should be enough to contain `index_set<LO,HI>`*
- typedef int(\* [intfn](#)) ()  
*For exception catching: pointer to function returning int.*
- typedef int(\* [intintfn](#)) (int)  
*For exception catching: pointer to function of int returning int.*
- using [tuning\\_slow](#)
- using [tuning\\_naive](#)
- using [tuning\\_fast](#)

## Functions

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [operator!=](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool  
*Test for inequality of multivectors.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [operator!=](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> bool  
*Test for inequality of multivector and scalar.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [operator!=](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool  
*Test for inequality of scalar and multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [error\\_squared\\_tol](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T  
*Quadratic norm error tolerance relative to a specific multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [error\\_squared](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs, const Scalar\_T threshold) -> Scalar\_T

*Relative or absolute error using the quadratic norm.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool`

*Test for approximate equality of multivectors.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`

*Test for approximate equality of multivectors.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric sum of multivector and scalar.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric sum of scalar and multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric sum.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric difference of multivector and scalar.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto operator- (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric difference of scalar and multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric difference.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto operator\* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Product of multivector and scalar.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto operator\* (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Product of scalar and multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto operator\* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator^ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Outer product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator& (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inner product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator% (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Left contraction.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto star (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`

*Hestenes scalar product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Quotient of multivector and scalar.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Quotient of scalar and multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric quotient.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Transformation via twisted adjoint action.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric multiplicative inverse.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Integer power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Multivector power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_←  
_T, LO, HI, Tune_P >`

*Outer product power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Real part: synonym for scalar part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Imaginary part: deprecated (always 0)*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Pure part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Even part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Odd part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >`

*Vector part of multivector, as a vector\_t with respect to frame()*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each {i} is replaced by -{i} in each term, eg. {1}\*{2} -> (-{2})\*(-{1})*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg. {1}\*{2} -> {2}\*{1}.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation, rev o invo == invo o rev.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`



*Scalar\_T quadratic form == (rev(x)\*x)(0)*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [norm](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Scalar\_T norm == sum of norm of coordinates.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [abs](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Absolute value == sqrt(norm)*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [max\\_abs](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [complexifier](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [elliptic](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [sqrt](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector with specified complexifier.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [sqrt](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [clifford\\_exp](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Exponential of multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [log](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector with specified complexifier.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [log](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [cos](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Cosine of multivector with specified complexifier.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [cos](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >



*Inverse hyperbolic sine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic sine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto tanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Hyperbolic tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`static void check\_complex (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false)`

*Check that i is a valid complexifier for val.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator\* (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Geometric product.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator^ (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Outer product.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator& (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Inner product.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator% (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Left contraction.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto star (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`  
*Hestenes scalar product.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator/ (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator| (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator>> (std::istream &s, framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`  
*Read multivector from input.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto operator<< (std::ostream &os, const framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &`  
*Write multivector to output.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI>`  
`auto operator<< (std::ostream &os, const std::pair< const index\_set< LO, HI >, Scalar_T > &term) -> std::ostream &`  
*Write term to output.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto exp (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Exponential of multivector.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI>`  
`static auto crd\_of\_mult (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`  
*Coordinate of product of terms.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI>`  
`auto operator\* (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index\_set< LO, HI >, Scalar_T >`  
*Product of terms.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto sqrt (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto log (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI>`  
`static auto crd\_of\_mult (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`

- Coordinate of product of terms.*

  - `_GLUCAT_CTAssert` (std::numeric\_limits< unsigned char >::radix==2, CannotDetermineBitsPerChar) const `index_t` BITS\_PER\_CHAR

*If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.*

  - `_GLUCAT_CTAssert` (\_GLUCAT\_BITS\_PER\_ULONG==BITS\_PER\_SET\_VALUE, BitsPerULongDoesNotMatchSetValueT) const `index_t` DEFAULT\_LO

*Default lowest index in an index set.*

  - template<typename LHS\_T, typename RHS\_T>  
auto `pos_mod` (LHS\_T lhs, RHS\_T rhs) -> LHS\_T

*Modulo function which works reliably for lhs < 0.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `operator^` (const `index_set`< LO, HI > &lhs, const `index_set`< LO, HI > &rhs) -> const `index_set`< LO, HI >

*Symmetric set difference: exclusive or.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `operator&` (const `index_set`< LO, HI > &lhs, const `index_set`< LO, HI > &rhs) -> const `index_set`< LO, HI >

*Set intersection: and.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `operator|` (const `index_set`< LO, HI > &lhs, const `index_set`< LO, HI > &rhs) -> const `index_set`< LO, HI >

*Set union: or.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `compare` (const `index_set`< LO, HI > &a, const `index_set`< LO, HI > &b) -> int

*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*

  - `_GLUCAT_CTAssert` (sizeof(set\_value\_t) >=sizeof(std::bitset< DEFAULT\_HI-DEFAULT\_LO >), Default\_index\_set\_too\_big\_for\_value) template< const `index_t` LO

*Size of set\_value\_t should be enough to contain bitset<DEFAULT\_HI-DEFAULT\_LO>*

  - const `index_t` HI auto `operator<<` (std::ostream &os, const `index_set`< LO, HI > &ist) -> std::ostream &
  - template<const `index_t` LO, const `index_t` HI>  
auto `operator>>` (std::istream &s, `index_set`< LO, HI > &ist) -> std::istream &

*Read in index set.*

  - auto `sign_of_square` (`index_t` j) -> int

*Square of generator {j}.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `min_neg` (const `index_set`< LO, HI > &ist) -> `index_t`

*Minimum negative index, or 0 if none.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `max_pos` (const `index_set`< LO, HI > &ist) -> `index_t`

*Maximum positive index, or 0 if none.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `operator<<` (std::ostream &os, const `index_set`< LO, HI > &ist) -> std::ostream &

*Write out index set.*

  - static auto `inverse_reversed_gray` (unsigned long x) -> unsigned long

*Inverse reversed Gray code.*

  - static auto `inverse_gray` (unsigned long x) -> unsigned long

*Inverse Gray code.*

  - template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P>  
auto `operator*` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >

*Geometric product.*

  - template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P>  
auto `operator^` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >

*Outer product.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto operator& (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Inner product.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto operator% (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Left contraction.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto star (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs) -> Scalar_T`

*Hestenes scalar product.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto operator/ (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Geometric quotient.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto operator| (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Transformation via twisted adjoint action.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto operator>> (std::istream &s, matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`

*Read multivector from input.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto operator<< (std::ostream &os, const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream`  
`&`

*Write multivector to output.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto reframe (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs, matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs_reframed, matrix_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs_reframed) -> const index_set< LO, HI >`

*Find a common frame for operands of a binary operator.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI,`  
`Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto matrix_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P`  
`<_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto matrix_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto exp (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI,`  
`Tune_P >`

*Exponential of multivector.*

- `auto offset_level (const index_t p, const index_t q) -> index_t`

*Determine the log2 dim corresponding to signature p, q.*

- template<typename Matrix\_Index\_T, const [index\\_t](#) LO, const [index\\_t](#) HI>  
static auto [folded\\_dim](#) (const [index\\_set](#)< LO, HI > &sub) -> Matrix\_Index\_T

*Determine the matrix dimension of the fold of a subalgebra.*

- template<typename Multivector\_T, typename Matrix\_T, typename Basis\_Matrix\_T>  
static auto [fast](#) (const Matrix\_T &X, [index\\_t](#) level) -> Multivector\_T

*Inverse generalized Fast Fourier Transform.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P, const size\_t Size>  
static auto [pade\\_approx](#) (const std::array< Scalar\_T, Size > &numer, const std::array< Scalar\_T, Size > &denom, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &X) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static void [db\\_step](#) ([matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &M, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &Y)

*Single step of product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [db\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 4)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [cr\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_Y\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 1)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Cyclic reduction square root iteration.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [pade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation of log.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [cascade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Incomplete square root cascade and Pade' approximation of log.*

- template<typename Scalar\_T>  
auto [log2](#) (const Scalar\_T &x) -> Scalar\_T

*Log base 2 of scalar.*

- template<typename Scalar\_T>  
auto [to\\_promote](#) (const Scalar\_T &val) -> typename [numeric\\_traits](#)< Scalar\_T >::promoted::type

*Cast to promote.*

- template<typename Scalar\_T>  
auto [to\\_demote](#) (const Scalar\_T &val) -> typename [numeric\\_traits](#)< Scalar\_T >::demoted::type

*Cast to demote.*

- int [try\\_catch](#) (intfn f)

*Exception catching for functions returning int.*

- int [try\\_catch](#) (intintfn f, int arg)

*Exception catching for functions of int returning int.*

## Variables

- template<typename Scalar\_T, typename Index\_Set\_T, typename Multivector\_T>  
const Scalar\_T [clifford\\_algebra](#)< Scalar\_T, Index\_Set\_T, Multivector\_T >::default\_truncation = std::numeric\_limits<Scalar\_T>::epsilon()

*Default for truncation.*

- const double [MS\\_PER\\_S](#) = 1000.0



*Timing constant: deprecated here - moved to [test/timing.h](#).*

- const [index\\_t](#) BITS\_PER\_SET\_VALUE = std::numeric\_limits<[set\\_value\\_t](#)>::digits

*Number of bits in [set\\_value\\_t](#).*

- const [index\\_t](#) DEFAULT\_HI = [index\\_t](#)(BITS\_PER\_SET\_VALUE / 2)

*Default highest index in an index set.*

- static const long double [I\\_pi](#) = 3.1415926535897932384626433832795029L
- static const long double [I\\_ln2](#) = 0.6931471805599453094172321214581766L
- const unsigned int [Tuning\\_Int\\_Digits](#) = std::numeric\_limits<int>::digits
- const unsigned int [Tuning\\_Max\\_Threshold](#) = 1 << [Tuning\\_Int\\_Digits](#)
- const unsigned int [Tuning\\_Slow\\_Mult\\_Matrix\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Slow\\_Basis\\_Max\\_Count](#) = 0
- const unsigned int [Tuning\\_Slow\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Slow\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Slow\\_Products\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Naive\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Naive\\_Basis\\_Max\\_Count](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Naive\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Naive\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Fast\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Fast\\_Div\\_Max\\_Steps](#) = 0
- const unsigned int [Tuning\\_Fast\\_CR\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [Tuning\\_Fast\\_DB\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [Tuning\\_Fast\\_Log\\_Max\\_Outer\\_Steps](#) = 16
- const unsigned int [Tuning\\_Fast\\_Log\\_Max\\_Inner\\_Steps](#) = 8
- const unsigned int [Tuning\\_Fast\\_Basis\\_Max\\_Count](#) = 1
- const unsigned int [Tuning\\_Fast\\_Fast\\_Size\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Fast\\_Inv\\_Fast\\_Dim\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Fast\\_Products\\_Size\\_Threshold](#) = 0

## 5.2.1 Typedef Documentation

### 5.2.1.1 [index\\_t](#)

```
using glucat::index\_t = int
```

Size of [index\\_t](#) should be enough to represent LO, HI.

Definition at line 77 of file [global.h](#).

### 5.2.1.2 [intfn](#)

```
typedef int(* glucat::intfn) ()
```

For exception catching: pointer to function returning int.

Definition at line 37 of file [try\\_catch.h](#).



### 5.2.1.3 intintfn

```
typedef int(* glucat::intintfn) (int)
```

For exception catching: pointer to function of int returning int.

Definition at line 40 of file [try\\_catch.h](#).

### 5.2.1.4 set\_value\_t

```
using glucat::set_value_t = unsigned long
```

Size of [set\\_value\\_t](#) should be enough to contain `index_set<LO,HI>`

Definition at line 79 of file [global.h](#).

### 5.2.1.5 tuning\_fast

```
using glucat::tuning_fast
```

**Initial value:**

```
tuning
<
    Tuning_Fast_Mult_Matrix_Threshold,
    Tuning_Fast_Div_Max_Steps,
    Tuning_Fast_CR_Sqrt_Max_Steps,
    Tuning_Fast_DB_Sqrt_Max_Steps,
    Tuning_Fast_Log_Max_Outer_Steps,
    Tuning_Fast_Log_Max_Inner_Steps,
    Tuning_Fast_Basis_Max_Count,
    Tuning_Fast_Fast_Size_Threshold,
    Tuning_Fast_Inv_Fast_Dim_Threshold,
    Tuning_Fast_Products_Size_Threshold,
    Tuning_Default_Denom_Different_Bits,
    Tuning_Default_Extra_Different_Bits,
    Tuning_Default_Function_Precision
>
```

Definition at line 97 of file [tuning.h](#).

### 5.2.1.6 tuning\_naive

```
using glucat::tuning_naive
```

**Initial value:**

```
tuning
<
    Tuning_Naive_Mult_Matrix_Threshold,
    Tuning_Default_Div_Max_Steps,
    Tuning_Default_CR_Sqrt_Max_Steps,
    Tuning_Default_DB_Sqrt_Max_Steps,
    Tuning_Default_Log_Max_Outer_Steps,
    Tuning_Default_Log_Max_Inner_Steps,
    Tuning_Naive_Basis_Max_Count,
    Tuning_Naive_Fast_Size_Threshold,
    Tuning_Naive_Inv_Fast_Dim_Threshold,
    Tuning_Default_Products_Size_Threshold,
    Tuning_Default_Denom_Different_Bits,
    Tuning_Default_Extra_Different_Bits,
    Tuning_Default_Function_Precision
>
```

Definition at line 69 of file [tuning.h](#).

### 5.2.1.7 tuning\_slow

using [glucat::tuning\\_slow](#)

#### Initial value:

```
tuning
<
    Tuning_Slow_Mult_Matrix_Threshold,
    Tuning_Default_Div_Max_Steps,
    Tuning_Default_CR_Sqrt_Max_Steps,
    Tuning_Default_DB_Sqrt_Max_Steps,
    Tuning_Default_Log_Max_Outer_Steps,
    Tuning_Default_Log_Max_Inner_Steps,
    Tuning_Slow_Basis_Max_Count,
    Tuning_Slow_Fast_Size_Threshold,
    Tuning_Slow_Inv_Fast_Dim_Threshold,
    Tuning_Slow_Products_Size_Threshold,
    Tuning_Default_Denom_Different_Bits,
    Tuning_Default_Extra_Different_Bits,
    Tuning_Default_Function_Precision
>
```

Definition at line 47 of file [tuning.h](#).

## 5.2.2 Function Documentation

### 5.2.2.1 \_GLUCAT\_CTAssert() [1/3]

```
glucat::_GLUCAT_CTAssert (
    _GLUCAT_BITS_PER_ULONG  = BITS\_PER\_SET\_VALUE,
    BitsPerULongDoesNotMatchSetValueT ) const
```

Default lowest index in an index set.

References [BITS\\_PER\\_SET\\_VALUE](#).

### 5.2.2.2 \_GLUCAT\_CTAssert() [2/3]

```
glucat::_GLUCAT_CTAssert (
    sizeof(set\_value\_t) >=sizeof(std::bitset< DEFAULT\_HI-DEFAULT\_LO >) ,
    Default_index_set_too_big_for_value ) const
```

Size of [set\\_value\\_t](#) should be enough to contain `bitset<DEFAULT_HI-DEFAULT_LO>`

Write out index set

References [\\_GLUCAT\\_CTAssert\(\)](#), and [DEFAULT\\_HI](#).

### 5.2.2.3 \_GLUCAT\_CTAssert() [3/3]

```
glucat::_GLUCAT_CTAssert (
    std::numeric_limits< unsigned char >::radix  = 2,
    CannotDetermineBitsPerChar ) const
```

If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.

Number of bits per char is used to determine number of bits in [set\\_value\\_t](#)

Referenced by [\\_GLUCAT\\_CTAssert\(\)](#).

#### 5.2.2.4 abs()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::abs (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Absolute value == sqrt(norm)

Definition at line 577 of file [clifford\\_algebra\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::sqrt\(\)](#).

Referenced by [PyClical.clifford::abs\(\)](#), [clifford\\_to\\_str\(\)](#), [matrix\\_log\(\)](#), and [matrix\\_sqrt\(\)](#).

#### 5.2.2.5 acos() [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::acos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Inverse cosine of multivector.

Definition at line 903 of file [clifford\\_algebra\\_imp.h](#).

References [acos\(\)](#), and [complexifier\(\)](#).

#### 5.2.2.6 acos() [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::acos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse cosine of multivector with specified complexifier.

Definition at line 883 of file [clifford\\_algebra\\_imp.h](#).

References [acosh\(\)](#), and [check\\_complex\(\)](#).

Referenced by [acos\(\)](#).

#### 5.2.2.7 acosh() [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::acosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic cosine of multivector.

Definition at line 844 of file [clifford\\_algebra\\_imp.h](#).

References [acosh\(\)](#), and [complexifier\(\)](#).

### 5.2.2.8 `acosh()` [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::acosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic cosine of multivector with specified complexifier.

Definition at line 825 of file `clifford_algebra_imp.h`.

References `check_complex()`, `log()`, `norm()`, and `sqrt()`.

Referenced by `acos()`, and `acosh()`.

### 5.2.2.9 `approx_equal()` [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::approx_equal (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> bool [inline]
```

Test for approximate equality of multivectors.

Definition at line 169 of file `clifford_algebra_imp.h`.

References `approx_equal()`, and `error_squared_tol()`.

### 5.2.2.10 `approx_equal()` [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::approx_equal (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
    const Scalar_T threshold,
    const Scalar_T tolerance) -> bool [inline]
```

Test for approximate equality of multivectors.

Definition at line 154 of file `clifford_algebra_imp.h`.

References `error_squared()`.

Referenced by `approx_equal()`, and `matrix_sqrt()`.

**5.2.2.11 asin()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::asin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Inverse sine of multivector.

Definition at line 1008 of file [clifford\\_algebra\\_imp.h](#).

References [asin\(\)](#), and [complexifier\(\)](#).

**5.2.2.12 asin()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::asin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse sine of multivector with specified complexifier.

Definition at line 988 of file [clifford\\_algebra\\_imp.h](#).

References [asinh\(\)](#), and [check\\_complex\(\)](#).

Referenced by [asin\(\)](#).

**5.2.2.13 asinh()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::asinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic sine of multivector.

Definition at line 949 of file [clifford\\_algebra\\_imp.h](#).

References [asinh\(\)](#), and [complexifier\(\)](#).

**5.2.2.14 asinh()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::asinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic sine of multivector with specified complexifier.

Definition at line 930 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [log\(\)](#), [norm\(\)](#), and [sqrt\(\)](#).

Referenced by [asin\(\)](#), and [asinh\(\)](#).

**5.2.2.15 atan() [1/2]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::atan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Inverse tangent of multivector.

Definition at line 1108 of file [clifford\\_algebra\\_imp.h](#).

References [atan\(\)](#), and [complexifier\(\)](#).

**5.2.2.16 atan() [2/2]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::atan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse tangent of multivector with specified complexifier.

Definition at line 1088 of file [clifford\\_algebra\\_imp.h](#).

References [atanh\(\)](#), and [check\\_complex\(\)](#).

Referenced by [atan\(\)](#).

**5.2.2.17 atanh() [1/2]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::atanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic tangent of multivector.

Definition at line 1052 of file [clifford\\_algebra\\_imp.h](#).

References [atanh\(\)](#), and [complexifier\(\)](#).

**5.2.2.18 atanh() [2/2]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::atanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic tangent of multivector with specified complexifier.

Definition at line 1035 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [log\(\)](#), and [norm\(\)](#).

Referenced by [atan\(\)](#), and [atanh\(\)](#).

### 5.2.2.19 cascade\_log()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
static auto glucat::cascade_log (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val) -> const matrix_multi<Scalar_T, LO, HI, Tune_P> [static]
```

Incomplete square root cascade and Pade' approximation of log.

Definition at line 1920 of file [matrix\\_multi\\_imp.h](#).

References [db\\_step\(\)](#), [epsilon](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), [norm\(\)](#), and [pade\\_log\(\)](#).

Referenced by [matrix\\_log\(\)](#).

### 5.2.2.20 check\_complex()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
static void glucat::check_complex (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) [inline], [static]
```

Check that i is a valid complexifier for val.

Definition at line 652 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#).

Referenced by [acos\(\)](#), [acosh\(\)](#), [asin\(\)](#), [asinh\(\)](#), [atan\(\)](#), [atanh\(\)](#), [cos\(\)](#), [log\(\)](#), [log\(\)](#), [sin\(\)](#), [sqrt\(\)](#), [sqrt\(\)](#), and [tan\(\)](#).

### 5.2.2.21 clifford\_exp()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::clifford_exp (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_T, LO, HI, Tune_P>
```

Exponential of multivector.

Definition at line 690 of file [clifford\\_algebra\\_imp.h](#).

References [log2\(\)](#).

Referenced by [exp\(\)](#), and [exp\(\)](#).

### 5.2.2.22 compare()

```
template<const index_t LO, const index_t HI>
auto glucat::compare (
    const index_set< LO, HI > & a,
    const index_set< LO, HI > & b) -> int [inline]
```

"lexicographic compare" eg. {3,4,5} is less than {3,7,8}

Lexicographic ordering of two sets: -1 if  $a < b$ , +1 if  $a > b$ , 0 if  $a == b$ .

Definition at line 574 of file [index\\_set\\_imp.h](#).

### 5.2.2.23 complexifier()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::complexifier (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P>
```

Square root of -1 which commutes with all members of the frame of the given multivector.

Definition at line 592 of file [clifford\\_algebra\\_imp.h](#).

References [pos\\_mod\(\)](#).

Referenced by [acos\(\)](#), [acosh\(\)](#), [asin\(\)](#), [asinh\(\)](#), [atan\(\)](#), [atanh\(\)](#), [check\\_complex\(\)](#), [cos\(\)](#), [elliptic\(\)](#), [log\(\)](#), [sin\(\)](#), [sqrt\(\)](#), and [tan\(\)](#).

### 5.2.2.24 conj()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::conj (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Conjugation,  $rev \circ inv = inv \circ rev$ .

Definition at line 553 of file [clifford\\_algebra\\_imp.h](#).

### 5.2.2.25 cos() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::cos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Cosine of multivector.

Definition at line 874 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [cos\(\)](#).



**5.2.2.26 cos()** [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::cos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Cosine of multivector with specified complexifier.

Definition at line 851 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), and [exp\(\)](#).

Referenced by [cos\(\)](#), and [tan\(\)](#).

**5.2.2.27 cosh()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::cosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Hyperbolic cosine of multivector.

Definition at line 807 of file [clifford\\_algebra\\_imp.h](#).

References [exp\(\)](#).

Referenced by [tanh\(\)](#).

**5.2.2.28 cr\_sqrt()**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
static auto glucat::cr_sqrt (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    Scalar_T norm_Y_tol = std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1)) ->
const matrix_multi<Scalar_T,LO,HI,Tune_P> [static]
```

Cyclic reduction square root iteration.

Definition at line 1349 of file [matrix\\_multi\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#), and [norm\(\)](#).

Referenced by [matrix\\_sqrt\(\)](#).

**5.2.2.29 crd\_of\_mult()** [1/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
static auto glucat::crd_of_mult (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs) -> Scalar_T [inline],
[static]
```

Coordinate of product of terms.

Referenced by [operator%\(\)](#), [operator&\(\)](#), [operator\\*\(\)](#), and [operator^\(\)](#).

**5.2.2.30 crd\_of\_mult()** [2/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
static auto glucat::crd_of_mult (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs) -> Scalar_T [inline],
[static]
```

Coordinate of product of terms.

Definition at line 1709 of file [framed\\_multi\\_imp.h](#).

**5.2.2.31 db\_sqrt()**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
static auto glucat::db_sqrt (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    Scalar_T norm_tol = std::pow(std::numeric_limits<Scalar_T>::epsilon(), 4)) ->
const matrix_multi<Scalar_T,LO,HI,Tune_P> [static]
```

Product form of Denman-Beavers square root iteration.

Definition at line 1320 of file [matrix\\_multi\\_imp.h](#).

References [db\\_step\(\)](#), [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#), and [norm\(\)](#).

Referenced by [matrix\\_sqrt\(\)](#).

**5.2.2.32 db\_step()**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
static void glucat::db_step (
    matrix_multi< Scalar_T, LO, HI, Tune_P > & M,
    matrix_multi< Scalar_T, LO, HI, Tune_P > & Y) [inline], [static]
```

Single step of product form of Denman-Beavers square root iteration.

Definition at line 1308 of file [matrix\\_multi\\_imp.h](#).

References [inv\(\)](#).

Referenced by [cascade\\_log\(\)](#), and [db\\_sqrt\(\)](#).

### 5.2.2.33 `elliptic()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::elliptic (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_←
_T, LO, HI, Tune_P> [inline]
```

Square root of -1 which commutes with all members of the frame of the given multivector The name "elliptic" is now deprecated: use "complexifier" instead.

Definition at line 643 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#).

### 5.2.2.34 `error_squared()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T,
const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::error_squared (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
    const Scalar_T threshold) -> Scalar_T [inline]
```

Relative or absolute error using the quadratic norm.

Definition at line 134 of file [clifford\\_algebra\\_imp.h](#).

References [norm\(\)](#).

Referenced by [approx\\_equal\(\)](#).

### 5.2.2.35 `error_squared_tol()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::error_squared_tol (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T
```

Quadratic norm error tolerance relative to a specific multivector.

Definition at line 112 of file [clifford\\_algebra\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::pow\(\)](#).

Referenced by [approx\\_equal\(\)](#).

### 5.2.2.36 even()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::even (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar↵
_T,LO,HI,Tune_P> [inline]
```

Even part.

Definition at line 513 of file [clifford\\_algebra\\_imp.h](#).

### 5.2.2.37 exp() [1/2]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::exp (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & val) -> const framed\_multi<Scalar↵
_T,LO,HI,Tune_P>
```

Exponential of multivector.

Definition at line 1750 of file [framed\\_multi\\_imp.h](#).

References [clifford\\_exp\(\)](#), [exp\(\)](#), and [scalar\(\)](#).

Referenced by [cos\(\)](#), [cosh\(\)](#), [exp\(\)](#), [matrix\\_log\(\)](#), [matrix\\_sqrt\(\)](#), [pow\(\)](#), [sin\(\)](#), and [sinh\(\)](#).

### 5.2.2.38 exp() [2/2]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::exp (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val) -> const matrix\_multi<Scalar↵
_T,LO,HI,Tune_P>
```

Exponential of multivector.

Definition at line 2086 of file [matrix\\_multi\\_imp.h](#).

References [clifford\\_exp\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::scalar\(\)](#).

### 5.2.2.39 fast()

```
template<typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T>
static auto glucat::fast (
    const Matrix_T & X,
    index\_t level) -> Multivector_T [static]
```

Inverse generalized Fast Fourier Transform.

Definition at line 1027 of file [matrix\\_multi\\_imp.h](#).

References [fast\(\)](#), [glucat::matrix::signed\\_perm\\_nork\(\)](#), and [glucat::matrix::unit\(\)](#).

Referenced by [fast\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

#### 5.2.2.40 folded\_dim()

```
template<typename Matrix_Index_T, const index_t LO, const index_t HI>
static auto glucat::folded_dim (
    const index_set< LO, HI > & sub) -> Matrix_Index_T    [inline], [static]
```

Determine the matrix dimension of the fold of a subalgebra.

Definition at line 101 of file [matrix\\_multi\\_imp.h](#).

References [offset\\_level\(\)](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

#### 5.2.2.41 imag()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::imag (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T    [inline]
```

Imaginary part: deprecated (always 0)

Definition at line 497 of file [clifford\\_algebra\\_imp.h](#).

#### 5.2.2.42 inv()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::inv (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P>    [inline]
```

Geometric multiplicative inverse.

Definition at line 400 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [db\\_step\(\)](#), and [matrix\\_log\(\)](#).

#### 5.2.2.43 inverse\_gray()

```
static auto glucat::inverse_gray (
    unsigned long x) -> unsigned long    [inline], [static]
```

Inverse Gray code.

Definition at line 863 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::sign\\_of\\_mult\(\)](#).

#### 5.2.2.44 `inverse_reversed_gray()`

```
static auto glucat::inverse_reversed_gray (
    unsigned long x) -> unsigned long    [inline], [static]
```

Inverse reversed Gray code.

Definition at line 846 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::sign\\_of\\_mult\(\)](#).

#### 5.2.2.45 `involute()`

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::involute (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P>    [inline]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1}\*{2} -> (-{2})\*(-{1})

Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.

Definition at line 537 of file [clifford\\_algebra\\_imp.h](#).

#### 5.2.2.46 `log()` [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & val,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
```

Natural logarithm of multivector with specified complexifier.

Definition at line 1800 of file [framed\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), and [log\(\)](#).

#### 5.2.2.47 `log()` [2/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
```

Natural logarithm of multivector with specified complexifier.

Definition at line 2045 of file [matrix\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [matrix\\_log\(\)](#).

**5.2.2.48 log()** [3/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::log (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_
_T,LO,HI,Tune_P> [inline]
```

Natural logarithm of multivector.

Definition at line 799 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [log\(\)](#).

**5.2.2.49 log()** [4/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::log (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Natural logarithm of multivector with specified complexifier.

Definition at line 791 of file [clifford\\_algebra\\_imp.h](#).

References [log\(\)](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [atanh\(\)](#), [log\(\)](#), [log\(\)](#), [log\(\)](#), and [pow\(\)](#).

**5.2.2.50 log2()**

```
template<typename Scalar_T>
auto glucat::log2 (
    const Scalar_T & x) -> Scalar_T [inline]
```

Log base 2 of scalar.

Definition at line 303 of file [scalar.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::log2\(\)](#).

Referenced by [clifford\\_exp\(\)](#).

**5.2.2.51 matrix\_log()**

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::matrix_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & i,
    const index\_t level) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Natural logarithm of multivector with specified complexifier.

Definition at line 1967 of file [matrix\\_multi\\_imp.h](#).

References [abs\(\)](#), [cascade\\_log\(\)](#), [glucat::matrix::classify\\_eigenvalues\(\)](#), [exp\(\)](#), [inv\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, matrix\\_log\(\), norm\(\), and glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::scalar\(\)](#).

Referenced by [log\(\)](#), and [matrix\\_log\(\)](#).

### 5.2.2.52 `matrix_sqrt()`

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::matrix_sqrt (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & i,
    const index\_t level) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Square root of multivector with specified complexifier.

Definition at line [1571](#) of file [matrix\\_multi\\_imp.h](#).

References [abs\(\)](#), [approx\\_equal\(\)](#), [glucat::matrix::classify\\_eigenvalues\(\)](#), [cr\\_sqrt\(\)](#), [db\\_sqrt\(\)](#), [pade::pade\\_sqrt\\_denom< Scalar\\_T >::exp\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), [matrix\\_sqrt\(\)](#), [norm\(\)](#), [pade::pade\\_sqrt\\_numer< Scalar\\_T >::pade\\_approx\(\)](#), [pow\(\)](#), and [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::scalar\(\)](#).

Referenced by [matrix\\_sqrt\(\)](#), and [sqrt\(\)](#).

### 5.2.2.53 `max_abs()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::max_abs (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

Definition at line [585](#) of file [clifford\\_algebra\\_imp.h](#).

### 5.2.2.54 `max_pos()`

```
template<const index\_t LO, const index\_t HI>
auto glucat::max_pos (
    const index\_set< LO, HI > & ist) -> index\_t [inline]
```

Maximum positive index, or 0 if none.

Definition at line [977](#) of file [index\\_set\\_imp.h](#).

Referenced by [operator<<\(\)](#).

### 5.2.2.55 `min_neg()`

```
template<const index\_t LO, const index\_t HI>
auto glucat::min_neg (
    const index\_set< LO, HI > & ist) -> index\_t [inline]
```

Minimum negative index, or 0 if none.

Definition at line [970](#) of file [index\\_set\\_imp.h](#).

Referenced by [operator<<\(\)](#).



**5.2.2.56 norm()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::norm (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Scalar\_T norm == sum of norm of coordinates.

Definition at line 569 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [atanh\(\)](#), [cascade\\_log\(\)](#), [cr\\_sqrt\(\)](#), [db\\_sqrt\(\)](#), [error\\_squared\(\)](#), [matrix\\_log\(\)](#), and [matrix\\_sqrt\(\)](#).

**5.2.2.57 odd()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::odd (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Odd part.

Definition at line 521 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.58 offset\_level()**

```
auto glucat::offset_level (
    const index_t p,
    const index_t q) -> index_t [inline]
```

Determine the log2 dim corresponding to signature p, q.

Definition at line 86 of file [matrix\\_multi\\_imp.h](#).

References [pos\\_mod\(\)](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), and [folded\\_dim\(\)](#).

**5.2.2.59 operator"!="() [1/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator!= (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> bool [inline]
```

Test for inequality of multivectors.

Definition at line 86 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.60 operator!=(()) [2/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator!=( (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> bool [inline]
```

Test for inequality of multivector and scalar.

Definition at line 94 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.61 operator!=(()) [3/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator!=( (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> bool [inline]
```

Test for inequality of scalar and multivector.

Definition at line 102 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.62 operator%() [1/3]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator% (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
```

Left contraction.

Definition at line 597 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), and [crd\\_of\\_mult\(\)](#).

**5.2.2.63 operator%() [2/3]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator% (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P> [inline]
```

Left contraction.

Definition at line 581 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.64 operator%() [3/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator% (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_↵
T,LO,HI,Tune_P> [inline]
```

Left contraction.

Definition at line 322 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.65 operator&() [1/4]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator& (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<Scalar_↵
_T,LO,HI,Tune_P>
```

Inner product.

Definition at line 495 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), and [crd\\_of\\_mult\(\)](#).

**5.2.2.66 operator&() [2/4]**

```
template<const index_t LO, const index_t HI>
auto glucat::operator& (
    const index_set< LO, HI > & lhs,
    const index_set< LO, HI > & rhs) -> const index_set<LO,HI> [inline]
```

Set intersection: and.

Definition at line 186 of file [index\\_set\\_imp.h](#).

**5.2.2.67 operator&() [3/4]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator& (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Inner product.

Definition at line 562 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.68 operator&() [4/4]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator& (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_↔
T,LO,HI,Tune_P> [inline]
```

Inner product.

Definition at line 307 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.69 operator\*() [1/6]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<Scalar_↔
_T,LO,HI,Tune_P>
```

Geometric product.

Definition at line 374 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#).

**5.2.2.70 operator\*() [2/6]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<Scalar_↔
_T,LO,HI,Tune_P> [inline]
```

Geometric product.

Definition at line 502 of file [matrix\\_multi\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#), and [reframe\(\)](#).

**5.2.2.71 operator\*() [3/6]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_↔
T,LO,HI,Tune_P> [inline]
```

Geometric product.

Definition at line 277 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.72 operator\*() [4/6]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Product of multivector and scalar.

Definition at line 251 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.73 operator\*() [5/6]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_↔
_T,LO,HI,Tune_P> [inline]
```

Product of scalar and multivector.

Definition at line 262 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.74 operator\*() [6/6]**

```
template<typename Scalar_T, const index_t LO, const index_t HI>
auto glucat::operator* (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs) -> const std::↔
::pair<const index_set<LO,HI>, Scalar_T> [inline]
```

Product of terms.

Definition at line 1717 of file [framed\\_multi\\_imp.h](#).

References [crd\\_of\\_mult\(\)](#).

**5.2.2.75 operator+() [1/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator+ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_↔
_T,LO,HI,Tune_P> [inline]
```

Geometric sum.

Definition at line 206 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.76 operator+()** [2/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator+ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric sum of multivector and scalar.

Definition at line 181 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.77 operator+()** [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator+ (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Geometric sum of scalar and multivector.

Definition at line 192 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.78 operator-()** [1/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator- (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_←
T,LO,HI,Tune_P> [inline]
```

Geometric difference.

Definition at line 240 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.79 operator-()** [2/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator- (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric difference of multivector and scalar.

Definition at line 217 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.80 operator-() [3/3]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator- (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Geometric difference of scalar and multivector.

Definition at line 228 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.81 operator/() [1/5]**

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator/ (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Geometric quotient.

Definition at line 734 of file [framed\\_multi\\_imp.h](#).

**5.2.2.82 operator/() [2/5]**

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator/ (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P>
```

Geometric quotient.

Definition at line 614 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix::isnan\(\)](#), and [reframe\(\)](#).

**5.2.2.83 operator/() [3/5]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T,
const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator/ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Geometric quotient.

Definition at line 374 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.84 operator/()** [4/5]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator/ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Quotient of multivector and scalar.

Definition at line 348 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.85 operator/()** [5/5]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator/ (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Quotient of scalar and multivector.

Definition at line 359 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.86 operator<<()** [1/5]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator<< (
    std::ostream & os,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::ostream&
```

Write multivector to output.

Definition at line 1148 of file [framed\\_multi\\_imp.h](#).

References [scalar\(\)](#), and [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::truncated\(\)](#).

**5.2.2.87 operator<<()** [2/5]

```
const index\_t HI auto glucat::operator<< (
    std::ostream & os,
    const index\_set< LO, HI > & ist) -> std::ostream &
```

References [max\\_pos\(\)](#), [min\\_neg\(\)](#), and [sign\\_of\\_square\(\)](#).

**5.2.2.88 operator<<()** [3/5]

```
template<const index\_t LO, const index\_t HI>
auto glucat::operator<< (
    std::ostream & os,
    const index\_set< LO, HI > & ist) -> std::ostream&
```

Write out index set.

Definition at line 611 of file [index\\_set\\_imp.h](#).



**5.2.2.89 operator<<() [4/5]**

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator<< (
    std::ostream & os,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::ostream& [inline]
```

Write multivector to output.

Definition at line [955](#) of file [matrix\\_multi\\_imp.h](#).

**5.2.2.90 operator<<<() [5/5]**

```
template<typename Scalar_T, const index\_t LO, const index\_t HI>
auto glucat::operator<<< (
    std::ostream & os,
    const std::pair< const index\_set< LO, HI >, Scalar_T > & term) -> std::ostream&
```

Write term to output.

Definition at line [1209](#) of file [framed\\_multi\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_double\(\)](#).

**5.2.2.91 operator>>() [1/3]**

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator>> (
    std::istream & s,
    framed\_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::istream&
```

Read multivector from input.

Definition at line [1248](#) of file [framed\\_multi\\_imp.h](#).

**5.2.2.92 operator>>>() [2/3]**

```
template<const index\_t LO, const index\_t HI>
auto glucat::operator>>> (
    std::istream & s,
    index\_set< LO, HI > & ist) -> std::istream&
```

Read in index set.

Definition at line [634](#) of file [index\\_set\\_imp.h](#).

**5.2.2.93 operator>>>>() [3/3]**

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator>>>> (
    std::istream & s,
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::istream& [inline]
```

Read multivector from input.

Definition at line [966](#) of file [matrix\\_multi\\_imp.h](#).

**5.2.2.94 operator<sup>^</sup>() [1/4]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator^ (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<Scalar_↵
_T,LO,HI,Tune_P>
```

Outer product.

Definition at line 416 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), and [crd\\_of\\_mult\(\)](#).

**5.2.2.95 operator<sup>^</sup>() [2/4]**

```
template<const index_t LO, const index_t HI>
auto glucat::operator^ (
    const index_set< LO, HI > & lhs,
    const index_set< LO, HI > & rhs) -> const index_set<LO,HI> [inline]
```

Symmetric set difference: exclusive or.

Definition at line 161 of file [index\\_set\\_imp.h](#).

**5.2.2.96 operator<sup>^</sup>() [3/4]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator^ (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Outer product.

Definition at line 543 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.97 operator<sup>^</sup>() [4/4]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator^ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Outer product.

Definition at line 292 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.98 operator" | () [1/4]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator| (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Transformation via twisted adjoint action.

Definition at line 760 of file [framed\\_multi\\_imp.h](#).

**5.2.2.99 operator" | () [2/4]**

```
template<const index_t LO, const index_t HI>
auto glucat::operator| (
    const index_set< LO, HI > & lhs,
    const index_set< LO, HI > & rhs) -> const index_set<LO,HI> [inline]
```

Set union: or.

Definition at line 211 of file [index\\_set\\_imp.h](#).

**5.2.2.100 operator" | () [3/4]**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator| (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Transformation via twisted adjoint action.

Definition at line 717 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.101 operator" | () [4/4]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator| (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Transformation via twisted adjoint action.

Definition at line 389 of file [clifford\\_algebra\\_imp.h](#).

### 5.2.2.102 `outer_pow()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::outer_pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    int rhs) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Outer product power of multivector.

Definition at line 470 of file [clifford\\_algebra\\_imp.h](#).

### 5.2.2.103 `pade_approx()`

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P, const size_t
Size>
static auto glucat::pade_approx (
    const std::array< Scalar_T, Size > & numer,
    const std::array< Scalar_T, Size > & denom,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & X) -> const matrix\_multi<Scalar_
_T,LO,HI,Tune_P> [inline], [static]
```

Pade' approximation.

Definition at line 1245 of file [matrix\\_multi\\_imp.h](#).

Referenced by [matrix\\_sqrt\(\)](#), and [pade\\_log\(\)](#).

### 5.2.2.104 `pade_log()`

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
static auto glucat::pade_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val) -> const matrix\_multi<Scalar_
_T,LO,HI,Tune_P> [static]
```

Pade' approximation of log.

Definition at line 1900 of file [matrix\\_multi\\_imp.h](#).

References [pade::pade\\_log\\_denom< Scalar\\_T >::denom](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), [pade::pade\\_log\\_numer< Scalar\\_T >::numer](#), and [pade\\_approx\(\)](#).

Referenced by [cascade\\_log\(\)](#).

### 5.2.2.105 `pos_mod()`

```
template<typename LHS_T, typename RHS_T>
auto glucat::pos_mod (
    LHS_T lhs,
    RHS_T rhs) -> LHS_T [inline]
```

Modulo function which works reliably for lhs < 0.

Definition at line 117 of file [global.h](#).

Referenced by [complexifier\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_vector\(\)](#), [offset\\_level\(\)](#), and [glucat::gen::generator\\_table< Matrix\\_T >::operator\(\)\(\)](#).

**5.2.2.106 pow()** [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector<Scalar_T,
T,LO,HI,Tune_P> [inline]
```

Multivector power of multivector.

Definition at line 446 of file [clifford\\_algebra\\_imp.h](#).

References [exp\(\)](#), and [log\(\)](#).

**5.2.2.107 pow()** [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    int rhs) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Integer power of multivector.

Definition at line 407 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [matrix\\_sqrt\(\)](#).

**5.2.2.108 pure()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::pure (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_
_T,LO,HI,Tune_P> [inline]
```

Pure part.

Definition at line 505 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.109 quad()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::quad (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Scalar\_T quadratic form == (rev(x)\*x)(0)

Definition at line 561 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.110 real()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::real (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Real part: synonym for scalar part.

Definition at line 486 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.111 reframe()**

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::reframe (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs,
    matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs_reframed,
    matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs_reframed) -> const index_set<LO,HI>
[inline]
```

Find a common frame for operands of a binary operator.

Definition at line 345 of file [matrix\\_multi\\_imp.h](#).

Referenced by [operator\\*\(\)](#), and [operator/\(\)](#).

**5.2.2.112 reverse()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::reverse (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar↔
_T,LO,HI,Tune_P> [inline]
```

Reversion, eg.  $\{1\}*\{2\} \rightarrow \{2\}*\{1\}$ .

Definition at line 545 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.113 scalar()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::scalar (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Scalar part.

Definition at line 478 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [exp\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\(\)](#), and [operator<<\(\)](#).

**5.2.2.114 sign\_of\_square()**

```
auto glucat::sign_of_square (
    index_t j) -> int [inline]
```

Square of generator {j}.

Square of generator index j.

Definition at line 963 of file [index\\_set\\_imp.h](#).

Referenced by [operator<<\(\)](#).

**5.2.2.115 sin() [1/2]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Sine of multivector.

Definition at line 979 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [sin\(\)](#).

**5.2.2.116 sin() [2/2]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Sine of multivector with specified complexifier.

Definition at line 956 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), and [exp\(\)](#).

Referenced by [sin\(\)](#), and [tan\(\)](#).

**5.2.2.117 sinh()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar←
_T,LO,HI,Tune_P> [inline]
```

Hyperbolic sine of multivector.

Definition at line 911 of file [clifford\\_algebra\\_imp.h](#).

References [exp\(\)](#).

Referenced by [tanh\(\)](#).

**5.2.2.118 sqrt()** [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & val,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
```

Square root of multivector with specified complexifier.

Definition at line 1727 of file [framed\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), and [sqrt\(\)](#).

**5.2.2.119 sqrt()** [2/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
```

Square root of multivector with specified complexifier.

Definition at line 1667 of file [matrix\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [matrix\\_sqrt\(\)](#).

**5.2.2.120 sqrt()** [3/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Square root of multivector.

Definition at line 683 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [sqrt\(\)](#).

**5.2.2.121 sqrt()** [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Square root of multivector with specified complexifier.

Definition at line 675 of file [clifford\\_algebra\\_imp.h](#).

References [sqrt\(\)](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [sqrt\(\)](#), [sqrt\(\)](#), and [sqrt\(\)](#).



**5.2.2.122 star()** [1/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::star (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> Scalar_T
```

Hestenes scalar product.

Definition at line 684 of file [framed\\_multi\\_imp.h](#).

**5.2.2.123 star()** [2/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::star (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> Scalar_T [inline]
```

Hestenes scalar product.

Definition at line 600 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.124 star()** [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::star (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> Scalar_T [inline]
```

Hestenes scalar product.

Definition at line 337 of file [clifford\\_algebra\\_imp.h](#).

References [star\(\)](#).

Referenced by [star\(\)](#).

**5.2.2.125 tan()** [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::tan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_
_T,LO,HI,Tune_P> [inline]
```

Tangent of multivector.

Definition at line 1079 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [tan\(\)](#).

**5.2.2.126 tan()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::tan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>  [inline]
```

Tangent of multivector with specified complexifier.

Definition at line 1060 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [cos\(\)](#), and [sin\(\)](#).

Referenced by [tan\(\)](#).

**5.2.2.127 tanh()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::tanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P>  [inline]
```

Hyperbolic tangent of multivector.

Definition at line 1016 of file [clifford\\_algebra\\_imp.h](#).

References [cosh\(\)](#), and [sinh\(\)](#).

**5.2.2.128 to\_demote()**

```
template<typename Scalar_T>
auto glucat::to_demote (
    const Scalar_T & val) -> typename numeric\_traits<Scalar_T>::demoted::type  [inline]
```

Cast to demote.

Definition at line 135 of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**5.2.2.129 to\_promote()**

```
template<typename Scalar_T>
auto glucat::to_promote (
    const Scalar_T & val) -> typename numeric\_traits<Scalar_T>::promoted::type  [inline]
```

Cast to promote.

Definition at line 125 of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**5.2.2.130 try\_catch() [1/2]**

```
int glucat::try_catch (
    intfn f)
```

Exception catching for functions returning int.

Definition at line 49 of file [try\\_catch.h](#).

Referenced by [glucat::control\\_t::call\(\)](#), and [glucat::control\\_t::call\(\)](#).

**5.2.2.131 try\_catch() [2/2]**

```
int glucat::try_catch (
    intintfn f,
    int arg)
```

Exception catching for functions of int returning int.

Definition at line 64 of file [try\\_catch.h](#).

**5.2.2.132 vector\_part()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::vector_part (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const std::vector<Scalar_←
_T> [inline]
```

Vector part of multivector, as a vector\_t with respect to frame()

Definition at line 529 of file [clifford\\_algebra\\_imp.h](#).

**5.2.3 Variable Documentation****5.2.3.1 BITS\_PER\_SET\_VALUE**

```
const index_t glucat::BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits
```

Number of bits in [set\\_value\\_t](#).

Definition at line 103 of file [global.h](#).

Referenced by [\\_GLUCAT\\_CTAssert\(\)](#).

**5.2.3.2 clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >::default\_truncation**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const Scalar_T glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_←
truncation = std::numeric_limits<Scalar_T>::epsilon()
```

Default for truncation.

Definition at line 74 of file [clifford\\_algebra\\_imp.h](#).

### 5.2.3.3 DEFAULT\_HI

```
const index_t glucat::DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2)
```

Default highest index in an index set.

Definition at line 111 of file [global.h](#).

Referenced by [\\_GLUCAT\\_CTAssert\(\)](#).

### 5.2.3.4 l\_ln2

```
const long double glucat::l_ln2 = 0.6931471805599453094172321214581766L [static]
```

Definition at line 44 of file [long\\_double.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::promoted::ln\\_2\(\)](#).

### 5.2.3.5 l\_pi

```
const long double glucat::l_pi = 3.1415926535897932384626433832795029L [static]
```

Definition at line 43 of file [long\\_double.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::promoted::pi\(\)](#).

### 5.2.3.6 MS\_PER\_S

```
const double glucat::MS_PER_S = 1000.0
```

Timing constant: deprecated here - moved to [test/timing.h](#).

Definition at line 83 of file [global.h](#).

### 5.2.3.7 Tuning\_Fast\_Basis\_Max\_Count

```
const unsigned int glucat::Tuning_Fast_Basis_Max_Count = 1
```

Definition at line 92 of file [tuning.h](#).

### 5.2.3.8 Tuning\_Fast\_CR\_Sqrt\_Max\_Steps

```
const unsigned int glucat::Tuning_Fast_CR_Sqrt_Max_Steps = 256
```

Definition at line 88 of file [tuning.h](#).

#### 5.2.3.9 Tuning\_Fast\_DB\_Sqrt\_Max\_Steps

```
const unsigned int glucat::Tuning_Fast_DB_Sqrt_Max_Steps = 256
```

Definition at line 89 of file [tuning.h](#).

#### 5.2.3.10 Tuning\_Fast\_Div\_Max\_Steps

```
const unsigned int glucat::Tuning_Fast_Div_Max_Steps = 0
```

Definition at line 87 of file [tuning.h](#).

#### 5.2.3.11 Tuning\_Fast\_Fast\_Size\_Threshold

```
const unsigned int glucat::Tuning_Fast_Fast_Size_Threshold = 0
```

Definition at line 93 of file [tuning.h](#).

#### 5.2.3.12 Tuning\_Fast\_Inv\_Fast\_Dim\_Threshold

```
const unsigned int glucat::Tuning_Fast_Inv_Fast_Dim_Threshold = 0
```

Definition at line 94 of file [tuning.h](#).

#### 5.2.3.13 Tuning\_Fast\_Log\_Max\_Inner\_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Inner_Steps = 8
```

Definition at line 91 of file [tuning.h](#).

#### 5.2.3.14 Tuning\_Fast\_Log\_Max\_Outer\_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Outer_Steps = 16
```

Definition at line 90 of file [tuning.h](#).

#### 5.2.3.15 Tuning\_Fast\_Mult\_Matrix\_Threshold

```
const unsigned int glucat::Tuning_Fast_Mult_Matrix_Threshold = 0
```

Definition at line 86 of file [tuning.h](#).

#### 5.2.3.16 Tuning\_Fast\_Products\_Size\_Threshold

```
const unsigned int glucat::Tuning_Fast_Products_Size_Threshold = 0
```

Definition at line 95 of file [tuning.h](#).

#### 5.2.3.17 Tuning\_Int\_Digits

```
const unsigned int glucat::Tuning_Int_Digits = std::numeric_limits<int>::digits
```

Definition at line 36 of file [tuning.h](#).

#### 5.2.3.18 Tuning\_Max\_Threshold

```
const unsigned int glucat::Tuning_Max_Threshold = 1 << Tuning_Int_Digits
```

Definition at line 37 of file [tuning.h](#).

#### 5.2.3.19 Tuning\_Naive\_Basis\_Max\_Count

```
const unsigned int glucat::Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold
```

Definition at line 65 of file [tuning.h](#).

#### 5.2.3.20 Tuning\_Naive\_Fast\_Size\_Threshold

```
const unsigned int glucat::Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 66 of file [tuning.h](#).

#### 5.2.3.21 Tuning\_Naive\_Inv\_Fast\_Dim\_Threshold

```
const unsigned int glucat::Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 67 of file [tuning.h](#).

#### 5.2.3.22 Tuning\_Naive\_Mult\_Matrix\_Threshold

```
const unsigned int glucat::Tuning_Naive_Mult_Matrix_Threshold = 0
```

Definition at line 64 of file [tuning.h](#).

#### 5.2.3.23 Tuning\_Slow\_Basis\_Max\_Count

```
const unsigned int glucat::Tuning_Slow_Basis_Max_Count = 0
```

Definition at line 42 of file [tuning.h](#).

#### 5.2.3.24 Tuning\_Slow\_Fast\_Size\_Threshold

```
const unsigned int glucat::Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 43 of file [tuning.h](#).

#### 5.2.3.25 Tuning\_Slow\_Inv\_Fast\_Dim\_Threshold

```
const unsigned int glucat::Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 44 of file [tuning.h](#).

#### 5.2.3.26 Tuning\_Slow\_Mult\_Matrix\_Threshold

```
const unsigned int glucat::Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold
```

Definition at line 41 of file [tuning.h](#).

#### 5.2.3.27 Tuning\_Slow\_Products\_Size\_Threshold

```
const unsigned int glucat::Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 45 of file [tuning.h](#).

## 5.3 glucat::gen Namespace Reference

### Classes

- class [generator\\_table](#)  
*Table of generators for specific signatures.*

### Typedefs

- using [signature\\_t](#) = std::pair<[index\\_t](#), [index\\_t](#)>  
*A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min()*

### Variables

- static const std::array< [index\\_t](#), 8 > [offset\\_to\\_super](#) = {0,-1, 0,-1,-2, 3, 2, 1}  
*Offsets between the current signature and that of the real superalgebra.*

## 5.3.1 Typedef Documentation

### 5.3.1.1 signature\_t

```
using glucat::gen::signature_t = std::pair<index_t, index_t>
```

A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min()

Definition at line 48 of file [generation.h](#).

## 5.3.2 Variable Documentation

### 5.3.2.1 offset\_to\_super

```
const std::array<index_t, 8> glucat::gen::offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1} [static]
```

Offsets between the current signature and that of the real superalgebra.

Definition at line 86 of file [generation.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), and [glucat::gen::generator\\_table< Matrix\\_T >::operator\(\)](#).

## 5.4 glucat::matrix Namespace Reference

### Classes

- struct [eig\\_genus](#)  
*Structure containing classification of eigenvalues.*

### Typedefs

- using [eig\\_case\\_t](#)  
*Classification of eigenvalues of a matrix.*

### Functions

- template<typename LHS\_T, typename RHS\_T>  
auto [kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Kronecker tensor product of matrices - as per Matlab kron.*
- template<typename LHS\_T, typename RHS\_T>  
auto [mono\\_kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Sparse Kronecker tensor product of monomial matrices.*
- template<typename LHS\_T, typename RHS\_T>  
auto [nork](#) (const LHS\_T &lhs, const RHS\_T &rhs, const bool mono=true) -> const RHS\_T  
*Left inverse of Kronecker product.*
- template<typename LHS\_T, typename RHS\_T>  
auto [signed\\_perm\\_nork](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*
- template<typename Matrix\_T>  
auto [nnz](#) (const Matrix\_T &m) -> typename Matrix\_T::size\_type  
*Number of non-zeros.*
- template<typename Matrix\_T>  
auto [isinf](#) (const Matrix\_T &m) -> bool  
*Infinite.*
- template<typename Matrix\_T>  
auto [isnan](#) (const Matrix\_T &m) -> bool  
*Not a Number.*
- template<typename Matrix\_T>  
auto [unit](#) (const typename Matrix\_T::size\_type n) -> const Matrix\_T



*Unit matrix - as per Matlab eye.*

- template<typename LHS\_T, typename RHS\_T>  
auto [mono\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type

*Product of monomial matrices.*

- template<typename LHS\_T, typename RHS\_T>  
auto [sparse\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type

*Product of sparse matrices.*

- template<typename LHS\_T, typename RHS\_T>  
auto [prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type

*Product of matrices.*

- template<typename Scalar\_T, typename LHS\_T, typename RHS\_T>  
auto [inner](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> Scalar\_T

*Inner product:  $\sum(x(i,j)*y(i,j))/x.nrows()$*

- template<typename Matrix\_T>  
auto [norm\\_frob2](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type

*Square of Frobenius norm.*

- template<typename Matrix\_T>  
auto [trace](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type

*Matrix trace.*

- template<typename Matrix\_T>  
auto [eigenvalues](#) (const Matrix\_T &val) -> std::vector< std::complex< double > >

*Eigenvalues of a matrix.*

- template<typename Matrix\_T>  
auto [classify\\_eigenvalues](#) (const Matrix\_T &val) -> [eig\\_genus](#)< Matrix\_T >

*Classify the eigenvalues of a matrix.*

- template<typename LHS\_T, typename RHS\_T>  
void [nork\\_range](#) (RHS\_T &result, const typename LHS\_T::const\_iterator2 lhs\_it2, const RHS\_T &rhs, const typename RHS\_T::size\_type res\_s1, const typename RHS\_T::size\_type res\_s2)

*Utility routine for nork: calculate result for a range of indices.*

- template<typename Matrix\_T>  
static auto [to\\_lapack](#) (const Matrix\_T &val) -> ublas::matrix< double, ublas::column\_major >

*Convert matrix to LAPACK format.*

## 5.4.1 Typedef Documentation

### 5.4.1.1 eig\_case\_t

```
using glucat::matrix::eig_case_t
```

**Initial value:**

```
enum {
    safe_eigs,
    neg_real_eigs,
    both_eigs}
```

Classification of eigenvalues of a matrix.

Definition at line 133 of file [matrix.h](#).

## 5.4.2 Function Documentation

### 5.4.2.1 `classify_eigenvalues()`

```
template<typename Matrix_T>
auto glucat::matrix::classify_eigenvalues (
    const Matrix_T & val) -> eig_genus<Matrix_T>
```

Classify the eigenvalues of a matrix.

Definition at line 548 of file [matrix\\_imp.h](#).

References [eigenvalues\(\)](#), [epsilon](#), [glucat::matrix::eig\\_genus< Matrix\\_T >::m\\_eig\\_case](#), [glucat::matrix::eig\\_genus< Matrix\\_T >::m\\_eig\\_genus< Matrix\\_T >::m\\_safe\\_arg](#), [glucat::numeric\\_traits< Scalar\\_T >::pi\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T](#)

Referenced by [glucat::matrix\\_log\(\)](#), and [glucat::matrix\\_sqrt\(\)](#).

### 5.4.2.2 `eigenvalues()`

```
template<typename Matrix_T>
auto glucat::matrix::eigenvalues (
    const Matrix_T & val) -> std::vector< std::complex<double> >
```

Eigenvalues of a matrix.

Definition at line 500 of file [matrix\\_imp.h](#).

References [to\\_lapack\(\)](#).

Referenced by [classify\\_eigenvalues\(\)](#).

### 5.4.2.3 `inner()`

```
template<typename Scalar_T, typename LHS_T, typename RHS_T>
auto glucat::matrix::inner (
    const LHS_T & lhs,
    const RHS_T & rhs) -> Scalar_T
```

Inner product:  $\text{sum}(x(i,j)*y(i,j))/x.\text{nrows}()$

Inner product:  $\text{sum}(lhs(i,j)*rhs(i,j))/lhs.\text{nrows}()$

Definition at line 373 of file [matrix\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

### 5.4.2.4 `isinf()`

```
template<typename Matrix_T>
auto glucat::matrix::isinf (
    const Matrix_T & m) -> bool
```

Infinite.

Definition at line 275 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::isInf\(\)](#).

#### 5.4.2.5 `isnan()`

```
template<typename Matrix_T>
auto glucat::matrix::isnan (
    const Matrix_T & m) -> bool
```

Not a Number.

Definition at line 292 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::isNaN\(\)](#).

Referenced by [glucat::operator/\(\)](#).

#### 5.4.2.6 `kron()`

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::kron (
    const LHS_T & lhs,
    const RHS_T & rhs) -> const RHS_T
```

Kronecker tensor product of matrices - as per Matlab `kron`.

Definition at line 83 of file [matrix\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\(\)](#).

#### 5.4.2.7 `mono_kron()`

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::mono_kron (
    const LHS_T & lhs,
    const RHS_T & rhs) -> const RHS_T
```

Sparse Kronecker tensor product of monomial matrices.

Definition at line 119 of file [matrix\\_imp.h](#).

Referenced by [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_pm1\\_qm1\(\)](#).

#### 5.4.2.8 `mono_prod()`

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::mono_prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs) -> const typename RHS_T::expression↔
_type
```

Product of monomial matrices.

Definition at line 320 of file [matrix\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_f](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_pp4\\_qm4\(\)](#), and [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_qp1\\_pm](#)

#### 5.4.2.9 nnz()

```
template<typename Matrix_T>
auto glucat::matrix::nnz (
    const Matrix_T & m) -> typename Matrix_T::size_type
```

Number of non-zeros.

Definition at line 258 of file [matrix\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

#### 5.4.2.10 nork()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::nork (
    const LHS_T & lhs,
    const RHS_T & rhs,
    const bool mono = true) -> const RHS_T
```

Left inverse of Kronecker product.

Definition at line 182 of file [matrix\\_imp.h](#).

References [nork\\_range\(\)](#), and [norm\\_frob2\(\)](#).

#### 5.4.2.11 nork\_range()

```
template<typename LHS_T, typename RHS_T>
void glucat::matrix::nork_range (
    RHS_T & result,
    const typename LHS_T::const_iterator2 lhs_it2,
    const RHS_T & rhs,
    const typename RHS_T::size_type res_s1,
    const typename RHS_T::size_type res_s2)
```

Utility routine for nork: calculate result for a range of indices.

Definition at line 152 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

Referenced by [nork\(\)](#), and [signed\\_perm\\_nork\(\)](#).

#### 5.4.2.12 norm\_frob2()

```
template<typename Matrix_T>
auto glucat::matrix::norm_frob2 (
    const Matrix_T & val) -> typename Matrix_T::value_type
```

Square of Frobenius norm.

Definition at line 395 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::isNaN\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#).

Referenced by [nork\(\)](#).

#### 5.4.2.13 prod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs) -> const typename RHS_T::expression↵
_type [inline]
```

Product of matrices.

Definition at line 361 of file [matrix\\_imp.h](#).

#### 5.4.2.14 signed\_perm\_nork()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::signed_perm_nork (
    const LHS_T & lhs,
    const RHS_T & rhs) -> const RHS_T
```

Left inverse of Kronecker product where lhs is a signed permutation matrix.

Definition at line 228 of file [matrix\\_imp.h](#).

References [nork\\_range\(\)](#).

Referenced by [glucat::fast\(\)](#).

#### 5.4.2.15 sparse\_prod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::sparse_prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs) -> const typename RHS_T::expression↵
_type [inline]
```

Product of sparse matrices.

Definition at line 350 of file [matrix\\_imp.h](#).

#### 5.4.2.16 to\_lapack()

```
template<typename Matrix_T>
static auto glucat::matrix::to_lapack (
    const Matrix_T & val) -> ublas::matrix<double, ublas::column_major> [static]
```

Convert matrix to LAPACK format.

Definition at line 440 of file [matrix\\_imp.h](#).

Referenced by [eigenvalues\(\)](#).

#### 5.4.2.17 trace()

```
template<typename Matrix_T>
auto glucat::matrix::trace (
    const Matrix_T & val) -> typename Matrix_T::value_type
```

Matrix trace.

Definition at line 416 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::isNaN\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#).

#### 5.4.2.18 unit()

```
template<typename Matrix_T>
auto glucat::matrix::unit (
    const typename Matrix_T::size_type n) -> const Matrix_T [inline]
```

Unit matrix - as per Matlab eye.

Definition at line 310 of file [matrix\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), [glucat::fast\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_pm1\\_qm1\(\)](#), and [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_vector\(\)](#).

## 5.5 glucat::timing Namespace Reference

### Functions

- static double [elapsed](#) (clock\_t cpu\_time)  
*Elapsed time in milliseconds.*

### Variables

- const double [MS\\_PER\\_SEC](#) = 1000.0  
*Timing constant: milliseconds per second.*
- const double [MS\\_PER\\_CLOCK](#) = [MS\\_PER\\_SEC](#) / double(CLOCKS\_PER\_SEC)  
*Timing constant: milliseconds per clock.*
- const int [EXTRA\\_TRIALS](#) = 2  
*Timing constant: trial expansion factor.*

### 5.5.1 Function Documentation

#### 5.5.1.1 elapsed()

```
static double glucat::timing::elapsed (
    clock_t cpu_time) [inline], [static]
```

Elapsed time in milliseconds.

Definition at line 51 of file [timing.h](#).

References [MS\\_PER\\_CLOCK](#).

## 5.5.2 Variable Documentation

### 5.5.2.1 EXTRA\_TRIALS

```
const int glucat::timing::EXTRA_TRIALS = 2
```

Timing constant: trial expansion factor.

Definition at line 45 of file [timing.h](#).

### 5.5.2.2 MS\_PER\_CLOCK

```
const double glucat::timing::MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC)
```

Timing constant: milliseconds per clock.

Definition at line 42 of file [timing.h](#).

Referenced by [elapsed\(\)](#).

### 5.5.2.3 MS\_PER\_SEC

```
const double glucat::timing::MS_PER_SEC = 1000.0
```

Timing constant: milliseconds per second.

Definition at line 39 of file [timing.h](#).

## 5.6 pade Namespace Reference

### Classes

- struct [pade\\_log\\_denom](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*
- struct [pade\\_log\\_denom< dd\\_real >](#)
- struct [pade\\_log\\_denom< float >](#)
- struct [pade\\_log\\_denom< long double >](#)
- struct [pade\\_log\\_denom< qd\\_real >](#)
- struct [pade\\_log\\_numer](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*
- struct [pade\\_log\\_numer< dd\\_real >](#)
- struct [pade\\_log\\_numer< float >](#)
- struct [pade\\_log\\_numer< long double >](#)
- struct [pade\\_log\\_numer< qd\\_real >](#)
- struct [pade\\_sqrt\\_denom](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*
- struct [pade\\_sqrt\\_denom< dd\\_real >](#)
- struct [pade\\_sqrt\\_denom< float >](#)
- struct [pade\\_sqrt\\_denom< long double >](#)
- struct [pade\\_sqrt\\_denom< qd\\_real >](#)
- struct [pade\\_sqrt\\_numer](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*
- struct [pade\\_sqrt\\_numer< dd\\_real >](#)
- struct [pade\\_sqrt\\_numer< float >](#)
- struct [pade\\_sqrt\\_numer< long double >](#)
- struct [pade\\_sqrt\\_numer< qd\\_real >](#)

## Variables

- `template<typename Scalar_T>`  
`const pade_sqrt_numer< Scalar_T >::array pade_sqrt_numer< Scalar_T >::numer`
- `template<typename Scalar_T>`  
`const pade_sqrt_denom< Scalar_T >::array pade_sqrt_denom< Scalar_T >::denom`
- `const pade_sqrt_numer< float >::array pade_sqrt_numer< float >::numer`
- `const pade_sqrt_denom< float >::array pade_sqrt_denom< float >::denom`
- `const pade_sqrt_numer< longdouble >::array pade_sqrt_numer< longdouble >::numer`
- `const pade_sqrt_denom< longdouble >::array pade_sqrt_denom< longdouble >::denom`
- `const pade_sqrt_numer< dd_real >::array pade_sqrt_numer< dd_real >::numer`
- `const pade_sqrt_denom< dd_real >::array pade_sqrt_denom< dd_real >::denom`
- `const pade_sqrt_numer< qd_real >::array pade_sqrt_numer< qd_real >::numer`
- `const pade_sqrt_denom< qd_real >::array pade_sqrt_denom< qd_real >::denom`
- `template<typename Scalar_T>`  
`const pade_log_numer< Scalar_T >::array pade_log_numer< Scalar_T >::numer`
- `template<typename Scalar_T>`  
`const pade_log_denom< Scalar_T >::array pade_log_denom< Scalar_T >::denom`
- `const pade_log_numer< float >::array pade_log_numer< float >::numer`
- `const pade_log_denom< float >::array pade_log_denom< float >::denom`
- `const pade_log_numer< longdouble >::array pade_log_numer< longdouble >::numer`
- `const pade_log_denom< longdouble >::array pade_log_denom< longdouble >::denom`
- `const pade_log_numer< dd_real >::array pade_log_numer< dd_real >::numer`
- `const pade_log_denom< dd_real >::array pade_log_denom< dd_real >::denom`
- `const pade_log_numer< qd_real >::array pade_log_numer< qd_real >::numer`
- `const pade_log_denom< qd_real >::array pade_log_denom< qd_real >::denom`

## 5.6.1 Variable Documentation

### 5.6.1.1 pade\_log\_denom< dd\_real >::denom

`const pade_log_denom<dd_real>::array pade::pade_log_denom< dd_real >::denom`

**Initial value:**

```
=
{
    dd_real("1"),
    dd_real("2100")/dd_real("41"),
    dd_real("341145")/dd_real("1066"),
    dd_real("11069856")/dd_real("19721"),
    dd_real("6918660")/dd_real("19721"),
    dd_real("1410864")/dd_real("16687"),
    dd_real("734825")/dd_real("94054"),
    dd_real("348840")/dd_real("1363783"),
    dd_real("6783")/dd_real("2727566"),
    dd_real("266")/dd_real("53187537"),
    dd_real("7")/dd_real("8155422340"),
    dd_real("21")/dd_real("2"),
    dd_real("12635")/dd_real("82"),
    dd_real("1037799")/dd_real("2132"),
    dd_real("9883800")/dd_real("19721"),
    dd_real("293930")/dd_real("1517"),
    dd_real("88179")/dd_real("3034"),
    dd_real("305235")/dd_real("188108"),
    dd_real("40698")/dd_real("1363783"),
    dd_real("9975")/dd_real("70916716"),
    dd_real("7")/dd_real("70916716"),
    dd_real("1")/dd_real("538257874440")
}
```

Definition at line 1825 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.2 pade\_log\_denom< float >::denom

`const pade_log_denom<float>::array pade::pade_log_denom< float >::denom`

**Initial value:**

```
=
{
    1.0,          9.0/2.0,          144.0/17.0,    147.0/17.0,
    441.0/85.0,    63.0/34.0,        84.0/221.0,    9.0/221.0,
    9.0/4862.0,    1.0/48620.0
}
```

Definition at line 1763 of file [matrix\\_multi\\_imp.h](#).



### 5.6.1.3 pade\_log\_denom< longdouble >::denom

```
const pade_log_denom<longdouble>::array pade::pade_log_denom< longdouble >::denom
```

**Initial value:**

```
=
{
    1.0L,                17.0L/2.0L,                1088.0L/33.0L,                850.0L/11.0L,
    41650.0L/341.0L,     140777.0L/1023.0L,         1126216.0L/9889.0L,
    63206.0L/899.0L,
    790075.0L/24273.0L,   60775.0L/5394.0L,                38896.0L/13485.0L,
    21658.0L/40455.0L,
    21658.0L/310155.0L,   4165.0L/682341.0L,                680.0L/2047023.0L,
    34.0L/3411705.0L,
    17.0L/129644790.0L,   1.0L/2333606220
}
```

Definition at line 1790 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.4 pade\_log\_denom< qd\_real >::denom

```
const pade_log_denom<qd_real>::array pade::pade_log_denom< qd_real >::denom
```

**Initial value:**

```
=
{
    qd_real("1"),
    qd_real("33")/qd_real("2"),
    qd_real("8448")/qd_real("65"),
    qd_real("42284")/qd_real("65"),
    qd_real("211420")/qd_real("91"),
    qd_real("573562")/qd_real("91"),
    qd_real("32119472")/qd_real("2379"),
    qd_real("92917044")/qd_real("3965"),
    qd_real("603960786")/qd_real("17995"),
    qd_real("144626625")/qd_real("3599"),
    qd_real("2776831200")/qd_real("68381"),
    qd_real("16692542100")/qd_real("478667"),
    qd_real("12241197540")/qd_real("478667"),
    qd_real("1098569010")/qd_real("68381"),
    qd_real("31387686000")/qd_real("3624193"),
    qd_real("9939433900")/qd_real("2479711"),
    qd_real("67091178825")/qd_real("42155087"),
    qd_real("2683647153")/qd_real("4959422"),
    qd_real("19083713088")/qd_real("121505839"),
    qd_real("4708152900")/qd_real("121505839"),
    qd_real("941630580")/qd_real("116546417"),
    qd_real("88704330")/qd_real("62755763"),
    qd_real("12902448")/qd_real("62755763"),
    qd_real("1542684")/qd_real("62755763"),
    qd_real("6427850")/qd_real("2698497809"),
    qd_real("3471039")/qd_real("18889484663"),
    qd_real("8544096")/qd_real("774468871183"),
    qd_real("39556")/qd_real("79027435835"),
    qd_real("118668")/qd_real("7191496660985"),
    qd_real("10230")/qd_real("27327687311743"),
    qd_real("5456")/qd_real("1011124430534491"),
    qd_real("44")/qd_real("1011124430534491"),
    qd_real("11")/qd_real("70778710137414370"),
    qd_real("1")/qd_real("7219428434016265740")
}
```

Definition at line 1872 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.5 pade\_log\_denom< Scalar\_T >::denom

```
template<typename Scalar_T>
const pade_log_denom<Scalar_T>::array pade::pade_log_denom< Scalar_T >::denom
```

**Initial value:**

```
=
{
    1.0,          13.0/2.0,          468.0/25.0,          1573.0/50.0,
    1573.0/46.0,    11583.0/460.0,    10296.0/805.0,    2574.0/575.0,
    11583.0/10925.0, 143.0/874.0,          572.0/37145.0,    117.0/148580.0,
    13.0/742900.0,          1.0/10400600.0
}
```

Definition at line 1737 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.6 pade\_log\_number< dd\_real >::number

```
const pade_log_number<dd_real>::array pade::pade_log_number< dd_real >::number
```

Initial value:

```
=
{
    dd_real("0"),          dd_real("1"),
    dd_real("10"),          dd_real("22781")/dd_real("492"),
    dd_real("21603")/dd_real("164"),    dd_real("5492649")/dd_real("21320"),
    dd_real("978724")/dd_real("2665"),    dd_real("4191605")/dd_real("10619"),
    dd_real("12874933")/dd_real("39442"),    dd_real("11473457")/dd_real("54612"),
    dd_real("2406734")/dd_real("22755"),    dd_real("166770367")/dd_real("4004880"),
    dd_real("30653165")/dd_real("2402928"),    dd_real("647746389")/dd_real("215195552"),
    dd_real("25346331")/dd_real("47074027"),    dd_real("278270613")/dd_real("3900419380"),
    dd_real("105689791")/dd_real("15601677520"),    dd_real("606046475")/dd_real("1379188292768"),
    dd_real("969715")/dd_real("53502994116"),    dd_real("11098301")/dd_real("26204577562592"),
    dd_real("118999")/dd_real("26204577562592"),    dd_real("18858053")/dd_real("1392249205900512960")
}
```

Definition at line 1805 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.7 pade\_log\_number< float >::number

```
const pade_log_number<float>::array pade::pade_log_number< float >::number
```

Initial value:

```
=
{
    0.0,          1.0,          4.0,          1337.0/204.0,
    385.0/68.0,    1879.0/680.0,    193.0/255.0,    197.0/1820.0,
    419.0/61880.0, 7129.0/61261200.0
}
```

Definition at line 1751 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.8 pade\_log\_number< longdouble >::number

```
const pade_log_number<longdouble>::array pade::pade_log_number< longdouble >::number
```

Initial value:

```
=
{
    0.0L,          1.0L,          8.0L,
    3835.0L/132.0L,          11363807.0L/122760.0L,          162981.0L/1705.0L,
    8365.0L/132.0L,          9036157.0L/125860.0L,
    18009875.0L/453096.0L,    44211925.0L/2718576.0L,          4149566.0L/849555.0L,
    16973929.0L/16020180.0L,
    172459.0L/1068012.0L,    116317061.0L/7025382936.0L,          19679783.0L/18441630207.0L,
    23763863.0L/614721006900.0L,
    50747.0L/79318839600.0L, 42142223.0L/14295951736466400.0L
}
```

Definition at line 1776 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.9 pade\_log\_numer< qd\_real >::numer

```
const pade_log_numer<qd_real>::array pade::pade_log_numer< qd_real >::numer
```

Initial value:

```
=
{
    qd_real("0"),
    qd_real("16"),
    qd_real("95201")/qd_real("780"),
    qd_real("30721")/qd_real("52"),
    qd_real("7416257")/qd_real("3640"),
    qd_real("1039099")/qd_real("195"),
    qd_real("6097772319")/qd_real("555100"),
    qd_real("1564058073")/qd_real("85400"),
    qd_real("30404640205")/qd_real("1209264"),
    qd_real("725351278")/qd_real("25193"),
    qd_real("4092322670789")/qd_real("147429436"),
    qd_real("4559713849589")/qd_real("201040140"),
    qd_real("5049361751189")/qd_real("320023080"),
    qd_real("74979677195")/qd_real("8000577"),
    qd_real("16569850691873")/qd_real("3481514244"),
    qd_real("1065906022369")/qd_real("515779888"),
    qd_real("335956770855841")/qd_real("438412904800"),
    qd_real("1462444287585964")/qd_real("6041877844275"),
    qd_real("397242326339851")/qd_real("6122436215532"),
    qd_real("64211291334131")/qd_real("4373168725380"),
    qd_real("142322343550859")/qd_real("51080680851480"),
    qd_real("154355972958659")/qd_real("351179680853925"),
    qd_real("167483568676259")/qd_real("2937139148960100"),
    qd_real("4230788929433")/qd_real("704913395750424"),
    qd_real("197968763176019")/qd_real("392923948371995600"),
    qd_real("10537522306718")/qd_real("319250708052246425"),
    qd_real("236648286272519")/qd_real("144249197475035425500"),
    qd_real("260715545088119")/qd_real("4375558990076074573500"),
    qd_real("289596255666839")/qd_real("192874640282553367199880"),
    qd_real("8802625510547")/qd_real("361639950529787563499775"),
    qd_real("373831661521439")/qd_real("1659204093030665341336967700"),
    qd_real("446033437968239")/qd_real("464577146048586295574350956000"),
    qd_real("53676090078349")/qd_real("47386868896955802148583797512000")
}
```

Definition at line 1846 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.10 pade\_log\_numer< Scalar\_T >::numer

```
template<typename Scalar_T>
const pade_log_numer<Scalar_T>::array pade::pade_log_numer< Scalar_T >::numer
```

Initial value:

```
=
{
    0.0,
    1.0,
    6.0,
    4741.0/300.0,
    1441.0/60.0,
    107091.0/4600.0,
    8638.0/575.0,
    263111.0/40250.0,
    153081.0/80500.0,
    395243.0/1101240.0,
    28549.0/688275.0,
    605453.0/228813200.0,
    785633.0/10296594000.0,
    1145993.0/1873980108000.0
}
```

Definition at line 1720 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.11 pade\_sqrt\_denom< dd\_real >::denom

```
const pade_sqrt_denom<dd_real>::array pade::pade_sqrt_denom< dd_real >::denom
```

Initial value:

```
=
{
    dd_real("1"),
    dd_real("41")/dd_real("4"),
    dd_real("195")/dd_real("4"),
    dd_real("9139")/dd_real("64"),
}
```

```

        dd_real("73815")/dd_real("256"),
        dd_real("121737")/dd_real("256"),
        dd_real("4539051")/dd_real("16384"),
        dd_real("4032015")/dd_real("65536"),
        dd_real("86493225")/dd_real("16777216"),
        dd_real("5014575")/dd_real("33554432"),
        dd_real("5311735")/dd_real("4294967296"),
        dd_real("33649")/dd_real("17179869184"),
        dd_real("231")/dd_real("1099511627776"),
        dd_real("435897")/dd_real("1024"),
        dd_real("840565")/dd_real("2048"),
        dd_real("9641775")/dd_real("65536"),
        dd_real("84672315")/dd_real("4194304"),
        dd_real("67863915")/dd_real("67108864"),
        dd_real("4345965")/dd_real("268435456"),
        dd_real("1081575")/dd_real("17179869184"),
        dd_real("8855")/dd_real("274877906944"),
        dd_real("1")/dd_real("4398046511104")
    }

```

Definition at line 1496 of file [matrix\\_multi\\_imp.h](#).

#### 5.6.1.12 `pade_sqrt_denom< float >::denom`

```
const pade_sqrt_denom<float>::array pade::pade_sqrt_denom< float >::denom
```

Initial value:

```

=
{
    1.0,          17.0/4.0,      15.0/2.0,      455.0/64.0,
    1001.0/256.0,  1287.0/1024.0,   231.0/1024.0,  165.0/8192.0,
    45.0/65536,   1.0/262144.0
}

```

Definition at line 1433 of file [matrix\\_multi\\_imp.h](#).

#### 5.6.1.13 `pade_sqrt_denom< longdouble >::denom`

```
const pade_sqrt_denom<longdouble>::array pade::pade_sqrt_denom< longdouble >::denom
```

Initial value:

```

=
{
    1.0L,          33.0L/4.0L,      31.0L,      4495.0L/64.0L,
    27405.0L/256.0L,  118755.0L/1024.0L,   94185.0L/1024.0L,  444015.0L/8192.0L,
    1562275.0L/65536.0L,  2042975.0L/262144.0L,   245157.0L/131072.0L,  676039.0L/2097152.0L,
    323323.0L/8388608.0L,  101745.0L/33554432.0L,   4845.0L/33554432.0L,  969.0L/268435456.0L,
    153.0L/4294967296.0L,  1.0L/17179869184.0L
}

```

Definition at line 1460 of file [matrix\\_multi\\_imp.h](#).

#### 5.6.1.14 `pade_sqrt_denom< qd_real >::denom`

```
const pade_sqrt_denom<qd_real>::array pade::pade_sqrt_denom< qd_real >::denom
```

Initial value:

```

=
{
    qd_real("1"),
    qd_real("126"),
    qd_real("557845")/qd_real("256"),
    qd_real("12515965")/qd_real("1024"),
    qd_real("1916797311")/qd_real("65536"),
    qd_real("4450881435")/qd_real("131072"),
    qd_real("171503444385")/qd_real("8388608"),
    qd_real("221120793075")/qd_real("33554432"),
    qd_real("4923689695575")/qd_real("4294967296"),
    qd_real("456864812569")/qd_real("4294967296"),
    qd_real("3486599885395")/qd_real("137438953472"),
    qd_real("2804116503573")/qd_real("549755813888"),
    qd_real("1886827875075")/qd_real("2199023255552"),
    qd_real("263012370465")/qd_real("2199023255552"),
    qd_real("240141729555")/qd_real("17592186044416"),
    qd_real("65")/qd_real("4"),
    qd_real("39711")/qd_real("64"),
    qd_real("5949147")/qd_real("1024"),
    qd_real("170574723")/qd_real("8192"),
    qd_real("8996462475")/qd_real("262144"),
    qd_real("59826782925")/qd_real("2097152"),
    qd_real("420696483235")/qd_real("33554432"),
    qd_real("797168807855")/qd_real("268435456"),
    qd_real("6499270398159")/qd_real("17179869184"),
}

```

```

qd_real("176848560525")/qd_real("140737488355328"),
  qd_real("51538723353")/qd_real("562949953421312"),
  qd_real("1450433115")/qd_real("281474976710656"),
  qd_real("977699359")/qd_real("4503599627370496"),
  qd_real("118183439")/qd_real("18014398509481984"),
  qd_real("9652005")/qd_real("72057594037927936"),
  qd_real("121737")/qd_real("72057594037927936"),
  qd_real("6545")/qd_real("576460752303423488"),
  qd_real("561")/qd_real("18446744073709551616"),
  qd_real("1")/qd_real("73786976294838206464")
}

```

Definition at line 1543 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.15 pade\_sqrt\_denom< Scalar\_T >::denom

```

template<typename Scalar_T>
const pade_sqrt_denom<Scalar_T>::array pade::pade_sqrt_denom< Scalar_T >::denom

```

Initial value:

```

=
{
    1.0,                25.0/4.0,                69.0/4.0,                1771.0/64.0,
    7315.0/256.0,       20349.0/1024.0,          4845.0/512.0,          12597.0/4096.0,
    21879.0/32768.0,    12155.0/131072.0,        1001.0/131072.0,        1365.0/4194304.0,
    91.0/16777216.0,   1.0/67108864.0
}

```

Definition at line 1407 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.16 pade\_sqrt\_numer< dd\_real >::numer

```

const pade_sqrt_numer<dd_real>::array pade::pade_sqrt_numer< dd_real >::numer

```

Initial value:

```

=
{
    dd_real("1"),
    dd_real("215")/dd_real("4"),
    dd_real("90687")/dd_real("256"),
    dd_real("168861")/dd_real("256"),
    dd_real("7228859")/dd_real("16384"),
    dd_real("7538115")/dd_real("65536"),
    dd_real("195747825")/dd_real("16777216"),
    dd_real("14375115")/dd_real("33554432"),
    dd_real("20764055")/dd_real("4294967296"),
    dd_real("206701")/dd_real("17179869184"),
    dd_real("3311")/dd_real("1099511627776"),
    dd_real("43")/dd_real("4"),
    dd_real("10621")/dd_real("64"),
    dd_real("567987")/dd_real("1024"),
    dd_real("1246355")/dd_real("2048"),
    dd_real("16583853")/dd_real("65536"),
    dd_real("173376645")/dd_real("4194304"),
    dd_real("171655785")/dd_real("67108864"),
    dd_real("14375115")/dd_real("268435456"),
    dd_real("5167525")/dd_real("17179869184"),
    dd_real("76153")/dd_real("274877906944"),
    dd_real("43")/dd_real("4398046511104")
}

```

Definition at line 1476 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.17 pade\_sqrt\_numer< float >::numer

```

const pade_sqrt_numer<float>::array pade::pade_sqrt_numer< float >::numer

```

Initial value:

```

=
{
    1.0,                19.0/4.0,                19.0/2.0,                665.0/64.0,
    1729.0/256.0,       2717.0/1024.0,          627.0/1024.0,          627.0/8192.0,
    285.0/65536.0,     19.0/262144.0
}

```

Definition at line 1421 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.18 pade\_sqrt\_numer< longdouble >::numer

```
const pade_sqrt_numer<longdouble>::array pade::pade_sqrt_numer< longdouble >::numer
```

Initial value:

```
=
{
    1.0L,                35.0L/4.0L,                35.0L,                5425.0L/64.0L,
    35525.0L/256.0L,     166257.0L/1024.0L,         143325.0L/1024.0L,         740025.0L/8192.0L,
    2877875.0L/65536.0L,  4206125.0L/262144.0L,         572033.0L/131072.0L,         1820105.0L/2097152.0L,
    1028755.0L/8388608.0L, 395675.0L/33554432.0L,         24225.0L/33554432.0L,         6783.0L/268435456.0L,
    1785.0L/4294967296.0L, 35.0L/17179869184.0L
}
```

Definition at line 1446 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.19 pade\_sqrt\_numer< qd\_real >::numer

```
const pade_sqrt_numer<qd_real>::array pade::pade_sqrt_numer< qd_real >::numer
```

Initial value:

```
=
{
    qd_real("1"),
    qd_real("134"),
    qd_real("633485")/qd_real("256"),
    qd_real("15246721")/qd_real("1024"),
    qd_real("2518145487")/qd_real("65536"),
    qd_real("6344873535")/qd_real("131072"),
    qd_real("267226297065")/qd_real("8388608"),
    qd_real("379874182975")/qd_real("33554432"),
    qd_real("9425348845815")/qd_real("4294967296"),
    qd_real("987417498133")/qd_real("4294967296"),
    qd_real("8055248011085")/qd_real("137438953472"),
    qd_real("6958363175533")/qd_real("549755813888"),
    qd_real("5056698705201")/qd_real("2199023255552"),
    qd_real("766166470485")/qd_real("2199023255552"),
    qd_real("766166470485")/qd_real("17592186044416"),
    qd_real("623623871325")/qd_real("140737488355328"),
    qd_real("203123203803")/qd_real("562949953421312"),
    qd_real("6478601247")/qd_real("281474976710656"),
    qd_real("5038912081")/qd_real("4503599627370496"),
    qd_real("719844583")/qd_real("18014398509481984"),
    qd_real("71853815")/qd_real("72057594037927936"),
    qd_real("1165197")/qd_real("72057594037927936"),
    qd_real("87703")/qd_real("576460752303423488"),
    qd_real("12529")/qd_real("18446744073709551616"),
    qd_real("67")/qd_real("73786976294838206464")
}
```

Definition at line 1517 of file [matrix\\_multi\\_imp.h](#).

### 5.6.1.20 pade\_sqrt\_numer< Scalar\_T >::numer

```
template<typename Scalar_T>
const pade_sqrt_numer<Scalar_T>::array pade::pade_sqrt_numer< Scalar_T >::numer
```

Initial value:

```
=
{
    1.0,                27.0/4.0,                81.0/4.0,                2277.0/64.0,
    10395.0/256.0,       32319.0/1024.0,           8721.0/512.0,           26163.0/4096.0,
    53703.0/32768.0,     36465.0/131072.0,          3861.0/131072.0,       7371.0/4194304.0,
    819.0/16777216.0,    27.0/67108864.0
}
```

Definition at line 1390 of file [matrix\\_multi\\_imp.h](#).

## 5.7 PyClical Namespace Reference

### Classes

- class [clifford](#)
- class [index\\_set](#)

### Functions

- [index\\_set\\_hidden\\_doctests](#) ()
- [clifford\\_hidden\\_doctests](#) ()
- [e](#) (obj)
- [istpq](#) (p, q)
- [\\_test](#) ()

### Variables

- [\\_\\_version\\_\\_](#) = str([glucat\\_package\\_version](#), 'utf-8')
- [lhs](#)
- [rhs](#)
- [threshold](#) = error\_squared\_tol([rhs](#)) if threshold is [None](#) else threshold
- [None](#)
- [tol](#) = error\_squared\_tol([rhs](#)) if tol is [None](#) else tol
- [obj](#)
- [i](#)
- [ixt](#)
- [fill](#)
- [scalar\\_epsilon](#) = [epsilon](#)
- float [pi](#) = atan([clifford](#)(1.0)) \* 4.0
- float [tau](#) = atan([clifford](#)(1.0)) \* 8.0
- [cl](#) = [clifford](#)
- [ist](#) = [index\\_set](#)
- [ninf3](#) = [e](#)(4) + [e](#)(-1)
- [nbar3](#) = [e](#)(4) - [e](#)(-1)

### 5.7.1 Function Documentation

#### 5.7.1.1 [\\_test\(\)](#)

`PyClical._test ()` [protected]

Definition at line [1962](#) of file [PyClical.pyx](#).

References [\\_test\(\)](#).

Referenced by [\\_test\(\)](#).

### 5.7.1.2 clifford\_hidden\_doctests()

PyClicl.al.clifford\_hidden\_doctests ()

Tests for functions that Doctest cannot see.

For clifford.\_\_cinit\_\_: Construct an object of type clifford.

```
>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}"))
-{1}
>>> print(clifford(2, index_set({1,2})))
2{1,2}
>>> print(clifford([2,3], index_set({1,2})))
2{1}+3{2}
>>> print(clifford([1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from <class 'list'>.
>>> print(clifford(None))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from <class 'NoneType'>.
>>> print(clifford(None, [1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
>>> print(clifford([1,2], [1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
>>> print(clifford(""))
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string ''.
>>> print(clifford("{")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{'.
>>> print(clifford("{1}")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1'.
>>> print(clifford("{1}")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '+'.
>>> print(clifford("-"))
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '-'.
>>> print(clifford("{1}+"))
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1}+'.

For clifford.__richcmp__: Compare objects of type clifford.

>>> clifford("{1}") == clifford("1{1}")
True
```



```
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True
```

Definition at line 1253 of file [PyClical.pyx](#).

### 5.7.1.3 e()

```
PyClical.e (
    obj)
```

Abbreviation for `clifford(index_set(obj))`.

```
>>> print(e(1))
{1}
>>> print(e(-1))
{-1}
>>> print(e(0))
1
```

Definition at line 1936 of file [PyClical.pyx](#).

### 5.7.1.4 index\_set\_hidden\_doctests()

```
PyClical.index_set_hidden_doctests ()
```

Tests for functions that Doctest cannot see.

For `index_set.__cinit__`: Construct `index_set`.

```
>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set(""))
{}
>>> print(index_set("{1}")
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1'.
>>> print(index_set("{1}")
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1'.
```

```
>>> print(index_set("{1,2,100}"))
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
>>> print(index_set({1,2,100}))
Traceback (most recent call last):
...
IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.
>>> print(index_set([1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize index_set object from <class 'list'>.

For index_set.__richcmp__: Compare two objects of class index_set.
```

```
>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
>>> None == index_set({1,2})
False
>>> None != index_set({1,2})
True
>>> None < index_set({1,2})
False
>>> None <= index_set({1,2})
False
>>> None > index_set({1,2})
False
>>> None >= index_set({1,2})
False
>>> index_set({1,2}) == None
False
>>> index_set({1,2}) != None
True
>>> index_set({1,2}) < None
False
>>> index_set({1,2}) <= None
False
>>> index_set({1,2}) > None
False
>>> index_set({1,2}) >= None
False
```

Definition at line 406 of file [PyClicl.pyx](#).

#### 5.7.1.5 istpq()

```
PyClicl.istpq (
    p,
    q)
```

Abbreviation for `index_set({-q,...p})`.

```
>>> print(istpq(2,3))
{-3,-2,-1,1,2}
```

Definition at line 1949 of file [PyClicl.pyx](#).

## 5.7.2 Variable Documentation

### 5.7.2.1 `__version__`

```
PyClical.__version__ = str(glucat_package_version, 'utf-8') [private]
```

Definition at line 35 of file [PyClical.pyx](#).

### 5.7.2.2 `cl`

```
PyClical.cl = clifford
```

Definition at line 1910 of file [PyClical.pyx](#).

### 5.7.2.3 `fill`

```
PyClical.fill
```

Definition at line 1864 of file [PyClical.pyx](#).

### 5.7.2.4 `i`

```
PyClical.i
```

Definition at line 1591 of file [PyClical.pyx](#).

### 5.7.2.5 `ist`

```
PyClical.ist = index_set
```

Definition at line 1928 of file [PyClical.pyx](#).

### 5.7.2.6 `ixt`

```
PyClical.ixt
```

Definition at line 1864 of file [PyClical.pyx](#).

### 5.7.2.7 `lhs`

```
PyClical.lhs
```

Definition at line 1359 of file [PyClical.pyx](#).

#### 5.7.2.8 nbar3

```
PyClical.nbar3 = e(4) - e(-1)
```

Definition at line 1959 of file [PyClical.pyx](#).

#### 5.7.2.9 ninf3

```
PyClical.ninf3 = e(4) + e(-1)
```

Definition at line 1958 of file [PyClical.pyx](#).

#### 5.7.2.10 None

```
PyClical.None
```

Definition at line 1359 of file [PyClical.pyx](#).

#### 5.7.2.11 obj

```
PyClical.obj
```

Definition at line 1591 of file [PyClical.pyx](#).

#### 5.7.2.12 pi

```
float PyClical.pi = atan(clifford(1.0)) * 4.0
```

Definition at line 1907 of file [PyClical.pyx](#).

#### 5.7.2.13 rhs

```
PyClical.rhs
```

Definition at line 1359 of file [PyClical.pyx](#).

#### 5.7.2.14 scalar\_epsilon

```
PyClical.scalar_epsilon = epsilon
```

Definition at line 1905 of file [PyClical.pyx](#).

#### 5.7.2.15 tau

```
float PyClical.tau = atan(clifford(1.0)) * 8.0
```

Definition at line 1908 of file [PyClical.pyx](#).

### 5.7.2.16 threshold

```
PyClical.threshold = error_squared_tol(rhs) if threshold is None else threshold
```

Definition at line 1359 of file [PyClical.pyx](#).

### 5.7.2.17 tol

```
PyClical.tol = error_squared_tol(rhs) if tol is None else tol
```

Definition at line 1359 of file [PyClical.pyx](#).

## 5.8 std Namespace Reference

### Classes

- struct [numeric\\_limits](#)< [glucat::framed\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) > >  
*Numeric limits for framed\_multi inherit limits for the corresponding scalar type.*
- struct [numeric\\_limits](#)< [glucat::matrix\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) > >  
*Numeric limits for matrix\_multi inherit limits for the corresponding scalar type.*



## Chapter 6

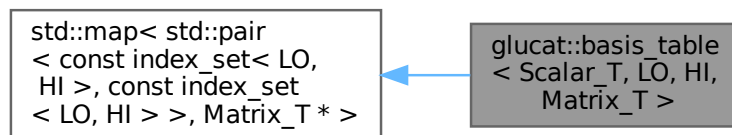
# Class Documentation

### 6.1 glucat::basis\_table< Scalar\_T, LO, HI, Matrix\_T > Class Template Reference

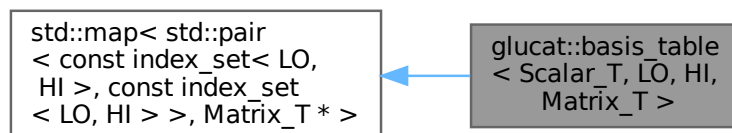
Table of basis elements used as a cache by basis\_element()

```
#include <matrix_multi_imp.h>
```

Inheritance diagram for glucat::basis\_table< Scalar\_T, LO, HI, Matrix\_T >:



Collaboration diagram for glucat::basis\_table< Scalar\_T, LO, HI, Matrix\_T >:



## Public Member Functions

- [basis\\_table](#) (const [basis\\_table](#) &)=delete
- auto [operator=](#) (const [basis\\_table](#) &) -> [basis\\_table](#) &=delete

## Static Public Member Functions

- static auto [basis](#) () -> [basis\\_table](#) &  
*Single instance of basis table.*

## Private Member Functions

- [basis\\_table](#) ()=default
- [~basis\\_table](#) ()=default

## Friends

- class [friend\\_for\\_private\\_destructor](#)

### 6.1.1 Detailed Description

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
class glucat::basis_table< Scalar_T, LO, HI, Matrix_T >
```

Table of basis elements used as a cache by [basis\\_element\(\)](#)

Definition at line [1162](#) of file [matrix\\_multi\\_imp.h](#).

### 6.1.2 Constructor & Destructor Documentation

#### 6.1.2.1 [basis\\_table\(\)](#) [1/2]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::basis_table () [private], [default]
```

Referenced by [basis\(\)](#), [basis\\_table\(\)](#), and [operator=\(\)](#).

#### 6.1.2.2 [~basis\\_table\(\)](#)

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::~~basis\_table () [private], [default]
```

#### 6.1.2.3 [basis\\_table\(\)](#) [2/2]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::basis_table (
    const basis\_table< Scalar_T, LO, HI, Matrix_T > & ) [delete]
```

References [basis\\_table\(\)](#).



## 6.1.3 Member Function Documentation

### 6.1.3.1 basis()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
static auto glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::basis () -> basis_table&    [inline],
[static]
```

Single instance of basis table.

Definition at line 1168 of file [matrix\\_multi\\_imp.h](#).

References [basis\\_table\(\)](#).

### 6.1.3.2 operator=()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
auto glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::operator= (
    const basis_table< Scalar_T, LO, HI, Matrix_T > & ) -> basis_table &=delete
[delete]
```

References [basis\\_table\(\)](#).

## 6.1.4 Friends And Related Symbol Documentation

### 6.1.4.1 friend\_for\_private\_destructor

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
friend class friend_for_private_destructor    [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 1173 of file [matrix\\_multi\\_imp.h](#).

References [friend\\_for\\_private\\_destructor](#).

Referenced by [friend\\_for\\_private\\_destructor](#).

The documentation for this class was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.2 glucat::bool\_to\_type< truth\_value > Class Template Reference

Bool to type.

```
#include <global.h>
```

## Private Types

- enum { `value` = `truth_value` }

### 6.2.1 Detailed Description

```
template<bool truth_value>
class glucat::bool_to_type< truth_value >
```

Bool to type.

Definition at line 69 of file [global.h](#).

### 6.2.2 Member Enumeration Documentation

#### 6.2.2.1 anonymous enum

```
template<bool truth_value>
anonymous enum [private]
```

#### Enumerator

value	
-------	--

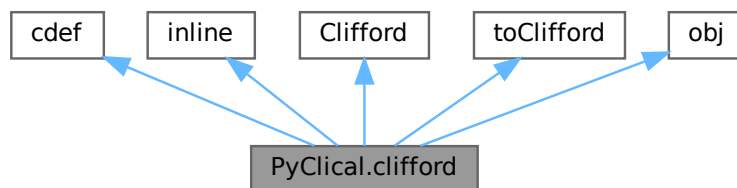
Definition at line 72 of file [global.h](#).

The documentation for this class was generated from the following file:

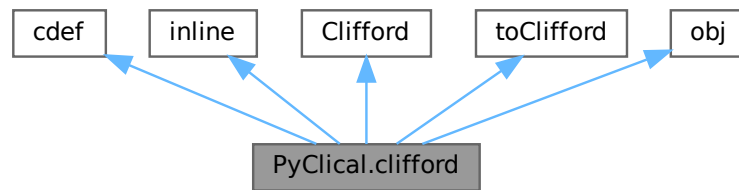
- [glucat/global.h](#)

## 6.3 PyClical.clifford Class Reference

Inheritance diagram for PyClical.clifford:



Collaboration diagram for PyClical.clifford:



### Public Member Functions

- `__cinit__` (self, other=0, `ixt=None`)
- `__dealloc__` (self)
- `__contains__` (self, x)
- `__iter__` (self)
- `reframe` (self, `ixt`)
- `__richcmp__` (lhs, rhs, int, op)
- `__getitem__` (self, `ixt`)
- `__neg__` (self)
- `__pos__` (self)
- `__add__` (lhs, rhs)
- `__iadd__` (self, rhs)
- `__sub__` (lhs, rhs)
- `__isub__` (self, rhs)
- `__mul__` (lhs, rhs)
- `__imul__` (self, rhs)
- `__mod__` (lhs, rhs)
- `__imod__` (self, rhs)
- `__and__` (lhs, rhs)
- `__iand__` (self, rhs)
- `__xor__` (lhs, rhs)
- `__ixor__` (self, rhs)
- `__truediv__` (lhs, rhs)
- `__idiv__` (self, rhs)
- `inv` (self)
- `__or__` (lhs, rhs)
- `__ior__` (self, rhs)
- `__pow__` (self, m, dummy)
- `pow` (self, m)
- `outer_pow` (self, m)
- `__call__` (self, grade)
- `scalar` (self)
- `pure` (self)
- `even` (self)
- `odd` (self)
- `vector_part` (self, frm=`None`)
- `involute` (self)
- `reverse` (self)

- [conj](#) (self)
- [quad](#) (self)
- [norm](#) (self)
- [abs](#) (self)
- [max\\_abs](#) (self)
- [truncated](#) (self, limit)
- [isinf](#) (self)
- [isnan](#) (self)
- [frame](#) (self)
- [\\_\\_repr\\_\\_](#) (self)
- [\\_\\_str\\_\\_](#) (self)

## Public Attributes

- [instance](#) = new [Clifford](#)((<[clifford](#)>other).unwrap())

## 6.3.1 Detailed Description

Python class `clifford` wraps C++ class `Clifford`.

Definition at line 537 of file [PyClical.pyx](#).

## 6.3.2 Member Function Documentation

### 6.3.2.1 `__add__()`

```
PyClical.clifford.__add__ (
    lhs,
    rhs)
```

Geometric sum.

```
>>> print(clifford(1) + clifford("{2}"))
1+{2}
>>> print(clifford("{1}") + clifford("{2}"))
{1}+{2}
```

Definition at line 740 of file [PyClical.pyx](#).

### 6.3.2.2 `__and__()`

```
PyClical.clifford.__and__ (
    lhs,
    rhs)
```

Inner product.

```
>>> print(clifford("{1}") & clifford("{2}"))
0
>>> print(clifford(2) & clifford("{2}"))
0
>>> print(clifford("{1}") & clifford("{1}"))
1
>>> print(clifford("{1}") & clifford("{1,2}"))
{2}
```

Definition at line 836 of file [PyClical.pyx](#).

### 6.3.2.3 `__call__()`

```
PyClical.clifford.__call__ (
    self,
    grade)
```

Pure grade-vector part.

```
>>> print(clifford("{1}") (1))
{1}
>>> print(clifford("{1}") (0))
0
>>> print(clifford("1+{1}+{1,2}") (0))
1
>>> print(clifford("1+{1}+{1,2}") (1))
{1}
>>> print(clifford("1+{1}+{1,2}") (2))
{1,2}
>>> print(clifford("1+{1}+{1,2}") (3))
0
```

Definition at line 1020 of file [PyClical.pyx](#).

References [instance](#), and [PyClical.index\\_set.instance](#).

### 6.3.2.4 `__cinit__()`

```
PyClical.clifford.__cinit__ (
    self,
    other = 0,
    ixt = None)
```

Construct an object of type clifford.

```
>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}") )
-{1}
>>> print(clifford(2, index_set ({1,2})))
2{1,2}
>>> print(clifford([2,3], index_set ({1,2})))
2{1}+3{2}
```

Definition at line 565 of file [PyClical.pyx](#).

### 6.3.2.5 `__contains__()`

```
PyClical.clifford.__contains__ (
    self,
    x)
```

Not applicable.

```
>>> x=clifford(index_set({-3,4,7})); -3 in x
Traceback (most recent call last):
...
TypeError: Not applicable.
```

Definition at line 627 of file [PyClical.pyx](#).

### 6.3.2.6 `__dealloc__()`

```
PyClical.clifford.__dealloc__ (
    self)
```

Clean up by deallocating the instance of C++ class Clifford.

Definition at line 621 of file [PyClical.pyx](#).

References [instance](#), and [PyClical.index\\_set.instance](#).

### 6.3.2.7 `__getitem__()`

```
PyClical.clifford.__getitem__ (
    self,
    ixt)
```

Subscripting: map from index set to scalar coordinate.

```
>>> clifford("{1}")[index_set(1)]
1.0
>>> clifford("{1}")[index_set({1})]
1.0
>>> clifford("{1}")[index_set({1,2})]
0.0
>>> clifford("2{1,2}")[index_set({1,2})]
2.0
```

Definition at line 707 of file [PyClical.pyx](#).

References [instance](#), and [PyClical.index\\_set.instance](#).

**6.3.2.8 `__iadd__()`**

```
PyClical.clifford.__iadd__ (
    self,
    rhs)
```

Geometric sum.

```
>>> x = clifford(1); x += clifford("{2}"); print(x)
1+{2}
```

Definition at line 751 of file [PyClical.pyx](#).

**6.3.2.9 `__iand__()`**

```
PyClical.clifford.__iand__ (
    self,
    rhs)
```

Inner product.

```
>>> x = clifford("{1}"); x &= clifford("{2}"); print(x)
0
>>> x = clifford(2); x &= clifford("{2}"); print(x)
0
>>> x = clifford("{1}"); x &= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
{2}
```

Definition at line 851 of file [PyClical.pyx](#).

**6.3.2.10 `__idiv__()`**

```
PyClical.clifford.__idiv__ (
    self,
    rhs)
```

Geometric quotient.

```
>>> x = clifford("{1}"); x /= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x /= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x /= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
-{2}
```

Definition at line 911 of file [PyClical.pyx](#).

**6.3.2.11 `__imod__()`**

```
PyClical.clifford.__imod__ (
    self,
    rhs)
```

Contraction.

```
>>> x = clifford("{1}"); x %= clifford("{2}"); print(x)
0
>>> x = clifford(2); x %= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x %= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
{2}
```

Definition at line 821 of file [PyClical.pyx](#).

**6.3.2.12 `__imul__()`**

```
PyClical.clifford.__imul__ (
    self,
    rhs)
```

Geometric product.

```
>>> x = clifford(2); x *= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x *= clifford("{2}"); print(x)
{1,2}
>>> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
{2}
```

Definition at line 793 of file [PyClical.pyx](#).

**6.3.2.13 `__ior__()`**

```
PyClical.clifford.__ior__ (
    self,
    rhs)
```

Transform left hand side, using right hand side as a transformation.

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
-{1}
```

Definition at line 950 of file [PyClical.pyx](#).



**6.3.2.14** `__isub__()`

```
PyClical.clifford.__isub__ (
    self,
    rhs)
```

Geometric difference.

```
>>> x = clifford(1); x -= clifford("{2}"); print(x)
1-{2}
```

Definition at line 771 of file [PyClical.pyx](#).

**6.3.2.15** `__iter__()`

```
PyClical.clifford.__iter__ (
    self)
```

Not applicable.

```
>>> for a in clifford(index_set({-3,4,7})):print(a, end=",")
Traceback (most recent call last):
...
TypeError: Not applicable.
```

Definition at line 638 of file [PyClical.pyx](#).

**6.3.2.16** `__ixor__()`

```
PyClical.clifford.__ixor__ (
    self,
    rhs)
```

Outer product.

```
>>> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x ^= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
0
>>> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
0
```

Definition at line 881 of file [PyClical.pyx](#).

**6.3.2.17 `__mod__()`**

```
PyClical.clifford.__mod__ (
    lhs,
    rhs)
```

Contraction.

```
>>> print(clifford("{1}") % clifford("{2}"))
0
>>> print(clifford(2) % clifford("{2}"))
2{2}
>>> print(clifford("{1}") % clifford("{1}"))
1
>>> print(clifford("{1}") % clifford("{1,2}"))
{2}
```

Definition at line 806 of file [PyClical.pyx](#).

**6.3.2.18 `__mul__()`**

```
PyClical.clifford.__mul__ (
    lhs,
    rhs)
```

Geometric product.

```
>>> print(clifford("{1}") * clifford("{2}"))
{1,2}
>>> print(clifford(2) * clifford("{2}"))
2{2}
>>> print(clifford("{1}") * clifford("{1,2}"))
{2}
```

Definition at line 780 of file [PyClical.pyx](#).

**6.3.2.19 `__neg__()`**

```
PyClical.clifford.__neg__ (
    self)
```

Unary `-`.

```
>>> print(-clifford("{1}"))
-{1}
```

Definition at line 722 of file [PyClical.pyx](#).

References [instance](#), and [PyClical.index\\_set.instance](#).

**6.3.2.20 `__or__()`**

```
PyClical.clifford.__or__ (
    lhs,
    rhs)
```

Transform left hand side, using right hand side as a transformation.

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
-{1}
```

Definition at line 939 of file [PyClical.pyx](#).

**6.3.2.21 `__pos__()`**

```
PyClical.clifford.__pos__ (
    self)
```

Unary +.

```
>>> print(+clifford("{1}"))
{1}
```

Definition at line 731 of file [PyClical.pyx](#).

**6.3.2.22 `__pow__()`**

```
PyClical.clifford.__pow__ (
    self,
    m,
    dummy)
```

Power: self to the m.

```
>>> x=clifford("{1}"); print(x ** 2)
1
>>> x=clifford("2"); print(x ** 2)
4
>>> x=clifford("2+{1}"); print(x ** 0)
1
>>> x=clifford("2+{1}"); print(x ** 1)
2+{1}
>>> x=clifford("2+{1}"); print(x ** 2)
5+4{1}
>>> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
1
```

Definition at line 961 of file [PyClical.pyx](#).

References [pow\(\)](#).

**6.3.2.23 `__repr__()`**

```
PyClical.clifford.__repr__ (
    self)
```

The "official" string representation of self.

```
>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
```

Definition at line 1235 of file [PyClical.pyx](#).

References [clifford\\_to\\_repr\(\)](#).

**6.3.2.24 `__richcmp__()`**

```
PyClical.clifford.__richcmp__ (
    lhs,
    rhs,
    int,
    op)
```

Compare objects of type clifford.

```
>>> clifford("{1}") == clifford("1{1}")
True
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True
```

Definition at line 672 of file [PyClical.pyx](#).

**6.3.2.25 `__str__()`**

```
PyClical.clifford.__str__ (
    self)
```

The "informal" string representation of self.

```
>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
'1+3{-1}+2{1,2}+4{-2,7}'
```

Definition at line 1244 of file [PyClical.pyx](#).

References [clifford\\_to\\_str\(\)](#).

**6.3.2.26 `__sub__()`**

```
PyClical.clifford.__sub__ (
    lhs,
    rhs)
```

Geometric difference.

```
>>> print(clifford(1) - clifford("{2}"))
1-{2}
>>> print(clifford("{1}") - clifford("{2}"))
{1}-{2}
```

Definition at line 760 of file [PyClical.pyx](#).

**6.3.2.27 `__truediv__()`**

```
PyClical.clifford.__truediv__ (
    lhs,
    rhs)
```

Geometric quotient.

```
>>> print(clifford("{1}") / clifford("{2}"))
{1,2}
>>> print(clifford(2) / clifford("{2}"))
2{2}
>>> print(clifford("{1}") / clifford("{1}"))
1
>>> print(clifford("{1}") / clifford("{1,2}"))
-{2}
```

Definition at line 896 of file [PyClical.pyx](#).

**6.3.2.28 `__xor__()`**

```
PyClical.clifford.__xor__ (
    lhs,
    rhs)
```

Outer product.

```
>>> print(clifford("{1}") ^ clifford("{2}"))
{1,2}
>>> print(clifford(2) ^ clifford("{2}"))
2{2}
>>> print(clifford("{1}") ^ clifford("{1}"))
0
>>> print(clifford("{1}") ^ clifford("{1,2}"))
0
```

Definition at line 866 of file [PyClical.pyx](#).

### 6.3.2.29 abs()

```
PyClical.clifford.abs (  
    self)
```

Absolute value: square root of norm.

```
>>> clifford("1+{-1}+{1,2}+{1,2,3}").abs()  
2.0
```

Definition at line 1175 of file [PyClical.pyx](#).

References [glucat.abs\(\)](#).

### 6.3.2.30 conj()

```
PyClical.clifford.conj (  
    self)
```

Conjugation, reverse o involute == involute o reverse.

```
>>> print((clifford("{1}")).conj())  
-{1}  
>>> print((clifford("{2}") * clifford("{1}")).conj())  
{1,2}  
>>> print((clifford("{1}") * clifford("{2}")).conj())  
-{1,2}  
>>> print(clifford("1+{1}+{1,2}").conj())  
1-{1}-{1,2}
```

Definition at line 1138 of file [PyClical.pyx](#).

References [conj\(\)](#), [instance](#), and [PyClical.index\\_set.instance](#).

Referenced by [conj\(\)](#).

### 6.3.2.31 even()

```
PyClical.clifford.even (  
    self)
```

Even part of multivector, sum of even grade terms.

```
>>> print(clifford("1+{1}+{1,2}").even())  
1+{1,2}
```

Definition at line 1061 of file [PyClical.pyx](#).

References [even\(\)](#), [instance](#), and [PyClical.index\\_set.instance](#).

Referenced by [even\(\)](#).

**6.3.2.32 frame()**

```
PyClical.clifford.frame (
    self)
```

Subalgebra generated by all generators of terms of given multivector.

```
>>> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
{-2,-1,1,2,7}
>>> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
<class 'PyClical.index_set'>
```

Definition at line 1224 of file [PyClical.pyx](#).

References [frame\(\)](#), [instance](#), and [PyClical.index\\_set.instance](#).

Referenced by [frame\(\)](#).

**6.3.2.33 inv()**

```
PyClical.clifford.inv (
    self)
```

Geometric multiplicative inverse.

```
>>> x = clifford("{1}"); print(x.inv())
{1}
>>> x = clifford(2); print(x.inv())
0.5
>>> x = clifford("{1,2}"); print(x.inv())
-1,2}
```

Definition at line 926 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [inv\(\)](#).

Referenced by [inv\(\)](#).

**6.3.2.34 involute()**

```
PyClical.clifford.involute (
    self)
```

Main involution, each {i} is replaced by -{i} in each term,  
eg. `clifford("{1}") -> -clifford("{1}")`.

```
>>> print(clifford("{1}").involute())
-1}
>>> print((clifford("{2}") * clifford("{1}")).involute())
-1,2}
>>> print((clifford("{1}") * clifford("{2}")).involute())
1,2}
>>> print(clifford("1+{1}+{1,2}").involute())
1-1}+{1,2}
```

Definition at line 1107 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [involute\(\)](#).

Referenced by [involute\(\)](#).

### 6.3.2.35 isinf()

```
PyClical.clifford.isinf (  
    self)
```

Check if a multivector contains any infinite values.

```
>>> clifford().isinf()  
False
```

Definition at line 1206 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [isnan\(\)](#).

### 6.3.2.36 isnan()

```
PyClical.clifford.isnan (  
    self)
```

Check if a multivector contains any IEEE NaN values.

```
>>> clifford().isnan()  
False
```

Definition at line 1215 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [isinf\(\)](#).

Referenced by [isinf\(\)](#), and [isnan\(\)](#).

### 6.3.2.37 max\_abs()

```
PyClical.clifford.max_abs (  
    self)
```

Maximum of absolute values of components of multivector: multivector infinity norm.

```
>>> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()  
1.0  
>>> clifford("3+2{1}+{1,2}").max_abs()  
3.0
```

Definition at line 1184 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [max\\_abs\(\)](#).

Referenced by [max\\_abs\(\)](#).



**6.3.2.38 norm()**

```
PyClical.clifford.norm (
    self)
```

Norm == sum of squares of coordinates.

```
>>> clifford("1+{1}+{1,2}").norm()
3.0
>>> clifford("1+{-1}+{1,2}+{1,2,3}").norm()
4.0
```

Definition at line 1164 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [norm\(\)](#).

Referenced by [norm\(\)](#).

**6.3.2.39 odd()**

```
PyClical.clifford.odd (
    self)
```

Odd part of multivector, sum of odd grade terms.

```
>>> print(clifford("1+{1}+{1,2}").odd())
{1}
```

Definition at line 1070 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [odd\(\)](#).

Referenced by [odd\(\)](#).

**6.3.2.40 outer\_pow()**

```
PyClical.clifford.outer_pow (
    self,
    m)
```

Outer product power.

```
>>> x=clifford("2+{1}"); print(x.outer_pow(0))
1
>>> x=clifford("2+{1}"); print(x.outer_pow(1))
2+{1}
>>> x=clifford("2+{1}"); print(x.outer_pow(2))
4+4{1}
>>> print(clifford("1+{1}+{1,2}").outer_pow(3))
1+3{1}+3{1,2}
```

Definition at line 1004 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [outer\\_pow\(\)](#).

Referenced by [outer\\_pow\(\)](#).

### 6.3.2.41 pow()

```
PyClical.clifford.pow (
    self,
    m)

Power: self to the m.

>>> x=clifford("{1}"); print(x.pow(2))
1
>>> x=clifford("2"); print(x.pow(2))
4
>>> x=clifford("2+{1}"); print(x.pow(0))
1
>>> x=clifford("2+{1}"); print(x.pow(1))
2+{1}
>>> x=clifford("2+{1}"); print(x.pow(2))
5+4{1}
>>> print(clifford("1+{1}+{1,2}").pow(3))
1+3{1}+3{1,2}
>>> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
1
```

Definition at line 980 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [pow\(\)](#).

Referenced by [\\_\\_pow\\_\\_\(\)](#), and [pow\(\)](#).

### 6.3.2.42 pure()

```
PyClical.clifford.pure (
    self)

Pure part.

>>> print(clifford("1+{1}+{1,2}").pure())
{1}+{1,2}
>>> print(clifford("{1,2}").pure())
{1,2}
```

Definition at line 1050 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [pure\(\)](#).

Referenced by [pure\(\)](#).

### 6.3.2.43 quad()

```
PyClical.clifford.quad (
    self)

Quadratic form == (rev(x)*x)(0).

>>> print(clifford("1+{1}+{1,2}").quad())
3.0
>>> print(clifford("1+{-1}+{1,2}+{1,2,3}").quad())
2.0
```

Definition at line 1153 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [quad\(\)](#).

Referenced by [quad\(\)](#).

**6.3.2.44 reframe()**

```
PyClical.clifford.reframe (
    self,
    ixt)
```

Put self into a larger frame, containing the union of self.frame() and index set ixt. This can be used to make multiplication faster, by multiplying within a common frame.

```
>>> clifford("2+3{1}").reframe(index_set({1,2,3}))
clifford("2+3{1}")
>>> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() == (s|t);
True
```

Definition at line 649 of file [PyClical.pyx](#).

**6.3.2.45 reverse()**

```
PyClical.clifford.reverse (
    self)
```

Reversion, eg. `clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}")`.

```
>>> print(clifford("{1}").reverse())
{1}
>>> print((clifford("{2}") * clifford("{1}")).reverse())
{1,2}
>>> print((clifford("{1}") * clifford("{2}")).reverse())
-{1,2}
>>> print(clifford("1+{1}+{1,2}").reverse())
1+{1}-{1,2}
```

Definition at line 1123 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [reverse\(\)](#).

Referenced by [reverse\(\)](#).

**6.3.2.46 scalar()**

```
PyClical.clifford.scalar (
    self)
```

Scalar part.

```
>>> clifford("1+{1}+{1,2}").scalar()
1.0
>>> clifford("{1,2}").scalar()
0.0
```

Definition at line 1039 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [scalar\(\)](#).

Referenced by [scalar\(\)](#).

### 6.3.2.47 truncated()

```
PyClical.clifford.truncated (
    self,
    limit)
```

Remove all terms of self with relative size smaller than limit.

```
>>> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
clifford("100000000")
>>> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
clifford("10000+{1}")
```

Definition at line 1195 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [truncated\(\)](#).

Referenced by [truncated\(\)](#).

### 6.3.2.48 vector\_part()

```
PyClical.clifford.vector_part (
    self,
    frm = None)
```

Vector part of multivector, as a Python list, with respect to frm.

```
>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
[2.0, 3.0]
>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set({-1,1,2})))
[0.0, 2.0, 3.0]
```

Definition at line 1079 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index\\_set.instance](#), and [vector\\_part\(\)](#).

Referenced by [vector\\_part\(\)](#).

## 6.3.3 Member Data Documentation

### 6.3.3.1 instance

```
PyClical.clifford.instance = new Clifford((<clifford>other).unwrap())
```

Definition at line 592 of file [PyClical.pyx](#).

Referenced by [\\_\\_call\\_\\_\(\)](#), [\\_\\_dealloc\\_\\_\(\)](#), [\\_\\_getitem\\_\\_\(\)](#), [\\_\\_neg\\_\\_\(\)](#), [conj\(\)](#), [even\(\)](#), [frame\(\)](#), [inv\(\)](#), [involute\(\)](#), [isinf\(\)](#), [isnan\(\)](#), [max\\_abs\(\)](#), [norm\(\)](#), [odd\(\)](#), [outer\\_pow\(\)](#), [pow\(\)](#), [pure\(\)](#), [quad\(\)](#), [reverse\(\)](#), [scalar\(\)](#), [truncated\(\)](#), and [vector\\_part\(\)](#).

The documentation for this class was generated from the following file:

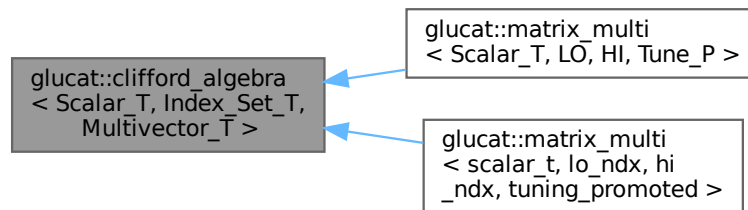
- [pyclical/PyClical.pyx](#)

## 6.4 glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T > Class Template Reference

clifford\_algebra<> declares the operations of a [Clifford](#) algebra

```
#include <clifford_algebra.h>
```

Inheritance diagram for glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >:



### Public Types

- using [scalar\\_t](#) = Scalar\_T
- using [index\\_set\\_t](#) = Index\_Set\_T
- using [multivector\\_t](#) = Multivector\_T
- using [pair\\_t](#) = std::pair<const [index\\_set\\_t](#), Scalar\_T>
- using [vector\\_t](#) = std::vector<Scalar\_T>

### Public Member Functions

- virtual [~clifford\\_algebra](#) ()=default
- virtual auto [operator==](#) (const [multivector\\_t](#) &val) const -> bool=0  
*Test for equality of multivectors.*
- virtual auto [operator==](#) (const Scalar\_T &scr) const -> bool=0  
*Test for equality of multivector and scalar.*
- virtual auto [operator+=](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Geometric sum.*
- virtual auto [operator+=](#) (const Scalar\_T &scr) -> [multivector\\_t](#) &=0  
*Geometric sum of multivector and scalar.*
- virtual auto [operator-=](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Geometric difference.*
- virtual auto [operator-=](#) (const Scalar\_T &scr) -> [multivector\\_t](#) &=0  
*Geometric difference of multivector and scalar.*
- virtual auto [operator-](#) () const -> const [multivector\\_t](#)=0  
*Unary -.*
- virtual auto [operator\\*=](#) (const Scalar\_T &scr) -> [multivector\\_t](#) &=0  
*Product of multivector and scalar.*
- virtual auto [operator\\*=](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0

- Geometric product.*
- virtual auto `operator%=(const multivector_t &rhs) -> multivector_t &=0`
- Contraction.*
- virtual auto `operator&=(const multivector_t &rhs) -> multivector_t &=0`
- Inner product.*
- virtual auto `operator^=(const multivector_t &rhs) -> multivector_t &=0`
- Outer product.*
- virtual auto `operator/=(const Scalar_T &scr) -> multivector_t &=0`
- Quotient of multivector and scalar.*
- virtual auto `operator/=(const multivector_t &rhs) -> multivector_t &=0`
- Geometric quotient.*
- virtual auto `operator|=(const multivector_t &rhs) -> multivector_t &=0`
- Transformation via twisted adjoint action.*
- virtual auto `inv () const -> const multivector_t=0`
- Geometric multiplicative inverse.*
- virtual auto `pow (int m) const -> const multivector_t=0`
- \*this to the m*
- virtual auto `outer_pow (int m) const -> const multivector_t=0`
- Outer product power.*
- virtual auto `frame () const -> const index_set_t=0`
- Subalgebra generated by all generators of terms of given multivector.*
- virtual auto `grade () const -> index_t=0`
- Maximum of the grades of each term.*
- virtual auto `operator[] (const index_set_t ist) const -> Scalar_T=0`
- Subscripting: map from index set to scalar coordinate.*
- virtual auto `operator() (index_t grade) const -> const multivector_t=0`
- Pure grade-vector part.*
- virtual auto `scalar () const -> Scalar_T=0`
- Scalar part.*
- virtual auto `pure () const -> const multivector_t=0`
- Pure part.*
- virtual auto `even () const -> const multivector_t=0`
- Even part of multivector, sum of even grade terms.*
- virtual auto `odd () const -> const multivector_t=0`
- Odd part of multivector, sum of odd grade terms.*
- virtual auto `vector_part () const -> const vector_t=0`
- Vector part of multivector, as a vector\_t with respect to frame()*
- virtual auto `vector_part (const index_set_t frm, const bool prechecked) const -> const vector_t=0`
- Vector part of multivector, as a vector\_t with respect to frm.*
- virtual auto `involute () const -> const multivector_t=0`
- Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.*
- virtual auto `reverse () const -> const multivector_t=0`
- Reversion, eg. {1}\*{2} -> {2}\*{1}.*
- virtual auto `conj () const -> const multivector_t=0`
- Conjugation, reverse o involute == involute o reverse.*
- virtual auto `quad () const -> Scalar_T=0`
- Scalar\_T quadratic form == (rev(x)\*x)(0)*
- virtual auto `norm () const -> Scalar_T=0`
- Scalar\_T norm == sum of norm of coordinates.*
- virtual auto `max_abs () const -> Scalar_T=0`
- Maximum of absolute values of components of multivector: multivector infinity norm.*

- virtual auto [truncated](#) (const Scalar\_T &limit=[default\\_truncation](#)) const -> const [multivector\\_t](#)=0  
*Remove all terms with relative size smaller than limit.*
- virtual auto [isinf](#) () const -> bool=0  
*Check if a multivector contains any infinite values.*
- virtual auto [isnan](#) () const -> bool=0  
*Check if a multivector contains any IEEE NaN values.*
- virtual void [write](#) (const std::string &msg="") const =0  
*Write formatted multivector to output.*
- virtual void [write](#) (std::ofstream &ofile, const std::string &msg="") const =0  
*Write formatted multivector to file.*

### Static Public Member Functions

- static auto [classname](#) () -> const std::string

### Static Public Attributes

- static const [index\\_t v\\_lo](#) = index\_set\_t::v\_lo
- static const [index\\_t v\\_hi](#) = index\_set\_t::v\_hi
- static const Scalar\_T [default\\_truncation](#)  
*Default for truncation.*

## 6.4.1 Detailed Description

**template<typename Scalar\_T, typename Index\_Set\_T, typename Multivector\_T>**  
**class glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >**

[clifford\\_algebra<>](#) declares the operations of a [Clifford](#) algebra

Definition at line [45](#) of file [clifford\\_algebra.h](#).

## 6.4.2 Member Typedef Documentation

### 6.4.2.1 index\_set\_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::index_set_t = Index_↵
Set_T
```

Definition at line [49](#) of file [clifford\\_algebra.h](#).

### 6.4.2.2 multivector\_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::multivector_t = Multivector_↵
_T
```

Definition at line [52](#) of file [clifford\\_algebra.h](#).

### 6.4.2.3 pair\_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pair_t = std::pair<const
index_set_t, Scalar_T>
```

Definition at line 53 of file [clifford\\_algebra.h](#).

### 6.4.2.4 scalar\_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar_t = Scalar_T
```

Definition at line 48 of file [clifford\\_algebra.h](#).

### 6.4.2.5 vector\_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_t = std::vector<Scalar↵
_T>
```

Definition at line 54 of file [clifford\\_algebra.h](#).

## 6.4.3 Constructor & Destructor Documentation

### 6.4.3.1 ~clifford\_algebra()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::~~clifford_algebra
() [virtual], [default]
```

## 6.4.4 Member Function Documentation

### 6.4.4.1 classname()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::classname () -> const
std::string [static]
```

Definition at line 66 of file [clifford\\_algebra\\_imp.h](#).

### 6.4.4.2 conj()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::conj () const
-> const multivector_t [pure virtual]
```

Conjugation, reverse o involute == involute o reverse.

References [conj\(\)](#).

Referenced by [conj\(\)](#).



#### 6.4.4.3 even()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::even () const
-> const multivector_t [pure virtual]
```

Even part of multivector, sum of even grade terms.

References [even\(\)](#).

Referenced by [even\(\)](#).

#### 6.4.4.4 frame()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame () const
-> const index_set_t [pure virtual]
```

Subalgebra generated by all generators of terms of given multivector.

References [frame\(\)](#).

Referenced by [frame\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

#### 6.4.4.5 grade()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::grade () const
-> index_t [pure virtual]
```

Maximum of the grades of each term.

References [grade\(\)](#).

Referenced by [grade\(\)](#), and [operator\\*\(\)](#).

#### 6.4.4.6 inv()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::inv () const
-> const multivector_t [pure virtual]
```

Geometric multiplicative inverse.

References [inv\(\)](#).

Referenced by [inv\(\)](#).

#### 6.4.4.7 involute()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::involute ()
const -> const multivector\_t [pure virtual]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.

References [involute\(\)](#).

Referenced by [involute\(\)](#).

#### 6.4.4.8 isinf()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::isinf () const
-> bool [pure virtual]
```

Check if a multivector contains any infinite values.

References [isinf\(\)](#).

Referenced by [isinf\(\)](#).

#### 6.4.4.9 isnan()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan () const
-> bool [pure virtual]
```

Check if a multivector contains any IEEE NaN values.

References [isnan\(\)](#).

Referenced by [glucat::cascade\\_log\(\)](#), [glucat::exp\(\)](#), [isnan\(\)](#), [glucat::log\(\)](#), [glucat::matrix\\_log\(\)](#), [glucat::matrix\\_sqrt\(\)](#), [glucat::pade\\_log\(\)](#), and [glucat::sqrt\(\)](#).

#### 6.4.4.10 max\_abs()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::max_abs ()
const -> Scalar_T [pure virtual]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

References [max\\_abs\(\)](#).

Referenced by [max\\_abs\(\)](#).

**6.4.4.11 norm()**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::norm () const
-> Scalar_T [pure virtual]
```

Scalar\_T norm == sum of norm of coordinates.

References [norm\(\)](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [norm\(\)](#).

**6.4.4.12 odd()**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::odd () const
-> const multivector_t [pure virtual]
```

Odd part of multivector, sum of odd grade terms.

References [odd\(\)](#).

Referenced by [odd\(\)](#).

**6.4.4.13 operator%=( )**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator%= (
    const multivector_t & rhs) -> multivector_t & [pure virtual]
```

Contraction.

**6.4.4.14 operator&=( )**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator&= (
    const multivector_t & rhs) -> multivector_t & [pure virtual]
```

Inner product.

**6.4.4.15 operator()( )**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator() (
    index_t grade) const -> const multivector_t [pure virtual]
```

Pure grade-vector part.

References [grade\(\)](#).

**6.4.4.16 operator\*=( ) [1/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
    const multivector\_t & rhs) -> multivector\_t & [pure virtual]
```

Geometric product.

**6.4.4.17 operator\*=( ) [2/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
    const Scalar_T & scr) -> multivector\_t & [pure virtual]
```

Product of multivector and scalar.

**6.4.4.18 operator+=( ) [1/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
    const multivector\_t & rhs) -> multivector\_t & [pure virtual]
```

Geometric sum.

**6.4.4.19 operator+=( ) [2/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
    const Scalar_T & scr) -> multivector\_t & [pure virtual]
```

Geometric sum of multivector and scalar.

**6.4.4.20 operator-( )**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator- (
    const -> const multivector\_t [pure virtual]
```

Unary -.

**6.4.4.21 operator-=( ) [1/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
    const multivector\_t & rhs) -> multivector\_t & [pure virtual]
```

Geometric difference.

**6.4.4.22 operator-=( ) [2/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
    const Scalar_T & scr) -> multivector_t & [pure virtual]
```

Geometric difference of multivector and scalar.

**6.4.4.23 operator/=( ) [1/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
    const multivector_t & rhs) -> multivector_t & [pure virtual]
```

Geometric quotient.

**6.4.4.24 operator/=( ) [2/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
    const Scalar_T & scr) -> multivector_t & [pure virtual]
```

Quotient of multivector and scalar.

**6.4.4.25 operator==( ) [1/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator== (
    const multivector_t & val) const -> bool [pure virtual]
```

Test for equality of multivectors.

**6.4.4.26 operator==( ) [2/2]**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator== (
    const Scalar_T & scr) const -> bool [pure virtual]
```

Test for equality of multivector and scalar.

**6.4.4.27 operator[]( )**

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator[] (
    const index_set_t ist) const -> Scalar_T [pure virtual]
```

Subscripting: map from index set to scalar coordinate.

#### 6.4.4.28 operator^=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator^= (
    const multivector_t & rhs) -> multivector_t & [pure virtual]
```

Outer product.

#### 6.4.4.29 operator" |=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator|= (
    const multivector_t & rhs) -> multivector_t & [pure virtual]
```

Transformation via twisted adjoint action.

#### 6.4.4.30 outer\_pow()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::outer_pow (
    int m) const -> const multivector_t [pure virtual]
```

Outer product power.

References [outer\\_pow\(\)](#).

Referenced by [outer\\_pow\(\)](#).

#### 6.4.4.31 pow()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pow (
    int m) const -> const multivector_t [pure virtual]
```

\*this to the m

References [pow\(\)](#).

Referenced by [pow\(\)](#).

#### 6.4.4.32 pure()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pure () const
-> const multivector_t [pure virtual]
```

Pure part.

References [pure\(\)](#).

Referenced by [pure\(\)](#).

#### 6.4.4.33 quad()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::quad () const
-> Scalar_T [pure virtual]
```

Scalar\_T quadratic form == (rev(x)\*x)(0)

References [quad\(\)](#).

Referenced by [quad\(\)](#).

#### 6.4.4.34 reverse()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::reverse ()
const -> const multivector_t [pure virtual]
```

Reversion, eg. {1}\*{2} -> {2}\*{1}.

References [reverse\(\)](#).

Referenced by [reverse\(\)](#).

#### 6.4.4.35 scalar()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar () const
-> Scalar_T [pure virtual]
```

Scalar part.

References [scalar\(\)](#).

Referenced by [glucat::exp\(\)](#), [glucat::matrix\\_log\(\)](#), [glucat::matrix\\_sqrt\(\)](#), and [scalar\(\)](#).

#### 6.4.4.36 truncated()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::truncated (
    const Scalar_T & limit = default_truncation) const -> const multivector_t [pure
virtual]
```

Remove all terms with relative size smaller than limit.

References [default\\_truncation](#), and [truncated\(\)](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::operator<<\(\)](#), and [truncated\(\)](#).

**6.4.4.37 vector\_part()** [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part ()
const -> const vector\_t [pure virtual]
```

Vector part of multivector, as a [vector\\_t](#) with respect to [frame\(\)](#)

References [vector\\_part\(\)](#).

Referenced by [vector\\_part\(\)](#), and [vector\\_part\(\)](#).

**6.4.4.38 vector\_part()** [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part (
    const index\_set\_t frm,
    const bool prechecked) const -> const vector\_t [pure virtual]
```

Vector part of multivector, as a [vector\\_t](#) with respect to frm.

References [vector\\_part\(\)](#).

**6.4.4.39 write()** [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual void glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
    const std::string & msg = "") const [pure virtual]
```

Write formatted multivector to output.

References [write\(\)](#).

Referenced by [write\(\)](#), and [write\(\)](#).

**6.4.4.40 write()** [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual void glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
    std::ofstream & ofile,
    const std::string & msg = "") const [pure virtual]
```

Write formatted multivector to file.

References [write\(\)](#).



## 6.4.5 Member Data Documentation

### 6.4.5.1 default\_truncation

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const Scalar_T glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_↵
truncation [static]
```

Default for truncation.

Definition at line 59 of file [clifford\\_algebra.h](#).

Referenced by [truncated\(\)](#).

### 6.4.5.2 v\_hi

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const index_t glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_hi = index↵
_set_t::v_hi [static]
```

Definition at line 51 of file [clifford\\_algebra.h](#).

### 6.4.5.3 v\_lo

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const index_t glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_lo = index↵
_set_t::v_lo [static]
```

Definition at line 50 of file [clifford\\_algebra.h](#).

The documentation for this class was generated from the following files:

- glucat/[clifford\\_algebra.h](#)
- glucat/[clifford\\_algebra\\_imp.h](#)

## 6.5 glucat::compare\_types< LHS\_T, RHS\_T > Class Template Reference

Type comparison.

```
#include <global.h>
```

### Public Types

- enum { [are\\_same](#) = false }

### 6.5.1 Detailed Description

```
template<typename LHS_T, typename RHS_T>  
class glucat::compare_types< LHS_T, RHS_T >
```

Type comparison.

Definition at line 54 of file [global.h](#).

### 6.5.2 Member Enumeration Documentation

#### 6.5.2.1 anonymous enum

```
template<typename LHS_T, typename RHS_T>  
anonymous enum
```

## Enumerator

are_same	
----------	--

Definition at line 57 of file [global.h](#).

The documentation for this class was generated from the following file:

- [glucat/global.h](#)

## 6.6 glucat::control\_t Class Reference

Parameters to control tests.

```
#include <control.h>
```

### Public Member Functions

- `int call (intfn f) const`  
*Call a function that returns int.*
- `int call (intintfn f, int arg) const`  
*Call a function of int that returns int.*

### Static Public Member Functions

- `static const control_t & control (int argc, char **argv)`
- `static bool verbose ()`  
*Produce more detailed output from tests.*

### Private Member Functions

- `bool valid () const`
- `bool catch_exceptions () const`
- `control_t (int argc, char **argv)`  
*Constructor from program arguments.*
- `control_t ()=default`
- `~control_t ()=default`
- `control_t (const control_t &)=delete`
- `control_t & operator= (const control_t &)=delete`

### Private Attributes

- `bool m_valid`  
*Test parameters are valid.*
- `bool m_catch_exceptions`  
*Catch exceptions.*

## Static Private Attributes

- static bool [m\\_verbose\\_output](#) = false  
*Produce more detailed output from tests.*

## Friends

- class [friend\\_for\\_private\\_destructor](#)

## 6.6.1 Detailed Description

Parameters to control tests.

Definition at line 39 of file [control.h](#).

## 6.6.2 Constructor & Destructor Documentation

### 6.6.2.1 [control\\_t\(\)](#) [1/3]

```
glucat::control_t::control_t (  
    int argc,  
    char ** argv) [private]
```

Constructor from program arguments.

Test control constructor from program arguments.

Definition at line 88 of file [control.h](#).

References [GLUCAT\\_PACKAGE\\_NAME](#), [GLUCAT\\_VERSION](#), [m\\_catch\\_exceptions](#), [m\\_valid](#), [m\\_verbose\\_output](#), and [valid\(\)](#).

Referenced by [control\(\)](#), [control\\_t\(\)](#), and [operator=\(\)](#).

### 6.6.2.2 [control\\_t\(\)](#) [2/3]

```
glucat::control_t::control_t () [private], [default]
```

### 6.6.2.3 [~control\\_t\(\)](#)

```
glucat::control_t::~~control_t () [private], [default]
```

### 6.6.2.4 [control\\_t\(\)](#) [3/3]

```
glucat::control_t::control_t (  
    const control\_t & ) [private], [delete]
```

References [control\\_t\(\)](#).

## 6.6.3 Member Function Documentation

### 6.6.3.1 call() [1/2]

```
int glucat::control_t::call (  
    intfn f) const [inline]
```

Call a function that returns int.

Definition at line 136 of file [control.h](#).

References [catch\\_exceptions\(\)](#), [glucat::try\\_catch\(\)](#), and [valid\(\)](#).

### 6.6.3.2 call() [2/2]

```
int glucat::control_t::call (  
    intintfn f,  
    int arg) const [inline]
```

Call a function of int that returns int.

Definition at line 150 of file [control.h](#).

References [catch\\_exceptions\(\)](#), [glucat::try\\_catch\(\)](#), and [valid\(\)](#).

### 6.6.3.3 catch\_exceptions()

```
bool glucat::control_t::catch_exceptions () const [inline], [private]
```

Definition at line 49 of file [control.h](#).

References [m\\_catch\\_exceptions](#).

Referenced by [call\(\)](#), and [call\(\)](#).

### 6.6.3.4 control()

```
static const control_t & glucat::control_t::control (  
    int argc,  
    char ** argv) [inline], [static]
```

Single instance Ref: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.

Definition at line 71 of file [control.h](#).

References [control\\_t\(\)](#).

### 6.6.3.5 operator=()

```
control_t & glucat::control_t::operator= (  
    const control_t & ) [private], [delete]
```

References [control\\_t\(\)](#).

#### 6.6.3.6 valid()

```
bool glucat::control_t::valid () const [inline], [private]
```

Definition at line 44 of file [control.h](#).

References [m\\_valid](#).

Referenced by [call\(\)](#), [call\(\)](#), and [control\\_t\(\)](#).

#### 6.6.3.7 verbose()

```
static bool glucat::control_t::verbose () [inline], [static]
```

Produce more detailed output from tests.

Definition at line 80 of file [control.h](#).

References [m\\_verbose\\_output](#).

### 6.6.4 Friends And Related Symbol Documentation

#### 6.6.4.1 friend\_for\_private\_destructor

```
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 67 of file [control.h](#).

References [friend\\_for\\_private\\_destructor](#).

Referenced by [friend\\_for\\_private\\_destructor](#).

### 6.6.5 Member Data Documentation

#### 6.6.5.1 m\_catch\_exceptions

```
bool glucat::control_t::m_catch_exceptions [private]
```

Catch exceptions.

Definition at line 48 of file [control.h](#).

Referenced by [catch\\_exceptions\(\)](#), and [control\\_t\(\)](#).

### 6.6.5.2 m\_valid

```
bool glucat::control_t::m_valid [private]
```

Test parameters are valid.

Definition at line 43 of file [control.h](#).

Referenced by [control\\_t\(\)](#), and [valid\(\)](#).

### 6.6.5.3 m\_verbose\_output

```
bool glucat::control_t::m_verbose_output = false [static], [private]
```

Produce more detailed output from tests.

Definition at line 53 of file [control.h](#).

Referenced by [control\\_t\(\)](#), and [verbose\(\)](#).

The documentation for this class was generated from the following file:

- [test/control.h](#)

## 6.7 glucat::CTAssertion< bool > Struct Template Reference

Compile time assertion.

### 6.7.1 Detailed Description

```
template<bool>
struct glucat::CTAssertion< bool >
```

Compile time assertion.

Definition at line 46 of file [global.h](#).

The documentation for this struct was generated from the following file:

- [glucat/global.h](#)

## 6.8 glucat::matrix::eig\_genus< Matrix\_T > Struct Template Reference

Structure containing classification of eigenvalues.

```
#include <matrix.h>
```

## Public Types

- using [Scalar\\_T](#) = typename Matrix\_T::value\_type

## Public Attributes

- bool [m\\_is\\_singular](#) = false  
*Is the matrix singular?*
- [eig\\_case\\_t](#) [m\\_eig\\_case](#) = safe\_eigs  
*What kind of eigenvalues does the matrix contain?*
- [Scalar\\_T](#) [m\\_safe\\_arg](#) = [Scalar\\_T](#)(0)  
*Argument such that  $\exp(\pi \cdot m\_safe\_arg)$  lies between arguments of eigenvalues.*

### 6.8.1 Detailed Description

```
template<typename Matrix_T>
struct glucat::matrix::eig_genus< Matrix_T >
```

Structure containing classification of eigenvalues.

Definition at line 140 of file [matrix.h](#).

### 6.8.2 Member Typedef Documentation

#### 6.8.2.1 Scalar\_T

```
template<typename Matrix_T>
using glucat::matrix::eig_genus< Matrix_T >::Scalar_T = typename Matrix_T::value_type
```

Definition at line 142 of file [matrix.h](#).

### 6.8.3 Member Data Documentation

#### 6.8.3.1 m\_eig\_case

```
template<typename Matrix_T>
eig_case_t glucat::matrix::eig_genus< Matrix_T >::m_eig_case = safe_eigs
```

What kind of eigenvalues does the matrix contain?

Definition at line 146 of file [matrix.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).



### 6.8.3.2 m\_is\_singular

```
template<typename Matrix_T>
bool glucat::matrix::eig_genus< Matrix_T >::m_is_singular = false
```

Is the matrix singular?

Definition at line 144 of file [matrix.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

### 6.8.3.3 m\_safe\_arg

```
template<typename Matrix_T>
Scalar_T glucat::matrix::eig_genus< Matrix_T >::m_safe_arg = Scalar_T(0)
```

Argument such that  $\exp(\pi \cdot m\_safe\_arg)$  lies between arguments of eigenvalues.

Definition at line 148 of file [matrix.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

The documentation for this struct was generated from the following file:

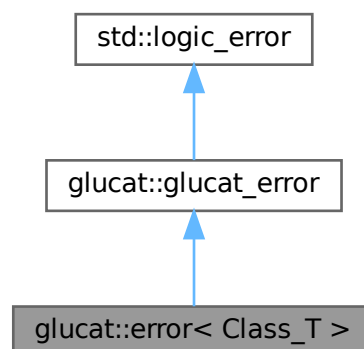
- [glucat/matrix.h](#)

## 6.9 glucat::error< Class\_T > Class Template Reference

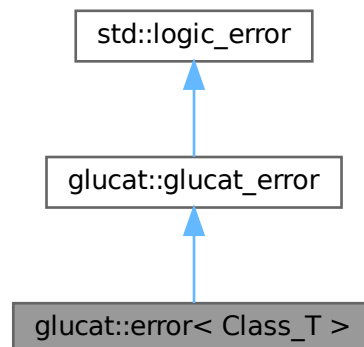
Specific exception class.

```
#include <errors.h>
```

Inheritance diagram for glucat::error< Class\_T >:



Collaboration diagram for `glucat::error< Class_T >`:



### Public Member Functions

- [error](#) (const std::string &msg)  
*Specific exception class.*
- [error](#) (const std::string &context, const std::string &msg)
- auto [heading](#) () const noexcept -> const std::string override
- auto [classname](#) () const noexcept -> const std::string override
- void [print\\_error\\_msg](#) () const override

### Public Member Functions inherited from [glucat::glucat\\_error](#)

- [glucat\\_error](#) (const std::string &context, const std::string &msg)
- [~glucat\\_error](#) () noexcept override=default

### Additional Inherited Members

### Public Attributes inherited from [glucat::glucat\\_error](#)

- std::string [name](#)

## 6.9.1 Detailed Description

```
template<class Class_T>
class glucat::error< Class_T >
```

Specific exception class.

Definition at line 56 of file [errors.h](#).

## 6.9.2 Constructor & Destructor Documentation

### 6.9.2.1 error() [1/2]

```
template<class Class_T>
glucat::error< Class_T >::error (
    const std::string & msg)
```

Specific exception class.

Definition at line 44 of file [errors\\_imp.h](#).

References [classname\(\)](#), and [glucat::glucat\\_error::glucat\\_error\(\)](#).

### 6.9.2.2 error() [2/2]

```
template<class Class_T>
glucat::error< Class_T >::error (
    const std::string & context,
    const std::string & msg)
```

Definition at line 50 of file [errors\\_imp.h](#).

References [glucat::glucat\\_error::glucat\\_error\(\)](#).

## 6.9.3 Member Function Documentation

### 6.9.3.1 classname()

```
template<class Class_T>
auto glucat::error< Class_T >::classname () const -> const std::string [override], [virtual],
[noexcept]
```

Implements [glucat::glucat\\_error](#).

Definition at line 63 of file [errors\\_imp.h](#).

References [glucat::glucat\\_error::name](#).

Referenced by [error\(\)](#), and [print\\_error\\_msg\(\)](#).

### 6.9.3.2 heading()

```
template<class Class_T>
auto glucat::error< Class_T >::heading () const -> const std::string [override], [virtual],
[noexcept]
```

Implements [glucat::glucat\\_error](#).

Definition at line 57 of file [errors\\_imp.h](#).

Referenced by [print\\_error\\_msg\(\)](#).

### 6.9.3.3 print\_error\_msg()

```
template<class Class_T>
void glucat::error< Class_T >::print_error_msg () const [override], [virtual]
```

Implements [glucat::glucat\\_error](#).

Definition at line 69 of file [errors\\_imp.h](#).

References [classname\(\)](#), and [heading\(\)](#).

The documentation for this class was generated from the following files:

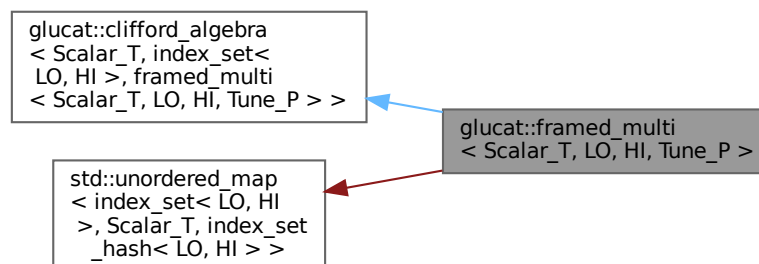
- [glucat/errors.h](#)
- [glucat/errors\\_imp.h](#)

## 6.10 glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > Class Template Reference

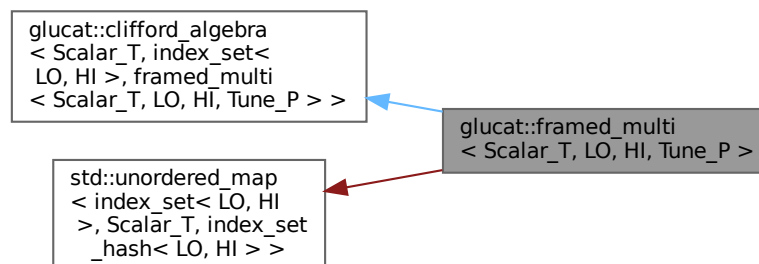
A framed\_multi<Scalar\_T,LO,HI,Tune\_P> is a framed approximation to a multivector.

```
#include <framed_multi.h>
```

Inheritance diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >:



Collaboration diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >:



## Classes

- class [hash\\_size\\_t](#)
- class [var\\_term](#)

*Variable term.*

## Public Types

- using [multivector\\_t](#) = [framed\\_multi](#)
- using [framed\\_multi\\_t](#) = [multivector\\_t](#)
- using [scalar\\_t](#) = [Scalar\\_T](#)
- using [tune\\_p](#) = [Tune\\_P](#)
- using [index\\_set\\_t](#) = [index\\_set](#)<[LO](#), [HI](#)>
- using [term\\_t](#) = std::pair<const [index\\_set\\_t](#), [Scalar\\_T](#)>
- using [vector\\_t](#) = std::vector<[Scalar\\_T](#)>
- using [error\\_t](#) = [error](#)<[multivector\\_t](#)>
- using [matrix\\_multi\\_t](#) = [matrix\\_multi](#)<[Scalar\\_T](#),[LO](#),[HI](#),[Tune\\_P](#) >

## Public Types inherited from

[glucat::clifford\\_algebra](#)< [Scalar\\_T](#), [index\\_set](#)< [LO](#), [HI](#) >, [framed\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) > >

- using [scalar\\_t](#)
- using [index\\_set\\_t](#)
- using [multivector\\_t](#)
- using [pair\\_t](#)
- using [vector\\_t](#)

## Public Member Functions

- [~framed\\_multi](#) () override=default  
*Destructor.*
- [framed\\_multi](#) ()  
*Default constructor.*
- template<typename Other\_Scalar\_T>  
[framed\\_multi](#) (const [framed\\_multi](#)< Other\_Scalar\_T, [LO](#), [HI](#), [Tune\\_P](#) > &val)  
*Construct a multivector from a multivector with a different scalar type.*
- template<typename Other\_Scalar\_T>  
[framed\\_multi](#) (const [framed\\_multi](#)< Other\_Scalar\_T, [LO](#), [HI](#), [Tune\\_P](#) > &val, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- [framed\\_multi](#) (const [framed\\_multi\\_t](#) &val, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- [framed\\_multi](#) (const [index\\_set\\_t](#) ist, const [Scalar\\_T](#) &crd=[Scalar\\_T](#)(1))  
*Construct a multivector from an index set and a scalar coordinate.*
- [framed\\_multi](#) (const [index\\_set\\_t](#) ist, const [Scalar\\_T](#) &crd, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from an index set and a scalar coordinate.*
- [framed\\_multi](#) (const [Scalar\\_T](#) &scr, const [index\\_set\\_t](#) frm=[index\\_set\\_t](#)())  
*Construct a multivector from a scalar (within a frame, if given)*
- [framed\\_multi](#) (const int scr, const [index\\_set\\_t](#) frm=[index\\_set\\_t](#)())  
*Construct a multivector from an int (within a frame, if given)*
- [framed\\_multi](#) (const [vector\\_t](#) &vec, const [index\\_set\\_t](#) frm, const bool prechecked=false)

- Construct a multivector, within a given frame, from a given vector.*

  - `framed_multi` (const std::string &str)

*Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*

  - `framed_multi` (const std::string &str, const `index_set_t` frm, const bool prechecked=false)

*Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*

  - `framed_multi` (const char \*str)

*Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".*

  - `framed_multi` (const char \*str, const `index_set_t` frm, const bool prechecked=false)

*Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".*

  - template<typename Other\_Scalar\_T>  
`framed_multi` (const `matrix_multi`< Other\_Scalar\_T, LO, HI, Tune\_P > &val)

*Construct a multivector from a `matrix_multi_t`.*

  - template<typename Other\_Scalar\_T>  
auto `fast_matrix_multi` (const `index_set_t` frm) const -> const `matrix_multi`< Other\_Scalar\_T, LO, HI, Tune\_P >  
>

*Use generalized FFT to construct a `matrix_multi_t`.*

  - auto `fast_framed_multi` () const -> const `framed_multi_t`

*Use inverse generalized FFT to construct a `framed_multi_t`.*

  - `_GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS` auto `nbr_terms` () const -> unsigned long

*Number of terms.*

  - auto `operator+=` (const `term_t` &term) -> `multivector_t` &

*Add a term, if non-zero.*

## Public Member Functions inherited from

`glucat::clifford_algebra`< `Scalar_T`, `index_set`< LO, HI >, `framed_multi`< `Scalar_T`, LO, HI, Tune\_P > >

- virtual `~clifford_algebra` ()=default
  - virtual auto `operator==` (const `multivector_t` &val) const -> bool=0
- Test for equality of multivectors.*
- virtual auto `operator==` (const `Scalar_T` &scr) const -> bool=0
- Test for equality of multivector and scalar.*
- virtual auto `operator+=` (const `multivector_t` &rhs) -> `multivector_t` &=0
- Geometric sum.*
- virtual auto `operator+=` (const `Scalar_T` &scr) -> `multivector_t` &=0
- Geometric sum of multivector and scalar.*
- virtual auto `operator-=` (const `multivector_t` &rhs) -> `multivector_t` &=0
- Geometric difference.*
- virtual auto `operator-=` (const `Scalar_T` &scr) -> `multivector_t` &=0
- Geometric difference of multivector and scalar.*
- virtual auto `operator-` () const -> const `multivector_t`=0
- Unary -.*
- virtual auto `operator*=` (const `Scalar_T` &scr) -> `multivector_t` &=0
- Product of multivector and scalar.*
- virtual auto `operator*=` (const `multivector_t` &rhs) -> `multivector_t` &=0
- Geometric product.*
- virtual auto `operator%=-` (const `multivector_t` &rhs) -> `multivector_t` &=0
- Contraction.*
- virtual auto `operator&=-` (const `multivector_t` &rhs) -> `multivector_t` &=0
- Inner product.*
- virtual auto `operator^=-` (const `multivector_t` &rhs) -> `multivector_t` &=0

- Outer product.*
- virtual auto `operator/` (const Scalar\_T &scr) -> `multivector_t` &=0
- Quotient of multivector and scalar.*
- virtual auto `operator/` (const `multivector_t` &rhs) -> `multivector_t` &=0
- Geometric quotient.*
- virtual auto `operator|` (const `multivector_t` &rhs) -> `multivector_t` &=0
- Transformation via twisted adjoint action.*
- virtual auto `inv` () const -> const `multivector_t`=0
- Geometric multiplicative inverse.*
- virtual auto `pow` (int m) const -> const `multivector_t`=0
- \*this to the m*
- virtual auto `outer_pow` (int m) const -> const `multivector_t`=0
- Outer product power.*
- virtual auto `frame` () const -> const `index_set_t`=0
- Subalgebra generated by all generators of terms of given multivector.*
- virtual auto `grade` () const -> `index_t`=0
- Maximum of the grades of each term.*
- virtual auto `operator[]` (const `index_set_t` ist) const -> Scalar\_T=0
- Subscripting: map from index set to scalar coordinate.*
- virtual auto `operator()` (`index_t` grade) const -> const `multivector_t`=0
- Pure grade-vector part.*
- virtual auto `scalar` () const -> Scalar\_T=0
- Scalar part.*
- virtual auto `pure` () const -> const `multivector_t`=0
- Pure part.*
- virtual auto `even` () const -> const `multivector_t`=0
- Even part of multivector, sum of even grade terms.*
- virtual auto `odd` () const -> const `multivector_t`=0
- Odd part of multivector, sum of odd grade terms.*
- virtual auto `vector_part` () const -> const `vector_t`=0
- Vector part of multivector, as a `vector_t` with respect to `frame()`*
- virtual auto `vector_part` (const `index_set_t` frm, const bool prechecked) const -> const `vector_t`=0
- Vector part of multivector, as a `vector_t` with respect to frm.*
- virtual auto `involute` () const -> const `multivector_t`=0
- Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.*
- virtual auto `reverse` () const -> const `multivector_t`=0
- Reversion, eg. {1}\*{2} -> {2}\*{1}.*
- virtual auto `conj` () const -> const `multivector_t`=0
- Conjugation, reverse o involute == involute o reverse.*
- virtual auto `quad` () const -> Scalar\_T=0
- Scalar\_T quadratic form == (rev(x)\*x)(0)*
- virtual auto `norm` () const -> Scalar\_T=0
- Scalar\_T norm == sum of norm of coordinates.*
- virtual auto `max_abs` () const -> Scalar\_T=0
- Maximum of absolute values of components of multivector: multivector infinity norm.*
- virtual auto `truncated` (const Scalar\_T &limit=default\_truncation) const -> const `multivector_t`=0
- Remove all terms with relative size smaller than limit.*
- virtual auto `isinf` () const -> bool=0
- Check if a multivector contains any infinite values.*
- virtual auto `isnan` () const -> bool=0
- Check if a multivector contains any IEEE NaN values.*

- virtual void [write](#) (const std::string &msg="") const=0  
*Write formatted multivector to output.*
- virtual void [write](#) (std::ofstream &ofile, const std::string &msg="") const=0  
*Write formatted multivector to file.*

### Static Public Member Functions

- static auto [classname](#) () -> const std::string  
*Class name used in messages.*
- static auto [random](#) (const [index\\_set\\_t](#) frm, Scalar\_T fill=Scalar\_T(1)) -> const [multivector\\_t](#)  
*Random multivector within a frame.*

### Static Public Member Functions inherited from

[glucat::clifford\\_algebra](#)< [Scalar\\_T](#), [index\\_set](#)< [LO](#), [HI](#) >, [framed\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) > >

- static auto [classname](#) () -> const std::string

### Private Types

- using [var\\_term\\_t](#) = class [var\\_term](#)
- using [matrix\\_t](#) = typename [matrix\\_multi\\_t::matrix\\_t](#)
- using [sorted\\_map\\_t](#) = std::map< [index\\_set\\_t](#), Scalar\_T, std::less<const [index\\_set\\_t](#)> >
- using [map\\_t](#) = std::unordered\_map<[index\\_set\\_t](#), Scalar\_T, [index\\_set\\_hash](#)<[LO](#), [HI](#)>>
- using [framed\\_pair\\_t](#) = std::pair<const [multivector\\_t](#), const [multivector\\_t](#)>
- using [size\\_type](#) = typename map\_t::size\_type
- using [iterator](#) = typename map\_t::iterator
- using [const\\_iterator](#) = typename map\_t::const\_iterator

### Private Member Functions

- [framed\\_multi](#) (const [hash\\_size\\_t](#) &hash\_size)  
*Private constructor using hash\_size.*
- auto [fold](#) (const [index\\_set\\_t](#) frm) const -> [multivector\\_t](#)  
*Subalgebra isomorphism: fold each term within the given frame.*
- auto [unfold](#) (const [index\\_set\\_t](#) frm) const -> [multivector\\_t](#)  
*Subalgebra isomorphism: unfold each term within the given frame.*
- auto [centre\\_pm4\\_qp4](#) ([index\\_t](#) &p, [index\\_t](#) &q) -> [multivector\\_t](#) &  
*Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p-4,q+4\}}$ .*
- auto [centre\\_pp4\\_qm4](#) ([index\\_t](#) &p, [index\\_t](#) &q) -> [multivector\\_t](#) &  
*Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p+4,q-4\}}$ .*
- auto [centre\\_qp1\\_pm1](#) ([index\\_t](#) &p, [index\\_t](#) &q) -> [multivector\\_t](#) &  
*Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{q+1,p-1\}}$ .*
- auto [divide](#) (const [index\\_set\\_t](#) ist) const -> const [framed\\_pair\\_t](#)  
*Divide multivector into part divisible by [index\\_set](#) and remainder.*
- auto [fast](#) (const [index\\_t](#) level, const bool odd) const -> const [matrix\\_t](#)  
*Generalized FFT from [multivector\\_t](#) to [matrix\\_t](#).*



## Friends

- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P>  
class [matrix\\_multi](#)
- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P>  
class [framed\\_multi](#)
- auto [operator\\*](#) (const [multivector\\_t](#) &lhs, const [multivector\\_t](#) &rhs) -> const [multivector\\_t](#)
- auto [operator^](#) (const [multivector\\_t](#) &lhs, const [multivector\\_t](#) &rhs) -> const [multivector\\_t](#)
- auto [operator&](#) (const [multivector\\_t](#) &lhs, const [multivector\\_t](#) &rhs) -> const [multivector\\_t](#)
- auto [operator%](#) (const [multivector\\_t](#) &lhs, const [multivector\\_t](#) &rhs) -> const [multivector\\_t](#)
- auto [star](#) (const [multivector\\_t](#) &lhs, const [multivector\\_t](#) &rhs) -> [Scalar\\_T](#)
- auto [operator/](#) (const [multivector\\_t](#) &lhs, const [multivector\\_t](#) &rhs) -> const [multivector\\_t](#)
- auto [operator|](#) (const [multivector\\_t](#) &lhs, const [multivector\\_t](#) &rhs) -> const [multivector\\_t](#)
- auto [operator>>](#) (std::istream &s, [multivector\\_t](#) &val) -> std::istream &
- auto [operator<<](#) (std::ostream &os, const [multivector\\_t](#) &val) -> std::ostream &
- auto [operator<<](#) (std::ostream &os, const [term\\_t](#) &term) -> std::ostream &
- auto [exp](#) (const [multivector\\_t](#) &val) -> const [multivector\\_t](#)

## Additional Inherited Members

### Static Public Attributes inherited from

[glucat::clifford\\_algebra](#)< [Scalar\\_T](#), [index\\_set](#)< [LO](#), [HI](#) >, [framed\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) > >

- static const [index\\_t](#) [v\\_lo](#)
- static const [index\\_t](#) [v\\_hi](#)
- static const [Scalar\\_T](#) [default\\_truncation](#)

*Default for truncation.*

## 6.10.1 Detailed Description

```
template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P>
class glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >
```

A [framed\\_multi](#)<[Scalar\\_T](#),[LO](#),[HI](#),[Tune\\_P](#)> is a framed approximation to a multivector.

Definition at line 55 of file [matrix\\_multi.h](#).

## 6.10.2 Member Typedef Documentation

### 6.10.2.1 const\_iterator

```
template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P>
using glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::const_iterator = typename map_t↔
::const_iterator [private]
```

Definition at line 167 of file [framed\\_multi.h](#).

### 6.10.2.2 error\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::error_t = error<multivector_t>
```

Definition at line 138 of file [framed\\_multi.h](#).

### 6.10.2.3 framed\_multi\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi_t = multivector_t
```

Definition at line 132 of file [framed\\_multi.h](#).

### 6.10.2.4 framed\_pair\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_pair_t = std::pair<const multivector_t,
const multivector_t> [private]
```

Definition at line 164 of file [framed\\_multi.h](#).

### 6.10.2.5 index\_set\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index_set<LO, HI>
```

Definition at line 135 of file [framed\\_multi.h](#).

### 6.10.2.6 iterator

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::iterator = typename map_t::iterator
[private]
```

Definition at line 166 of file [framed\\_multi.h](#).

### 6.10.2.7 map\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::map_t = std::unordered_map<index_set_t,
Scalar_T, index_set_hash<LO, HI>> [private]
```

Definition at line 150 of file [framed\\_multi.h](#).

### 6.10.2.8 matrix\_multi\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = matrix_multi<Scalar_←
_T, LO, HI, Tune_P >
```

Definition at line 139 of file [framed\\_multi.h](#).

### 6.10.2.9 matrix\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = typename matrix_multi_t::matrix_t
[private]
```

Definition at line 148 of file [framed\\_multi.h](#).

### 6.10.2.10 multivector\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = framed_multi
```

Definition at line 131 of file [framed\\_multi.h](#).

### 6.10.2.11 scalar\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 133 of file [framed\\_multi.h](#).

### 6.10.2.12 size\_type

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::size_type = typename map_t::size_type
[private]
```

Definition at line 165 of file [framed\\_multi.h](#).

### 6.10.2.13 sorted\_map\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::sorted_map_t = std::map< index_set_t,
Scalar_T, std::less<const index_set_t> > [private]
```

Definition at line 149 of file [framed\\_multi.h](#).

#### 6.10.2.14 term\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index_set_t,
Scalar_T>
```

Definition at line 136 of file [framed\\_multi.h](#).

#### 6.10.2.15 tune\_p

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 134 of file [framed\\_multi.h](#).

#### 6.10.2.16 var\_term\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term_t = class var_term [private]
```

Definition at line 147 of file [framed\\_multi.h](#).

#### 6.10.2.17 vector\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 137 of file [framed\\_multi.h](#).

### 6.10.3 Constructor & Destructor Documentation

#### 6.10.3.1 ~framed\_multi()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::~~framed_multi () [override], [default]
```

Destructor.

References [framed\\_multi](#).

#### 6.10.3.2 framed\_multi() [1/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi ()
```

Default constructor.

Definition at line 59 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#).

**6.10.3.3 framed\_multi()** [2/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const hash_size_t & hash_size) [private]
```

Private constructor using hash\_size.

Definition at line 66 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#).

**6.10.3.4 framed\_multi()** [3/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 74 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#), [framed\\_multi](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**6.10.3.5 framed\_multi()** [4/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index_set_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 85 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::frame\(\)](#), [framed\\_multi](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**6.10.3.6 framed\_multi()** [5/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed_multi_t & val,
    const index_set_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 98 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#), and [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::frame\(\)](#).

**6.10.3.7 framed\_multi()** [6/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const index\_set\_t ist,
    const Scalar_T & crd = Scalar_T(1))
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 111 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#).

**6.10.3.8 framed\_multi()** [7/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const index\_set\_t ist,
    const Scalar_T & crd,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 121 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#).

**6.10.3.9 framed\_multi()** [8/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const Scalar_T & scr,
    const index\_set\_t frm = index\_set\_t())
```

Construct a multivector from a scalar (within a frame, if given)

Definition at line 134 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#).

**6.10.3.10 framed\_multi()** [9/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const int scr,
    const index\_set\_t frm = index\_set\_t())
```

Construct a multivector from an int (within a frame, if given)

Definition at line 144 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#).

**6.10.3.11 framed\_multi()** [10/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const vector\_t & vec,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 154 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#), [glucat::index\\_set< LO, HI >::count\(\)](#), [glucat::index\\_set< LO, HI >::max\(\)](#), and [glucat::index\\_set< LO, HI >::min\(\)](#).

**6.10.3.12 framed\_multi()** [11/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const std::string & str)
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 176 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#).

**6.10.3.13 framed\_multi()** [12/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const std::string & str,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 192 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#).

**6.10.3.14 framed\_multi()** [13/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const char * str) \[inline\]
```

Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 209 of file [framed\\_multi.h](#).

References [framed\\_multi](#).

### 6.10.3.15 framed\_multi() [14/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const char * str,
    const index_set_t frm,
    const bool prechecked = false) [inline]
```

Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 212 of file [framed\\_multi.h](#).

References [framed\\_multi](#).

### 6.10.3.16 framed\_multi() [15/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a [matrix\\_multi\\_t](#).

Definition at line 205 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_N](#), [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), [fast\\_framed\\_multi\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::frame\(\)](#), [glucat::matrix::inner\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#), [matrix\\_multi](#), [glucat::matrix::nnz\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::truncated\(\)](#), and [glucat::clifford\\_algebra< Scalar\\_T, index\\_set< LO, HI >, framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P > >::truncated\(\)](#).

## 6.10.4 Member Function Documentation

### 6.10.4.1 centre\_pm4\_qp4()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pm4_qp4 (
    index_t & p,
    index_t & q) -> multivector_t& [private]
```

Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p-4,q+4\}}$ .

Definition at line 1469 of file [framed\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra< Scalar\\_T, index\\_set< LO, HI >, framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P > >::frame\(\)](#).

Referenced by [exp](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).



#### 6.10.4.2 centre\_pp4\_qm4()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pp4_qm4 (
    index_t & p,
    index_t & q) -> multivector_t& [private]
```

Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p+4,q-4\}}$ .

Definition at line 1511 of file [framed\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra< Scalar\\_T, index\\_set< LO, HI >, framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P > >::frame\(\)](#).

Referenced by [exp](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

#### 6.10.4.3 centre\_qp1\_pm1()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_qp1_pm1 (
    index_t & p,
    index_t & q) -> multivector_t& [private]
```

Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{q+1,p-1\}}$ .

Definition at line 1553 of file [framed\\_multi\\_imp.h](#).

Referenced by [exp](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

#### 6.10.4.4 classname()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::classname () -> const std::string
[static]
```

Class name used in messages.

Definition at line 50 of file [framed\\_multi\\_imp.h](#).

#### 6.10.4.5 divide()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::divide (
    const index_set_t ist) const -> const framed_pair_t [private]
```

Divide multivector into part divisible by [index\\_set](#) and remainder.

Divide multivector into quotient with terms divisible by index set, and remainder.

Definition at line 1586 of file [framed\\_multi\\_imp.h](#).

Referenced by [exp](#), and [fast\(\)](#).

#### 6.10.4.6 fast()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast (
    const index_t level,
    const bool odd) const -> const matrix_t [private]
```

Generalized FFT from [multivector\\_t](#) to [matrix\\_t](#).

Definition at line 1602 of file [framed\\_multi\\_imp.h](#).

References [divide\(\)](#), [glucat::matrix::kron\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, index\\_set< LO, HI >, framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\(\)](#), [glucat::scalar\(\)](#), and [glucat::matrix::unit\(\)](#).

Referenced by [exp](#).

#### 6.10.4.7 fast\_framed\_multi()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi () const -> const
framed_multi_t [inline]
```

Use inverse generalized FFT to construct a [framed\\_multi\\_t](#).

Definition at line 1700 of file [framed\\_multi\\_imp.h](#).

Referenced by [framed\\_multi\(\)](#).

#### 6.10.4.8 fast\_matrix\_multi()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
    const index_set_t frm) const -> const matrix_multi<Other_Scalar_T,LO,HI,Tune_P >
```

Use generalized FFT to construct a [matrix\\_multi\\_t](#).

Definition at line 1668 of file [framed\\_multi\\_imp.h](#).

References [fold\(\)](#), [matrix\\_multi](#), [glucat::gen::offset\\_to\\_super](#), and [glucat::pos\\_mod\(\)](#).

#### 6.10.4.9 fold()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fold (
    const index_set_t frm) const -> multivector_t [private]
```

Subalgebra isomorphism: fold each term within the given frame.

Definition at line 1434 of file [framed\\_multi\\_imp.h](#).

Referenced by [exp](#), and [fast\\_matrix\\_multi\(\)](#).

#### 6.10.4.10 nbr\_terms()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::nbr_terms () const -> unsigned long
```

Number of terms.

Definition at line 1356 of file [framed\\_multi\\_imp.h](#).

#### 6.10.4.11 operator+=(())

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
    const term_t & term) -> multivector_t& [inline]
```

Add a term, if non-zero.

Insert a term into a multivector, add terms with same index set.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line 295 of file [framed\\_multi\\_imp.h](#).

#### 6.10.4.12 random()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::random (
    const index_set_t frm,
    Scalar_T fill = Scalar_T(1)) -> const multivector_t [static]
```

Random multivector within a frame.

Definition at line 1058 of file [framed\\_multi\\_imp.h](#).

References [framed\\_multi](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::random\(\)](#).

#### 6.10.4.13 unfold()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::unfold (
    const index_set_t frm) const -> multivector_t [private]
```

Subalgebra isomorphism: unfold each term within the given frame.

Definition at line 1451 of file [framed\\_multi\\_imp.h](#).

Referenced by [exp](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

## 6.10.5 Friends And Related Symbol Documentation

### 6.10.5.1 exp

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto exp (
    const multivector_t & val) -> const multivector_t [friend]
```

References [centre\\_pm4\\_qp4\(\)](#), [centre\\_pp4\\_qm4\(\)](#), [centre\\_qp1\\_pm1\(\)](#), [divide\(\)](#), [exp](#), [fast\(\)](#), [fold\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, i](#), and [unfold\(\)](#).

Referenced by [exp](#).

### 6.10.5.2 framed\_multi

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P>
friend class framed_multi [friend]
```

Definition at line 143 of file [framed\\_multi.h](#).

References [framed\\_multi](#).

Referenced by [framed\\_multi](#), [framed\\_multi\(\)](#), [framed\\_multi\(\)](#), [framed\\_multi\(\)](#), [framed\\_multi\(\)](#), [random\(\)](#), and [~framed\\_multi\(\)](#).

### 6.10.5.3 matrix\_multi

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P>
friend class matrix_multi [friend]
```

Definition at line 141 of file [framed\\_multi.h](#).

References [matrix\\_multi](#).

Referenced by [fast\\_matrix\\_multi\(\)](#), [framed\\_multi\(\)](#), and [matrix\\_multi](#).

### 6.10.5.4 operator%

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto operator% (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

### 6.10.5.5 operator&

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto operator& (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

#### 6.10.5.6 operator\*

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto operator* (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

#### 6.10.5.7 operator/

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto operator/ (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

#### 6.10.5.8 operator<< [1/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto operator<< (
    std::ostream & os,
    const multivector_t & val) -> std::ostream & [friend]
```

#### 6.10.5.9 operator<< [2/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto operator<< (
    std::ostream & os,
    const term_t & term) -> std::ostream & [friend]
```

#### 6.10.5.10 operator>>

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto operator>> (
    std::istream & s,
    multivector_t & val) -> std::istream & [friend]
```

#### 6.10.5.11 operator^

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto operator^ (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

#### 6.10.5.12 operator"|

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto operator| (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

### 6.10.5.13 star

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto star (
    const multivector_t & lhs,
    const multivector_t & rhs) -> Scalar_T [friend]
```

References [star](#).

Referenced by [star](#).

The documentation for this class was generated from the following files:

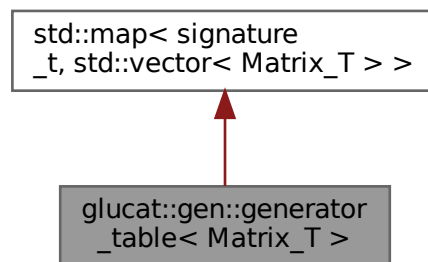
- [glucat/framed\\_multi.h](#)
- [glucat/matrix\\_multi.h](#)
- [glucat/framed\\_multi\\_imp.h](#)

## 6.11 glucat::gen::generator\_table< Matrix\_T > Class Template Reference

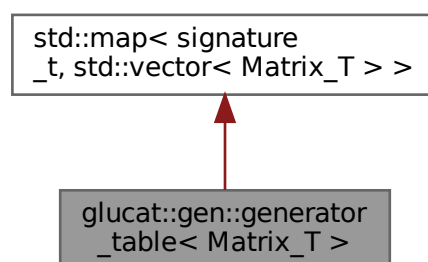
Table of generators for specific signatures.

```
#include <generation.h>
```

Inheritance diagram for glucat::gen::generator\_table< Matrix\_T >:



Collaboration diagram for glucat::gen::generator\_table< Matrix\_T >:



## Public Member Functions

- auto [operator\(\)](#) (const [index\\_t](#) p, const [index\\_t](#) q) -> const Matrix\_T \*  
*Pointer to generators for a specific signature.*
- [generator\\_table](#) (const [generator\\_table](#) &)=delete
- auto [operator=](#) (const [generator\\_table](#) &) -> [generator\\_table](#) &=delete

## Static Public Member Functions

- static auto [generator](#) () -> [generator\\_table](#)< Matrix\_T > &  
*Single instance of generator table.*

## Private Member Functions

- auto [gen\\_vector](#) (const [index\\_t](#) p, const [index\\_t](#) q) -> const std::vector< Matrix\_T > &  
*Construct a vector of generators for a specific signature.*
- void [gen\\_from\\_pm1\\_qm1](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for p-1,q-1.*
- void [gen\\_from\\_pm4\\_qp4](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for p-4,q+4.*
- void [gen\\_from\\_pp4\\_qm4](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for p+4,q-4.*
- void [gen\\_from\\_qp1\\_pm1](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for q+1,p-1.*
- [generator\\_table](#) ()=default
- [~generator\\_table](#) ()=default

## Friends

- class [friend\\_for\\_private\\_destructor](#)

### 6.11.1 Detailed Description

```
template<class Matrix_T>
class glucat::gen::generator_table< Matrix_T >
```

Table of generators for specific signatures.

Definition at line 52 of file [generation.h](#).

### 6.11.2 Constructor & Destructor Documentation

#### 6.11.2.1 generator\_table() [1/2]

```
template<class Matrix_T>
glucat::gen::generator_table< Matrix_T >::generator_table () [private], [default]
```

Referenced by [generator\(\)](#), [generator\\_table\(\)](#), and [operator=\(\)](#).

### 6.11.2.2 `~generator_table()`

```
template<class Matrix_T>
glucat::gen::generator_table< Matrix_T >::~~generator_table () [private], [default]
```

### 6.11.2.3 `generator_table()` [2/2]

```
template<class Matrix_T>
glucat::gen::generator_table< Matrix_T >::generator_table (
    const generator_table< Matrix_T > & ) [delete]
```

References [generator\\_table\(\)](#).

## 6.11.3 Member Function Documentation

### 6.11.3.1 `gen_from_pm1_qm1()`

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_pm1_qm1 (
    const std::vector< Matrix_T > & old,
    const signature_t sig) [private]
```

Construct generators for p,q given generators for p-1,q-1.

Definition at line 127 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_kron\(\)](#), and [glucat::matrix::unit\(\)](#).

Referenced by [gen\\_vector\(\)](#).

### 6.11.3.2 `gen_from_pm4_qp4()`

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_pm4_qp4 (
    const std::vector< Matrix_T > & old,
    const signature_t sig) [private]
```

Construct generators for p,q given generators for p-4,q+4.

Definition at line 165 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_prod\(\)](#).

Referenced by [gen\\_vector\(\)](#).



### 6.11.3.3 gen\_from\_pp4\_qm4()

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_pp4_qm4 (
    const std::vector< Matrix_T > & old,
    const signature_t sig) [private]
```

Construct generators for p,q given generators for p+4,q-4.

Definition at line 198 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_prod\(\)](#).

Referenced by [gen\\_vector\(\)](#).

### 6.11.3.4 gen\_from\_qp1\_pm1()

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_qp1_pm1 (
    const std::vector< Matrix_T > & old,
    const signature_t sig) [private]
```

Construct generators for p,q given generators for q+1,p-1.

Definition at line 231 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_prod\(\)](#).

Referenced by [gen\\_vector\(\)](#).

### 6.11.3.5 gen\_vector()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::gen_vector (
    const index_t p,
    const index_t q) -> const std::vector<Matrix_T>& [private]
```

Construct a vector of generators for a specific signature.

Definition at line 79 of file [generation\\_imp.h](#).

References [gen\\_from\\_pm1\\_qm1\(\)](#), [gen\\_from\\_pm4\\_qp4\(\)](#), [gen\\_from\\_pp4\\_qm4\(\)](#), [gen\\_from\\_qp1\\_pm1\(\)](#), [gen\\_vector\(\)](#), [glucat::pos\\_mod\(\)](#), and [glucat::matrix::unit\(\)](#).

Referenced by [gen\\_vector\(\)](#), and [operator>\(\)\(\)](#).

### 6.11.3.6 generator()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::generator () -> generator_table<Matrix_T>&
[static]
```

Single instance of generator table.

Definition at line 49 of file [generation\\_imp.h](#).

References [generator\\_table\(\)](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#).

### 6.11.3.7 operator()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::operator() (
    const index_t p,
    const index_t q) -> const Matrix_T* [inline]
```

Pointer to generators for a specific signature.

Definition at line 58 of file [generation\\_imp.h](#).

References [gen\\_vector\(\)](#), [glucat::gen::offset\\_to\\_super](#), and [glucat::pos\\_mod\(\)](#).

### 6.11.3.8 operator=()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::operator= (
    const generator_table< Matrix_T > & ) -> generator_table &=delete [delete]
```

References [generator\\_table\(\)](#).

## 6.11.4 Friends And Related Symbol Documentation

### 6.11.4.1 friend\_for\_private\_destructor

```
template<class Matrix_T>
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 75 of file [generation.h](#).

References [friend\\_for\\_private\\_destructor](#).

Referenced by [friend\\_for\\_private\\_destructor](#).

The documentation for this class was generated from the following files:

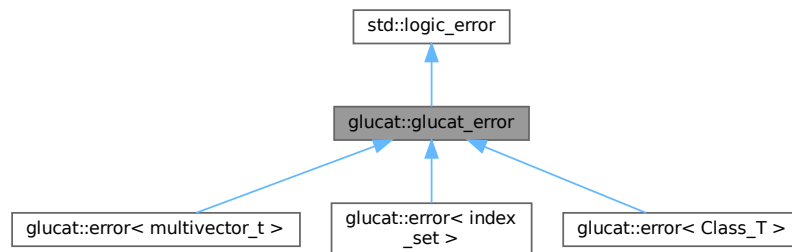
- [glucat/generation.h](#)
- [glucat/generation\\_imp.h](#)

## 6.12 glucat::glucat\_error Class Reference

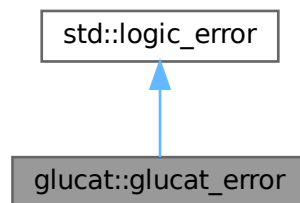
Abstract exception class.

```
#include <errors.h>
```

Inheritance diagram for glucat::glucat\_error:



Collaboration diagram for glucat::glucat\_error:



### Public Member Functions

- `glucat_error` (const std::string &context, const std::string &msg)
- `~glucat_error` () noexcept override=default
- virtual auto `heading` () const noexcept -> const std::string=0
- virtual auto `classname` () const noexcept -> const std::string=0
- virtual void `print_error_msg` () const =0

### Public Attributes

- std::string `name`

### 6.12.1 Detailed Description

Abstract exception class.

Definition at line 41 of file [errors.h](#).

### 6.12.2 Constructor & Destructor Documentation

#### 6.12.2.1 `glucat_error()`

```
glucat::glucat_error::glucat_error (
    const std::string & context,
    const std::string & msg) [inline]
```

Definition at line 44 of file [errors.h](#).

References [name](#).

Referenced by [glucat::error< Class\\_T >::error\(\)](#), and [glucat::error< Class\\_T >::error\(\)](#).

#### 6.12.2.2 `~glucat_error()`

```
glucat::glucat_error::~~glucat_error () [override], [default], [noexcept]
```

### 6.12.3 Member Function Documentation

#### 6.12.3.1 `classname()`

```
virtual auto glucat::glucat_error::classname () const -> const std::string [pure virtual],
[noexcept]
```

Implemented in [glucat::error< Class\\_T >](#), [glucat::error< index\\_set >](#), [glucat::error< index\\_set >](#), [glucat::error< multivector\\_t >](#), and [glucat::error< multivector\\_t >](#).

References [classname\(\)](#).

Referenced by [classname\(\)](#).

#### 6.12.3.2 `heading()`

```
virtual auto glucat::glucat_error::heading () const -> const std::string [pure virtual],
[noexcept]
```

Implemented in [glucat::error< Class\\_T >](#), [glucat::error< index\\_set >](#), [glucat::error< index\\_set >](#), [glucat::error< multivector\\_t >](#), and [glucat::error< multivector\\_t >](#).

References [heading\(\)](#).

Referenced by [heading\(\)](#).

### 6.12.3.3 print\_error\_msg()

```
virtual void glucat::glucat_error::print_error_msg () const [pure virtual]
```

Implemented in [glucat::error< Class\\_T >](#), [glucat::error< index\\_set >](#), [glucat::error< index\\_set >](#), [glucat::error< multivector\\_t >](#), and [glucat::error< multivector\\_t >](#).

References [print\\_error\\_msg\(\)](#).

Referenced by [print\\_error\\_msg\(\)](#).

## 6.12.4 Member Data Documentation

### 6.12.4.1 name

```
std::string glucat::glucat_error::name
```

Definition at line 51 of file [errors.h](#).

Referenced by [glucat::error< Class\\_T >::classname\(\)](#), and [glucat\\_error\(\)](#).

The documentation for this class was generated from the following file:

- [glucat/errors.h](#)

## 6.13 glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::hash\_size\_t Class Reference

### Public Member Functions

- [hash\\_size\\_t](#) (size\_t hash\_size)
- auto [operator\(\)](#) () const -> size\_t

### Private Attributes

- size\_t [n](#)

### 6.13.1 Detailed Description

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t
```

Definition at line 152 of file [framed\\_multi.h](#).

## 6.13.2 Constructor & Destructor Documentation

### 6.13.2.1 hash\_size\_t()

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::hash_size_t (
    size_t hash_size) [inline]
```

Definition at line [155](#) of file [framed\\_multi.h](#).

References [n](#).

## 6.13.3 Member Function Documentation

### 6.13.3.1 operator()()

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::operator() () const ->
size_t [inline]
```

Definition at line [158](#) of file [framed\\_multi.h](#).

References [n](#).

## 6.13.4 Member Data Documentation

### 6.13.4.1 n

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
size_t glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::n [private]
```

Definition at line [161](#) of file [framed\\_multi.h](#).

Referenced by [hash\\_size\\_t\(\)](#), and [operator\(\)\(\)](#).

The documentation for this class was generated from the following file:

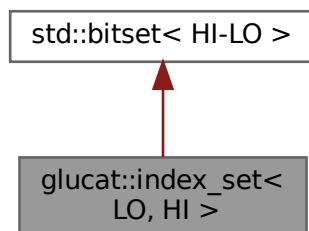
- [glucat/framed\\_multi.h](#)

## 6.14 glucat::index\_set< LO, HI > Class Template Reference

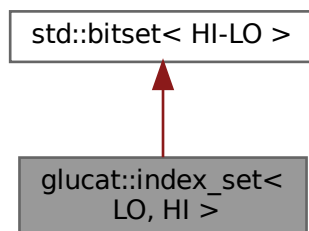
Index set class based on std::bitset<> in Gnu standard C++ library.

```
#include <index_set.h>
```

Inheritance diagram for glucat::index\_set< LO, HI >:



Collaboration diagram for glucat::index\_set< LO, HI >:



### Classes

- class [reference](#)  
*Index set member reference.*

### Public Types

- using [index\\_set\\_t](#) = [index\\_set](#)
- using [index\\_pair\\_t](#) = std::pair<[index\\_t](#), [index\\_t](#)>

## Public Member Functions

- [index\\_set](#) ()=default  
*Default constructor creates an empty set.*
- [index\\_set](#) (const [bitset\\_t](#) bst)  
*Constructor from [bitset\\_t](#).*
- [index\\_set](#) (const [index\\_t](#) idx)  
*Constructor from index.*
- [index\\_set](#) (const [set\\_value\\_t](#) folded\_val, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Constructor from set value of an index set folded within the given frame.*
- [index\\_set](#) (const [index\\_pair\\_t](#) &range, const bool prechecked=false)  
*Constructor from range of indices from range.first to range.second.*
- [index\\_set](#) (const std::string &str)  
*Constructor from string.*
- auto [operator==](#) (const [index\\_set\\_t](#) rhs) const -> bool  
*Equality.*
- auto [operator!=](#) (const [index\\_set\\_t](#) rhs) const -> bool  
*Inequality.*
- auto [operator~](#) () const -> [index\\_set\\_t](#)  
*Set complement: not.*
- auto [operator^](#) = (const [index\\_set\\_t](#) rhs) -> [index\\_set\\_t](#) &  
*Symmetric set difference: exclusive or.*
- auto [operator&](#) = (const [index\\_set\\_t](#) rhs) -> [index\\_set\\_t](#) &  
*Set intersection: and.*
- auto [operator|](#) = (const [index\\_set\\_t](#) rhs) -> [index\\_set\\_t](#) &  
*Set union: or.*
- auto [operator\[\]](#) (const [index\\_t](#) idx) const -> bool  
*Subscripting: Test idx for membership: test value of bit idx.*
- auto [test](#) (const [index\\_t](#) idx) const -> bool  
*Test idx for membership: test value of bit idx.*
- auto [set](#) () -> [index\\_set\\_t](#) &  
*Include all indices except 0: set all bits except 0.*
- auto [set](#) (const [index\\_t](#) idx) -> [index\\_set\\_t](#) &  
*Include idx: Set bit at idx if idx != 0.*
- auto [set](#) (const [index\\_t](#) idx, const int val) -> [index\\_set\\_t](#) &  
*Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.*
- auto [reset](#) () -> [index\\_set\\_t](#) &  
*Make set empty: Set all bits to 0.*
- auto [reset](#) (const [index\\_t](#) idx) -> [index\\_set\\_t](#) &  
*Exclude idx: Set bit at idx to 0.*
- auto [flip](#) () -> [index\\_set\\_t](#) &  
*Set complement, except 0: flip all bits, except 0.*
- auto [flip](#) (const [index\\_t](#) idx) -> [index\\_set\\_t](#) &  
*Complement membership of idx if idx != 0: flip bit at idx if idx != 0.*
- auto [count](#) () const -> [index\\_t](#)  
*Cardinality: Number of indices included in set.*
- auto [count\\_neg](#) () const -> [index\\_t](#)  
*Number of negative indices included in set.*
- auto [count\\_pos](#) () const -> [index\\_t](#)  
*Number of positive indices included in set.*
- auto [min](#) () const -> [index\\_t](#)



*Minimum member.*

- auto `max` () const -> `index_t`

*Maximum member.*

- auto `operator<` (const `index_set_t` rhs) const -> bool

*Less than operator used for comparisons, map, etc.*

- auto `is_contiguous` () const -> bool

*Determine if the index set is contiguous, ie. has no gaps.*

- auto `fold` () const -> const `index_set_t`

*Fold this index set within itself as a frame.*

- auto `fold` (const `index_set_t` frm, const bool prechecked=false) const -> const `index_set_t`

*Fold this index set within the given frame.*

- auto `unfold` (const `index_set_t` frm, const bool prechecked=false) const -> const `index_set_t`

*Unfold this index set within the given frame.*

- auto `value_of_fold` (const `index_set_t` frm) const -> `set_value_t`

*The set value of the fold of this index set within the given frame.*

- auto `sign_of_mult` (const `index_set_t` ist) const -> int

*Sign of geometric product of two *Clifford* basis elements.*

- auto `sign_of_square` () const -> int

*Sign of geometric square of a *Clifford* basis element.*

- auto `hash_fn` () const -> `size_t`

*Hash function.*

- auto `operator[]` (`index_t` idx) -> `reference`

*Subscripting: Element access.*

### Static Public Member Functions

- static auto `classname` () -> const std::string

### Static Public Attributes

- static const `index_t v_lo` = LO
- static const `index_t v_hi` = HI

### Private Types

- using `bitset_t` = std::bitset<HI - LO>
- using `error_t` = `error<index_set>`

### Private Member Functions

- `BOOST_STATIC_ASSERT` ((LO<=0) &&(0<=HI) &&(LO< HI) &&(-LO< \_GLUCAT\_BITS\_PER\_ULONG) &&(HI< \_GLUCAT\_BITS\_PER\_ULONG) &&(HI-LO<= \_GLUCAT\_BITS\_PER\_ULONG))
  - auto `lex_less_than` (const `index_set_t` rhs) const -> bool
- Lexicographic ordering of two sets: \*this < rhs.*

## Friends

- class [reference](#)
- auto [operator^](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> const [index\\_set\\_t](#)
- auto [operator&](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> const [index\\_set\\_t](#)
- auto [operator|](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> const [index\\_set\\_t](#)
- auto [compare](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> int

## 6.14.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index_set< LO, HI >
```

Index set class based on `std::bitset<>` in Gnu standard C++ library.

Definition at line 73 of file [index\\_set.h](#).

## 6.14.2 Member Typedef Documentation

### 6.14.2.1 [bitset\\_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index_set< LO, HI >::bitset_t = std::bitset<HI - LO> [private]
```

Definition at line 81 of file [index\\_set.h](#).

### 6.14.2.2 [error\\_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index_set< LO, HI >::error_t = error<index\_set> [private]
```

Definition at line 82 of file [index\\_set.h](#).

### 6.14.2.3 [index\\_pair\\_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index_set< LO, HI >::index_pair_t = std::pair<index\_t, index\_t>
```

Definition at line 85 of file [index\\_set.h](#).

### 6.14.2.4 [index\\_set\\_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index_set< LO, HI >::index_set_t = index\_set
```

Definition at line 84 of file [index\\_set.h](#).

### 6.14.3 Constructor & Destructor Documentation

#### 6.14.3.1 index\_set() [1/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set () [default]
```

Default constructor creates an empty set.

Referenced by [flip\(\)](#), [fold\(\)](#), and [fold\(\)](#).

#### 6.14.3.2 index\_set() [2/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const bitset_t bst)
```

Constructor from [bitset\\_t](#).

Definition at line 61 of file [index\\_set\\_imp.h](#).

#### 6.14.3.3 index\_set() [3/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const index_t idx)
```

Constructor from index.

Constructor from index value.

Definition at line 55 of file [index\\_set\\_imp.h](#).

References [set\(\)](#).

#### 6.14.3.4 index\_set() [4/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const set_value_t folded_val,
    const index_set_t frm,
    const bool prechecked = false)
```

Constructor from set value of an index set folded within the given frame.

Definition at line 68 of file [index\\_set\\_imp.h](#).

References [count\(\)](#), [fold\(\)](#), [min\(\)](#), and [unfold\(\)](#).

#### 6.14.3.5 `index_set()` [5/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const index_pair_t & range,
    const bool prechecked = false)
```

Constructor from range of indices from `range.first` to `range.second`.

Definition at line 82 of file `index_set_imp.h`.

#### 6.14.3.6 `index_set()` [6/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const std::string & str)
```

Constructor from string.

Definition at line 102 of file `index_set_imp.h`.

### 6.14.4 Member Function Documentation

#### 6.14.4.1 `BOOST_STATIC_ASSERT()`

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::BOOST_STATIC_ASSERT (
    (LO<=0) && (0<=HI) && (LO< HI) && (-LO< _GLUCAT_BITS_PER_ULONG) && (HI< _GLUCAT_↵
    BITS_PER_ULONG) && (HI-LO<=_GLUCAT_BITS_PER_ULONG) ) [private]
```

#### 6.14.4.2 `classname()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::classname () -> const std::string [inline], [static]
```

Definition at line 49 of file `index_set_imp.h`.

#### 6.14.4.3 `count()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count () const -> index_t [inline]
```

Cardinality: Number of indices included in set.

Definition at line 344 of file `index_set_imp.h`.

Referenced by `count_neg()`, `count_pos()`, `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi()`, `index_set()`, `is_contiguous()`, `operator<()`, and `sign_of_square()`.

#### 6.14.4.4 count\_neg()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_neg () const -> index_t [inline]
```

Number of negative indices included in set.

Definition at line 364 of file [index\\_set\\_imp.h](#).

References [count\(\)](#).

Referenced by [sign\\_of\\_square\(\)](#).

#### 6.14.4.5 count\_pos()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_pos () const -> index_t [inline]
```

Number of positive indices included in set.

Definition at line 376 of file [index\\_set\\_imp.h](#).

References [count\(\)](#).

#### 6.14.4.6 flip() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip () -> index_set_t& [inline]
```

Set complement, except 0: flip all bits, except 0.

Definition at line 319 of file [index\\_set\\_imp.h](#).

References [index\\_set\(\)](#).

#### 6.14.4.7 flip() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip (
    const index_t idx) -> index_set_t& [inline]
```

Complement membership of idx if idx != 0: flip bit at idx if idx != 0.

Definition at line 330 of file [index\\_set\\_imp.h](#).

#### 6.14.4.8 fold() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold () const -> const index_set_t [inline]
```

Fold this index set within itself as a frame.

Definition at line 747 of file [index\\_set\\_imp.h](#).

References [fold\(\)](#), and [index\\_set\(\)](#).

Referenced by [fold\(\)](#), [index\\_set\(\)](#), and [value\\_of\\_fold\(\)](#).

**6.14.4.9 fold()** [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold (
    const index_set_t frm,
    const bool prechecked = false) const -> const index_set_t
```

Fold this index set within the given frame.

Definition at line 755 of file [index\\_set\\_imp.h](#).

References [index\\_set\(\)](#), and [test\(\)](#).

**6.14.4.10 hash\_fn()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::hash_fn () const -> size_t [inline]
```

Hash function.

Definition at line 950 of file [index\\_set\\_imp.h](#).

**6.14.4.11 is\_contiguous()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::is_contiguous () const -> bool [inline]
```

Determine if the index set is contiguous, ie. has no gaps.

Determine if the index set is contiguous, ie. has no gaps when 0 is included.

Definition at line 732 of file [index\\_set\\_imp.h](#).

References [count\(\)](#), [max\(\)](#), and [min\(\)](#).

**6.14.4.12 lex\_less\_than()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::lex_less_than (
    const index_set_t rhs) const -> bool [inline], [private]
```

Lexicographic ordering of two sets: \*this < rhs.

Definition at line 588 of file [index\\_set\\_imp.h](#).

Referenced by [operator<\(\)](#).

**6.14.4.13 max()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::max () const -> index_t
```

Maximum member.

Maximum member, or 0 if none.

Definition at line 550 of file [index\\_set\\_imp.h](#).

References [test\(\)](#).

Referenced by [PyClical.index\\_set::\\_\\_iter\\_\\_\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [is\\_contiguous\(\)](#), and [unfold\(\)](#).

**6.14.4.14 min()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::min () const -> index_t
```

Minimum member.

Minimum member, or 0 if none.

Definition at line 461 of file [index\\_set\\_imp.h](#).

References [test\(\)](#).

Referenced by [PyClical.index\\_set::\\_\\_iter\\_\\_\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [index\\_set\(\)](#), [is\\_contiguous\(\)](#), and [unfold\(\)](#).

**6.14.4.15 operator"!="()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator!= (
    const index_set_t rhs) const -> bool [inline]
```

Inequality.

Definition at line 130 of file [index\\_set\\_imp.h](#).

**6.14.4.16 operator&=()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator&= (
    const index_set_t rhs) -> index_set_t& [inline]
```

Set intersection: and.

Definition at line 174 of file [index\\_set\\_imp.h](#).

**6.14.4.17 operator<()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator< (
    const index_set_t rhs) const -> bool [inline]
```

Less than operator used for comparisons, map, etc.

Definition at line 596 of file [index\\_set\\_imp.h](#).

References [count\(\)](#), and [lex\\_less\\_than\(\)](#).

**6.14.4.18 operator==(())**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator==( (
    const index_set_t rhs) const -> bool [inline]
```

Equality.

Definition at line 119 of file [index\\_set\\_imp.h](#).

**6.14.4.19 operator[]() [1/2]**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
    const index_t idx) const -> bool [inline]
```

Subscripting: Test idx for membership: test value of bit idx.

Definition at line 232 of file [index\\_set\\_imp.h](#).

References [test\(\)](#).

**6.14.4.20 operator[]() [2/2]**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
    index_t idx) -> reference [inline]
```

Subscripting: Element access.

Definition at line 224 of file [index\\_set\\_imp.h](#).

**6.14.4.21 operator^=()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator^= (
    const index_set_t rhs) -> index_set_t& [inline]
```

Symmetric set difference: exclusive or.

Definition at line 149 of file [index\\_set\\_imp.h](#).



#### 6.14.4.22 operator" |=()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator|= (
    const index_set_t rhs) -> index_set_t& [inline]
```

Set union: or.

Definition at line 199 of file [index\\_set\\_imp.h](#).

#### 6.14.4.23 operator~()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator~ () const -> index_set_t [inline]
```

Set complement: not.

Definition at line 141 of file [index\\_set\\_imp.h](#).

#### 6.14.4.24 reset() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset () -> index_set_t& [inline]
```

Make set empty: Set all bits to 0.

Definition at line 294 of file [index\\_set\\_imp.h](#).

#### 6.14.4.25 reset() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset (
    const index_t idx) -> index_set_t& [inline]
```

Exclude idx: Set bit at idx to 0.

Definition at line 305 of file [index\\_set\\_imp.h](#).

#### 6.14.4.26 set() [1/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set () -> index_set_t& [inline]
```

Include all indices except 0: set all bits except 0.

Definition at line 255 of file [index\\_set\\_imp.h](#).

Referenced by [index\\_set\(\)](#).

**6.14.4.27 set()** [2/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
    const index_t idx) -> index_set_t& [inline]
```

Include idx: Set bit at idx if idx != 0.

Definition at line 266 of file [index\\_set\\_imp.h](#).

**6.14.4.28 set()** [3/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
    const index_t idx,
    const int val) -> index_set_t& [inline]
```

Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.

Definition at line 280 of file [index\\_set\\_imp.h](#).

**6.14.4.29 sign\_of\_mult()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_mult (
    const index_set_t ist) const -> int
```

Sign of geometric product of two [Clifford](#) basis elements.

Definition at line 880 of file [index\\_set\\_imp.h](#).

References [glucat::inverse\\_gray\(\)](#), and [glucat::inverse\\_reversed\\_gray\(\)](#).

**6.14.4.30 sign\_of\_square()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_square () const -> int [inline]
```

Sign of geometric square of a [Clifford](#) basis element.

Definition at line 930 of file [index\\_set\\_imp.h](#).

References [count\(\)](#), and [count\\_neg\(\)](#).

**6.14.4.31 test()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::test (
    const index_t idx) const -> bool [inline]
```

Test idx for membership: test value of bit idx.

Definition at line 240 of file [index\\_set\\_imp.h](#).

Referenced by [fold\(\)](#), [max\(\)](#), [min\(\)](#), [operator\[\]\(\)](#), and [unfold\(\)](#).

#### 6.14.4.32 unfold()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::unfold (
    const index_set_t frm,
    const bool prechecked = false) const -> const index_set_t
```

Unfold this index set within the given frame.

Definition at line 794 of file [index\\_set\\_imp.h](#).

References [max\(\)](#), [min\(\)](#), and [test\(\)](#).

Referenced by [index\\_set\(\)](#).

#### 6.14.4.33 value\_of\_fold()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::value_of_fold (
    const index_set_t frm) const -> set_value_t [inline]
```

The set value of the fold of this index set within the given frame.

Definition at line 829 of file [index\\_set\\_imp.h](#).

References [fold\(\)](#).

### 6.14.5 Friends And Related Symbol Documentation

#### 6.14.5.1 compare

```
template<const index_t LO, const index_t HI>
auto compare (
    const index_set_t & lhs,
    const index_set_t & rhs) -> int [friend]
```

#### 6.14.5.2 operator&

```
template<const index_t LO, const index_t HI>
auto operator& (
    const index_set_t & lhs,
    const index_set_t & rhs) -> const index_set_t [friend]
```

#### 6.14.5.3 operator^

```
template<const index_t LO, const index_t HI>
auto operator^ (
    const index_set_t & lhs,
    const index_set_t & rhs) -> const index_set_t [friend]
```

#### 6.14.5.4 operator" |

```
template<const index_t LO, const index_t HI>
auto operator| (
    const index_set_t & lhs,
    const index_set_t & rhs) -> const index_set_t [friend]
```

#### 6.14.5.5 reference

```
template<const index_t LO, const index_t HI>
friend class reference [friend]
```

Definition at line 174 of file [index\\_set.h](#).

### 6.14.6 Member Data Documentation

#### 6.14.6.1 v\_hi

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_hi = HI [static]
```

Definition at line 88 of file [index\\_set.h](#).

#### 6.14.6.2 v\_lo

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_lo = LO [static]
```

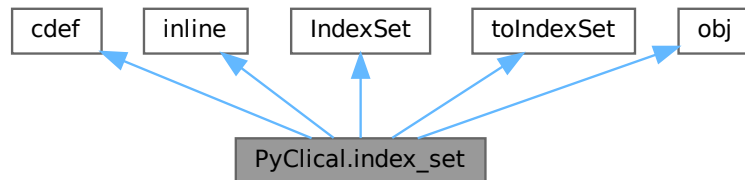
Definition at line 87 of file [index\\_set.h](#).

The documentation for this class was generated from the following files:

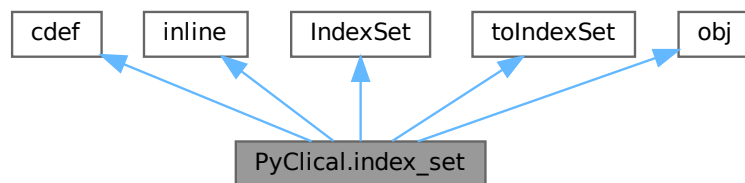
- [glucat/index\\_set.h](#)
- [glucat/index\\_set\\_imp.h](#)

## 6.15 PyClical.index\_set Class Reference

Inheritance diagram for PyClical.index\_set:



Collaboration diagram for PyClical.index\_set:



### Public Member Functions

- [\\_\\_cinit\\_\\_](#) (self, other=0)
- [\\_\\_dealloc\\_\\_](#) (self)
- [\\_\\_richcmp\\_\\_](#) (lhs, rhs, int, op)
- [\\_\\_setitem\\_\\_](#) (self, idx, val)
- [\\_\\_getitem\\_\\_](#) (self, idx)
- [\\_\\_contains\\_\\_](#) (self, idx)
- [\\_\\_iter\\_\\_](#) (self)
- [\\_\\_invert\\_\\_](#) (self)
- [\\_\\_xor\\_\\_](#) (lhs, rhs)
- [\\_\\_ixor\\_\\_](#) (self, rhs)
- [\\_\\_and\\_\\_](#) (lhs, rhs)
- [\\_\\_iand\\_\\_](#) (self, rhs)
- [\\_\\_or\\_\\_](#) (lhs, rhs)
- [\\_\\_ior\\_\\_](#) (self, rhs)
- [count](#) (self)
- [count\\_neg](#) (self)
- [count\\_pos](#) (self)
- [min](#) (self)
- [max](#) (self)

- `hash_fn` (self)
- `sign_of_mult` (self, *rhs*)
- `sign_of_square` (self)
- `__repr__` (self)
- `__str__` (self)

### Public Attributes

- `instance` = new `IndexSet`((<`index_set`>*other*).unwrap())

## 6.15.1 Detailed Description

Return the C++ `IndexSet` instance wrapped by `index_set(obj)`.

Python class `index_set` wraps C++ class `IndexSet`.

Definition at line 46 of file [PyClical.pyx](#).

## 6.15.2 Member Function Documentation

### 6.15.2.1 `__and__()`

```
PyClical.index_set.__and__ (
    lhs,
    rhs)
```

Set intersection: `and`.

```
>>> print(index_set({1}) & index_set({2}))
{}
>>> print(index_set({1,2}) & index_set({2}))
{2}
```

Definition at line 271 of file [PyClical.pyx](#).

### 6.15.2.2 `__cinit__()`

```
PyClical.index_set.__cinit__ (
    self,
    other = 0)
```

Construct an object of type `index_set`.

```
>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set("{1,2,1}"))
{1,2}
>>> print(index_set(""))
{}
```

Definition at line 74 of file [PyClical.pyx](#).

### 6.15.2.3 `__contains__()`

```
PyClical.index_set.__contains__ (
    self,
    idx)
```

Check that an `index_set` object contains the index `idx`: `idx in self`.

```
>>> 1 in index_set({1})
True
>>> 2 in index_set({1})
False
>>> -1 in index_set({2})
False
>>> 1 in index_set({2})
False
>>> 2 in index_set({2})
True
>>> 33 in index_set({2})
False
```

Definition at line 210 of file [PyClical.pyx](#).

References [instance](#).

### 6.15.2.4 `__dealloc__()`

```
PyClical.index_set.__dealloc__ (
    self)
```

Clean up by deallocating the instance of C++ class `IndexSet`.

Definition at line 116 of file [PyClical.pyx](#).

References [instance](#).

### 6.15.2.5 `__getitem__()`

```
PyClical.index_set.__getitem__ (
    self,
    idx)
```

Get the value of an `index_set` object at an index.

```
>>> index_set({1})[1]
True
>>> index_set({1})[2]
False
>>> index_set({2})[-1]
False
>>> index_set({2})[1]
False
>>> index_set({2})[2]
True
>>> index_set({2})[33]
False
```

Definition at line 191 of file [PyClical.pyx](#).

References [instance](#).

### 6.15.2.6 `__iand__()`

```
PyClical.index_set.__iand__ (
    self,
    rhs)
```

Set intersection: and.

```
>>> x = index_set({1}); x &= index_set({2}); print(x)
{}
>>> x = index_set({1,2}); x &= index_set({2}); print(x)
{2}
```

Definition at line 282 of file [PyClical.pyx](#).

### 6.15.2.7 `__invert__()`

```
PyClical.index_set.__invert__ (
    self)
```

Set complement: not.

```
>>> print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,
{-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,
```

Definition at line 240 of file [PyClical.pyx](#).

References [instance](#).

### 6.15.2.8 `__ior__()`

```
PyClical.index_set.__ior__ (
    self,
    rhs)
```

Set union: or.

```
>>> x = index_set({1}); x |= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x |= index_set({2}); print(x)
{1,2}
```

Definition at line 304 of file [PyClical.pyx](#).

### 6.15.2.9 `__iter__()`

```
PyClical.index_set.__iter__ (
    self)
```

Iterate over the indices of an `index_set`.

```
>>> for i in index_set({-3,4,7}):print(i, end=",")
-3,4,7,
```

Definition at line 229 of file [PyClical.pyx](#).

References [glucat::index\\_set< LO, HI >.max\(\)](#), [glucat::index\\_set< lo\\_ndx, hi\\_ndx >.max\(\)](#), [max\(\)](#), [glucat::index\\_set< LO, HI >.min\(\)](#), [glucat::index\\_set< lo\\_ndx, hi\\_ndx >.min\(\)](#), and [min\(\)](#).



### 6.15.2.10 `__ixor__()`

```
PyClical.index_set.__ixor__ (
    self,
    rhs)
```

Symmetric set difference: exclusive or.

```
>>> x = index_set({1}); x ^= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x ^= index_set({2}); print(x)
{1}
```

Definition at line 260 of file [PyClical.pyx](#).

### 6.15.2.11 `__or__()`

```
PyClical.index_set.__or__ (
    lhs,
    rhs)
```

Set union: or.

```
>>> print(index_set({1}) | index_set({2}))
{1,2}
>>> print(index_set({1,2}) | index_set({2}))
{1,2}
```

Definition at line 293 of file [PyClical.pyx](#).

### 6.15.2.12 `__repr__()`

```
PyClical.index_set.__repr__ (
    self)
```

The “official” string representation of self.

```
>>> index_set({1,2}).__repr__()
'index_set({1,2})'
>>> repr(index_set({1,2}))
'index_set({1,2})'
```

Definition at line 384 of file [PyClical.pyx](#).

References [index\\_set\\_to\\_repr\(\)](#).

### 6.15.2.13 `__richcmp__()`

```
PyClical.index_set.__richcmp__ (
    lhs,
    rhs,
    int,
    op)
```

Compare two objects of class `index_set`.

```
>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
```

Definition at line 122 of file [PyClical.pyx](#).

### 6.15.2.14 `__setitem__()`

```
PyClical.index_set.__setitem__ (
    self,
    idx,
    val)
```

Set the value of an `index_set` object at index `idx` to value `val`.

```
>>> s=index_set({1}); s[2] = True; print(s)
{1,2}
>>> s=index_set({1,2}); s[1] = False; print(s)
{2}
```

Definition at line 179 of file [PyClical.pyx](#).

References [instance](#).

### 6.15.2.15 `__str__()`

```
PyClical.index_set.__str__ (
    self)
```

The "informal" string representation of self.

```
>>> index_set({1,2}).__str__()
'{1,2}'
>>> str(index_set({1,2}))
'{1,2}'
```

Definition at line 395 of file [PyClical.pyx](#).

References [index\\_set\\_to\\_str\(\)](#).

### 6.15.2.16 `__xor__()`

```
PyClical.index_set.__xor__ (  
    lhs,  
    rhs)
```

Symmetric set difference: exclusive or.

```
>>> print(index_set({1}) ^ index_set({2}))  
{1,2}  
>>> print(index_set({1,2}) ^ index_set({2}))  
{1}
```

Definition at line 249 of file [PyClical.pyx](#).

### 6.15.2.17 `count()`

```
PyClical.index_set.count (  
    self)
```

Cardinality: Number of indices included in set.

```
>>> index_set({-1,1,2}).count()  
3
```

Definition at line 315 of file [PyClical.pyx](#).

References [count\(\)](#), and [instance](#).

Referenced by [count\(\)](#).

### 6.15.2.18 `count_neg()`

```
PyClical.index_set.count_neg (  
    self)
```

Number of negative indices included in set.

```
>>> index_set({-1,1,2}).count_neg()  
1
```

Definition at line 324 of file [PyClical.pyx](#).

References [count\\_neg\(\)](#), and [instance](#).

Referenced by [count\\_neg\(\)](#).

### 6.15.2.19 count\_pos()

```
PyClical.index_set.count_pos (  
    self)
```

Number of positive indices included in set.

```
>>> index_set({-1,1,2}).count_pos()  
2
```

Definition at line 333 of file [PyClical.pyx](#).

References [count\\_pos\(\)](#), and [instance](#).

Referenced by [count\\_pos\(\)](#).

### 6.15.2.20 hash\_fn()

```
PyClical.index_set.hash_fn (  
    self)
```

Hash function.

Definition at line 360 of file [PyClical.pyx](#).

References [hash\\_fn\(\)](#), and [instance](#).

Referenced by [hash\\_fn\(\)](#).

### 6.15.2.21 max()

```
PyClical.index_set.max (  
    self)
```

Maximum member.

```
>>> index_set({-1,1,2}).max()  
2
```

Definition at line 351 of file [PyClical.pyx](#).

References [instance](#), and [max\(\)](#).

Referenced by [\\_\\_iter\\_\\_\(\)](#), and [max\(\)](#).

### 6.15.2.22 min()

```
PyClical.index_set.min (  
    self)
```

Minimum member.

```
>>> index_set ({-1,1,2}).min()  
-1
```

Definition at line 342 of file [PyClical.pyx](#).

References [instance](#), and [min\(\)](#).

Referenced by [\\_\\_iter\\_\\_\(\)](#), and [min\(\)](#).

### 6.15.2.23 sign\_of\_mult()

```
PyClical.index_set.sign_of_mult (  
    self,  
    rhs)
```

Sign of geometric product of two Clifford basis elements.

```
>>> s = index_set ({1,2}); t=index_set ({-1}); s.sign_of_mult(t)  
1
```

Definition at line 366 of file [PyClical.pyx](#).

References [instance](#), and [sign\\_of\\_mult\(\)](#).

Referenced by [sign\\_of\\_mult\(\)](#).

### 6.15.2.24 sign\_of\_square()

```
PyClical.index_set.sign_of_square (  
    self)
```

Sign of geometric square of a Clifford basis element.

```
>>> s = index_set ({1,2}); s.sign_of_square()  
-1
```

Definition at line 375 of file [PyClical.pyx](#).

References [instance](#), and [sign\\_of\\_square\(\)](#).

Referenced by [sign\\_of\\_square\(\)](#).

### 6.15.3 Member Data Documentation

#### 6.15.3.1 instance

`PyClical.index_set.instance = new IndexSet((<index_set>other).unwrap())`

Definition at line 95 of file [PyClical.pyx](#).

Referenced by [PyClical.clifford.\\_\\_call\\_\\_\(\)](#), [\\_\\_contains\\_\\_\(\)](#), [PyClical.clifford.\\_\\_dealloc\\_\\_\(\)](#), [\\_\\_dealloc\\_\\_\(\)](#), [PyClical.clifford.\\_\\_getitem\\_\\_\(\)](#), [\\_\\_getitem\\_\\_\(\)](#), [\\_\\_invert\\_\\_\(\)](#), [PyClical.clifford.\\_\\_neg\\_\\_\(\)](#), [\\_\\_setitem\\_\\_\(\)](#), [PyClical.clifford.conj\(\)](#), [count\(\)](#), [count\\_neg\(\)](#), [count\\_pos\(\)](#), [PyClical.clifford.even\(\)](#), [PyClical.clifford.frame\(\)](#), [hash\\_fn\(\)](#), [PyClical.clifford.inv\(\)](#), [PyClical.clifford.involute\(\)](#), [PyClical.clifford.isinf\(\)](#), [PyClical.clifford.isnan\(\)](#), [max\(\)](#), [PyClical.clifford.max\\_abs\(\)](#), [min\(\)](#), [PyClical.clifford.norm\(\)](#), [PyClical.clifford.odd\(\)](#), [PyClical.clifford.outer\\_pow\(\)](#), [PyClical.clifford.pow\(\)](#), [PyClical.clifford.pure\(\)](#), [PyClical.clifford.quad\(\)](#), [PyClical.clifford.reverse\(\)](#), [PyClical.clifford.scalar\(\)](#), [sign\\_of\\_mult\(\)](#), [sign\\_of\\_square\(\)](#), [PyClical.clifford.truncated\(\)](#), and [PyClical.clifford.vector\\_part\(\)](#).

The documentation for this class was generated from the following file:

- [pyclical/PyClical.pyx](#)

## 6.16 `glucat::index_set_hash< LO, HI >` Class Template Reference

```
#include <framed_multi.h>
```

### Public Types

- using `index_set_t` = `index_set<LO, HI>`

### Public Member Functions

- auto `operator()` (`index_set_t` val) const -> `size_t`

#### 6.16.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index_set_hash< LO, HI >
```

Definition at line 117 of file [framed\\_multi.h](#).

### 6.16.2 Member Typedef Documentation

#### 6.16.2.1 `index_set_t`

```
template<const index\_t LO, const index\_t HI>
using glucat::index\_set\_hash< LO, HI >::index_set_t = index\_set<LO, HI>
```

Definition at line 120 of file [framed\\_multi.h](#).

### 6.16.3 Member Function Documentation

#### 6.16.3.1 operator()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set_hash< LO, HI >::operator() (
    index_set_t val) const -> size_t    [inline]
```

Definition at line 121 of file [framed\\_multi.h](#).

The documentation for this class was generated from the following file:

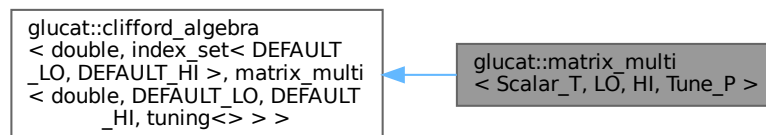
- [glucat/framed\\_multi.h](#)

## 6.17 glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P > Class Template Reference

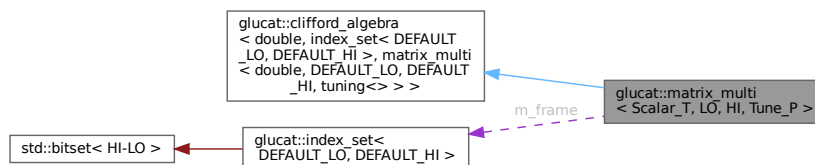
A [matrix\\_multi<Scalar\\_T,LO,HI,Tune\\_P>](#) is a matrix approximation to a multivector.

```
#include <matrix_multi.h>
```

Inheritance diagram for glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >:



Collaboration diagram for glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >:



## Public Types

- using `multivector_t` = `matrix_multi`
- using `matrix_multi_t` = `multivector_t`
- using `scalar_t` = `Scalar_T`
- using `tune_p` = `Tune_P`
- using `index_set_t` = `index_set`<LO, HI>
- using `term_t` = `std::pair`<const `index_set_t`, `Scalar_T`>
- using `vector_t` = `std::vector`<`Scalar_T`>
- using `error_t` = `error`<`multivector_t`>
- using `framed_multi_t` = `framed_multi`<`Scalar_T`,LO,HI,`Tune_P`>

## Public Types inherited from

`glucat::clifford_algebra`< `double`, `index_set`< `DEFAULT_LO`, `DEFAULT_HI` >, `matrix_multi`< `double`, `DE`

- using `scalar_t`
- using `index_set_t`
- using `multivector_t`
- using `pair_t`
- using `vector_t`

## Public Member Functions

- `~matrix_multi` () override=default  
*Destructor.*
- `matrix_multi` ()  
*Default constructor.*
- `template<typename Other_Scalar_T>`  
`matrix_multi` (const `matrix_multi`< `Other_Scalar_T`, LO, HI, `Tune_P` > &val)  
*Construct a multivector from a multivector with a different scalar type.*
- `template<typename Other_Scalar_T>`  
`matrix_multi` (const `matrix_multi`< `Other_Scalar_T`, LO, HI, `Tune_P` > &val, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- `matrix_multi` (const `multivector_t` &val, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- `matrix_multi` (const `index_set_t` ist, const `Scalar_T` &crd=`Scalar_T`(1))  
*Construct a multivector from an index set and a scalar coordinate.*
- `matrix_multi` (const `index_set_t` ist, const `Scalar_T` &crd, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from an index set and a scalar coordinate.*
- `matrix_multi` (const `Scalar_T` &scr, const `index_set_t` frm=`index_set_t`())  
*Construct a multivector from a scalar (within a frame, if given)*
- `matrix_multi` (const int scr, const `index_set_t` frm=`index_set_t`())  
*Construct a multivector from an int (within a frame, if given)*
- `matrix_multi` (const `vector_t` &vec, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given vector.*
- `matrix_multi` (const `std::string` &str)  
*Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*
- `matrix_multi` (const `std::string` &str, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*
- `matrix_multi` (const char \*str)



- Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".
- `matrix_multi` (const char \*str, const `index_set_t` frm, const bool prechecked=false)  
Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".
- template<typename Other\_Scalar\_T>  
`matrix_multi` (const `framed_multi`< Other\_Scalar\_T, LO, HI, Tune\_P > &val)  
Construct a multivector from a `framed_multi_t`.
- template<typename Other\_Scalar\_T>  
`matrix_multi` (const `framed_multi`< Other\_Scalar\_T, LO, HI, Tune\_P > &val, const `index_set_t` frm, const bool prechecked=false)  
Construct a multivector, within a given frame, from a `framed_multi_t`.
- auto `fast_matrix_multi` (const `index_set_t` frm) const -> const `matrix_multi_t`  
Use generalized FFT to construct a `matrix_multi_t`.
- template<typename Other\_Scalar\_T>  
auto `fast_framed_multi` () const -> const `framed_multi`< Other\_Scalar\_T, LO, HI, Tune\_P >  
Use inverse generalized FFT to construct a `framed_multi_t`.
- `_GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS` auto `operator=` (const `multivector_t` &rhs) -> `multivector_t` &  
Assignment operator.
- auto `operator+=` (const `term_t` &rhs) -> `multivector_t` &  
Add a term, if non-zero.

## Public Member Functions inherited from

`glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DE`

- virtual `~clifford_algebra` ()=default
- virtual auto `operator==` (const `multivector_t` &val) const -> bool=0  
Test for equality of multivectors.
- virtual auto `operator==` (const double &scr) const -> bool=0  
Test for equality of multivector and scalar.
- virtual auto `operator+=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
Geometric sum.
- virtual auto `operator+=` (const double &scr) -> `multivector_t` &=0  
Geometric sum of multivector and scalar.
- virtual auto `operator-=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
Geometric difference.
- virtual auto `operator-=` (const double &scr) -> `multivector_t` &=0  
Geometric difference of multivector and scalar.
- virtual auto `operator-` () const -> const `multivector_t`=0  
Unary -.
- virtual auto `operator*=` (const double &scr) -> `multivector_t` &=0  
Product of multivector and scalar.
- virtual auto `operator*=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
Geometric product.
- virtual auto `operator%=-` (const `multivector_t` &rhs) -> `multivector_t` &=0  
Contraction.
- virtual auto `operator&=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
Inner product.
- virtual auto `operator^=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
Outer product.
- virtual auto `operator/=` (const double &scr) -> `multivector_t` &=0  
Quotient of multivector and scalar.

- virtual auto `operator/=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Geometric quotient.*
- virtual auto `operator|=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Transformation via twisted adjoint action.*
- virtual auto `inv` () const -> const `multivector_t`=0  
*Geometric multiplicative inverse.*
- virtual auto `pow` (int m) const -> const `multivector_t`=0  
*\*this to the m*
- virtual auto `outer_pow` (int m) const -> const `multivector_t`=0  
*Outer product power.*
- virtual auto `frame` () const -> const `index_set_t`=0  
*Subalgebra generated by all generators of terms of given multivector.*
- virtual auto `grade` () const -> `index_t`=0  
*Maximum of the grades of each term.*
- virtual auto `operator[]` (const `index_set_t` ist) const -> double=0  
*Subscripting: map from index set to scalar coordinate.*
- virtual auto `operator()` (`index_t` grade) const -> const `multivector_t`=0  
*Pure grade-vector part.*
- virtual auto `scalar` () const -> double=0  
*Scalar part.*
- virtual auto `pure` () const -> const `multivector_t`=0  
*Pure part.*
- virtual auto `even` () const -> const `multivector_t`=0  
*Even part of multivector, sum of even grade terms.*
- virtual auto `odd` () const -> const `multivector_t`=0  
*Odd part of multivector, sum of odd grade terms.*
- virtual auto `vector_part` () const -> const `vector_t`=0  
*Vector part of multivector, as a `vector_t` with respect to `frame()`*
- virtual auto `vector_part` (const `index_set_t` frm, const bool prechecked) const -> const `vector_t`=0  
*Vector part of multivector, as a `vector_t` with respect to frm.*
- virtual auto `involute` () const -> const `multivector_t`=0  
*Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.*
- virtual auto `reverse` () const -> const `multivector_t`=0  
*Reversion, eg. {1}\*{2} -> {2}\*{1}.*
- virtual auto `conj` () const -> const `multivector_t`=0  
*Conjugation, reverse o involute == involute o reverse.*
- virtual auto `quad` () const -> double=0  
*Scalar\_T quadratic form == (rev(x)\*x)(0)*
- virtual auto `norm` () const -> double=0  
*Scalar\_T norm == sum of norm of coordinates.*
- virtual auto `max_abs` () const -> double=0  
*Maximum of absolute values of components of multivector: multivector infinity norm.*
- virtual auto `truncated` (const double &limit=`default_truncation`) const -> const `multivector_t`=0  
*Remove all terms with relative size smaller than limit.*
- virtual auto `isinf` () const -> bool=0  
*Check if a multivector contains any infinite values.*
- virtual auto `isnan` () const -> bool=0  
*Check if a multivector contains any IEEE NaN values.*
- virtual void `write` (const std::string &msg="") const=0  
*Write formatted multivector to output.*
- virtual void `write` (std::ofstream &ofile, const std::string &msg="") const=0  
*Write formatted multivector to file.*

## Static Public Member Functions

- static auto [classname](#) () -> const std::string  
*Class name used in messages.*
- static auto [random](#) (const [index\\_set\\_t](#) frm, Scalar\_T fill=Scalar\_T(1)) -> const [matrix\\_multi\\_t](#)  
*Random multivector within a frame.*

## Static Public Member Functions inherited from

[glucat::clifford\\_algebra< double, index\\_set< DEFAULT\\_LO, DEFAULT\\_HI >, matrix\\_multi< double, DE](#)

- static auto [classname](#) () -> const std::string

## Private Types

- using [orientation\\_t](#) = ublas::row\_major
- using [basis\\_matrix\\_t](#) = ublas::compressed\_matrix<int, [orientation\\_t](#)>
- using [matrix\\_t](#) = ublas::matrix<Scalar\_T, [orientation\\_t](#)>
- using [matrix\\_index\\_t](#) = typename matrix\_t::size\_type

## Private Member Functions

- template<typename Matrix\_T>  
[matrix\\_multi](#) (const Matrix\_T &mtx, const [index\\_set\\_t](#) frm)  
*Construct a multivector within a given frame from a given matrix.*
- [matrix\\_multi](#) (const [matrix\\_t](#) &mtx, const [index\\_set\\_t](#) frm)  
*Construct a multivector within a given frame from a given matrix.*
- auto [basis\\_element](#) (const [index\\_set](#)< LO, HI > &ist) const -> const [basis\\_matrix\\_t](#)  
*Create a basis element matrix within the current frame.*

## Private Attributes

- [index\\_set\\_t m\\_frame](#)  
*Index set representing the frame for the subalgebra which contains the multivector.*
- [matrix\\_t m\\_matrix](#)  
*Matrix value representing the multivector within the folded frame.*

## Friends

- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P>  
class [framed\\_multi](#)
- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P>  
class [matrix\\_multi](#)
- auto [operator\\*](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [operator^](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [operator&](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [operator%](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [star](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> Scalar\_T
- auto [operator/](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [operator|](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)

- auto [operator>>](#) (std::istream &s, [multivector\\_t](#) &val) -> std::istream &
- auto [operator<<](#) (std::ostream &os, const [multivector\\_t](#) &val) -> std::ostream &
- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P>  
auto [reframe](#) (const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &lhs, const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &rhs, [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &lhs\_reframed, [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &rhs\_reframed) -> const [index\\_set](#)< Other\_LO, Other\_HI >
- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P>  
auto [matrix\\_sqrt](#) (const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &val, const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P >
- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P>  
auto [matrix\\_log](#) (const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &val, const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P >

## Additional Inherited Members

### Static Public Attributes inherited from

[glucat::clifford\\_algebra](#)< [double](#), [index\\_set](#)< [DEFAULT\\_LO](#), [DEFAULT\\_HI](#) >, [matrix\\_multi](#)< [double](#), [DE](#)

- static const [index\\_t](#) [v\\_lo](#)
- static const [index\\_t](#) [v\\_hi](#)
- static const [double](#) [default\\_truncation](#)

*Default for truncation.*

## 6.17.1 Detailed Description

template<typename Scalar\_T = [double](#), const [index\\_t](#) LO = [DEFAULT\\_LO](#), const [index\\_t](#) HI = [DEFAULT\\_HI](#),  
typename Tune\_P = [tuning](#)<>>

class [glucat::matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

A [matrix\\_multi](#)<Scalar\_T,LO,HI,Tune\_P> is a matrix approximation to a multivector.

Definition at line 137 of file [matrix\\_multi.h](#).

## 6.17.2 Member Typedef Documentation

### 6.17.2.1 basis\_matrix\_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT\_LO, const index\_t HI = DEFAULT\_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::basis_matrix_t = ublas::compressed\_matrix<int, orientation\_t> [private]
```

Definition at line 157 of file [matrix\\_multi.h](#).

### 6.17.2.2 error\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::error_t = error<multivector_t>
```

Definition at line 148 of file [matrix\\_multi.h](#).

### 6.17.2.3 framed\_multi\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::framed_multi_t = framed_multi<Scalar_T, LO, HI, Tune_P>
```

Definition at line 149 of file [matrix\\_multi.h](#).

### 6.17.2.4 index\_set\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index_set<LO, HI>
```

Definition at line 145 of file [matrix\\_multi.h](#).

### 6.17.2.5 matrix\_index\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_index_t = typename matrix_t<Scalar_T, LO, HI, Tune_P>::size_type [private]
```

Definition at line 159 of file [matrix\\_multi.h](#).

### 6.17.2.6 matrix\_multi\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = multivector_t
```

Definition at line 142 of file [matrix\\_multi.h](#).

### 6.17.2.7 matrix\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = ublas::matrix<Scalar_T, LO, HI, Tune_P, orientation_t> [private]
```

Definition at line 158 of file [matrix\\_multi.h](#).

### 6.17.2.8 multivector\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = matrix_multi
```

Definition at line 141 of file [matrix\\_multi.h](#).

### 6.17.2.9 orientation\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::orientation_t = ublas::row_major
[private]
```

Definition at line 156 of file [matrix\\_multi.h](#).

### 6.17.2.10 scalar\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 143 of file [matrix\\_multi.h](#).

### 6.17.2.11 term\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index_set_t,
Scalar_T>
```

Definition at line 146 of file [matrix\\_multi.h](#).

### 6.17.2.12 tune\_p

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 144 of file [matrix\\_multi.h](#).

### 6.17.2.13 vector\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 147 of file [matrix\\_multi.h](#).

## 6.17.3 Constructor & Destructor Documentation

### 6.17.3.1 ~matrix\_multi()

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::~matrix\_multi () [override], [default]
```

Destructor.

### 6.17.3.2 matrix\_multi() [1/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi ()
```

Default constructor.

Definition at line 106 of file [matrix\\_multi\\_imp.h](#).

References [m\\_frame](#), and [m\\_matrix](#).

### 6.17.3.3 matrix\_multi() [2/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 115 of file [matrix\\_multi\\_imp.h](#).

References [m\\_frame](#), [m\\_matrix](#), [matrix\\_multi](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

### 6.17.3.4 matrix\_multi() [3/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 134 of file [matrix\\_multi\\_imp.h](#).

References [glucat::folded\\_dim\(\)](#), [m\\_frame](#), [m\\_matrix](#), [matrix\\_multi](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**6.17.3.5 matrix\_multi()** [4/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const multivector\_t & val,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 159 of file [matrix\\_multi\\_imp.h](#).

References [m\\_frame](#).

**6.17.3.6 matrix\_multi()** [5/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const index\_set\_t ist,
    const Scalar_T & crd = Scalar_T(1))
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 171 of file [matrix\\_multi\\_imp.h](#).

**6.17.3.7 matrix\_multi()** [6/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const index\_set\_t ist,
    const Scalar_T & crd,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 183 of file [matrix\\_multi\\_imp.h](#).

**6.17.3.8 matrix\_multi()** [7/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const Scalar_T & scr,
    const index\_set\_t frm = index\_set\_t() )
```

Construct a multivector from a scalar (within a frame, if given)

Definition at line 197 of file [matrix\\_multi\\_imp.h](#).

References [glucat::folded\\_dim\(\)](#), [m\\_frame](#), and [m\\_matrix](#).



**6.17.3.9 matrix\_multi()** [8/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const int scr,
    const index\_set\_t frm = index\_set\_t() )
```

Construct a multivector from an int (within a frame, if given)

Definition at line 209 of file [matrix\\_multi\\_imp.h](#).

**6.17.3.10 matrix\_multi()** [9/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const vector\_t & vec,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 215 of file [matrix\\_multi\\_imp.h](#).

**6.17.3.11 matrix\_multi()** [10/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const std::string & str)
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 240 of file [matrix\\_multi\\_imp.h](#).

**6.17.3.12 matrix\_multi()** [11/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const std::string & str,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 246 of file [matrix\\_multi\\_imp.h](#).

**6.17.3.13 matrix\_multi()** [12/17]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const char * str) [inline]
```

Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 196 of file [matrix\\_multi.h](#).

**6.17.3.14 matrix\_multi()** [13/17]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const char * str,
    const index\_set\_t frm,
    const bool prechecked = false) [inline]
```

Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 199 of file [matrix\\_multi.h](#).

**6.17.3.15 matrix\_multi()** [14/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a [framed\\_multi\\_t](#).

Definition at line 253 of file [matrix\\_multi\\_imp.h](#).

References [glucat::folded\\_dim\(\)](#), [glucat::clifford\\_algebra< double, index\\_set< DEFAULT\\_LO, DEFAULT\\_HI >, matrix\\_multi< double framed\\_multi, m\\_frame, and m\\_matrix](#).

**6.17.3.16 matrix\_multi()** [15/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a [framed\\_multi\\_t](#).

Definition at line 277 of file [matrix\\_multi\\_imp.h](#).

References [fast\\_matrix\\_multi\(\)](#), [glucat::folded\\_dim\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::frame\(\)](#), [framed\\_multi](#), [m\\_frame](#), [m\\_matrix](#), and [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::truncated\(\)](#).

**6.17.3.17 matrix\_multi()** [16/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Matrix_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const Matrix_T & mtx,
    const index\_set\_t frm) [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 303 of file [matrix\\_multi\\_imp.h](#).

References [m\\_frame](#), [m\\_matrix](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

### 6.17.3.18 matrix\_multi() [17/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix_t & mtx,
    const index_set_t frm) [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 322 of file [matrix\\_multi\\_imp.h](#).

References [m\\_frame](#), and [m\\_matrix](#).

## 6.17.4 Member Function Documentation

### 6.17.4.1 basis\_element()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element (
    const index_set< LO, HI > & ist) const -> const basis_matrix_t [private]
```

Create a basis element matrix within the current frame.

Definition at line 1186 of file [matrix\\_multi\\_imp.h](#).

References [glucat::gen::generator\\_table< Matrix\\_T >::generator\(\)](#), [m\\_frame](#), [glucat::matrix::mono\\_prod\(\)](#), [glucat::offset\\_level\(\)](#), and [glucat::matrix::unit\(\)](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

### 6.17.4.2 classname()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::classname () -> const std::string
[static]
```

Class name used in messages.

Definition at line 78 of file [matrix\\_multi\\_imp.h](#).

### 6.17.4.3 fast\_framed\_multi()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi () const -> const
framed_multi<Other_Scalar_T,LO,HI,Tune_P>
```

Use inverse generalized FFT to construct a [framed\\_multi\\_t](#).

Definition at line 1109 of file [matrix\\_multi\\_imp.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::centre\\_pm4\\_qp4\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::centre\\_qp1\\_pm1\(\)](#), [glucat::fast\(\)](#), [framed\\_multi](#), [m\\_frame](#), [m\\_matrix](#), [glucat::gen::offset\\_to\\_super](#), [glucat::pos\\_mod\(\)](#), and [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::unfold\(\)](#).

Referenced by [fast\\_matrix\\_multi\(\)](#).

#### 6.17.4.4 fast\_matrix\_multi()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
    const index_set_t frm) const -> const matrix_multi_t [inline]
```

Use generalized FFT to construct a [matrix\\_multi\\_t](#).

Definition at line 1096 of file [matrix\\_multi\\_imp.h](#).

References [fast\\_framed\\_multi\(\)](#), [fast\\_matrix\\_multi\(\)](#), and [m\\_frame](#).

Referenced by [fast\\_matrix\\_multi\(\)](#), and [matrix\\_multi\(\)](#).

#### 6.17.4.5 operator+=()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
    const term_t & rhs) -> multivector_t& [inline]
```

Add a term, if non-zero.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line 416 of file [matrix\\_multi\\_imp.h](#).

#### 6.17.4.6 operator=()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::operator= (
    const multivector_t & rhs) -> multivector_t&
```

Assignment operator.

Definition at line 330 of file [matrix\\_multi\\_imp.h](#).

References [m\\_frame](#), and [m\\_matrix](#).

#### 6.17.4.7 random()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::random (
    const index_set_t frm,
    Scalar_T fill = Scalar_T(1)) -> const matrix_multi_t [static]
```

Random multivector within a frame.

Definition at line 926 of file [matrix\\_multi\\_imp.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::random\(\)](#).

## 6.17.5 Friends And Related Symbol Documentation

### 6.17.5.1 framed\_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
friend class framed_multi [friend]
```

Definition at line 151 of file [matrix\\_multi.h](#).

Referenced by [fast\\_framed\\_multi\(\)](#), [matrix\\_multi\(\)](#), and [matrix\\_multi\(\)](#).

### 6.17.5.2 matrix\_log

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
auto matrix_log (
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
    const index_t level) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > [friend]
```

### 6.17.5.3 matrix\_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
friend class matrix_multi [friend]
```

Definition at line 153 of file [matrix\\_multi.h](#).

Referenced by [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), and [matrix\\_multi\(\)](#).

### 6.17.5.4 matrix\_sqrt

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
auto matrix_sqrt (
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
    const index_t level) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > [friend]
```

#### 6.17.5.5 operator%

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator% (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

#### 6.17.5.6 operator&

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator& (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

#### 6.17.5.7 operator\*

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator* (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

#### 6.17.5.8 operator/

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator/ (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

#### 6.17.5.9 operator<<

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const multivector_t & val) -> std::ostream & [friend]
```

#### 6.17.5.10 operator>>

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator>> (
    std::istream & s,
    multivector_t & val) -> std::istream & [friend]
```

### 6.17.5.11 operator^

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator^ (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

### 6.17.5.12 operator"|

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator| (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

### 6.17.5.13 reframe

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
auto reframe (
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs,
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs,
    matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs_reframed,
    matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs_reframed)
-> const index_set< Other_LO, Other_HI > [friend]
```

### 6.17.5.14 star

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto star (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> Scalar_T [friend]
```

## 6.17.6 Member Data Documentation

### 6.17.6.1 m\_frame

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
index_set_t glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_frame [private]
```

Index set representing the frame for the subalgebra which contains the multivector.

Definition at line 278 of file [matrix\\_multi.h](#).

Referenced by [basis\\_element\(\)](#), [fast\\_framed\\_multi\(\)](#), [fast\\_matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), and [operator=\(\)](#).

### 6.17.6.2 m\_matrix

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
matrix_t glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_matrix [private]
```

Matrix value representing the multivector within the folded frame.

Definition at line 280 of file [matrix\\_multi.h](#).

Referenced by [fast\\_framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), [matrix\\_multi\(\)](#), and [operator=\(\)](#).

The documentation for this class was generated from the following files:

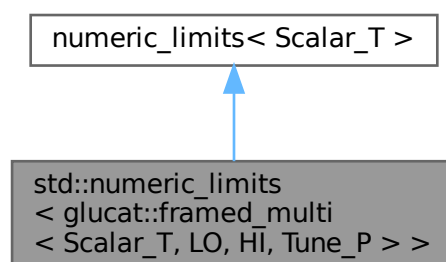
- [glucat/framed\\_multi.h](#)
- [glucat/matrix\\_multi.h](#)
- [glucat/matrix\\_multi\\_imp.h](#)

## 6.18 std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > > Struct Template Reference

Numeric limits for framed\_multi inherit limits for the corresponding scalar type.

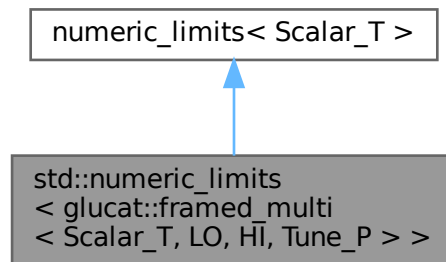
```
#include <framed_multi.h>
```

Inheritance diagram for std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > >:





Collaboration diagram for std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > >:



### 6.18.1 Detailed Description

```
template<typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P>
struct std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >
```

Numeric limits for framed\_multi inherit limits for the corresponding scalar type.

Definition at line 345 of file [framed\\_multi.h](#).

The documentation for this struct was generated from the following file:

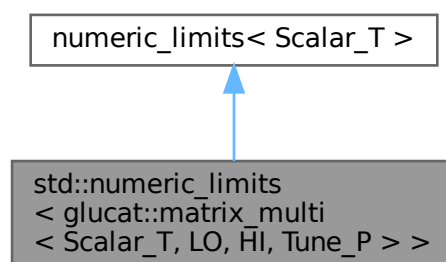
- [glucat/framed\\_multi.h](#)

## 6.19 std::numeric\_limits< glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P > > Struct Template Reference

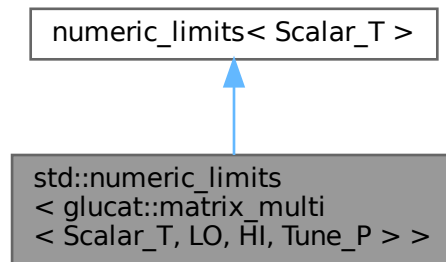
Numeric limits for matrix\_multi inherit limits for the corresponding scalar type.

```
#include <matrix_multi.h>
```

Inheritance diagram for std::numeric\_limits< glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P > >:



Collaboration diagram for `std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >`:



### 6.19.1 Detailed Description

```
template<typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P>
struct std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >
```

Numeric limits for `matrix_multi` inherit limits for the corresponding scalar type.

Definition at line 296 of file [matrix\\_multi.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi.h](#)

## 6.20 glucat::numeric\_traits< Scalar\_T > Class Template Reference

Extra traits which extend numeric limits.

```
#include <scalar.h>
```

### Classes

- struct [demoted](#)  
*Demoted type for long double.*
- struct [promoted](#)  
*Extra traits which extend numeric limits.*

## Public Member Functions

- auto [pi](#) () -> long double  
*Pi for long double.*
- auto [ln\\_2](#) () -> long double  
*log(2) for long double*
- auto [to\\_scalar\\_t](#) (const Other\_Scalar\_T &val) -> float  
*Extra traits which extend numeric limits.*
- auto [to\\_scalar\\_t](#) (const Other\_Scalar\_T &val) -> double  
*Cast to double.*
- auto [to\\_scalar\\_t](#) (const dd\_real &val) -> long double  
*Cast to long double.*
- auto [to\\_scalar\\_t](#) (const qd\_real &val) -> long double  
*Cast to long double.*
- auto [to\\_scalar\\_t](#) (const long double &val) -> dd\_real  
*Cast to dd\_real.*
- auto [to\\_scalar\\_t](#) (const qd\_real &val) -> dd\_real  
*Cast to dd\_real.*
- auto [to\\_scalar\\_t](#) (const long double &val) -> qd\_real  
*Cast to qd\_real.*
- auto [to\\_scalar\\_t](#) (const dd\_real &val) -> qd\_real  
*Cast to qd\_real.*

## Static Public Member Functions

- static auto [isInf](#) (const Scalar\_T &val) -> bool  
*Smart isinf.*
- static auto [isNaN](#) (const Scalar\_T &val) -> bool  
*Smart isnan.*
- static auto [isNaN\\_or\\_isInf](#) (const Scalar\_T &val) -> bool  
*Smart isnan or isinf.*
- static auto [NaN](#) () -> Scalar\_T  
*Smart NaN.*
- static auto [to\\_int](#) (const Scalar\_T &val) -> int  
*Cast to int.*
- static auto [to\\_double](#) (const Scalar\_T &val) -> double  
*Cast to double.*
- template<typename Other\_Scalar\_T>  
static auto [to\\_scalar\\_t](#) (const Other\_Scalar\_T &val) -> Scalar\_T  
*Cast to Scalar\_T.*
- static auto [fmod](#) (const Scalar\_T &lhs, const Scalar\_T &rhs) -> Scalar\_T  
*Modulo function for scalar.*
- static auto [conj](#) (const Scalar\_T &val) -> Scalar\_T  
*Complex conjugate of scalar.*
- static auto [real](#) (const Scalar\_T &val) -> Scalar\_T  
*Real part of scalar.*
- static auto [imag](#) (const Scalar\_T &val) -> Scalar\_T  
*Imaginary part of scalar.*
- static auto [abs](#) (const Scalar\_T &val) -> Scalar\_T  
*Absolute value of scalar.*

- static auto [pi](#) () -> Scalar\_T  
*Pi.*
- static auto [ln\\_2](#) () -> Scalar\_T  
*log(2)*
- static auto [pow](#) (const Scalar\_T &val, int n) -> Scalar\_T  
*Integer power.*
- static auto [sqrt](#) (const Scalar\_T &val) -> Scalar\_T  
*Square root of scalar.*
- static auto [exp](#) (const Scalar\_T &val) -> Scalar\_T  
*Exponential.*
- static auto [log](#) (const Scalar\_T &val) -> Scalar\_T  
*Logarithm of scalar.*
- static auto [log2](#) (const Scalar\_T &val) -> Scalar\_T  
*Log base 2.*
- static auto [cos](#) (const Scalar\_T &val) -> Scalar\_T  
*Cosine of scalar.*
- static auto [acos](#) (const Scalar\_T &val) -> Scalar\_T  
*Inverse cosine of scalar.*
- static auto [cosh](#) (const Scalar\_T &val) -> Scalar\_T  
*Hyperbolic cosine of scalar.*
- static auto [sin](#) (const Scalar\_T &val) -> Scalar\_T  
*Sine of scalar.*
- static auto [asin](#) (const Scalar\_T &val) -> Scalar\_T  
*Inverse sine of scalar.*
- static auto [sinh](#) (const Scalar\_T &val) -> Scalar\_T  
*Hyperbolic sine of scalar.*
- static auto [tan](#) (const Scalar\_T &val) -> Scalar\_T  
*Tangent of scalar.*
- static auto [atan](#) (const Scalar\_T &val) -> Scalar\_T  
*Inverse tangent of scalar.*
- static auto [tanh](#) (const Scalar\_T &val) -> Scalar\_T  
*Hyperbolic tangent of scalar.*

### Static Private Member Functions

- static auto [isInf](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< false >) -> bool  
*Smart isinf specialised for Scalar\_T without infinity.*
- static auto [isInf](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< true >) -> bool  
*Smart isinf specialised for Scalar\_T with infinity.*
- static auto [isNaN](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< false >) -> bool  
*Smart isnan specialised for Scalar\_T without quiet NaN.*
- static auto [isNaN](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< true >) -> bool  
*Smart isnan specialised for Scalar\_T with quiet NaN.*

### 6.20.1 Detailed Description

```
template<typename Scalar_T>
class glucat::numeric_traits< Scalar_T >
```

Extra traits which extend numeric limits.

Definition at line 47 of file [scalar.h](#).

## 6.20.2 Member Function Documentation

### 6.20.2.1 abs()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::abs (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Absolute value of scalar.

Definition at line 182 of file [scalar.h](#).

### 6.20.2.2 acos()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::acos (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Inverse cosine of scalar.

Definition at line 245 of file [scalar.h](#).

### 6.20.2.3 asin()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::asin (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Inverse sine of scalar.

Definition at line 266 of file [scalar.h](#).

### 6.20.2.4 atan()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::atan (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Inverse tangent of scalar.

Definition at line 287 of file [scalar.h](#).

### 6.20.2.5 conj()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::conj (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Complex conjugate of scalar.

Definition at line 161 of file [scalar.h](#).

### 6.20.2.6 cos()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::cos (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Cosine of scalar.

Definition at line 238 of file [scalar.h](#).

### 6.20.2.7 cosh()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::cosh (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Hyperbolic cosine of scalar.

Definition at line 252 of file [scalar.h](#).

### 6.20.2.8 exp()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::exp (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Exponential.

Definition at line 217 of file [scalar.h](#).

### 6.20.2.9 fmod()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::fmod (
    const Scalar_T & lhs,
    const Scalar_T & rhs) -> Scalar_T    [inline], [static]
```

Modulo function for scalar.

Definition at line 154 of file [scalar.h](#).

### 6.20.2.10 imag()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::imag (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Imaginary part of scalar.

Definition at line 175 of file [scalar.h](#).

**6.20.2.11** `isInf()` [1/3]

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val) -> bool    [inline], [static]
```

Smart `isinf`.

Definition at line 83 of file `scalar.h`.

References `isInf()`.

**6.20.2.12** `isInf()` [2/3]

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val,
    bool_to_type< false > ) -> bool    [inline], [static], [private]
```

Smart `isinf` specialised for `Scalar_T` without infinity.

Definition at line 54 of file `scalar.h`.

Referenced by `isInf()`, `glucat::matrix::isinf()`, and `isNaN_or_isInf()`.

**6.20.2.13** `isInf()` [3/3]

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val,
    bool_to_type< true > ) -> bool    [inline], [static], [private]
```

Smart `isinf` specialised for `Scalar_T` with infinity.

Definition at line 61 of file `scalar.h`.

References `_GLUCAT_ISINF`.

**6.20.2.14** `isNaN()` [1/3]

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val) -> bool    [inline], [static]
```

Smart `isnan`.

Definition at line 93 of file `scalar.h`.

References `isNaN()`.

**6.20.2.15 isNaN()** [2/3]

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val,
    bool_to_type< false > ) -> bool    [inline], [static], [private]
```

Smart isnan specialised for Scalar\_T without quiet NaN.

Definition at line 68 of file [scalar.h](#).

Referenced by [isNaN\(\)](#), [glucat::matrix::isnan\(\)](#), [isNaN\\_or\\_isInf\(\)](#), [glucat::matrix::norm\\_frob2\(\)](#), and [glucat::matrix::trace\(\)](#).

**6.20.2.16 isNaN()** [3/3]

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val,
    bool_to_type< true > ) -> bool    [inline], [static], [private]
```

Smart isnan specialised for Scalar\_T with quiet NaN.

Definition at line 75 of file [scalar.h](#).

References [\\_GLUCAT\\_ISNAN](#).

**6.20.2.17 isNaN\_or\_isInf()**

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::isNaN_or_isInf (
    const Scalar_T & val) -> bool    [inline], [static]
```

Smart isnan or isinf.

Definition at line 103 of file [scalar.h](#).

References [isInf\(\)](#), and [isNaN\(\)](#).

**6.20.2.18 ln\_2()** [1/2]

```
auto glucat::numeric_traits< longdouble >::ln_2 () -> long double    [inline]
```

log(2) for long double

Definition at line 59 of file [long\\_double.h](#).

**6.20.2.19 ln\_2()** [2/2]

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::ln_2 () -> Scalar_T    [inline], [static]
```

log(2)

Definition at line 196 of file [scalar.h](#).

Referenced by [log2\(\)](#).



### 6.20.2.20 log()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::log (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Logarithm of scalar.

Definition at line 224 of file [scalar.h](#).

Referenced by [log2\(\)](#).

### 6.20.2.21 log2()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::log2 (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Log base 2.

Definition at line 231 of file [scalar.h](#).

References [ln\\_2\(\)](#), and [log\(\)](#).

Referenced by [glucat::log2\(\)](#).

### 6.20.2.22 NaN()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::NaN () -> Scalar_T    [inline], [static]
```

Smart NaN.

Definition at line 115 of file [scalar.h](#).

Referenced by [glucat::cr\\_sqrt\(\)](#), [glucat::db\\_sqrt\(\)](#), [glucat::matrix::norm\\_frob2\(\)](#), [glucat::operator\\*\(\)](#), and [glucat::matrix::trace\(\)](#).

### 6.20.2.23 pi() [1/2]

```
auto glucat::numeric_traits< longdouble >::pi () -> long double    [inline]
```

Pi for long double.

Definition at line 51 of file [long\\_double.h](#).

#### 6.20.2.24 pi() [2/2]

```
template<typename Scalar_T>
static auto glucat::numeric\_traits< Scalar_T >::pi () -> Scalar_T    [inline], [static]
```

Pi.

Definition at line 189 of file [scalar.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

#### 6.20.2.25 pow()

```
template<typename Scalar_T>
static auto glucat::numeric\_traits< Scalar_T >::pow (
    const Scalar_T & val,
    int n) -> Scalar_T    [inline], [static]
```

Integer power.

Definition at line 203 of file [scalar.h](#).

Referenced by [glucat::error\\_squared\\_tol\(\)](#).

#### 6.20.2.26 real()

```
template<typename Scalar_T>
static auto glucat::numeric\_traits< Scalar_T >::real (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Real part of scalar.

Definition at line 168 of file [scalar.h](#).

#### 6.20.2.27 sin()

```
template<typename Scalar_T>
static auto glucat::numeric\_traits< Scalar_T >::sin (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Sine of scalar.

Definition at line 259 of file [scalar.h](#).

#### 6.20.2.28 sinh()

```
template<typename Scalar_T>
static auto glucat::numeric\_traits< Scalar_T >::sinh (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Hyperbolic sine of scalar.

Definition at line 273 of file [scalar.h](#).

### 6.20.2.29 sqrt()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::sqrt (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Square root of scalar.

Definition at line 210 of file [scalar.h](#).

Referenced by [glucat::abs\(\)](#).

### 6.20.2.30 tan()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::tan (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Tangent of scalar.

Definition at line 280 of file [scalar.h](#).

### 6.20.2.31 tanh()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::tanh (
    const Scalar_T & val) -> Scalar_T    [inline], [static]
```

Hyperbolic tangent of scalar.

Definition at line 294 of file [scalar.h](#).

### 6.20.2.32 to\_double()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::to_double (
    const Scalar_T & val) -> double    [inline], [static]
```

Cast to double.

Definition at line 133 of file [scalar.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#), [glucat::operator<<\(\)](#), [PyFloat\\_FromDouble\(\)](#), [glucat::numeric\\_traits< Scalar\\_T >::promoted::to\\_scalar\\_t\(\)](#) and [glucat::numeric\\_traits< Scalar\\_T >::promoted::to\\_scalar\\_t\(\)](#).

### 6.20.2.33 to\_int()

```
template<typename Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::to_int (
    const Scalar_T & val) -> int    [inline], [static]
```

Cast to int.

Definition at line 126 of file [scalar.h](#).

**6.20.2.34 to\_scalar\_t()** [1/9]

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
    const dd_real & val) -> long double    [inline]
```

Cast to long double.

Definition at line 71 of file [scalar\\_imp.h](#).

**6.20.2.35 to\_scalar\_t()** [2/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
    const dd_real & val) -> qd_real    [inline]
```

Cast to qd\_real.

Definition at line 116 of file [scalar\\_imp.h](#).

**6.20.2.36 to\_scalar\_t()** [3/9]

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
    const long double & val) -> dd_real    [inline]
```

Cast to dd\_real.

Definition at line 89 of file [scalar\\_imp.h](#).

**6.20.2.37 to\_scalar\_t()** [4/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
    const long double & val) -> qd_real    [inline]
```

Cast to qd\_real.

Definition at line 107 of file [scalar\\_imp.h](#).

**6.20.2.38 to\_scalar\_t()** [5/9]

```
auto glucat::numeric_traits< double >::to_scalar_t (
    const Other_Scalar_T & val) -> double    [inline]
```

Cast to double.

Definition at line 61 of file [scalar\\_imp.h](#).

**6.20.2.39 to\_scalar\_t()** [6/9]

```
auto glucat::numeric_traits< float >::to_scalar_t (
    const Other_Scalar_T & val) -> float    [inline]
```

Extra traits which extend numeric limits.

Cast to float

Definition at line 52 of file [scalar\\_imp.h](#).

**6.20.2.40 to\_scalar\_t()** [7/9]

```
template<typename Scalar_T>
template<typename Other_Scalar_T>
static auto glucat::numeric_traits< Scalar_T >::to_scalar_t (
    const Other_Scalar_T & val) -> Scalar_T    [inline], [static]
```

Cast to Scalar\_T.

Definition at line 141 of file [scalar.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix::nork\\_range\(\)](#), [glucat::to\\_demote\(\)](#), and [glucat::to\\_promote\(\)](#).

**6.20.2.41 to\_scalar\_t()** [8/9]

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
    const qd_real & val) -> dd_real    [inline]
```

Cast to dd\_real.

Definition at line 98 of file [scalar\\_imp.h](#).

**6.20.2.42 to\_scalar\_t()** [9/9]

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
    const qd_real & val) -> long double    [inline]
```

Cast to long double.

Definition at line 80 of file [scalar\\_imp.h](#).

The documentation for this class was generated from the following file:

- [glucat/scalar.h](#)

## 6.21 pade::pade\_log\_denom< Scalar\_T > Struct Template Reference

Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array<Scalar\_T, 14>

### Static Public Attributes

- static const [array](#) [denom](#)

### 6.21.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_log_denom< Scalar_T >
```

Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

Definition at line 1731 of file [matrix\\_multi\\_imp.h](#).

### 6.21.2 Member Typedef Documentation

#### 6.21.2.1 array

```
template<typename Scalar_T>
using pade::pade\_log\_denom< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1733 of file [matrix\\_multi\\_imp.h](#).

### 6.21.3 Member Data Documentation

#### 6.21.3.1 denom

```
template<typename Scalar_T>
const array pade::pade\_log\_denom< Scalar_T >::denom [static]
```

Definition at line 1734 of file [matrix\\_multi\\_imp.h](#).

Referenced by [glucat::pade\\_log\(\)](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.22 `pade::pade_log_number< Scalar_T >` Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by `Pade1(log(1+x),x,n,n)`

```
#include <matrix_multi_imp.h>
```

### Public Types

- using `array` = `std::array<Scalar_T, 14>`

### Static Public Attributes

- static const `array` `numer`

### 6.22.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_log_number< Scalar_T >
```

Coefficients of numerator polynomials of Pade approximations produced by `Pade1(log(1+x),x,n,n)`

Definition at line 1714 of file `matrix_multi_imp.h`.

### 6.22.2 Member Typedef Documentation

#### 6.22.2.1 `array`

```
template<typename Scalar_T>
using pade::pade_log_number< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1716 of file `matrix_multi_imp.h`.

### 6.22.3 Member Data Documentation

#### 6.22.3.1 `numer`

```
template<typename Scalar_T>
const array pade::pade_log_number< Scalar_T >::numer [static]
```

Definition at line 1717 of file `matrix_multi_imp.h`.

Referenced by `glucat::pade_log()`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.23 pade::pade\_sqrt\_denom< Scalar\_T > Struct Template Reference

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array<Scalar\_T, 14>

### Static Public Attributes

- static const [array](#) [denom](#)

### 6.23.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_sqrt_denom< Scalar_T >
```

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)

Definition at line 1401 of file [matrix\\_multi\\_imp.h](#).

### 6.23.2 Member Typedef Documentation

#### 6.23.2.1 array

```
template<typename Scalar_T>
using pade::pade\_sqrt\_denom< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1403 of file [matrix\\_multi\\_imp.h](#).

### 6.23.3 Member Data Documentation

#### 6.23.3.1 denom

```
template<typename Scalar_T>
const array pade::pade\_sqrt\_denom< Scalar_T >::denom [static]
```

Definition at line 1404 of file [matrix\\_multi\\_imp.h](#).

Referenced by [glucat::matrix\\_sqrt\(\)](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)



## 6.24 `pade::pade_sqrt_numer< Scalar_T >` Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by `Pade1(sqrt(1+x),x,n,n)`

```
#include <matrix_multi_imp.h>
```

### Public Types

- using `array` = `std::array<Scalar_T, 14>`

### Static Public Attributes

- static const `array numer`

### 6.24.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_sqrt_numer< Scalar_T >
```

Coefficients of numerator polynomials of Pade approximations produced by `Pade1(sqrt(1+x),x,n,n)`

Definition at line 1384 of file `matrix_multi_imp.h`.

### 6.24.2 Member Typedef Documentation

#### 6.24.2.1 `array`

```
template<typename Scalar_T>
using pade::pade_sqrt_numer< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1386 of file `matrix_multi_imp.h`.

### 6.24.3 Member Data Documentation

#### 6.24.3.1 `numer`

```
template<typename Scalar_T>
const array pade::pade_sqrt_numer< Scalar_T >::numer [static]
```

Definition at line 1387 of file `matrix_multi_imp.h`.

Referenced by `glucat::matrix_sqrt()`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.25 `glucat::random_generator< Scalar_T >` Class Template Reference

Random number generator with single instance per `Scalar_T`.

```
#include <random.h>
```

### Public Member Functions

- `random_generator` (const `random_generator` &)=delete
- auto `operator=` (const `random_generator` &) -> `random_generator` &=delete
- auto `uniform` () -> `Scalar_T`
- auto `normal` () -> `Scalar_T`

### Static Public Member Functions

- static auto `generator` () -> `random_generator` &  
*Single instance of Random number generator.*

### Private Member Functions

- `random_generator` ()
- `~random_generator` ()=default

### Private Attributes

- `std::mt19937` `uint_gen`
- `std::uniform_real_distribution< double >` `uniform_dist`
- `std::normal_distribution< double >` `normal_dist`

### Static Private Attributes

- static const unsigned long `seed` = 19590921UL

### Friends

- class `friend_for_private_destructor`

### 6.25.1 Detailed Description

```
template<typename Scalar_T>
class glucat::random_generator< Scalar_T >
```

Random number generator with single instance per `Scalar_T`.

Definition at line 42 of file `random.h`.

## 6.25.2 Constructor & Destructor Documentation

### 6.25.2.1 random\_generator() [1/2]

```
template<typename Scalar_T>
glucat::random_generator< Scalar_T >::random_generator (
    const random_generator< Scalar_T > & ) [delete]
```

References [random\\_generator\(\)](#).

Referenced by [generator\(\)](#), [operator=\(\)](#), and [random\\_generator\(\)](#).

### 6.25.2.2 random\_generator() [2/2]

```
template<typename Scalar_T>
glucat::random_generator< Scalar_T >::random_generator () [inline], [private]
```

Definition at line 61 of file [random.h](#).

References [normal\\_dist](#), [seed](#), [uint\\_gen](#), and [uniform\\_dist](#).

### 6.25.2.3 ~random\_generator()

```
template<typename Scalar_T>
glucat::random_generator< Scalar_T >::~~random_generator () [private], [default]
```

## 6.25.3 Member Function Documentation

### 6.25.3.1 generator()

```
template<typename Scalar_T>
static auto glucat::random_generator< Scalar_T >::generator () -> random_generator& [inline],
[static]
```

Single instance of Random number generator.

Definition at line 51 of file [random.h](#).

References [random\\_generator\(\)](#).

### 6.25.3.2 normal()

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::normal () -> Scalar_T [inline]
```

Definition at line 70 of file [random.h](#).

References [normal\\_dist](#).

### 6.25.3.3 operator=()

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::operator= (
    const random_generator< Scalar_T > & ) -> random_generator &=delete [delete]
```

References [random\\_generator\(\)](#).

### 6.25.3.4 uniform()

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::uniform () -> Scalar_T [inline]
```

Definition at line 68 of file [random.h](#).

References [uniform\\_dist](#).

## 6.25.4 Friends And Related Symbol Documentation

### 6.25.4.1 friend\_for\_private\_destructor

```
template<typename Scalar_T>
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 48 of file [random.h](#).

References [friend\\_for\\_private\\_destructor](#).

Referenced by [friend\\_for\\_private\\_destructor](#).

## 6.25.5 Member Data Documentation

### 6.25.5.1 normal\_dist

```
template<typename Scalar_T>
std::normal_distribution<double> glucat::random_generator< Scalar_T >::normal_dist [private]
```

Definition at line 59 of file [random.h](#).

Referenced by [normal\(\)](#), and [random\\_generator\(\)](#).

### 6.25.5.2 seed

```
template<typename Scalar_T>
const unsigned long glucat::random_generator< Scalar_T >::seed = 19590921UL [static], [private]
```

Definition at line 55 of file [random.h](#).

Referenced by [random\\_generator\(\)](#).

### 6.25.5.3 uint\_gen

```
template<typename Scalar_T>
std::mt19937 glucat::random_generator< Scalar_T >::uint_gen [private]
```

Definition at line 57 of file [random.h](#).

Referenced by [random\\_generator\(\)](#).

### 6.25.5.4 uniform\_dist

```
template<typename Scalar_T>
std::uniform_real_distribution<double> glucat::random_generator< Scalar_T >::uniform_dist
[private]
```

Definition at line 58 of file [random.h](#).

Referenced by [random\\_generator\(\)](#), and [uniform\(\)](#).

The documentation for this class was generated from the following file:

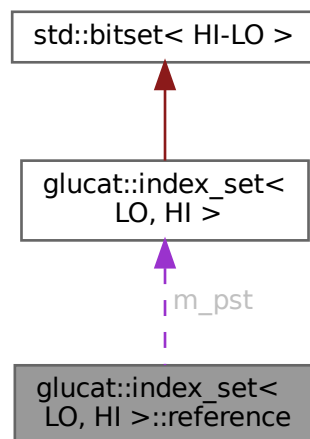
- [glucat/random.h](#)

## 6.26 glucat::index\_set< LO, HI >::reference Class Reference

Index set member reference.

```
#include <index_set.h>
```

Collaboration diagram for glucat::index\_set< LO, HI >::reference:



## Public Member Functions

- [reference](#) ()=delete  
*Default constructor is deleted.*
- [reference](#) ([index\\_set\\_t](#) &ist, [index\\_t](#) idx)  
*[index\\_set](#) reference*
- [~reference](#) ()=default
- auto [operator==](#) (const [reference](#) &c\_j) const -> bool  
*for b[i] == c[j];*
- auto [operator=](#) (const bool x) -> [reference](#) &  
*for b[i] = x;*
- auto [operator=](#) (const [reference](#) &c\_j) -> [reference](#) &  
*for b[i] = c[j];*
- auto [operator~](#) () const -> bool  
*Flips a bit.*
- [operator bool](#) () const  
*for x = b[i];*
- auto [flip](#) () -> [reference](#) &  
*for b[i].flip();*

## Private Attributes

- [index\\_set\\_t](#) \* m\_pst
- [index\\_t](#) m\_idx

## Friends

- class [index\\_set](#)

## 6.26.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index_set< LO, HI >::reference
```

Index set member reference.

Definition at line 177 of file [index\\_set.h](#).

## 6.26.2 Constructor & Destructor Documentation

### 6.26.2.1 [reference\(\)](#) [1/2]

```
template<const index\_t LO, const index\_t HI>
glucat::index_set< LO, HI >::reference::reference () [delete]
```

Default constructor is deleted.

Referenced by [flip\(\)](#), [operator=\(\)](#), [operator=\(\)](#), [operator==\(\)](#), and [~reference\(\)](#).

### 6.26.2.2 reference() [2/2]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::reference (
    index_set_t & ist,
    index_t idx) [inline]
```

[index\\_set](#) [reference](#)

Definition at line 985 of file [index\\_set\\_imp.h](#).

References [m\\_idx](#), and [m\\_pst](#).

### 6.26.2.3 ~reference()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::~reference () [default]
```

References [flip\(\)](#), and [reference\(\)](#).

## 6.26.3 Member Function Documentation

### 6.26.3.1 flip()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::flip () -> reference& [inline]
```

for [b\[i\].flip\(\)](#);

Definition at line 1049 of file [index\\_set\\_imp.h](#).

References [m\\_idx](#), [m\\_pst](#), and [reference\(\)](#).

Referenced by [~reference\(\)](#).

### 6.26.3.2 operator bool()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::operator bool () const [inline]
```

for [x = b\[i\]](#);

Definition at line 1041 of file [index\\_set\\_imp.h](#).

References [m\\_idx](#), and [m\\_pst](#).

**6.26.3.3 operator=()** [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
    const bool x) -> reference& [inline]
```

for b[i] = x;

Definition at line 1003 of file [index\\_set\\_imp.h](#).

References [m\\_idx](#), [m\\_pst](#), and [reference\(\)](#).

**6.26.3.4 operator=()** [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
    const reference & c_j) -> reference& [inline]
```

for b[i] = c[j];

Definition at line 1017 of file [index\\_set\\_imp.h](#).

References [m\\_idx](#), [m\\_pst](#), and [reference\(\)](#).

**6.26.3.5 operator==()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator== (
    const reference & c_j) const -> bool [inline]
```

for b[i] == c[j];

Definition at line 995 of file [index\\_set\\_imp.h](#).

References [m\\_idx](#), [m\\_pst](#), and [reference\(\)](#).

**6.26.3.6 operator~()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator~ () const -> bool [inline]
```

Flips a bit.

flips the bit

Definition at line 1034 of file [index\\_set\\_imp.h](#).

References [m\\_idx](#), and [m\\_pst](#).



## 6.26.4 Friends And Related Symbol Documentation

### 6.26.4.1 index\_set

```
template<const index_t LO, const index_t HI>
friend class index_set [friend]
```

Definition at line 178 of file [index\\_set.h](#).

References [index\\_set](#).

Referenced by [index\\_set](#).

## 6.26.5 Member Data Documentation

### 6.26.5.1 m\_idx

```
template<const index_t LO, const index_t HI>
index_t glucat::index_set< LO, HI >::reference::m_idx [private]
```

Definition at line 200 of file [index\\_set.h](#).

Referenced by [flip\(\)](#), [operator bool\(\)](#), [operator=\(\)](#), [operator=\(\)](#), [operator==\(\)](#), [operator~\(\)](#), and [reference\(\)](#).

### 6.26.5.2 m\_pst

```
template<const index_t LO, const index_t HI>
index_set_t* glucat::index_set< LO, HI >::reference::m_pst [private]
```

Definition at line 199 of file [index\\_set.h](#).

Referenced by [flip\(\)](#), [operator bool\(\)](#), [operator=\(\)](#), [operator=\(\)](#), [operator==\(\)](#), [operator~\(\)](#), and [reference\(\)](#).

The documentation for this class was generated from the following files:

- [glucat/index\\_set.h](#)
- [glucat/index\\_set\\_imp.h](#)

## 6.27 glucat::sorted\_range< Map\_T, Sorted\_Map\_T > Class Template Reference

Sorted range for use with output.

```
#include <framed_multi_imp.h>
```

## Public Types

- using `map_t` = `Map_T`
- using `sorted_map_t` = `Sorted_Map_T`
- using `sorted_iterator` = `typename Sorted_Map_T::const_iterator`

## Public Member Functions

- `sorted_range` (`Sorted_Map_T &sorted_val, const Map_T &val`)

## Public Attributes

- `sorted_iterator sorted_begin`
- `sorted_iterator sorted_end`

### 6.27.1 Detailed Description

```
template<typename Map_T, typename Sorted_Map_T>
class glucat::sorted_range< Map_T, Sorted_Map_T >
```

Sorted range for use with output.

Definition at line 1112 of file [framed\\_multi\\_imp.h](#).

### 6.27.2 Member Typedef Documentation

#### 6.27.2.1 `map_t`

```
template<typename Map_T, typename Sorted_Map_T>
using glucat::sorted_range< Map_T, Sorted_Map_T >::map_t = Map_T
```

Definition at line 1115 of file [framed\\_multi\\_imp.h](#).

#### 6.27.2.2 `sorted_iterator`

```
template<typename Map_T, typename Sorted_Map_T>
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_iterator = typename Sorted_Map_T↵
::const_iterator
```

Definition at line 1117 of file [framed\\_multi\\_imp.h](#).

#### 6.27.2.3 `sorted_map_t`

```
template<typename Map_T, typename Sorted_Map_T>
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_map_t = Sorted_Map_T
```

Definition at line 1116 of file [framed\\_multi\\_imp.h](#).

## 6.27.3 Constructor & Destructor Documentation

### 6.27.3.1 sorted\_range()

```
template<typename Map_T, typename Sorted_Map_T>
glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_range (
    Sorted_Map_T & sorted_val,
    const Map_T & val) [inline]
```

Definition at line 1119 of file [framed\\_multi\\_imp.h](#).

References [sorted\\_begin](#), and [sorted\\_end](#).

## 6.27.4 Member Data Documentation

### 6.27.4.1 sorted\_begin

```
template<typename Map_T, typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_begin
```

Definition at line 1126 of file [framed\\_multi\\_imp.h](#).

Referenced by [sorted\\_range\(\)](#).

### 6.27.4.2 sorted\_end

```
template<typename Map_T, typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_end
```

Definition at line 1127 of file [framed\\_multi\\_imp.h](#).

Referenced by [sorted\\_range\(\)](#).

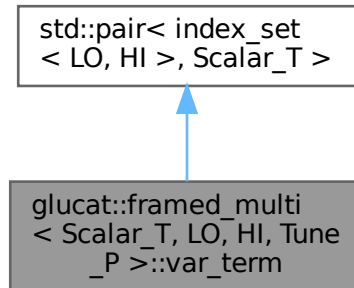
The documentation for this class was generated from the following file:

- [glucat/framed\\_multi\\_imp.h](#)

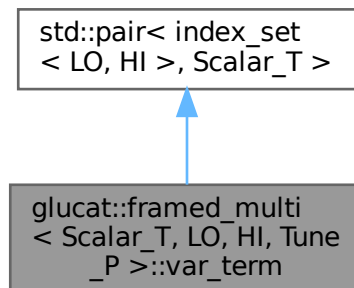
## 6.28 glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term Class Reference

Variable term.

Inheritance diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term:



Collaboration diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term:



### Public Types

- using `var_pair_t` = `std::pair<index_set<LO, HI>, Scalar_T>`

### Public Member Functions

- `~var_term()` = default  
*Destructor.*
- `var_term()`

*Default constructor.*

- `var_term` (const `index_set_t` ist, const `Scalar_T` &crd=`Scalar_T`(1))  
*Construct a variable term from an index set and a scalar coordinate.*
- `auto operator*=` (const `term_t` &rhs) -> `var_term_t` &  
*Product of variable term and term.*

## Static Public Member Functions

- `static auto classname` () -> const std::string  
*Class name used in messages.*

## 6.28.1 Detailed Description

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term
```

Variable term.

Definition at line 279 of file [framed\\_multi.h](#).

## 6.28.2 Member Typedef Documentation

### 6.28.2.1 var\_pair\_t

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_pair_t = std::pair<index_set<LO,
HI>, Scalar_T>
```

Definition at line 283 of file [framed\\_multi.h](#).

## 6.28.3 Constructor & Destructor Documentation

### 6.28.3.1 ~var\_term()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::~~var_term () [default]
```

Destructor.

### 6.28.3.2 var\_term() [1/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term () [inline]
```

Default constructor.

Definition at line 291 of file [framed\\_multi.h](#).

### 6.28.3.3 `var_term()` [2/2]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term (
    const index\_set\_t ist,
    const Scalar_T & crd = Scalar_T(1))  [inline]
```

Construct a variable term from an index set and a scalar coordinate.

Definition at line [295](#) of file [framed\\_multi.h](#).

## 6.28.4 Member Function Documentation

### 6.28.4.1 `classname()`

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
static auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::var_term::classname () -> const
std::string  [inline], [static]
```

Class name used in messages.

Definition at line [286](#) of file [framed\\_multi.h](#).

### 6.28.4.2 `operator*=( )`

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::var_term::operator*= (
    const term\_t & rhs) -> var\_term\_t&  [inline]
```

Product of variable term and term.

Definition at line [299](#) of file [framed\\_multi.h](#).

The documentation for this class was generated from the following file:

- [glucat/framed\\_multi.h](#)

## Chapter 7

# File Documentation

### 7.1 glucat/clifford\_algebra.h File Reference

```
#include "glucat/global.h"
```

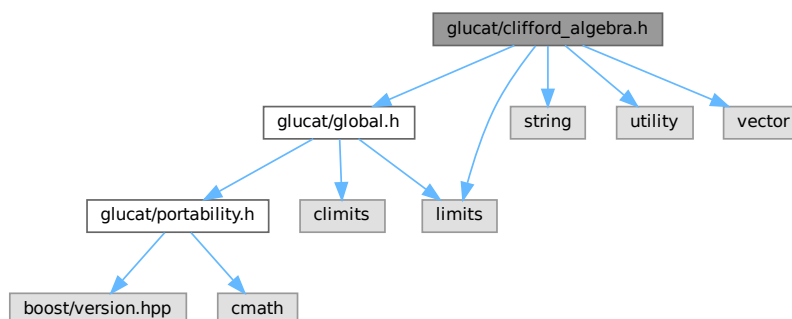
```
#include <limits>
```

```
#include <string>
```

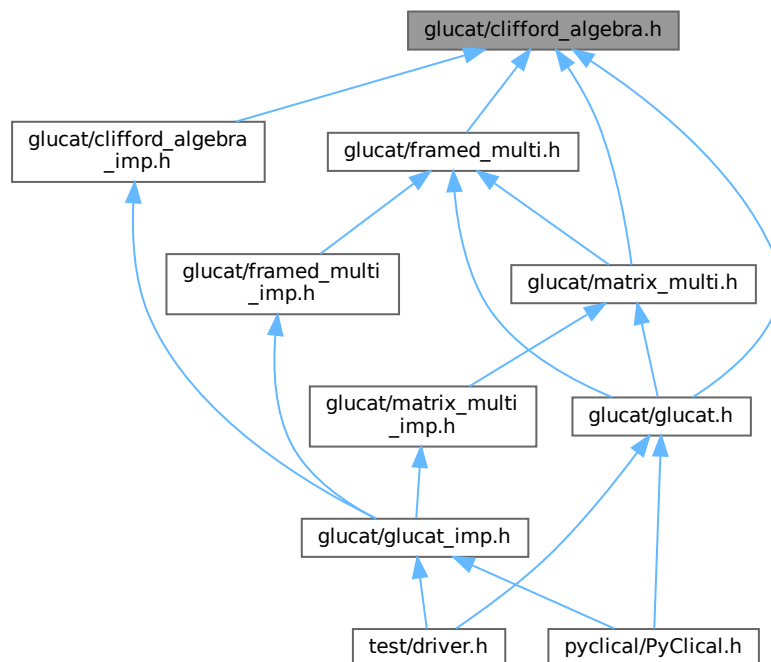
```
#include <utility>
```

```
#include <vector>
```

Include dependency graph for clifford\_algebra.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`  
*clifford\_algebra<> declares the operations of a [Clifford algebra](#)*

## Namespaces

- namespace `glucat`

## Macros

- `#define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS`

## Functions

- template<template< typename, const `index_t`, const `index_t`, typename > class Multivector, template< typename, const `index_t`, const `index_t`, typename > class RHS, typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P>  
 auto `glucat::operator!=` (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool  
*Test for inequality of multivectors.*
- template<template< typename, const `index_t`, const `index_t`, typename > class Multivector, typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P>  
 auto `glucat::operator!=` (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> bool



*Test for inequality of multivector and scalar.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator!=](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool

*Test for inequality of scalar and multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::error\\_squared\\_tol](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Quadratic norm error tolerance relative to a specific multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::error\\_squared](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs, const Scalar\_T threshold) -> Scalar\_T

*Relative or absolute error using the quadratic norm.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::approx\\_equal](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs, const Scalar\_T threshold, const Scalar\_T tolerance) -> bool

*Test for approximate equality of multivectors.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::approx\\_equal](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool

*Test for approximate equality of multivectors.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator+](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric sum of multivector and scalar.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator+](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric sum of scalar and multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator+](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric sum.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator-](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric difference of multivector and scalar.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator-](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric difference of scalar and multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator-](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric difference.*



- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric multiplicative inverse.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Integer power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Multivector power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Outer product power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Scalar part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Real part: synonym for scalar part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Imaginary part: deprecated (always 0)*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Pure part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Even part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Odd part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >`  
*Vector part of multivector, as a vector\_t with respect to frame()*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each  $\{i\}$  is replaced by  $-\{i\}$  in each term, eg.  $\{1\}*\{2\} \rightarrow (-\{2\})*(-\{1\})$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg.  $\{1\}*\{2\} \rightarrow \{2\}*\{1\}$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation, rev o invo == invo o rev.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T quadratic form ==  $(rev(x)*x)(0)$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T norm == sum of norm of coordinates.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Absolute value ==  $\sqrt{\text{norm}}$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::max\_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::clifford\_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Exponential of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucatt::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector.*

- ```

• template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO,
const index\_t HI, typename Tune_P>
auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI,
Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

```

*Cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>  
auto glucat::cosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Hyperbolic cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucatt::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucatt::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucatt::sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Sine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucats::sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Sine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`



## 7.1.1 Macro Definition Documentation

### 7.1.1.1 \_GLUCAT\_CLIFFORD\_ALGEBRA\_OPERATIONS

#define \_GLUCAT\_CLIFFORD\_ALGEBRA\_OPERATIONS

Definition at line 145 of file clifford\_algebra.h.

Referenced by `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element()`.

## 7.2 clifford\_algebra.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   clifford_algebra.h : Declare the operations of a Clifford algebra
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright            : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035
00036 #include <limits>
00037 #include <string>
00038 #include <utility>
00039 #include <vector>
00040
00041 namespace glucat
00042 {
00043     template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00044     class clifford_algebra
00045     {
00046     public:
00047         using scalar_t = Scalar_T;
00048         using index_set_t = Index_Set_T;
00049         static const index_t v_lo = index_set_t::v_lo;
00050         static const index_t v_hi = index_set_t::v_hi;
00051         using multivector_t = Multivector_T;
00052         using pair_t = std::pair<const index_set_t, Scalar_T>;
00053         using vector_t = std::vector<Scalar_T>;
00054
00055         static auto classname() -> const std::string;
00056
00057         static const Scalar_T default_truncation;
00058
00059         virtual ~clifford_algebra() = default;
00060
00061         // clifford_algebra operations
00062         virtual auto operator==(const multivector_t& val) const -> bool = 0;

```



```

00067     virtual auto operator== (const Scalar_T& scr) const -> bool = 0;
00069     virtual auto operator+= (const multivector_t& rhs) -> multivector_t& = 0;
00071     virtual auto operator+= (const Scalar_T& scr) -> multivector_t& = 0;
00073     virtual auto operator-= (const multivector_t& rhs) -> multivector_t& = 0;
00075     virtual auto operator-= (const Scalar_T& scr) -> multivector_t& = 0;
00077     virtual auto operator- () const -> const multivector_t = 0;
00079     virtual auto operator*= (const Scalar_T& scr) -> multivector_t& = 0;
00081     virtual auto operator*= (const multivector_t& rhs) -> multivector_t& = 0;
00083     virtual auto operator%= (const multivector_t& rhs) -> multivector_t& = 0;
00085     virtual auto operator&= (const multivector_t& rhs) -> multivector_t& = 0;
00087     virtual auto operator^= (const multivector_t& rhs) -> multivector_t& = 0;
00089     virtual auto operator/= (const Scalar_T& scr) -> multivector_t& = 0;
00091     virtual auto operator/= (const multivector_t& rhs) -> multivector_t& = 0;
00093     virtual auto operator|= (const multivector_t& rhs) -> multivector_t& = 0;
00095     virtual auto inv () const -> const multivector_t = 0;
00097     virtual auto pow (int m) const -> const multivector_t = 0;
00099     virtual auto outer_pow (int m) const -> const multivector_t = 0;
00101     virtual auto frame () const -> const index_set_t = 0;
00103     virtual auto grade () const -> index_t = 0;
00105     virtual auto operator[] (const index_set_t ist) const -> Scalar_T = 0;
00107     virtual auto operator() (index_t grade) const -> const multivector_t = 0;
00109     virtual auto scalar () const -> Scalar_T = 0;
00111     virtual auto pure () const -> const multivector_t = 0;
00113     virtual auto even () const -> const multivector_t = 0;
00115     virtual auto odd () const -> const multivector_t = 0;
00117     virtual auto vector_part () const -> const vector_t = 0;
00119     virtual auto vector_part (const index_set_t frm, const bool prechecked) const -> const vector_t =
0;
00121     virtual auto involute () const -> const multivector_t = 0;
00123     virtual auto reverse () const -> const multivector_t = 0;
00125     virtual auto conj () const -> const multivector_t = 0;
00127     virtual auto quad () const -> Scalar_T = 0;
00129     virtual auto norm () const -> Scalar_T = 0;
00131     virtual auto max_abs () const -> Scalar_T = 0;
00133     virtual auto truncated (const Scalar_T& limit = default_truncation) const -> const multivector_t
= 0;
00135     virtual auto isinf () const -> bool = 0;
00137     virtual auto isnan () const -> bool = 0;
00139     virtual void write (const std::string& msg="") const = 0;
00141     virtual void write (std::ofstream& ofile, const std::string& msg="") const = 0;
00142 };
00143
00144 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00145 #define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS \
00146     auto operator== (const multivector_t& val) const -> bool override;\
00147     auto operator== (const Scalar_T& scr) const -> bool override;\
00148     auto operator+= (const multivector_t& rhs) -> multivector_t& override;\
00149     auto operator+= (const Scalar_T& scr) -> multivector_t& override;\
00150     auto operator-= (const multivector_t& rhs) -> multivector_t& override;\
00151     auto operator-= (const Scalar_T& scr) -> multivector_t& override;\
00152     auto operator- () const -> const multivector_t override;\
00153     auto operator*= (const Scalar_T& scr) -> multivector_t& override;\
00154     auto operator*= (const multivector_t& rhs) -> multivector_t& override;\
00155     auto operator%= (const multivector_t& rhs) -> multivector_t& override;\
00156     auto operator&= (const multivector_t& rhs) -> multivector_t& override;\
00157     auto operator^= (const multivector_t& rhs) -> multivector_t& override;\
00158     auto operator/= (const Scalar_T& scr) -> multivector_t& override;\
00159     auto operator/= (const multivector_t& rhs) -> multivector_t& override;\
00160     auto operator|= (const multivector_t& rhs) -> multivector_t& override;\
00161     auto inv () const -> const multivector_t override;\
00162     auto pow (int m) const -> const multivector_t override;\
00163     auto outer_pow (int m) const -> const multivector_t override;\
00164     auto frame () const -> const index_set_t override;\
00165     auto grade () const -> index_t override;\
00166     auto operator[] (const index_set_t ist) const -> Scalar_T override;\
00167     auto operator() (index_t grade) const -> const multivector_t override;\
00168     auto scalar () const -> Scalar_T override;\
00169     auto pure () const -> const multivector_t override;\
00170     auto even () const -> const multivector_t override;\
00171     auto odd () const -> const multivector_t override;\
00172     auto vector_part () const -> const vector_t override;\
00173     auto vector_part (const index_set_t frm, const bool prechecked = false) const \
00174     -> const vector_t override;\
00175     auto involute () const -> const multivector_t override;\
00176     auto reverse () const -> const multivector_t override;\
00177     auto conj () const -> const multivector_t override;\
00178     auto quad () const -> Scalar_T override;\
00179     auto norm () const -> Scalar_T override;\
00180     auto max_abs () const -> Scalar_T override;\
00181     auto truncated (const Scalar_T& limit = multivector_t::default_truncation) const \
00182     -> const multivector_t override;\
00183     auto isinf () const -> bool override;\
00184     auto isnan () const -> bool override;\
00185     void write (const std::string& msg="") const override;\
00186     void write (std::ofstream& ofile, const std::string& msg="") const override;\
00187 #endif // _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00188

```



```

00190     template
00191     <
00192         template<typename, const index_t, const index_t, typename> class Multivector,
00193         template<typename, const index_t, const index_t, typename> class RHS,
00194         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00195     >
00196     auto
00197     operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
bool;
00198
00200     template
00201     <
00202         template<typename, const index_t, const index_t, typename> class Multivector,
00203         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00204     >
00205     auto
00206     operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> bool;
00207
00209     template
00210     <
00211         template<typename, const index_t, const index_t, typename> class Multivector,
00212         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00213     >
00214     auto
00215     operator!= (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00216
00218     template
00219     <
00220         template<typename, const index_t, const index_t, typename> class Multivector,
00221         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00222     >
00223     auto
00224     error_squared_tol(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00225
00227     template
00228     <
00229         template<typename, const index_t, const index_t, typename> class Multivector,
00230         template<typename, const index_t, const index_t, typename> class RHS,
00231         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00232     >
00233     auto
00234     error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00235                   const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00236                   const Scalar_T threshold) -> Scalar_T;
00237
00239     template
00240     <
00241         template<typename, const index_t, const index_t, typename> class Multivector,
00242         template<typename, const index_t, const index_t, typename> class RHS,
00243         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00244     >
00245     auto
00246     approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00247                 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00248                 const Scalar_T threshold,
00249                 const Scalar_T tolerance) -> bool;
00250
00252     template
00253     <
00254         template<typename, const index_t, const index_t, typename> class Multivector,
00255         template<typename, const index_t, const index_t, typename> class RHS,
00256         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00257     >
00258     auto
00259     approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00260                 const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00261
00263     template
00264     <
00265         template<typename, const index_t, const index_t, typename> class Multivector,
00266         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00267     >
00268     auto
00269     operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00270
00272     template
00273     <
00274         template<typename, const index_t, const index_t, typename> class Multivector,
00275         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00276     >
00277     auto
00278     operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00279
00281     template
00282     <

```

```

00283     template<typename, const index_t, const index_t, typename> class Multivector,
00284     template<typename, const index_t, const index_t, typename> class RHS,
00285     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00286 >
00287     auto
00288     operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00289
00291     template
00292     <
00293         template<typename, const index_t, const index_t, typename> class Multivector,
00294         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00295     >
00296     auto
00297     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00298
00300     template
00301     <
00302         template<typename, const index_t, const index_t, typename> class Multivector,
00303         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00304     >
00305     auto
00306     operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00307
00309     template
00310     <
00311         template<typename, const index_t, const index_t, typename> class Multivector,
00312         template<typename, const index_t, const index_t, typename> class RHS,
00313         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00314     >
00315     auto
00316     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00317
00319     template
00320     <
00321         template<typename, const index_t, const index_t, typename> class Multivector,
00322         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00323     >
00324     auto
00325     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00326
00328     template
00329     <
00330         template<typename, const index_t, const index_t, typename> class Multivector,
00331         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00332     >
00333     auto
00334     operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00335
00337     template
00338     <
00339         template<typename, const index_t, const index_t, typename> class Multivector,
00340         template<typename, const index_t, const index_t, typename> class RHS,
00341         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00342     >
00343     auto
00344     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00345
00347     template
00348     <
00349         template<typename, const index_t, const index_t, typename> class Multivector,
00350         template<typename, const index_t, const index_t, typename> class RHS,
00351         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00352     >
00353     auto
00354     operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00355
00357     template
00358     <
00359         template<typename, const index_t, const index_t, typename> class Multivector,
00360         template<typename, const index_t, const index_t, typename> class RHS,
00361         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00362     >
00363     auto
00364     operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00365
00367     template
00368     <
00369         template<typename, const index_t, const index_t, typename> class Multivector,

```

```

00370     template<typename, const index_t, const index_t, typename> class RHS,
00371     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00372 >
00373 auto
00374 operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00375
00376 template
00377 <
00378     template<typename, const index_t, const index_t, typename> class Multivector,
00379     template<typename, const index_t, const index_t, typename> class RHS,
00380     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00381 >
00382 auto
00383 star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
Scalar_T;

00385
00386 template
00387 <
00388     template<typename, const index_t, const index_t, typename> class Multivector,
00389     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00390 >
00391 auto
00392 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00394
00395 template
00396 <
00397     template<typename, const index_t, const index_t, typename> class Multivector,
00398     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00399 >
00400 auto
00401 operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00403
00404 template
00405 <
00406     template<typename, const index_t, const index_t, typename> class Multivector,
00407     template<typename, const index_t, const index_t, typename> class RHS,
00408     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00409 >
00410 auto
00411 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00413
00414 template
00415 <
00416     template<typename, const index_t, const index_t, typename> class Multivector,
00417     template<typename, const index_t, const index_t, typename> class RHS,
00418     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00419 >
00420 auto
00421 operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00423
00424 template
00425 <
00426     template<typename, const index_t, const index_t, typename> class Multivector,
00427     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00428 >
00429 auto
00430 inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;

00432
00433 template
00434 <
00435     template<typename, const index_t, const index_t, typename> class Multivector,
00436     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00437 >
00438 auto
00439 pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00441
00442 template
00443 <
00444     template<typename, const index_t, const index_t, typename> class Multivector,
00445     template<typename, const index_t, const index_t, typename> class RHS,
00446     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00447 >
00448 auto
00449 pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00451
00452 template< template<typename, const index_t, const index_t, typename> class Multivector,
00453     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00454 auto
00455 outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

```

```

00457
00459     template
00460     <
00461         template<typename, const index_t, const index_t, typename> class Multivector,
00462         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00463     >
00464     auto
00465     scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00466
00468     template
00469     <
00470         template<typename, const index_t, const index_t, typename> class Multivector,
00471         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00472     >
00473     auto
00474     real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00475
00477     template
00478     <
00479         template<typename, const index_t, const index_t, typename> class Multivector,
00480         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00481     >
00482     auto
00483     imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00484
00486     template
00487     <
00488         template<typename, const index_t, const index_t, typename> class Multivector,
00489         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00490     >
00491     auto
00492     pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00493
00495     template
00496     <
00497         template<typename, const index_t, const index_t, typename> class Multivector,
00498         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00499     >
00500     auto
00501     even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00502
00504     template
00505     <
00506         template<typename, const index_t, const index_t, typename> class Multivector,
00507         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00508     >
00509     auto
00510     odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00511
00513     template
00514     <
00515         template<typename, const index_t, const index_t, typename> class Multivector,
00516         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00517     >
00518     auto
00519     vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>;
00520
00522     template
00523     <
00524         template<typename, const index_t, const index_t, typename> class Multivector,
00525         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00526     >
00527     auto
00528     involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00529
00531     template
00532     <
00533         template<typename, const index_t, const index_t, typename> class Multivector,
00534         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00535     >
00536     auto
00537     reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00538
00540     template
00541     <
00542         template<typename, const index_t, const index_t, typename> class Multivector,
00543         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00544     >
00545     auto
00546     conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00547
00549     template
00550     <
00551         template<typename, const index_t, const index_t, typename> class Multivector,
00552         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00553     >
00554     auto

```

```

00555     quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00556
00557     template
00558     <
00559         template<typename, const index_t, const index_t, typename> class Multivector,
00560         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00561     >
00562     auto
00563     norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00564
00565     template
00566     <
00567         template<typename, const index_t, const index_t, typename> class Multivector,
00568         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00569     >
00570     auto
00571     abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00572
00573     template
00574     <
00575         template<typename, const index_t, const index_t, typename> class Multivector,
00576         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00577     >
00578     auto
00579     max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00580
00581     template
00582     <
00583         template<typename, const index_t, const index_t, typename> class Multivector,
00584         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00585     >
00586     auto
00587     complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00588     Multivector<Scalar_T,LO,HI,Tune_P>;
00589
00590     template
00591     <
00592         template<typename, const index_t, const index_t, typename> class Multivector,
00593         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00594     >
00595     auto
00596     elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00597
00598     template
00599     <
00600         template<typename, const index_t, const index_t, typename> class Multivector,
00601         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00602     >
00603     auto
00604     sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00605           const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00606           const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00607
00608     template
00609     <
00610         template<typename, const index_t, const index_t, typename> class Multivector,
00611         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00612     >
00613     auto
00614     sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00615
00616     // Transcendental functions
00617
00618     template
00619     <
00620         template<typename, const index_t, const index_t, typename> class Multivector,
00621         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00622     >
00623     auto
00624     clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00625     Multivector<Scalar_T,LO,HI,Tune_P>;
00626
00627     template
00628     <
00629         template<typename, const index_t, const index_t, typename> class Multivector,
00630         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00631     >
00632     auto
00633     log(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00634         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00635         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00636
00637     template
00638     <
00639         template<typename, const index_t, const index_t, typename> class Multivector,
00640         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00641     >
00642     auto

```

```

00651     log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00652
00653     template
00654     <
00655         template<typename, const index_t, const index_t, typename> class Multivector,
00656         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00657     >
00658     auto
00659     cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00660         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00661         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00662
00663     template
00664     <
00665         template<typename, const index_t, const index_t, typename> class Multivector,
00666         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00667     >
00668     auto
00669     cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00670
00671     template
00672     <
00673         template<typename, const index_t, const index_t, typename> class Multivector,
00674         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00675     >
00676     auto
00677     acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00678         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00679         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00680
00681     template
00682     <
00683         template<typename, const index_t, const index_t, typename> class Multivector,
00684         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00685     >
00686     auto
00687     acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00688
00689     template
00690     <
00691         template<typename, const index_t, const index_t, typename> class Multivector,
00692         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00693     >
00694     auto
00695     acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00696
00697     template
00698     <
00699         template<typename, const index_t, const index_t, typename> class Multivector,
00700         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00701     >
00702     auto
00703     cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00704
00705     template
00706     <
00707         template<typename, const index_t, const index_t, typename> class Multivector,
00708         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00709     >
00710     auto
00711     acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00712         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00713         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00714
00715     template
00716     <
00717         template<typename, const index_t, const index_t, typename> class Multivector,
00718         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00719     >
00720     auto
00721     acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00722
00723     template
00724     <
00725         template<typename, const index_t, const index_t, typename> class Multivector,
00726         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00727     >
00728     auto
00729     sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00730         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00731         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00732
00733     template
00734     <
00735         template<typename, const index_t, const index_t, typename> class Multivector,
00736         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00737     >
00738     auto
00739     sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00740
00741     template
00742     <
00743         template<typename, const index_t, const index_t, typename> class Multivector,
00744         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00745     >
00746     auto
00747     sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;

```

```

00748     auto
00749     asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00750          const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00751          const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00752
00753     template
00754     <
00755         template<typename, const index_t, const index_t, typename> class Multivector,
00756         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00757     >
00758     auto
00759     asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00760
00761     template
00762     <
00763         template<typename, const index_t, const index_t, typename> class Multivector,
00764         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00765     >
00766     auto
00767     sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00768
00769     template
00770     <
00771         template<typename, const index_t, const index_t, typename> class Multivector,
00772         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00773     >
00774     auto
00775     asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00776           const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00777           const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00778
00779     template
00780     <
00781         template<typename, const index_t, const index_t, typename> class Multivector,
00782         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00783     >
00784     auto
00785     asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00786
00787     template
00788     <
00789         template<typename, const index_t, const index_t, typename> class Multivector,
00790         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00791     >
00792     auto
00793     tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00794         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00795         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00796
00797     template
00798     <
00799         template<typename, const index_t, const index_t, typename> class Multivector,
00800         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00801     >
00802     auto
00803     tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00804
00805     template
00806     <
00807         template<typename, const index_t, const index_t, typename> class Multivector,
00808         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00809     >
00810     auto
00811     atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00812         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00813         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00814
00815     template
00816     <
00817         template<typename, const index_t, const index_t, typename> class Multivector,
00818         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00819     >
00820     auto
00821     atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00822
00823     template
00824     <
00825         template<typename, const index_t, const index_t, typename> class Multivector,
00826         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00827     >
00828     auto
00829     atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00830
00831     template
00832     <
00833         template<typename, const index_t, const index_t, typename> class Multivector,
00834         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00835     >
00836     auto
00837     tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00838
00839     template
00840     <
00841         template<typename, const index_t, const index_t, typename> class Multivector,
00842         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P

```

```

00845 >
00846 auto
00847 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00848       const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00849       const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00850
00852 template
00853 <
00854     template<typename, const index_t, const index_t, typename> class Multivector,
00855     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00856 >
00857 auto
00858 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00859 }
00860 #endif // _GLUCAT_CLIFFORD_ALGEBRA_H

```

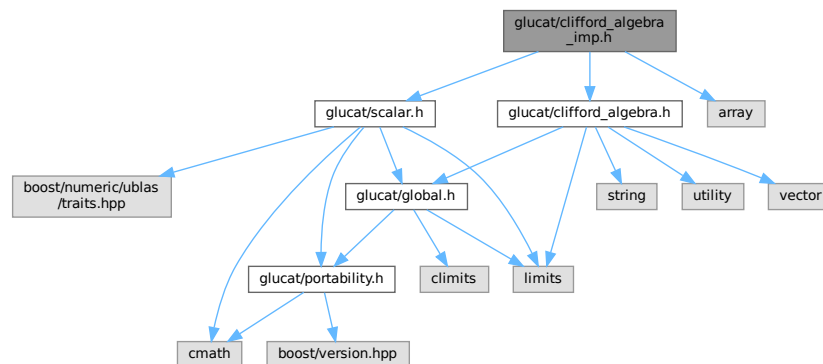
### 7.3 glucat/clifford\_algebra\_imp.h File Reference

```

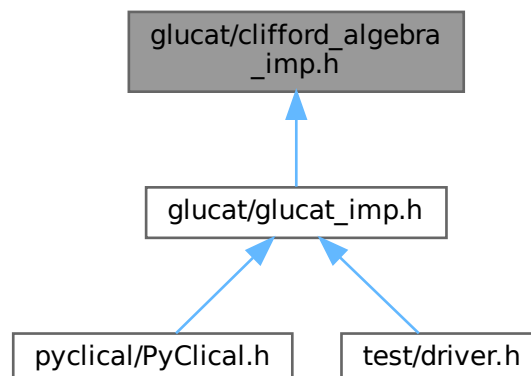
#include "glucat/clifford_algebra.h"
#include "glucat/scalar.h"
#include <array>

```

Include dependency graph for clifford\_algebra\_imp.h:



This graph shows which files directly or indirectly include this file:





## Namespaces

- namespace [glucat](#)

## Functions

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of multivectors.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool`  
*Test for inequality of multivector and scalar.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator!= (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of scalar and multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::error\_squared\_tol (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Quadratic norm error tolerance relative to a specific multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::error\_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T`  
*Relative or absolute error using the quadratic norm.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool`  
*Test for approximate equality of multivectors.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for approximate equality of multivectors.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric sum of multivector and scalar.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric sum of scalar and multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric sum.*



- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Quotient of scalar and multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric multiplicative inverse.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Integer power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Multivector power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Outer product power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Scalar part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Real part: synonym for scalar part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Imaginary part: deprecated (always 0)*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Pure part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Even part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Odd part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T, LO, HI, Tune_P >`

*Vector part of multivector, as a `vector_t` with respect to frame()*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each  $\{i\}$  is replaced by  $-\{i\}$  in each term, eg.  $\{1\}*\{2\} \rightarrow (-\{2\})*(-\{1\})$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg.  $\{1\}*\{2\} \rightarrow \{2\}*\{1\}$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation,  $rev \circ invo == invo \circ rev$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T quadratic form ==  $(rev(x)*x)(0)$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T norm == sum of norm of coordinates.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Absolute value ==  $\sqrt{norm}$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::max\_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`static void glucat::check\_complex (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false)`

*Check that  $i$  is a valid complexifier for  $val$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::clifford\_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Exponential of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::cosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Hyperbolic cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`



*Inverse hyperbolic tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector.*

## Variables

- `template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>`  
`const Scalar_T glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_truncation = std::`  
`::numeric_limits<Scalar_T>::epsilon()`

*Default for truncation.*

## 7.4 clifford\_algebra\_imp.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     clifford_algebra_imp.h : Implement common Clifford algebra functions
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
```



```

00032  *****/
00033
00034 // References for algorithms:
00035 // [AS]:
00036 // Milton Abramowicz and Irene A. Stegun, "Handbook of mathematical functions",
00037 // Dover 1972, first published 1965.
00038 // [CHKL]:
00039 // Sheung Hun Cheng, Nicholas J. Higham, Charles S. Kenney and Alan J. Laub,
00040 // "Approximating the Logarithm of a Matrix to Specified Accuracy", 1999.
00041 // ftp://ftp.ma.man.ac.uk/pub/narep/narep353.ps.gz
00042 // [GL]:
00043 // Gene H. Golub and Charles F. van Loan,
00044 // "Matrix Computations", 3rd ed., Johns Hopkins UP, 1996.
00045 // [GW]:
00046 // C.F. Gerald and P.O. Wheatley, "Applied Numerical Analysis",
00047 // 6th Edition, Addison-Wesley, 1999.
00048 // [H]:
00049 // Nicholas J. Higham
00050 // "The Scaling and Squaring Method for the Matrix Exponential Revisited",
00051 // SIAM Journal on Matrix Analysis and Applications,
00052 // Vol. 26, Issue 4 (2005), pp. 1179-1193.
00053 // [Z]:
00054 // Doron Zeilberger, "PADE" (Maple code), 2002.
00055 // http://www.math.rutgers.edu/~zeilberg/tokhniot/PADE
00056
00057 #include "glucat/clifford_algebra.h"
00058 #include "glucat/scalar.h"
00059
00060 #include <array>
00061
00062 namespace glucat
00063 {
00064     template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00065     auto
00066     clifford_algebra<Scalar_T, Index_Set_T, Multivector_T>::
00067     classname() -> const std::string
00068     { return "clifford_algebra"; }
00069
00071     template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00072     const
00073     Scalar_T
00074     clifford_algebra<Scalar_T, Index_Set_T, Multivector_T>::
00075     default_truncation = std::numeric_limits<Scalar_T>::epsilon();
00076
00078     template
00079     <
00080         template<typename, const index_t, const index_t, typename> class Multivector,
00081         template<typename, const index_t, const index_t, typename> class RHS,
00082         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00083     >
00084     inline
00085     auto
00086     operator!= (const Multivector<Scalar_T, LO, HI, Tune_P>& lhs, const RHS<Scalar_T, LO, HI, Tune_P>& rhs) ->
bool
00087     { return !(lhs == rhs); }
00088
00090     template< template<typename, const index_t, const index_t, typename> class Multivector,
00091               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092     inline
00093     auto
00094     operator!= (const Multivector<Scalar_T, LO, HI, Tune_P>& lhs, const Scalar_T& scr) -> bool
00095     { return !(lhs == scr); }
00096
00098     template< template<typename, const index_t, const index_t, typename> class Multivector,
00099               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00100     inline
00101     auto
00102     operator!= (const Scalar_T& scr, const Multivector<Scalar_T, LO, HI, Tune_P>& rhs) -> bool
00103     { return !(rhs == scr); }
00104
00106     template
00107     <
00108         template<typename, const index_t, const index_t, typename> class Multivector,
00109         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00110     >
00111     auto
00112     error_squared_tol(const Multivector<Scalar_T, LO, HI, Tune_P>& val) -> Scalar_T
00113     {
00114         using multivector_t = Multivector<Scalar_T, LO, HI, Tune_P>;
00115         static const auto scalar_eps = std::numeric_limits<Scalar_T>::epsilon();
00116         static const auto nbr_different_bits =
00117             std::numeric_limits<Scalar_T>::digits / Tune_P::denom_different_bits +
Tune_P::extra_different_bits;
00118         static const auto abs_tol = scalar_eps *
00119             numeric_traits<Scalar_T>::pow(Scalar_T(2), nbr_different_bits);
00120         using framed_multi_t = typename multivector_t::framed_multi_t;
00121         const auto nbr_terms = double(framed_multi_t(val).truncated(scalar_eps).nbr_terms());

```



```

00122     return abs_tol * abs_tol * std::max(Scalar_T(nbr_terms), Scalar_T(1));
00123 }
00124
00126 template
00127 <
00128     template<typename, const index_t, const index_t, typename> class Multivector,
00129     template<typename, const index_t, const index_t, typename> class RHS,
00130     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00131 >
00132 inline
00133 auto
00134 error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00135               const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00136               const Scalar_T threshold) -> Scalar_T
00137 {
00138     const auto relative = norm(rhs) > threshold;
00139     const auto abs_norm_diff = norm(rhs-lhs);
00140     return (relative)
00141         ? abs_norm_diff/norm(rhs)
00142         : abs_norm_diff;
00143 }
00144
00146 template
00147 <
00148     template<typename, const index_t, const index_t, typename> class Multivector,
00149     template<typename, const index_t, const index_t, typename> class RHS,
00150     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00151 >
00152 inline
00153 auto
00154 approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00155              const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00156              const Scalar_T threshold,
00157              const Scalar_T tolerance) -> bool
00158 { return error_squared(lhs, rhs, threshold) < tolerance; }
00159
00161 template
00162 <
00163     template<typename, const index_t, const index_t, typename> class Multivector,
00164     template<typename, const index_t, const index_t, typename> class RHS,
00165     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00166 >
00167 inline
00168 auto
00169 approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00170              const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool
00171 {
00172     const Scalar_T rhs_tol = error_squared_tol(rhs);
00173     return approx_equal(lhs, rhs, rhs_tol, rhs_tol);
00174 }
00175
00177 template< template<typename, const index_t, const index_t, typename> class Multivector,
00178           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00179 inline
00180 auto
00181 operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00182 {
00183     auto result = lhs;
00184     return result += scr;
00185 }
00186
00188 template< template<typename, const index_t, const index_t, typename> class Multivector,
00189           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00190 inline
00191 auto
00192 operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00193 {
00194     return rhs + scr;
00195 }
00196
00198 template
00199 <
00200     template<typename, const index_t, const index_t, typename> class Multivector,
00201     template<typename, const index_t, const index_t, typename> class RHS,
00202     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00203 >
00204 inline
00205 auto
00206 operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00207 {
00208     auto result = lhs;
00209     return result += rhs;
00210 }
00211

```

```

00213     template< template<typename, const index_t, const index_t, typename> class Multivector,
00214               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00215     inline
00216     auto
00217     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00218     {
00219         auto result = lhs;
00220         return result -= scr;
00221     }
00222
00224     template< template<typename, const index_t, const index_t, typename> class Multivector,
00225               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00226     inline
00227     auto
00228     operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00229     { return -rhs + scr; }
00230
00232     template
00233     <
00234         template<typename, const index_t, const index_t, typename> class Multivector,
00235         template<typename, const index_t, const index_t, typename> class RHS,
00236         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00237     >
00238     inline
00239     auto
00240     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00241     {
00242         auto result = lhs;
00243         return result -= rhs;
00244     }
00245
00247     template< template<typename, const index_t, const index_t, typename> class Multivector,
00248               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00249     inline
00250     auto
00251     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00252     {
00253         auto result = lhs;
00254         return result *= scr;
00255     }
00256
00258     template< template<typename, const index_t, const index_t, typename> class Multivector,
00259               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00260     inline
00261     auto
00262     operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00263     { // Note: this assumes that scalar commutes with multivector.
00264       // This excludes Clifford algebras over non-commuting rings.
00265         return rhs * scr;
00266     }
00267
00269     template
00270     <
00271         template<typename, const index_t, const index_t, typename> class Multivector,
00272         template<typename, const index_t, const index_t, typename> class RHS,
00273         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00274     >
00275     inline
00276     auto
00277     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00278     {
00279         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00280         return lhs * multivector_t(rhs);
00281     }
00282
00284     template
00285     <
00286         template<typename, const index_t, const index_t, typename> class Multivector,
00287         template<typename, const index_t, const index_t, typename> class RHS,
00288         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00289     >
00290     inline
00291     auto
00292     operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00293     {
00294         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00295         return lhs ^ multivector_t(rhs);
00296     }
00297
00299     template

```

```

00300 <
00301     template<typename, const index_t, const index_t, typename> class Multivector,
00302     template<typename, const index_t, const index_t, typename> class RHS,
00303     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00304 >
00305 inline
00306 auto
00307 operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00308 {
00309     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00310     return lhs & multivector_t(rhs);
00311 }
00312
00314 template
00315 <
00316     template<typename, const index_t, const index_t, typename> class Multivector,
00317     template<typename, const index_t, const index_t, typename> class RHS,
00318     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00319 >
00320 inline
00321 auto
00322 operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00323 {
00324     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00325     return lhs % multivector_t(rhs);
00326 }
00327
00329 template
00330 <
00331     template<typename, const index_t, const index_t, typename> class Multivector,
00332     template<typename, const index_t, const index_t, typename> class RHS,
00333     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00334 >
00335 inline
00336 auto
00337 star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
Scalar_T
00338 {
00339     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00340     return star(lhs, multivector_t(rhs));
00341 }
00342
00344 template< template<typename, const index_t, const index_t, typename> class Multivector,
00345           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00346 inline
00347 auto
00348 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00349 {
00350     auto result = lhs;
00351     return result /= scr;
00352 }
00353
00355 template< template<typename, const index_t, const index_t, typename> class Multivector,
00356           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00357 inline
00358 auto
00359 operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00360 {
00361     Multivector<Scalar_T,LO,HI,Tune_P> result = scr;
00362     return result /= rhs;
00363 }
00364
00366 template
00367 <
00368     template<typename, const index_t, const index_t, typename> class Multivector,
00369     template<typename, const index_t, const index_t, typename> class RHS,
00370     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00371 >
00372 inline
00373 auto
00374 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00375 {
00376     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00377     return lhs / multivector_t(rhs);
00378 }
00379
00381 template
00382 <
00383     template<typename, const index_t, const index_t, typename> class Multivector,
00384     template<typename, const index_t, const index_t, typename> class RHS,
00385     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00386 >

```

```

00387     inline
00388     auto
00389     operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00390     {
00391         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00392         return lhs | multivector_t(rhs);
00393     }
00394
00396     template< template<typename, const index_t, const index_t, typename> class Multivector,
00397               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00398     inline
00399     auto
00400     inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00401     { return val.inv(); }
00402
00404     template< template<typename, const index_t, const index_t, typename> class Multivector,
00405               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00406     auto
00407     pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00408     {
00409         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00410         if (lhs == Scalar_T(0))
00411         {
00412             using traits_t = numeric_traits<Scalar_T>;
00413             return
00414                 (rhs < 0)
00415                 ? traits_t::NaN()
00416                 : (rhs == 0)
00417                   ? Scalar_T(1)
00418                   : Scalar_T(0);
00419         }
00420         auto result = multivector_t(Scalar_T(1));
00421         auto power =
00422             (rhs < 0)
00423             ? lhs.inv()
00424             : lhs;
00425         for (auto
00426             k = std::abs(rhs);
00427             k != 0;
00428             k /= 2)
00429         {
00430             if (k % 2)
00431                 result *= power;
00432             power *= power;
00433         }
00434         return result;
00435     }
00436
00438     template
00439     <
00440         template<typename, const index_t, const index_t, typename> class Multivector,
00441         template<typename, const index_t, const index_t, typename> class RHS,
00442         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00443     >
00444     inline
00445     auto
00446     pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00447     {
00448         using traits_t = numeric_traits<Scalar_T>;
00449
00450         if (lhs == Scalar_T(0))
00451         {
00452             const Scalar_T m = rhs.scalar();
00453             if (rhs == m)
00454                 return
00455                     (m < 0)
00456                     ? traits_t::NaN()
00457                     : (m == 0)
00458                       ? Scalar_T(1)
00459                       : Scalar_T(0);
00460             else
00461                 return Scalar_T(0);
00462         }
00463         return exp(log(lhs) * rhs);
00464     }
00465
00467     template< template<typename, const index_t, const index_t, typename> class Multivector,
00468               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00469     auto
00470     outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00471     { return lhs.outer_pow(rhs); }
00472
00474     template< template<typename, const index_t, const index_t, typename> class Multivector,

```

```

00475         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00476     inline
00477     auto
00478     scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00479     { return val.scalar(); }
00480
00482     template< template<typename, const index_t, const index_t, typename> class Multivector,
00483               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00484     inline
00485     auto
00486     real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00487     { return val.scalar(); }
00488
00490     template
00491     <
00492         template<typename, const index_t, const index_t, typename> class Multivector,
00493         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00494     >
00495     inline
00496     auto
00497     imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00498     { return Scalar_T(0); }
00499
00501     template< template<typename, const index_t, const index_t, typename> class Multivector,
00502               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00503     inline
00504     auto
00505     pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00506     { return val - val.scalar(); }
00507
00509     template< template<typename, const index_t, const index_t, typename> class Multivector,
00510               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00511     inline
00512     auto
00513     even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00514     { return val.even(); }
00515
00517     template< template<typename, const index_t, const index_t, typename> class Multivector,
00518               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00519     inline
00520     auto
00521     odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00522     { return val.odd(); }
00523
00525     template< template<typename, const index_t, const index_t, typename> class Multivector,
00526               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00527     inline
00528     auto
00529     vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>
00530     { return val.vector_part(); }
00531
00533     template< template<typename, const index_t, const index_t, typename> class Multivector,
00534               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00535     inline
00536     auto
00537     involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00538     { return val.involute(); }
00539
00541     template< template<typename, const index_t, const index_t, typename> class Multivector,
00542               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00543     inline
00544     auto
00545     reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00546     { return val.reverse(); }
00547
00549     template< template<typename, const index_t, const index_t, typename> class Multivector,
00550               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00551     inline
00552     auto
00553     conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00554     { return val.conj(); }
00555
00557     template< template<typename, const index_t, const index_t, typename> class Multivector,
00558               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00559     inline
00560     auto
00561     quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00562     { return val.quad(); }
00563
00565     template< template<typename, const index_t, const index_t, typename> class Multivector,
00566               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00567     inline
00568     auto
00569     norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00570     { return val.norm(); }
00571
00573     template< template<typename, const index_t, const index_t, typename> class Multivector,

```

```

00574         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00575     inline
00576     auto
00577     abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00578     { return numeric_traits<Scalar_T>::sqrt(val.norm()); }
00579
00581     template< template<typename, const index_t, const index_t, typename> class Multivector,
00582             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00583     inline
00584     auto
00585     max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00586     { return val.max_abs(); }
00587
00589     template< template<typename, const index_t, const index_t, typename> class Multivector,
00590             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00591     auto
00592     complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00593     Multivector<Scalar_T,LO,HI,Tune_P>
00594     {
00595         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00596         using traits_t = numeric_traits<Scalar_T>;
00597
00598         auto frm = val.frame();
00599         using array_t = std::array<index_t, 4>;
00600         auto incp = array_t{0, 2, 1, 0};
00601         auto incq = array_t{1, 0, 0, 0};
00602         auto bott = pos_mod((frm.count_pos() - frm.count_neg()), 4);
00603         for (auto
00604             k = index_t(0);
00605             k != incp[bott];
00606             k++)
00607             for (auto
00608                 idx = index_t(1);
00609                 idx != HI+1;
00610                 ++idx)
00611                 if (!frm[idx])
00612                 {
00613                     frm.set(idx);
00614                     break;
00615                 }
00616         for (auto
00617             k = index_t(0);
00618             k != incq[bott];
00619             k++)
00620             for (auto
00621                 idx = index_t(-1);
00622                 idx != LO-1;
00623                 --idx)
00624                 if (!frm[idx])
00625                 {
00626                     frm.set(idx);
00627                     break;
00628                 }
00629         auto new_bott = pos_mod(frm.count_pos() - frm.count_neg(), 4);
00630         if ((incp[new_bott] == 0) && (incq[new_bott] == 0))
00631             return multivector_t(frm, Scalar_T(1));
00632         else
00633             // Return IEEE NaN or -Inf
00634             return traits_t::NaN();
00635     }
00636
00639     template< template<typename, const index_t, const index_t, typename> class Multivector,
00640             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00641     inline
00642     auto
00643     elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00644     { return complexifier(val); }
00645
00647     template< template<typename, const index_t, const index_t, typename> class Multivector,
00648             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00649     inline
00650     static
00651     void
00652     check_complex(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00653                  const Multivector<Scalar_T,LO,HI,Tune_P>& i, const bool prechecked = false)
00654     {
00655         if (!prechecked)
00656         {
00657             using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00658             using index_set_t = typename multivector_t::index_set_t;
00659             using error_t = typename multivector_t::error_t;
00660
00661             const auto i_frame = i.frame();
00662             // We need i to be a complexifier whose frame is large enough to represent val
00663             if (complexifier(i) != i ||
00664                 (val.frame() | i_frame) != i_frame ||

```

```

00665         complexifier(val).frame().count() > i_frame.count())
00666         throw error_t("check_complex(val, i): i is not a valid complexifier for val");
00667     }
00668 }
00669
00671 template< template<typename, const index_t, const index_t, typename> class Multivector,
00672           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00673 inline
00674 auto
00675 sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00676 { return sqrt(val, i, prechecked); }
00677
00679 template< template<typename, const index_t, const index_t, typename> class Multivector,
00680           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00681 inline
00682 auto
00683 sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00684 { return sqrt(val, complexifier(val), true); }
00685
00687 template< template<typename, const index_t, const index_t, typename> class Multivector,
00688           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00689 auto
00690 clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00691 {
00692     // Scaling and squaring Pade' approximation of matrix exponential
00693     // Reference: [GL], Section 11.3, p572-576
00694     // Reference: [H]
00695
00696     using traits_t = numeric_traits<Scalar_T>;
00697
00698     const auto scalar_val = val.scalar();
00699     const auto scalar_exp = traits_t::exp(scalar_val);
00700     if (traits_t::isNaN_or_isInf(scalar_exp))
00701         return traits_t::NaN();
00702     if (val == scalar_val)
00703         return scalar_exp;
00704
00705     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00706     auto A = val - scalar_val;
00707     const auto pure_scale2 = A.norm();
00708
00709     if (traits_t::isNaN_or_isInf(pure_scale2))
00710         return traits_t::NaN();
00711     if (pure_scale2 == Scalar_T(0))
00712         return scalar_exp;
00713
00714     const auto ilog2_scale =
00715         std::max(0, traits_t::to_int(ceil((log2(pure_scale2) +
Scalar_T(A.frame().count())/Scalar_T(2))) - 3);
00716     const auto i_scale = traits_t::pow(Scalar_T(2), ilog2_scale);
00717     if (traits_t::isNaN_or_isInf(i_scale))
00718         return traits_t::NaN();
00719
00720     A /= i_scale;
00721     multivector_t pure_exp;
00722     {
00723         using limits_t = std::numeric_limits<Scalar_T>;
00724         const auto nbr_even_powers = 2*(limits_t::digits / 32) + 4;
00725         using nbr_t = decltype(nbr_even_powers);
00726
00727         // Create an array of coefficients
00728         const auto max_power = 2*nbr_even_powers + 1;
00729         static std::array<Scalar_T, max_power+1> c;
00730         if (c[0] != Scalar_T(1))
00731         {
00732             c[0] = Scalar_T(1);
00733             for (auto
00734                 k = decltype(max_power)(0);
00735                 k != max_power;
00736                 ++k)
00737                 c[k+1] = c[k]*(max_power-k) / ((2*max_power-k)*(k+1));
00738         }
00739
00740         // Create an array of even powers
00741         std::array<multivector_t, nbr_even_powers> AA;
00742         AA[0] = A * A;
00743         AA[1] = AA[0] * AA[0];
00744         for (auto
00745             k = nbr_t(2);
00746             k != nbr_even_powers;
00747             ++k)
00748             AA[k] = AA[k-2] * AA[1];
00749
00750         // Use compensated summation to calculate U and AV
00751         auto residual = multivector_t();

```

```

00752     auto U = multivector_t(c[0]);
00753     for (auto
00754         k = nbr_t(0);
00755         k != nbr_even_powers;
00756         ++k)
00757     {
00758         const auto& term = AA[k]*c[2*k + 2] - residual;
00759         const auto& sum = U + term;
00760         residual = (sum - U) - term;
00761         U = sum;
00762     }
00763     residual = multivector_t();
00764     auto AV = multivector_t(c[1]);
00765     for (auto
00766         k = nbr_t(0);
00767         k != nbr_even_powers;
00768         ++k)
00769     {
00770         const auto& term = AA[k]*c[2*k + 3] - residual;
00771         const auto& sum = AV + term;
00772         residual = (sum - AV) - term;
00773         AV = sum;
00774     }
00775     AV *= A;
00776     pure_exp = (U+AV) / (U-AV);
00777 }
00778 for (auto
00779     k = decltype(ilog2_scale)(0);
00780     k != ilog2_scale;
00781     ++k)
00782     pure_exp *= pure_exp;
00783 return pure_exp * scalar_exp;
00784 }
00785
00786 template< template<typename, const index_t, const index_t, typename> class Multivector,
00787           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00788 inline
00789 auto
00791 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00792 { return log(val, i, prechecked); }
00793
00794 template< template<typename, const index_t, const index_t, typename> class Multivector,
00795           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00796 inline
00797 auto
00798 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00799 { return log(val, complexifier(val), true); }
00800
00801 template< template<typename, const index_t, const index_t, typename> class Multivector,
00802           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00803 inline
00804 auto
00805 cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00806 {
00807     using traits_t = numeric_traits<Scalar_T>;
00808     if (val.isnan())
00809         return traits_t::NaN();
00810
00811     const auto& s = val.scalar();
00812     if (val == s)
00813         return traits_t::cosh(s);
00814     return (exp(val)+exp(-val)) / Scalar_T(2);
00815 }
00816
00817 // Reference: [AS], Section 4.6, p86-89
00818 template< template<typename, const index_t, const index_t, typename> class Multivector,
00819           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00820 inline
00821 auto
00822 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00823 {
00824     using traits_t = numeric_traits<Scalar_T>;
00825     check_complex(val, i, prechecked);
00826     if (val.isnan())
00827         return traits_t::NaN();
00828
00829     const auto radical = sqrt(val*val - Scalar_T(1), i, true);
00830     return (norm(val + radical) >= norm(val))
00831         ? log(val + radical, i, true)
00832         : -log(val - radical, i, true);
00833 }
00834
00835 // Reference: [AS], Section 4.6, p86-89
00836 template< template<typename, const index_t, const index_t, typename> class Multivector,
00837           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >

```



```

00842     inline
00843     auto
00844     acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00845     { return acosh(val, complexifier(val), true); }
00846
00847     template< template<typename, const index_t, const index_t, typename> class Multivector,
00848             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00849     auto
00850     cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
00851     prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00852     {
00853         using traits_t = numeric_traits<Scalar_T>;
00854         if (val.isnan())
00855             return traits_t::NaN();
00856
00857         const auto& s = val.scalar();
00858         if (val == s)
00859             return traits_t::cos(s);
00860
00861         check_complex(val, i, prechecked);
00862
00863         static const auto& twopi = Scalar_T(2) * traits_t::pi();
00864         const auto& z = i *
00865             (val - s + traits_t::fmod(s, twopi));
00866         return (exp(z)+exp(-z)) / Scalar_T(2);
00867     }
00868
00869     template< template<typename, const index_t, const index_t, typename> class Multivector,
00870             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00871     inline
00872     auto
00873     cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00874     { return cos(val, complexifier(val), true); }
00875
00876     // Reference: [AS], Section 4.4, p79-83
00877     template< template<typename, const index_t, const index_t, typename> class Multivector,
00878             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00879     inline
00880     auto
00881     acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00882     bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00883     {
00884         using traits_t = numeric_traits<Scalar_T>;
00885         if (val.isnan())
00886             return traits_t::NaN();
00887
00888         const auto& s = val.scalar();
00889         if (val == s && traits_t::abs(s) <= Scalar_T(1))
00890             return traits_t::acos(s);
00891
00892         check_complex(val, i, prechecked);
00893         return i * acosh(val, i, true);
00894     }
00895
00896     // Reference: [AS], Section 4.4, p79-83
00897     template< template<typename, const index_t, const index_t, typename> class Multivector,
00898             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00899     inline
00900     auto
00901     acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00902     { return acos(val, complexifier(val), true); }
00903
00904     template< template<typename, const index_t, const index_t, typename> class Multivector,
00905             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00906     inline
00907     auto
00908     sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00909     {
00910         using traits_t = numeric_traits<Scalar_T>;
00911         if (val.isnan())
00912             return traits_t::NaN();
00913
00914         const auto& s = val.scalar();
00915         if (val == s)
00916             return traits_t::sinh(s);
00917
00918         return (exp(val)-exp(-val)) / Scalar_T(2);
00919     }
00920
00921     // Reference: [AS], Section 4.6, p86-89
00922     template< template<typename, const index_t, const index_t, typename> class Multivector,
00923             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00924     inline
00925     auto
00926     asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00927     bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00928     {
00929

```

```

00932     using traits_t = numeric_traits<Scalar_T>;
00933     check_complex(val, i, prechecked);
00934     if (val.isnan())
00935         return traits_t::NaN();
00936
00937     const auto radical = sqrt(val*val + Scalar_T(1), i, true);
00938     return (norm(val + radical) >= norm(val))
00939         ? log(val + radical, i, true)
00940         : -log(-val + radical, i, true);
00941 }
00942
00943 // Reference: [AS], Section 4.6, p86-89
00944 template< template<typename, const index_t, const index_t, typename> class Multivector,
00945           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00946 inline
00947 auto
00948 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00949 { return asinh(val, complexifier(val), true); }
00950
00951 template< template<typename, const index_t, const index_t, typename> class Multivector,
00952           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00953 auto
00954 sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
00955 prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00956 {
00957     using traits_t = numeric_traits<Scalar_T>;
00958     if (val.isnan())
00959         return traits_t::NaN();
00960
00961     const auto& s = val.scalar();
00962     if (val == s)
00963         return traits_t::sin(s);
00964
00965     check_complex(val, i, prechecked);
00966
00967     static const auto& twopi = Scalar_T(2) * traits_t::pi();
00968     const auto& z = i *
00969         (val - s + traits_t::fmod(s, twopi));
00970     return i * (exp(-z)-exp(z)) / Scalar_T(2);
00971 }
00972
00973 template< template<typename, const index_t, const index_t, typename> class Multivector,
00974           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00975 inline
00976 auto
00977 sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00978 { return sin(val, complexifier(val), true); }
00979
00980 // Reference: [AS], Section 4.4, p79-83
00981 template< template<typename, const index_t, const index_t, typename> class Multivector,
00982           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00983 inline
00984 auto
00985 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00986 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00987 {
00988     using traits_t = numeric_traits<Scalar_T>;
00989     if (val.isnan())
00990         return traits_t::NaN();
00991
00992     const auto& s = val.scalar();
00993     if (val == s && traits_t::abs(s) <= Scalar_T(1))
00994         return traits_t::asin(s);
00995
00996     check_complex(val, i, prechecked);
00997     return -i * asinh(i * val, i, true);
00998 }
00999
01000 // Reference: [AS], Section 4.4, p79-83
01001 template< template<typename, const index_t, const index_t, typename> class Multivector,
01002           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01003 inline
01004 auto
01005 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01006 { return asin(val, complexifier(val), true); }
01007
01008 template< template<typename, const index_t, const index_t, typename> class Multivector,
01009           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01010 inline
01011 auto
01012 tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01013 {
01014     using traits_t = numeric_traits<Scalar_T>;
01015     if (val.isnan())
01016         return traits_t::NaN();
01017
01018     const auto& s = val.scalar();

```

```

01023     if (val == s)
01024         return traits_t::tanh(s);
01025
01026     return sinh(val) / cosh(val);
01027 }
01028
01030 // Reference: [AS], Section 4.6, p86-89
01031 template< template<typename, const index_t, const index_t, typename> class Multivector,
01032           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01033 inline
01034 auto
01035 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
01036 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01037 {
01038     using traits_t = numeric_traits<Scalar_T>;
01039     check_complex(val, i, prechecked);
01040     return val.isnan()
01041         ? traits_t::NaN()
01042         : (norm(val + Scalar_T(1)) > norm(val - Scalar_T(1)))
01043           ? (log(val + Scalar_T(1), i, true) - log(-val + Scalar_T(1), i, true)) / Scalar_T(2)
01044           : log((val + Scalar_T(1)) / (-val + Scalar_T(1)), i, true) / Scalar_T(2);
01045 }
01047 // Reference: [AS], Section 4.6, p86-89
01048 template< template<typename, const index_t, const index_t, typename> class Multivector,
01049           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01050 inline
01051 auto
01052 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01053 { return atanh(val, complexifier(val), true); }
01054
01056 template< template<typename, const index_t, const index_t, typename> class Multivector,
01057           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01058 inline
01059 auto
01060 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
01061 prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01062 {
01063     using traits_t = numeric_traits<Scalar_T>;
01064     if (val.isnan())
01065         return traits_t::NaN();
01066
01067     const auto& s = val.scalar();
01068     if (val == s)
01069         return traits_t::tan(s);
01070
01071     check_complex(val, i, prechecked);
01072     return sin(val, i, true) / cos(val, i, true);
01073 }
01075 template< template<typename, const index_t, const index_t, typename> class Multivector,
01076           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01077 inline
01078 auto
01079 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01080 { return tan(val, complexifier(val), true); }
01081
01083 // Reference: [AS], Section 4.4, p79-83
01084 template< template<typename, const index_t, const index_t, typename> class Multivector,
01085           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01086 inline
01087 auto
01088 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
01089 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01090 {
01091     using traits_t = numeric_traits<Scalar_T>;
01092     if (val.isnan())
01093         return traits_t::NaN();
01094
01095     const auto& s = val.scalar();
01096     if (val == s)
01097         return traits_t::atan(s);
01098
01099     check_complex(val, i, prechecked);
01100     return -i * atanh(i * val, i, true);
01101 }
01103 // Reference: [AS], Section 4.4, p79-83
01104 template< template<typename, const index_t, const index_t, typename> class Multivector,
01105           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01106 inline
01107 auto
01108 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01109 { return atan(val, complexifier(val), true); }
01110 }
01112 #endif // _GLUCAT_CLIFFORD_ALGEBRA_IMP_H

```



## 7.6 errors.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_ERRORS_H
00002 #define _GLUCAT_ERRORS_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     errors.h : Declare error classes and functions
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009 *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include <string>
00035 #include <exception>
00036 #include <stdexcept>
00037
00038 namespace glucat
00039 {
00040     class glucat_error : public std::logic_error
00041     {
00042     public:
00043         glucat_error(const std::string& context, const std::string& msg)
00044             : logic_error(msg), name(context)
00045         { }
00046         ~glucat_error() noexcept override = default;
00047         virtual auto heading() const noexcept -> const std::string =0;
00048         virtual auto classname() const noexcept -> const std::string =0;
00049         virtual void print_error_msg() const =0;
00050         std::string name;
00051     };
00052
00053     template< class Class_T >
00054     class error : public glucat_error
00055     {
00056     public:
00057         error(const std::string& msg);
00058         error(const std::string& context, const std::string& msg);
00059         auto heading() const noexcept -> const std::string override;
00060         auto classname() const noexcept -> const std::string override;
00061         void print_error_msg() const override;
00062     };
00063
00064 }
00065 #endif // _GLUCAT_ERRORS_H

```

## 7.7 glucat/errors\_imp.h File Reference

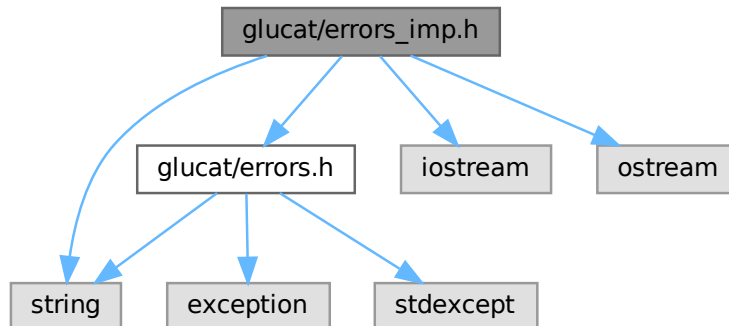
```

#include "glucate/errors.h"
#include <string>
#include <iostream>

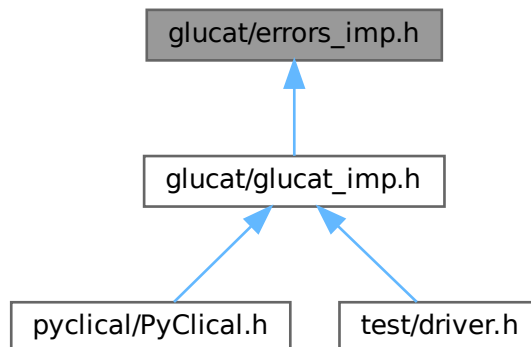
```

```
#include <ostream>
```

Include dependency graph for errors\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`

## 7.8 errors\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_ERRORS_IMP_H
00002 #define _GLUCAT_ERRORS_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     errors_imp.h : Define error functions
  
```

```

00006      -----
00007      begin                : Sun 2001-12-20
00008      copyright            : (C) 2001-2007 by Paul C. Leopardi
00009      *****
00010
00011      This library is free software: you can redistribute it and/or modify
00012      it under the terms of the GNU Lesser General Public License as published
00013      by the Free Software Foundation, either version 3 of the License, or
00014      (at your option) any later version.
00015
00016      This library is distributed in the hope that it will be useful,
00017      but WITHOUT ANY WARRANTY; without even the implied warranty of
00018      MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019      GNU Lesser General Public License for more details.
00020
00021      You should have received a copy of the GNU Lesser General Public License
00022      along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024      *****
00025      This library is based on a prototype written by Arvind Raja and was
00026      licensed under the LGPL with permission of the author. See Arvind Raja,
00027      "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028      in Ablamowicz, Lounesto and Parra (eds.)
00029      "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030      *****
00031      See also Arvind Raja's original header comments in glucat.h
00032      *****/
00033
00034      #include "glucat/errors.h"
00035
00036      #include <string>
00037      #include <iostream>
00038      #include <ostream>
00039
00040      namespace glucat
00041      {
00042      template< class Class_T >
00043      error<Class_T>::
00044      error(const std::string& msg)
00045      : glucat_error(Class_T::classname(), msg)
00046      { }
00047
00048      template< class Class_T >
00049      error<Class_T>::
00050      error(const std::string& context, const std::string& msg)
00051      : glucat_error(context, msg)
00052      { }
00053
00054      template< class Class_T >
00055      auto
00056      error<Class_T>::
00057      heading() const noexcept -> const std::string
00058      { return "Error in glucat:."; }
00059
00060      template< class Class_T >
00061      auto
00062      error<Class_T>::
00063      classname() const noexcept -> const std::string
00064      { return name; }
00065
00066      template< class Class_T >
00067      void
00068      error<Class_T>::
00069      print_error_msg() const
00070      { std::cerr << heading() << classname() << std::endl << what() << std::endl; }
00071      }
00072      #endif // _GLUCAT_ERRORS_IMP_H

```

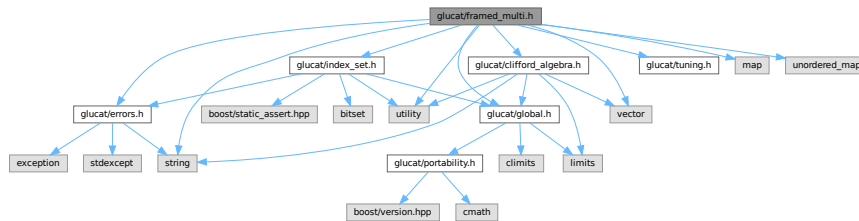
## 7.9 glucat/framed\_multi.h File Reference

```

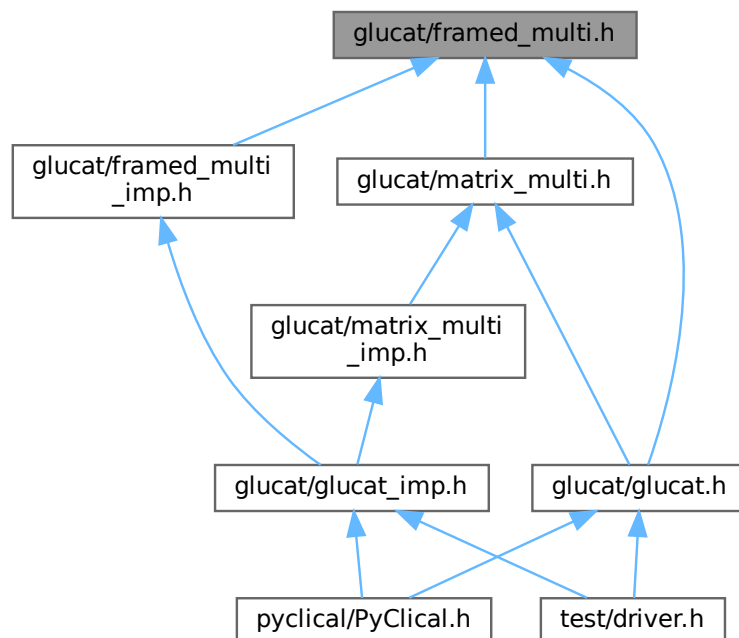
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include <string>
#include <utility>
#include <map>

```

```
#include <unordered_map>
#include <vector>
Include dependency graph for framed_multi.h:
```



This graph shows which files directly or indirectly include this file:



## Classes

- class [glucat::index\\_set\\_hash< LO, HI >](#)
- class [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >](#)  
*A framed\_multi<Scalar\_T,LO,HI,Tune\_P> is a framed approximation to a multivector.*
- class [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::hash\\_size\\_t](#)
- class [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::var\\_term](#)  
*Variable term.*
- struct [std::numeric\\_limits< glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P > >](#)  
*Numeric limits for framed\_multi inherit limits for the corresponding scalar type.*



## Namespaces

- namespace [glucat](#)
- namespace [std](#)

## Functions

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator\\*](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Geometric product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator^](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Outer product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator&](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Inner product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator%](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Left contraction.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::star](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T  
*Hestenes scalar product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator/](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Geometric quotient.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator|](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Transformation via twisted adjoint action.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator>>](#) (std::istream &s, [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::istream &  
*Read multivector from input.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator<<](#) (std::ostream &os, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::ostream &  
*Write multivector to output.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI>  
auto [glucat::operator<<](#) (std::ostream &os, const std::pair< const [index\\_set](#)< LO, HI >, Scalar\_T > &term) -> std::ostream &  
*Write term to output.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::exp](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Exponential of multivector.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI>  
static auto [glucat::crd\\_of\\_mult](#) (const std::pair< const [index\\_set](#)< LO, HI >, Scalar\_T > &lhs, const std::pair< const [index\\_set](#)< LO, HI >, Scalar\_T > &rhs) -> Scalar\_T

*Coordinate of product of terms.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI>`  
`auto glucat::operator\* (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index\_set< LO, HI >, Scalar_T >`

*Product of terms.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::sqrt (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::log (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

## 7.10 [framed\\_multi.h](#)

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_FRAMED_MULTI_H
00002 #define _GLUCAT_FRAMED_MULTI_H
00003 /*****
00004  GluCat : Generic library of universal Clifford algebra templates
00005  framed_multi.h : Declare a class for the framed representation of a multivector
00006  -----
00007  begin                : Sun 2001-12-09
00008  copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  *****/
00010
00011  This library is free software: you can redistribute it and/or modify
00012  it under the terms of the GNU Lesser General Public License as published
00013  by the Free Software Foundation, either version 3 of the License, or
00014  (at your option) any later version.
00015
00016  This library is distributed in the hope that it will be useful,
00017  but WITHOUT ANY WARRANTY; without even the implied warranty of
00018  MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019  GNU Lesser General Public License for more details.
00020
00021  You should have received a copy of the GNU Lesser General Public License
00022  along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024  *****/
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  *****/
00031  See also Arvind Raja's original header comments in glucat.h
00032  *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039
00040 #if defined(_GLUCAT_USE_BOOST_POOL_ALLOC)
00041 // Use the Boost pool allocator
00042 #include <boost/pool/poolfwd.hpp>
00043 #endif
00044
00045 #include <string>
00046 #include <utility>
00047 #include <map>
00048 #include <unordered_map>
00049 #include <vector>
00050
00051 namespace glucat
00052 {
00053     // Forward declarations for friends
00054
00055     template< typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >
00056     class framed\_multi; // forward

```

```

00057
00058     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00059     class matrix_multi; // forward
00060
00062     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00063     auto
00064     operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00065     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00066
00067     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00068     auto
00069     operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00070     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00071
00072     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00073     auto
00074     operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00075     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00076
00077     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00078     auto
00079     operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00080     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00081
00082     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00083     auto
00084     star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
00085     -> Scalar_T;
00086
00087     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00088     auto
00089     operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00090     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00091
00092     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00093     auto
00094     operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00095     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00096
00097     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00098     auto
00099     operator> (std::istream& s, framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00100
00102     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00103     auto
00104     operator<< (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00105
00107     template< typename Scalar_T, const index_t LO, const index_t HI >
00108     auto
00109     operator<< (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
00110     std::ostream&;
00111
00112     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00113     auto
00114     exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00115
00116     template< const index_t LO, const index_t HI>
00117     class index_set_hash
00118     {
00119     public:
00120         using index_set_t = index_set<LO, HI>;
00121         inline auto operator()(index_set_t val) const -> size_t { return val.hash_fn(); }
00122     };
00123
00125     template< typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
00126     typename Tune_P = tuning<> >
00127     class framed_multi :
00128     public clifford_algebra< Scalar_T, index_set<LO,HI>, framed_multi<Scalar_T,LO,HI,Tune_P> >,
00129     private std::unordered_map< index_set<LO,HI>, Scalar_T, index_set_hash<LO,HI> >
00130     {
00131     public:
00132         using multivector_t = framed_multi;
00133         using framed_multi_t = multivector_t;
00134         using scalar_t = Scalar_T;
00135         using tune_p = Tune_P;
00136         using index_set_t = index_set<LO, HI>;
00137         using term_t = std::pair<const index_set_t, Scalar_T>;
00138         using vector_t = std::vector<Scalar_T>;
00139         using error_t = error<multivector_t>;
00140         using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P >;
00141         template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
00142         Other_Tune_P >
00143         friend class matrix_multi;
00144         template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
00145         Other_Tune_P >
00146         friend class framed_multi;

```

```

00145 private:
00146     class var_term; // forward
00147     using var_term_t = class var_term;
00148     using matrix_t = typename matrix_multi_t::matrix_t;
00149     using sorted_map_t = std::map< index_set_t, Scalar_T, std::less<const index_set_t> >;
00150     using map_t = std::unordered_map<index_set_t, Scalar_T, index_set_hash<LO, HI>;
00151
00152     class hash_size_t
00153     {
00154     public:
00155         hash_size_t(size_t hash_size)
00156             : n(hash_size)
00157             { };
00158         auto operator() () const -> size_t
00159             { return n; }
00160     private:
00161         size_t n;
00162     };
00163
00164     using framed_pair_t = std::pair<const multivector_t, const multivector_t>;
00165     using size_type = typename map_t::size_type;
00166     using iterator = typename map_t::iterator;
00167     using const_iterator = typename map_t::const_iterator;
00168
00169     public:
00170         static auto classname() -> const std::string;
00171         ~framed_multi() override = default;
00172         framed_multi();
00173
00174     private:
00175         framed_multi(const hash_size_t& hash_size);
00176
00177     public:
00178         template< typename Other_Scalar_T >
00179         framed_multi(const framed_multi<Other_Scalar_T, LO, HI, Tune_P>& val);
00180         template< typename Other_Scalar_T >
00181         framed_multi(const framed_multi<Other_Scalar_T, LO, HI, Tune_P>& val,
00182                     const index_set_t frm, const bool prechecked = false);
00183         framed_multi(const framed_multi_t& val,
00184                     const index_set_t frm, const bool prechecked = false);
00185         framed_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00186         framed_multi(const index_set_t ist, const Scalar_T& crd,
00187                     const index_set_t frm, const bool prechecked = false);
00188         framed_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());
00189         framed_multi(const int scr, const index_set_t frm = index_set_t());
00190         framed_multi(const vector_t& vec,
00191                     const index_set_t frm, const bool prechecked = false);
00192         framed_multi(const std::string& str);
00193         framed_multi(const std::string& str,
00194                     const index_set_t frm, const bool prechecked = false);
00195         framed_multi(const char* str)
00196         { *this = framed_multi(std::string(str)); };
00197         framed_multi(const char* str,
00198                     const index_set_t frm, const bool prechecked = false)
00199         { *this = framed_multi(std::string(str), frm, prechecked); };
00200         template< typename Other_Scalar_T >
00201         framed_multi(const matrix_multi<Other_Scalar_T, LO, HI, Tune_P >& val);
00202         template< typename Other_Scalar_T >
00203         auto fast_matrix_multi(const index_set_t frm) const -> const
00204             matrix_multi<Other_Scalar_T, LO, HI, Tune_P >;
00205         auto fast_framed_multi() const -> const framed_multi_t;
00206
00207     _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00208
00209     auto nbr_terms() const -> unsigned long;
00210
00211     static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const multivector_t;
00212
00213     // Friend declarations
00214
00215     friend auto
00216         operator* <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00217     friend auto
00218         operator^ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00219     friend auto
00220         operator& <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00221     friend auto
00222         operator% <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00223     friend auto
00224         star <>(const multivector_t& lhs, const multivector_t& rhs) -> Scalar_T;
00225     friend auto
00226         operator/ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00227     friend auto
00228         operator| <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00229
00230     friend auto
00231         operator<> <>(std::istream& s, multivector_t& val) -> std::istream&;
00232     friend auto

```

```

00252     operator<< (<>(std::ostream& os, const multivector_t& val) -> std::ostream&;
00253 friend auto
00254     operator<< (<>(std::ostream& os, const term_t& term) -> std::ostream&;
00255
00256 friend auto
00257     exp (<>(const multivector_t& val) -> const multivector_t;
00258
00260 auto     operator+= (const term_t& term) -> multivector_t&;
00261
00262 private:
00264 auto     fold(const index_set_t frm) const -> multivector_t;
00266 auto     unfold(const index_set_t frm) const -> multivector_t;
00268 auto     centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&;
00270 auto     centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&;
00272 auto     centre_qp1_pm1(index_t& p, index_t& q) -> multivector_t&;
00274 auto     divide(const index_set_t ist) const -> const framed_pair_t;
00276 auto     fast(const index_t level, const bool odd) const -> const matrix_t;
00277
00279 class var_term :
00280 public std::pair<index_set<LO,HI>, Scalar_T>
00281 {
00282 public:
00283     using var_pair_t = std::pair<index_set<LO, HI>, Scalar_T>;
00284
00286     static auto classname() -> const std::string
00287     { return "var_term"; };
00289     ~var_term() = default;
00291     var_term()
00292     : var_pair_t(index_set_t(), Scalar_T(1))
00293     { };
00295     var_term(const index_set_t ist, const Scalar_T& crd = Scalar_T(1))
00296     : var_pair_t(ist, crd)
00297     { };
00299     auto operator*= (const term_t& rhs) -> var_term_t&
00300     {
00301         this->second *= rhs.second * this->first.sign_of_mult(rhs.first);
00302         this->first ^= rhs.first;
00303         return *this;
00304     }
00305 };
00306 };
00307
00308 // Non-members
00309
00311 template< typename Scalar_T, const index_t LO, const index_t HI >
00312 inline
00313 static
00314 auto
00315 crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00316             const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T;
00317
00319 template< typename Scalar_T, const index_t LO, const index_t HI >
00320 auto
00321 operator*
00322 (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00323  const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const index_set<LO,HI>,
00324 Scalar_T>;
00326 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327 auto
00328 sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
00329 bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00331 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00332 auto
00333 exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00334
00336 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00337 auto
00338 log(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
00339 bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00340 }
00341 namespace std
00342 {
00344     template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00345     struct numeric_limits< glucat::framed_multi<Scalar_T,LO,HI,Tune_P> > :
00346     public numeric_limits<Scalar_T>
00347     { };
00348 }
00349 #endif // _GLUCAT_FRAMED_MULTI_H

```



## Macros

- `#define _GLUCAT_HASH_N(x)`
- `#define _GLUCAT_HASH_SIZE_T(x)`

## Functions

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator* (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator^ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Outer product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator& (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Inner product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator% (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi<`  
`Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Left contraction.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::star (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> Scalar_T`  
*Hestenes scalar product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator/ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator| (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator<< (std::ostream &os, const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std_↵`  
`::ostream &`  
*Write multivector to output.*
- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`auto glucat::operator<< (std::ostream &os, const std::pair< const index_set< LO, HI >, Scalar_T > &term)`  
`-> std::ostream &`  
*Write term to output.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator>> (std::istream &s, framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream`  
`&`  
*Read multivector from input.*
- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`static auto glucat::crd_of_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std_↵`  
`::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`  
*Coordinate of product of terms.*
- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`auto glucat::operator* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const`  
`index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >`

*Product of terms.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::sqrt (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::exp (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed\_multi< Scalar_T,`  
`LO, HI, Tune_P >`

*Exponential of multivector.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::log (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

## 7.11.1 Macro Definition Documentation

### 7.11.1.1 `_GLUCAT_HASH_N`

```
#define _GLUCAT_HASH_N(  
    x)
```

**Value:**

(x)

Definition at line 54 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

### 7.11.1.2 `_GLUCAT_HASH_SIZE_T`

```
#define _GLUCAT_HASH_SIZE_T(  
    x)
```

**Value:**

([typename multivector\\_t::hash\\_size\\_t](#)) (x)

Definition at line 55 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::operator%\(\)](#), [glucat::operator&\(\)](#), [glucat::operator\\*\(\)](#), and [glucat::operator^\(\)](#).



## 7.12 framed\_multi\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_FRAMED_MULTI_IMP_H
00002 #define _GLUCAT_FRAMED_MULTI_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     framed_multi_imp.h : Implement the coordinate map representation of a
00006     Clifford algebra element
00007     -----
00008     begin                : Sun 2001-12-09
00009     copyright             : (C) 2001-2021 by Paul C. Leopardi
00010     *****/
00011
00012     This library is free software: you can redistribute it and/or modify
00013     it under the terms of the GNU Lesser General Public License as published
00014     by the Free Software Foundation, either version 3 of the License, or
00015     (at your option) any later version.
00016
00017     This library is distributed in the hope that it will be useful,
00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
00023     along with this library. If not, see <http://www.gnu.org/licenses/>.
00024
00025     *****/
00026     This library is based on a prototype written by Arvind Raja and was
00027     licensed under the LGPL with permission of the author. See Arvind Raja,
00028     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029     in Ablamowicz, Lounesto and Parra (eds.)
00030     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031     *****/
00032     See also Arvind Raja's original header comments in glucat.h
00033     *****/
00034
00035 #include "glucat/framed_multi.h"
00036
00037 #include "glucat/scalar.h"
00038 #include "glucat/random.h"
00039 #include "glucat/generation.h"
00040 #include "glucat/matrix.h"
00041
00042 #include <sstream>
00043 #include <fstream>
00044
00045 namespace glucat
00046 {
00047     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00048     auto
00049     framed_multi<Scalar_T,LO,HI,Tune_P>::
00050     classname() -> const std::string
00051     { return "framed_multi"; }
00052
00053 #define _GLUCAT_HASH_N(x) (x)
00054 #define _GLUCAT_HASH_SIZE_T(x) (typename multivector_t::hash_size_t)(x)
00055
00056     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00057     framed_multi<Scalar_T,LO,HI,Tune_P>::
00058     framed_multi()
00059     : map_t(_GLUCAT_HASH_N(0))
00060     { }
00061
00062     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00063     framed_multi<Scalar_T,LO,HI,Tune_P>::
00064     framed_multi(const hash_size_t& hash_size)
00065     : map_t(_GLUCAT_HASH_N(hash_size()))
00066     { }
00067
00068     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00069     template< typename Other_Scalar_T >
00070     framed_multi<Scalar_T,LO,HI,Tune_P>::
00071     framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00072     : map_t(_GLUCAT_HASH_N(val.size()))
00073     {
00074         for (auto& val_term : val)
00075             this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00076     }
00077
00078     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00079     template< typename Other_Scalar_T >
00080     framed_multi<Scalar_T,LO,HI,Tune_P>::
00081     framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00082                 const index_set_t frm, const bool prechecked)

```

```

00088 : map_t(_GLUCAT_HASH_N(val.size()))
00089 {
00090     if (!prechecked && (val.frame() | frm) != frm)
00091         throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00092     for (auto& val_term : val)
00093         this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00094 }
00095
00097 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00098 framed_multi<Scalar_T,LO,HI,Tune_P>::
00099 framed_multi(const multivector_t& val,
00100             const index_set_t frm, const bool prechecked)
00101 : map_t(_GLUCAT_HASH_N(val.size()))
00102 {
00103     if (!prechecked && (val.frame() | frm) != frm)
00104         throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00105     for (auto& val_term : val)
00106         this->insert(val_term);
00107 }
00108
00110 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00111 framed_multi<Scalar_T,LO,HI,Tune_P>::
00112 framed_multi(const index_set_t ist, const Scalar_T& crd)
00113 : map_t(_GLUCAT_HASH_N(1))
00114 {
00115     if (crd != Scalar_T(0))
00116         this->insert(term_t(ist, crd));
00117 }
00118
00120 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00121 framed_multi<Scalar_T,LO,HI,Tune_P>::
00122 framed_multi(const index_set_t ist, const Scalar_T& crd,
00123             const index_set_t frm, const bool prechecked)
00124 : map_t(_GLUCAT_HASH_N(1))
00125 {
00126     if (!prechecked && (ist | frm) != frm)
00127         throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00128     if (crd != Scalar_T(0))
00129         this->insert(term_t(ist, crd));
00130 }
00131
00133 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00134 framed_multi<Scalar_T,LO,HI,Tune_P>::
00135 framed_multi(const Scalar_T& scr, const index_set_t frm)
00136 : map_t(_GLUCAT_HASH_N(1))
00137 {
00138     if (scr != Scalar_T(0))
00139         this->insert(term_t(index_set_t(), scr));
00140 }
00141
00143 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00144 framed_multi<Scalar_T,LO,HI,Tune_P>::
00145 framed_multi(const int scr, const index_set_t frm)
00146 : map_t(_GLUCAT_HASH_N(1))
00147 {
00148     if (scr != Scalar_T(0))
00149         this->insert(term_t(index_set_t(), Scalar_T(scr)));
00150 }
00151
00153 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00154 framed_multi<Scalar_T,LO,HI,Tune_P>::
00155 framed_multi(const vector_t& vec,
00156             const index_set_t frm, const bool prechecked)
00157 : map_t(_GLUCAT_HASH_N(vec.size()))
00158 {
00159     if (!prechecked && index_t(vec.size()) != frm.count())
00160         throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00161     auto idx = frm.min();
00162     const auto frm_end = frm.max()+1;
00163     for (auto& crd : vec)
00164     {
00165         *this += term_t(index_set_t(idx), crd);
00166         for (
00167             ++idx;
00168             idx != frm_end && !frm[idx];
00169             ++idx)
00170         ;
00171     }
00172 }
00173
00175 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00176 framed_multi<Scalar_T,LO,HI,Tune_P>::
00177 framed_multi(const std::string& str)
00178 : map_t(_GLUCAT_HASH_N(0))
00179 {
00180     std::istringstream ss(str);
00181     ss » *this;

```

```

00182     if (!ss)
00183         throw error_t("multivector_t(str): could not parse string");
00184     // Peek to see if the end of the string has been reached.
00185     ss.peek();
00186     if (!ss.eof())
00187         throw error_t("multivector_t(str): could not parse entire string");
00188 }
00189
00191 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00192 framed_multi<Scalar_T,LO,HI,Tune_P>::
00193 framed_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00194 : map_t(_GLUCAT_HASH_N(0))
00195 {
00196     if (prechecked)
00197         *this = multivector_t(str);
00198     else
00199         *this = multivector_t(multivector_t(str), frm, false);
00200 }
00201
00203 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00204 template< typename Other_Scalar_T >
00205 framed_multi<Scalar_T,LO,HI,Tune_P>::
00206 framed_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00207 : map_t(_GLUCAT_HASH_N(1))
00208 {
00209     if (val == Other_Scalar_T(0))
00210         return;
00211
00212     const auto dim = val.m_matrix.size();
00213     using traits_t = numeric_traits<Scalar_T>;
00214     if (dim == 1)
00215     {
00216         this->insert(term_t(index_set_t(), traits_t::to_scalar_t(val.m_matrix(0, 0))));
00217         return;
00218     }
00219     if (dim >= Tune_P::inv_fast_dim_threshold)
00220     try
00221     {
00222         *this = (val.template fast_framed_multi<Scalar_T>()).truncated();
00223         return;
00224     }
00225     catch (const glucat_error& e)
00226     { }
00227
00228     const auto val_norm = traits_t::to_scalar_t(val.norm());
00229     if (traits_t::isNaN_or_isInf(val_norm))
00230     {
00231         *this = traits_t::NaN();
00232         return;
00233     }
00234     const auto frm = val.frame();
00235     const auto algebra_dim = set_value_t(1) << frm.count();
00236     auto result = multivector_t(
00237         _GLUCAT_HASH_SIZE_T(std::min<size_t>(algebra_dim, matrix::nnz(val.m_matrix))));
00238     for (auto
00239         stv = set_value_t(0);
00240         stv != algebra_dim;
00241         stv++)
00242     {
00243         const auto ist = index_set_t(stv, frm, true);
00244         const auto crd =
00245             traits_t::to_scalar_t(matrix::inner<Other_Scalar_T>(val.basis_element(ist), val.m_matrix));
00246         if (crd != Scalar_T(0))
00247             result.insert(term_t(ist, crd));
00248     }
00249     *this = result.truncated();
00250 }
00251
00253 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00254 auto
00255 framed_multi<Scalar_T,LO,HI,Tune_P>::
00256 operator==(const multivector_t& rhs) const -> bool
00257 {
00258     if (this->size() != rhs.size())
00259         return false;
00260     const auto rhs_end = rhs.end();
00261     for (auto& this_term : *this)
00262     {
00263         const const_iterator& rhs_it = rhs.find(this_term.first);
00264         if (rhs_it == rhs_end || rhs_it->second != this_term.second)
00265             return false;
00266     }
00267     return true;
00268 }
00269
00271 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00272 inline

```

```

00273     auto
00274     framed_multi<Scalar_T,LO,HI,Tune_P>::
00275     operator== (const Scalar_T& scr) const -> bool
00276     {
00277         switch (this->size())
00278         {
00279             case 0:
00280                 return scr == Scalar_T(0);
00281             case 1:
00282                 {
00283                     const auto& this_it = this->begin();
00284                     return this_it->first == index_set_t() && this_it->second == scr;
00285                 }
00286             default:
00287                 return false;
00288         }
00289     }
00290
00291     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00292     inline
00293     auto
00294     framed_multi<Scalar_T,LO,HI,Tune_P>::
00295     operator+= (const Scalar_T& scr) -> multivector_t&
00296     {
00297         *this += term_t(index_set_t(), scr);
00298         return *this;
00299     }
00300
00301     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00302     inline
00303     auto
00304     framed_multi<Scalar_T,LO,HI,Tune_P>::
00305     operator+= (const multivector_t& rhs) -> multivector_t&
00306     { // simply add terms
00307         for (auto& rhs_term : rhs)
00308             *this += rhs_term;
00309         return *this;
00310     }
00311
00312     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00313     inline
00314     auto
00315     framed_multi<Scalar_T,LO,HI,Tune_P>::
00316     operator-= (const Scalar_T& scr) -> multivector_t&
00317     {
00318         *this += term_t(index_set_t(), -scr);
00319         return *this;
00320     }
00321
00322     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00323     inline
00324     auto
00325     framed_multi<Scalar_T,LO,HI,Tune_P>::
00326     operator-= (const multivector_t& rhs) -> multivector_t&
00327     {
00328         for (auto& rhs_term : rhs)
00329             *this += term_t(rhs_term.first, -(rhs_term.second));
00330         return *this;
00331     }
00332
00333     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00334     inline
00335     auto
00336     framed_multi<Scalar_T,LO,HI,Tune_P>::
00337     operator- () const -> const multivector_t
00338     { // multiply coordinates of all terms by -1
00339         auto result = *this;
00340         for (auto& result_term : result)
00341             result_term.second *= Scalar_T(-1);
00342         return result;
00343     }
00344
00345     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00346     auto
00347     framed_multi<Scalar_T,LO,HI,Tune_P>::
00348     operator*= (const Scalar_T& scr) -> multivector_t&
00349     { // multiply coordinates of all terms by scalar
00350         using traits_t = numeric_traits<Scalar_T>;
00351
00352         if (traits_t::isNaN_or_isInf(scr))
00353             return *this = traits_t::NaN();
00354         if (scr == Scalar_T(0))
00355             if (this->isnan())
00356                 *this = traits_t::NaN();
00357             else
00358                 this->clear();
00359         else
00360             *this *= scr;
00361     }

```

```

00366         for (auto& this_term : *this)
00367             this_term.second *= scr;
00368         return *this;
00369     }
00370
00371     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00372     auto
00373     operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00374     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00375     {
00376         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00377         using traits_t = numeric_traits<Scalar_T>;
00378
00379         if (lhs.isnan() || rhs.isnan())
00380             return traits_t::NaN();
00381
00382         const double lhs_size = lhs.size();
00383         const double rhs_size = rhs.size();
00384         const auto our_frame = lhs.frame() | rhs.frame();
00385         const auto frm_count = our_frame.count();
00386         const auto algebra_dim = set_value_t(1) << frm_count;
00387         const auto direct_mult = lhs_size * rhs_size <= double(algebra_dim);
00388         if (direct_mult)
00389         { // If we have a sparse multiply, store the result directly
00390             auto result = multivector_t(
00391                 _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00392             for (auto& lhs_term : lhs)
00393                 for (auto& rhs_term : rhs)
00394                     result += lhs_term * rhs_term;
00395             return result;
00396         }
00397         else
00398         { // Past a certain threshold, the matrix algorithm is fastest
00399             using matrix_multi_t = typename multivector_t::matrix_multi_t;
00400             return matrix_multi_t(lhs, our_frame, true) *
00401                 matrix_multi_t(rhs, our_frame, true);
00402         }
00403     }
00404
00405     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00406     inline
00407     auto
00408     framed_multi<Scalar_T,LO,HI,Tune_P>::
00409     operator*= (const multivector_t& rhs) -> multivector_t&
00410     { return *this = *this * rhs; }
00411
00412     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00413     auto
00414     operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00415     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00416     { // Arvind Raja's original reference:
00417         // "old clical, outerproduct(p,q:pterm):pterm in file compmod.pas"
00418
00419         if (lhs.empty() || rhs.empty())
00420             return Scalar_T(0);
00421
00422         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00423         using index_set_t = typename multivector_t::index_set_t;
00424         using term_t = typename multivector_t::term_t;
00425
00426         const auto empty_set = index_set_t();
00427
00428         const double lhs_size = lhs.size();
00429         const double rhs_size = rhs.size();
00430         const auto lhs_frame = lhs.frame();
00431         const auto rhs_frame = rhs.frame();
00432         const auto our_frame = lhs_frame | rhs_frame;
00433         const auto algebra_dim = set_value_t(1) << our_frame.count();
00434         auto result = multivector_t(
00435             _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00436         const auto lhs_end = lhs.end();
00437         const auto rhs_end = rhs.end();
00438
00439         if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00440         {
00441             for (auto
00442                 result_stv = set_value_t(0);
00443                 result_stv != algebra_dim;
00444                 ++result_stv)
00445             {
00446                 const auto result_ist = index_set_t(result_stv, our_frame, true);
00447                 const auto lhs_result_frame = lhs_frame & result_ist;
00448                 const auto lhs_result_dim = set_value_t(1) << lhs_result_frame.count();
00449                 auto result_crd = Scalar_T(0);
00450                 for (auto
00451                     lhs_stv = set_value_t(0);
00452                     lhs_stv != lhs_result_dim;

```

```

00454         ++lhs_stv)
00455     {
00456         const auto lhs_ist = index_set_t(lhs_stv, lhs_result_frame, true);
00457         const auto rhs_ist = result_ist ^ lhs_ist;
00458         if ((rhs_ist | rhs_frame) == rhs_frame)
00459         {
00460             const auto lhs_it = lhs.find(lhs_ist);
00461             if (lhs_it != lhs_end)
00462             {
00463                 const auto rhs_it = rhs.find(rhs_ist);
00464                 if (rhs_it != rhs_end)
00465                     result_crd += crd_of_mult(*lhs_it, *rhs_it);
00466             }
00467         }
00468     }
00469     if (result_crd != Scalar_T(0))
00470         result.insert(term_t(result_ist, result_crd));
00471 }
00472 return result;
00473 }
00474 else
00475 {
00476     for (auto& lhs_term : lhs)
00477     for (auto& rhs_term : rhs)
00478     if ((lhs_term.first & rhs_term.first) == empty_set)
00479         result += lhs_term * rhs_term;
00480 return result;
00481 }
00482 }
00483
00484 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00485 inline
00486 auto
00487 framed_multi<Scalar_T,LO,HI,Tune_P>::
00488 operator^= (const multivector_t& rhs) -> multivector_t&
00489 { return *this = *this ^ rhs; }
00490
00491 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00492 auto
00493 operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00494 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00495 { // Arvind Raja's original reference:
00496 // "old clical, innerproduct(p,q:pterm):pterm in file compmod.pas"
00497
00498 if (lhs.empty() || rhs.empty())
00499     return Scalar_T(0);
00500
00501 using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00502 using index_set_t = typename multivector_t::index_set_t;
00503 using term_t = typename multivector_t::term_t;
00504
00505 const auto lhs_end = lhs.end();
00506 const auto rhs_end = rhs.end();
00507 const double lhs_size = lhs.size();
00508 const double rhs_size = rhs.size();
00509
00510 const auto lhs_frame = lhs.frame();
00511 const auto rhs_frame = rhs.frame();
00512
00513 const auto our_frame = lhs_frame | rhs_frame;
00514 const auto algebra_dim = set_value_t(1) << our_frame.count();
00515 auto result = multivector_t(
00516     _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00517 if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00518 {
00519     for (auto
00520         result_stv = set_value_t(0);
00521         result_stv != algebra_dim;
00522         ++result_stv)
00523     {
00524         const auto result_ist = index_set_t(result_stv, our_frame, true);
00525         const auto comp_frame = our_frame & ~result_ist;
00526         const auto comp_dim = set_value_t(1) << comp_frame.count();
00527         auto result_crd = Scalar_T(0);
00528         for (auto
00529             comp_stv = set_value_t(1);
00530             comp_stv != comp_dim;
00531             ++comp_stv)
00532         {
00533             const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00534             const auto our_ist = result_ist ^ comp_ist;
00535             if ((our_ist | lhs_frame) == lhs_frame)
00536             {
00537                 const auto lhs_it = lhs.find(our_ist);
00538                 if (lhs_it != lhs_end)
00539                 {
00540                     const auto rhs_it = rhs.find(comp_ist);

```

```

00542         if (rhs_it != rhs_end)
00543             result_crd += crd_of_mult(*lhs_it, *rhs_it);
00544     }
00545 }
00546 if (result_stv != 0)
00547 {
00548     if ((our_ist | rhs_frame) == rhs_frame)
00549     {
00550         const auto rhs_it = rhs.find(our_ist);
00551         if (rhs_it != rhs_end)
00552         {
00553             const auto lhs_it = lhs.find(comp_ist);
00554             if (lhs_it != lhs_end)
00555                 result_crd += crd_of_mult(*lhs_it, *rhs_it);
00556         }
00557     }
00558 }
00559 }
00560 if (result_crd != Scalar_T(0))
00561     result.insert(term_t(result_ist, result_crd));
00562 }
00563 }
00564 else
00565 {
00566     const auto empty_set = index_set_t();
00567     for (auto& lhs_term : lhs)
00568     {
00569         const auto lhs_ist = lhs_term.first;
00570         if (lhs_ist != empty_set)
00571             for (auto& rhs_term : rhs)
00572             {
00573                 const auto rhs_ist = rhs_term.first;
00574                 if (rhs_ist != empty_set)
00575                 {
00576                     const auto our_ist = lhs_ist | rhs_ist;
00577                     if ((lhs_ist == our_ist) || (rhs_ist == our_ist))
00578                         result += lhs_term * rhs_term;
00579                 }
00580             }
00581     }
00582 }
00583 return result;
00584 }
00585
00586 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00587 inline
00588 auto
00589 framed_multi<Scalar_T,LO,HI,Tune_P>::
00590 operator*= (const multivector_t& rhs) -> multivector_t&
00591 { return *this = *this & rhs; }
00592
00593 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00594 auto
00595 operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00596 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00597 {
00598     // Reference: Leo Dorst, "Honing geometric algebra for its use in the computer sciences",
00599     // in Geometric Computing with Clifford Algebras, ed. G. Sommer,
00600     // Springer 2001, Chapter 6, pp. 127-152.
00601     // http://staff.science.uva.nl/~leo/clifford/index.html
00602
00603     if (lhs.empty() || rhs.empty())
00604         return Scalar_T(0);
00605
00606     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00607     using index_set_t = typename multivector_t::index_set_t;
00608     using term_t = typename multivector_t::term_t;
00609     using map_t = typename multivector_t::map_t;
00610
00611     const auto lhs_end = lhs.end();
00612     const auto rhs_end = rhs.end();
00613     const double lhs_size = lhs.size();
00614     const double rhs_size = rhs.size();
00615     const auto lhs_frame = lhs.frame();
00616     const auto rhs_frame = rhs.frame();
00617
00618     const auto our_frame = lhs_frame | rhs_frame;
00619     const auto algebra_dim = set_value_t(1) << our_frame.count();
00620     auto result = multivector_t(
00621         _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00622
00623     if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00624     {
00625         for (auto
00626             result_stv = set_value_t(0);
00627             result_stv != algebra_dim;
00628             ++result_stv)

```

```

00630     {
00631         const auto result_ist = index_set_t(result_stv, our_frame, true);
00632         const auto comp_frame = lhs_frame & ~result_ist;
00633         const auto comp_dim = set_value_t(1) << comp_frame.count();
00634         auto result_crd = Scalar_T(0);
00635         for (auto
00636             comp_stv = set_value_t(0);
00637             comp_stv != comp_dim;
00638             ++comp_stv)
00639         {
00640             const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00641             const auto rhs_ist = result_ist ^ comp_ist;
00642             if ((rhs_ist | rhs_frame) == rhs_frame)
00643             {
00644                 const auto rhs_it = rhs.find(rhs_ist);
00645                 if (rhs_it != rhs_end)
00646                 {
00647                     const auto lhs_it = lhs.find(comp_ist);
00648                     if (lhs_it != lhs_end)
00649                         result_crd += crd_of_mult(*lhs_it, *rhs_it);
00650                 }
00651             }
00652         }
00653         if (result_crd != Scalar_T(0))
00654             result.insert(term_t(result_ist, result_crd));
00655     }
00656 }
00657 else
00658 {
00659     for (auto& rhs_term : rhs)
00660     {
00661         const auto rhs_ist = rhs_term.first;
00662         for (auto& lhs_term : lhs)
00663         {
00664             const index_set_t lhs_ist = lhs_term.first;
00665             if ((lhs_ist | rhs_ist) == rhs_ist)
00666                 result += lhs_term * rhs_term;
00667         }
00668     }
00669 }
00670 return result;
00671 }
00672
00673 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00674 inline
00675 auto
00676 framed_multi<Scalar_T,LO,HI,Tune_P>::
00677 operator%=(const multivector_t& rhs) -> multivector_t&
00678 { return *this = *this % rhs; }
00679
00680 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00681 auto
00682 star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
00683 -> Scalar_T
00684 {
00685     {
00686         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00687
00688         auto result = Scalar_T(0);
00689         const auto small_star_large = lhs.size() < rhs.size();
00690         const auto* smallp =
00691             small_star_large
00692             ? &lhs
00693             : &rhs;
00694         const auto* largep =
00695             small_star_large
00696             ? &rhs
00697             : &lhs;
00698
00699         for (auto& small_term : *smallp)
00700         {
00701             const auto small_ist = small_term.first;
00702             const auto large_crd = (*largep)[small_ist];
00703             if (large_crd != Scalar_T(0))
00704                 result += small_ist.sign_of_square() * small_term.second * large_crd;
00705         }
00706         return result;
00707     }
00708
00709 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00710 auto
00711 framed_multi<Scalar_T,LO,HI,Tune_P>::
00712 operator/=(const Scalar_T& scr) -> multivector_t&
00713 { // Divide coordinates of all terms by scr
00714     using traits_t = numeric_traits<Scalar_T>;
00715
00716     if (traits_t::isNaN(scr))
00717         return *this = traits_t::NaN();
00718 }

```



```

00719     if (traits_t::isInf(scr))
00720     if (this->isnan())
00721         *this = traits_t::NaN();
00722     else
00723         this->clear();
00724     else
00725         for (auto& this_term : *this)
00726             this_term.second /= scr;
00727     return *this;
00728 }
00729
00731 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00732 inline
00733 auto
00734 operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00735 {
00736     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00737     using traits_t = numeric_traits<Scalar_T>;
00738     using index_set_t = typename multivector_t::index_set_t;
00739     using matrix_multi_t = typename multivector_t::matrix_multi_t;
00740
00741     if (rhs == Scalar_T(0))
00742         return traits_t::NaN();
00743
00744     const auto our_frame = lhs.frame() | rhs.frame();
00745     return matrix_multi_t(lhs, our_frame, true) / matrix_multi_t(rhs, our_frame, true);
00746 }
00747
00749 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00750 inline
00751 auto
00752 framed_multi<Scalar_T,LO,HI,Tune_P>::
00753 operator/= (const multivector_t& rhs) -> multivector_t&
00754 { return *this = *this / rhs; }
00755
00757 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00758 inline
00759 auto
00760 operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00761 {
00762     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00763     using matrix_multi_t = typename multivector_t::matrix_multi_t;
00764
00765     return matrix_multi_t(rhs) * matrix_multi_t(lhs) / matrix_multi_t(rhs.involute());
00766 }
00767
00769 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00770 inline
00771 auto
00772 framed_multi<Scalar_T,LO,HI,Tune_P>::
00773 operator|= (const multivector_t& rhs) -> multivector_t&
00774 { return *this = *this | rhs; }
00775
00777 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00778 inline
00779 auto
00780 framed_multi<Scalar_T,LO,HI,Tune_P>::
00781 inv() const -> const multivector_t
00782 {
00783     auto result = matrix_multi_t(Scalar_T(1), this->frame());
00784     return result /= matrix_multi_t(*this);
00785 }
00786
00788 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00789 auto
00790 framed_multi<Scalar_T,LO,HI,Tune_P>::
00791 pow(int m) const -> const multivector_t
00792 { return glucat::pow(*this, m); }
00793
00795 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00796 auto
00797 framed_multi<Scalar_T,LO,HI,Tune_P>::
00798 outer_pow(int m) const -> const multivector_t
00799 {
00800     if (m < 0)
00801         throw error_t("outer_pow(int): negative exponent");
00802     auto result = multivector_t(Scalar_T(1));
00803     auto a = *this;
00804     for (;
00805         m != 0;
00806         m >= 1, a = a ^ a)
00807         if (m & 1)
00808             result ^= a;
00809     return result;
00810 }

```

```

00811
00813 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00814 inline
00815 auto
00816 framed_multi<Scalar_T,LO,HI,Tune_P>::
00817 frame() const -> const index_set_t
00818 {
00819     auto result = index_set_t();
00820     for (auto& this_term : *this)
00821         result |= this_term.first;
00822     return result;
00823 }
00824
00826 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00827 inline
00828 auto
00829 framed_multi<Scalar_T,LO,HI,Tune_P>::
00830 grade() const -> index_t
00831 {
00832     auto result = index_t(0);
00833     for (auto& this_term : *this)
00834         result = std::max(result, this_term.first.count());
00835     return result;
00836 }
00837
00839 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00840 inline
00841 auto
00842 framed_multi<Scalar_T,LO,HI,Tune_P>::
00843 operator[] (const index_set_t ist) const -> Scalar_T
00844 {
00845     const auto& this_it = this->find(ist);
00846     if (this_it == this->end())
00847         return Scalar_T(0);
00848     else
00849         return this_it->second;
00850 }
00851
00853 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00854 auto
00855 framed_multi<Scalar_T,LO,HI,Tune_P>::
00856 operator() (index_t grade) const -> const multivector_t
00857 {
00858     if ((grade < 0) || (grade > HI-LO))
00859         return Scalar_T(0);
00860     else
00861     {
00862         auto result = multivector_t();
00863         for (auto& this_term : *this)
00864             if (this_term.first.count() == grade)
00865                 result += this_term;
00866         return result;
00867     }
00868 }
00869
00871 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00872 inline
00873 auto
00874 framed_multi<Scalar_T,LO,HI,Tune_P>::
00875 scalar() const -> Scalar_T
00876 { return (*this)[index_set_t()]; }
00877
00879 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00880 inline
00881 auto
00882 framed_multi<Scalar_T,LO,HI,Tune_P>::
00883 pure() const -> const multivector_t
00884 { return *this - this->scalar(); }
00885
00887 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00888 auto
00889 framed_multi<Scalar_T,LO,HI,Tune_P>::
00890 even() const -> const multivector_t
00891 { // even part of x, sum of the pure(count) with even count
00892     auto result = multivector_t();
00893     for (auto& this_term : *this)
00894         if ((this_term.first.count() % 2) == 0)
00895             result.insert(this_term);
00896     return result;
00897 }
00898
00900 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00901 auto
00902 framed_multi<Scalar_T,LO,HI,Tune_P>::
00903 odd() const -> const multivector_t
00904 { // even part of x, sum of the pure(count) with even count
00905     auto result = multivector_t();

```

```

00906     for (auto& this_term : *this)
00907         if ((this_term.first.count() % 2) == 1)
00908             result.insert(this_term);
00909     return result;
00910 }
00911
00913 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00914 auto
00915 framed_multi<Scalar_T,LO,HI,Tune_P>::
00916 vector_part() const -> const vector_t
00917 { return this->vector_part(this->frame(), true); }
00918
00920 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00921 auto
00922 framed_multi<Scalar_T,LO,HI,Tune_P>::
00923 vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00924 {
00925     if (!prechecked && (this->frame() | frm) != frm)
00926         throw error_t("vector_part(frm): value is outside of requested frame");
00927     auto result = vector_t();
00928     result.reserve(frm.count());
00929     const auto frm_end = frm.max()+1;
00930     for (auto
00931         idx = frm.min();
00932         idx != frm_end;
00933         ++idx)
00934         // Frame may contain indices which do not correspond to a grade 1 term but
00935         // frame cannot omit any index corresponding to a grade 1 term
00936         if (frm[idx])
00937             result.push_back((*this)[index_set_t(idx)]);
00938     return result;
00939 }
00940
00942 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00943 auto
00944 framed_multi<Scalar_T,LO,HI,Tune_P>::
00945 involute() const -> const multivector_t
00946 {
00947     auto result = *this;
00948     for (auto& result_term : result)
00949     { // for a k-vector u, involute(u) == (-1)^k * u
00950         if ((result_term.first.count() % 2) == 1)
00951             result_term.second *= Scalar_T(-1);
00952     }
00953     return result;
00954 }
00955
00957 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00958 auto
00959 framed_multi<Scalar_T,LO,HI,Tune_P>::
00960 reverse() const -> const multivector_t
00961 {
00962     auto result = *this;
00963     for (auto& result_term : result)
00964         // For a k-vector u, reverse(u) = { -u, k == 2,3 (mod 4)
00965         // { u, k == 0,1 (mod 4)
00966         switch (result_term.first.count() % 4)
00967         {
00968             case 2:
00969             case 3:
00970                 result_term.second *= Scalar_T(-1);
00971                 break;
00972             default:
00973                 break;
00974         }
00975     return result;
00976 }
00977
00979 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00980 auto
00981 framed_multi<Scalar_T,LO,HI,Tune_P>::
00982 conj() const -> const multivector_t
00983 {
00984     auto result = *this;
00985     for (auto& result_term : result)
00986         // For a k-vector u, conj(u) = { -u, k == 1,2 (mod 4)
00987         // { u, k == 0,3 (mod 4)
00988         switch (result_term.first.count() % 4)
00989         {
00990             case 1:
00991             case 2:
00992                 result_term.second *= Scalar_T(-1);
00993                 break;
00994             default:
00995                 break;
00996         }
00997     return result;

```

```

00998     }
00999
01001     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01002     auto
01003     framed_multi<Scalar_T,LO,HI,Tune_P>::
01004     quad() const -> Scalar_T
01005     {
01006         // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
01007         // ref: old clical: quadfunction(p:pterm):pterm in file compmod.pas
01008         auto result = Scalar_T(0);
01009         for (auto& this_term : *this)
01010         {
01011             const auto sign =
01012                 (this_term.first.count_neg() % 2)
01013                 ? -Scalar_T(1)
01014                 : Scalar_T(1);
01015             result += sign * (this_term.second) * (this_term.second);
01016         }
01017         return result;
01018     }
01019
01021     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01022     auto
01023     framed_multi<Scalar_T,LO,HI,Tune_P>::
01024     norm() const -> Scalar_T
01025     {
01026         using traits_t = numeric_traits<Scalar_T>;
01027
01028         auto result = Scalar_T(0);
01029         for (auto& this_term : *this)
01030         {
01031             const auto abs_crd = traits_t::abs(this_term.second);
01032             result += abs_crd * abs_crd;
01033         }
01034         return result;
01035     }
01036
01038     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01039     auto
01040     framed_multi<Scalar_T,LO,HI,Tune_P>::
01041     max_abs() const -> Scalar_T
01042     {
01043         using traits_t = numeric_traits<Scalar_T>;
01044
01045         auto result = Scalar_T(0);
01046         for (auto& this_term : *this)
01047         {
01048             const auto abs_crd = traits_t::abs(this_term.second);
01049             if (abs_crd > result)
01050                 result = abs_crd;
01051         }
01052         return result;
01053     }
01054
01056     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01057     auto
01058     framed_multi<Scalar_T,LO,HI,Tune_P>::
01059     random(const index_set_t frm, Scalar_T fill) -> const multivector_t
01060     {
01061         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01062         using index_set_t = typename multivector_t::index_set_t;
01063         using term_t = typename multivector_t::term_t;
01064
01065         using random_generator_t = random_generator<Scalar_T>;
01066         auto& generator = random_generator_t::generator();
01067
01068         fill =
01069             (fill < Scalar_T(0))
01070             ? Scalar_T(0)
01071             : (fill > Scalar_T(1))
01072             ? Scalar_T(1)
01073             : fill;
01074         const auto algebra_dim = set_value_t(1) « frm.count();
01075         using traits_t = numeric_traits<Scalar_T>;
01076         const auto mean_abs = traits_t::sqrt(Scalar_T(double(algebra_dim)));
01077         auto result = multivector_t();
01078         for (auto
01079             stv = set_value_t(0);
01080             stv != algebra_dim;
01081             ++stv)
01082             if (generator.uniform() < fill)
01083             {
01084                 const auto& result_crd = generator.normal() / mean_abs;
01085                 result.insert(term_t(index_set_t(stv, frm, true), result_crd));
01086             }
01087         return result;
01088     }

```

```

01089
01091 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01092 inline
01093 void
01094 framed_multi<Scalar_T,LO,HI,Tune_P>::
01095 write(const std::string& msg) const
01096 { std::cout << msg << std::endl << " " << (*this) << std::endl; }
01097
01099 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01100 inline
01101 void
01102 framed_multi<Scalar_T,LO,HI,Tune_P>::
01103 write(std::ofstream& ofile, const std::string& msg) const
01104 {
01105     if (!ofile)
01106         throw error_t("write(ofile,msg): cannot write to output file");
01107     ofile << msg << std::endl << " " << (*this) << std::endl;
01108 }
01109
01111 template< typename Map_T,typename Sorted_Map_T >
01112 class sorted_range
01113 {
01114 public:
01115     using map_t = Map_T;
01116     using sorted_map_t = Sorted_Map_T;
01117     using sorted_iterator = typename Sorted_Map_T::const_iterator;
01118
01119     sorted_range (Sorted_Map_T &sorted_val, const Map_T& val)
01120     {
01121         for (auto& val_term : val)
01122             sorted_val.insert(val_term);
01123         sorted_begin = sorted_val.begin();
01124         sorted_end = sorted_val.end();
01125     }
01126     sorted_iterator sorted_begin;
01127     sorted_iterator sorted_end;
01128 };
01129
01130 template< typename Sorted_Map_T >
01131 class sorted_range< Sorted_Map_T, Sorted_Map_T >
01132 {
01133 public:
01134     using map_t = Sorted_Map_T;
01135     using sorted_map_t = Sorted_Map_T;
01136     using sorted_iterator = typename Sorted_Map_T::const_iterator;
01137
01138     sorted_range (Sorted_Map_T &sorted_val, const Sorted_Map_T& val)
01139     : sorted_begin( val.begin() ),
01140       sorted_end( val.end() )
01141     { }
01142     sorted_iterator sorted_begin;
01143     sorted_iterator sorted_end;
01144 };
01145
01147 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01148 auto
01149 operator<< (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
01150 {
01151     using limits_t = std::numeric_limits<Scalar_T>;
01152     if (val.empty())
01153         os << 0;
01154     else if (val.isnan())
01155         os << limits_t::quiet_NaN();
01156     else if (val.isinf())
01157     {
01158         const Scalar_T& inf = limits_t::infinity();
01159         os << (scalar(val) < 0.0 ? -inf : inf);
01160     }
01161     else
01162     {
01163         using traits_t = numeric_traits<Scalar_T>;
01164         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01165         Scalar_T truncation;
01166         switch (os.flags() & std::ios::floatfield)
01167         {
01168             case std::ios_base::scientific:
01169                 truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()) + 1);
01170                 break;
01171             case std::ios_base::fixed:
01172                 truncation = Scalar_T(1) / (traits_t::pow(Scalar_T(10), int(os.precision())) *
01173 val.max_abs());
01174                 break;
01175             case std::ios_base::fixed | std::ios_base::scientific:
01176                 truncation = multivector_t::default_truncation;
01177                 break;
01178             default:
01179                 truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()));
01180         }
01181     }

```

```

01179         break;
01180     }
01181     auto truncated_val = val.truncated(truncation);
01182     if (truncated_val.empty())
01183         os << 0;
01184     else
01185     {
01186         using map_t = typename multivector_t::map_t;
01187         using sorted_map_t = typename multivector_t::sorted_map_t;
01188         using sorted_iterator = typename sorted_map_t::const_iterator;
01189         auto sorted_val = sorted_map_t();
01190         const auto sorted_val_range = sorted_range< map_t, sorted_map_t >(sorted_val, truncated_val);
01191         auto sorted_it = sorted_val_range.sorted_begin();
01192         os << *sorted_it;
01193         for (++sorted_it;
01194             sorted_it != sorted_val_range.sorted_end();
01195             ++sorted_it)
01196         {
01197             const Scalar_T& scr = sorted_it->second;
01198             if (scr >= 0.0)
01199                 os << '+';
01200             os << *sorted_it;
01201         }
01202     }
01203 }
01204 return os;
01205 }
01206
01207 template< typename Scalar_T, const index_t LO, const index_t HI >
01208 auto
01209 operator<< (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
01210 std::ostream&
01211 {
01212     const auto second_as_double = numeric_traits<Scalar_T>::to_double(term.second);
01213     const auto use_double =
01214         (os.precision() <= std::numeric_limits<double>::digits10) ||
01215         (term.second == Scalar_T(second_as_double));
01216     if (term.first.count() == 0)
01217         if (use_double)
01218             os << second_as_double;
01219         else
01220             os << term.second;
01221     else if (term.second == Scalar_T(-1))
01222     {
01223         os << '-';
01224         os << term.first;
01225     }
01226     else if (term.second != Scalar_T(1))
01227     {
01228         if (use_double)
01229         {
01230             auto tol = std::pow(10.0, -os.precision());
01231             if ( std::fabs(second_as_double + 1.0) < tol )
01232                 os << '-';
01233             else if ( std::fabs(second_as_double - 1.0) >= tol )
01234                 os << second_as_double;
01235         }
01236         else
01237             os << term.second;
01238         os << term.first;
01239     }
01240     else
01241         os << term.first;
01242     return os;
01243 }
01244
01245 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01246 auto
01247 operator>> (std::istream& s, framed_multi<Scalar_T,LO,HI,Tune_P> & val) -> std::istream&
01248 { // Input looks like 1.0-2.0{1,2}+3.2{3,4}.
01249     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01250     // Parsing variables.
01251     auto local_val = multivector_t();
01252     auto c = 0;
01253     // Parsing control variables.
01254     auto negative = false;
01255     auto expect_term = true;
01256     // The multivector may begin with '+' or '-'. Check for this.
01257     c = s.peek();
01258     if (s.good() && (c == int('+') || c == int('-')))
01259     { // A '-' here negates the following term.
01260         negative = (c == int('-'));
01261         // Consume the '+' or '-'.
01262         s.get();
01263     }
01264     while (s.good())
01265     { // Parse a term.

```

```

01267     // A term consists of an optional scalar, followed by an optional index set.
01268     // At least one of the two must be present.
01269     // Default coordinate is Scalar_T(1).
01270     auto coordinate = Scalar_T(1);
01271     // Default index set is empty.
01272     auto ist = index_set<LO,HI>();
01273     // First, check for an opening brace.
01274     c = s.peek();
01275     if (s.good())
01276     { // If the character is not an opening brace,
01277       // a coordinate value is expected here.
01278       if (c != int('{'))
01279       { // Try to read a coordinate value.
01280         double coordinate_as_double;
01281         s » coordinate_as_double;
01282         // Reading the coordinate may have resulted in an end of file condition.
01283         // This is not a failure.
01284         if (s)
01285           coordinate = Scalar_T(coordinate_as_double);
01286       }
01287     }
01288     else
01289     { // End of file here ends parsing while a term may still be expected.
01290       break;
01291     }
01292     // Coordinate is now Scalar_T(1) or a Scalar_T value.
01293     // Parse an optional index set.
01294     if (s.good())
01295     {
01296       c = s.peek();
01297       if (s.good() && c == int('{'))
01298       { // Try to read index set.
01299         s » ist;
01300       }
01301     }
01302     // Reading the term may have resulted in an end of file condition.
01303     // This is not a failure.
01304     if (s)
01305     {
01306       // Immediately after parsing a term, another term is not expected.
01307       expect_term = false;
01308       if (coordinate != Scalar_T(0))
01309       {
01310         // Add the term to the local multivector.
01311         coordinate =
01312             negative
01313             ? -coordinate
01314             : coordinate;
01315         using term_t = typename multivector_t::term_t;
01316         local_val += term_t(ist, coordinate);
01317       }
01318     }
01319     // Check if anything follows the current term.
01320     if (s.good())
01321     {
01322       c = s.peek();
01323       if (s.good())
01324       { // Only '+' and '-' are valid here.
01325         if (c == int('+') || c == int('-'))
01326         { // A '-' here negates the following term.
01327           negative = (c == int('-'));
01328           // Consume the '+' or '-'.
01329           s.get();
01330           // Immediately after '+' or '-',
01331           // expect another term.
01332           expect_term = true;
01333         }
01334         else
01335         { // Any other character here is a not failure,
01336           // but still ends the parsing of the multivector.
01337           break;
01338         }
01339       }
01340     }
01341     }
01342     // If a term is still expected, this is a failure.
01343     if (expect_term)
01344       s.clear(std::istream::failbit);
01345     // End of file is not a failure.
01346     if (s)
01347     { // The multivector has been successfully parsed.
01348       val = local_val;
01349     }
01350     return s;
01351   }
01352
01353   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >

```

```

01355     auto
01356     framed_multi<Scalar_T, LO, HI, Tune_P>::
01357     nbr_terms () const -> unsigned long
01358     { return this->size(); }
01359
01360     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01361     inline
01362     auto
01363     framed_multi<Scalar_T, LO, HI, Tune_P>::
01364     operator+= (const term_t& term) -> multivector_t&
01365     { // Do not insert terms with 0 coordinate
01366       if (term.second != Scalar_T(0))
01367       {
01368         const auto& this_it = this->find(term.first);
01369         if (this_it == this->end())
01370           this->insert(term);
01371         else if (this_it->second + term.second == Scalar_T(0))
01372           // Erase term if resulting coordinate is 0
01373           this->erase(this_it);
01374         else
01375           this_it->second += term.second;
01376       }
01377       return *this;
01378     }
01379
01380     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01381     auto
01382     framed_multi<Scalar_T, LO, HI, Tune_P>::
01383     isinf() const -> bool
01384     {
01385       using traits_t = numeric_traits<Scalar_T>;
01386
01387       if (std::numeric_limits<Scalar_T>::has_infinity)
01388         for (auto& this_term : *this)
01389           if (traits_t::isInf(this_term.second))
01390             return true;
01391       return false;
01392     }
01393
01394     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01395     auto
01396     framed_multi<Scalar_T, LO, HI, Tune_P>::
01397     isnan() const -> bool
01398     {
01399       using traits_t = numeric_traits<Scalar_T>;
01400
01401       if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
01402         for (auto& this_term : *this)
01403           if (traits_t::isNaN(this_term.second))
01404             return true;
01405       return false;
01406     }
01407
01408     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01409     auto
01410     framed_multi<Scalar_T, LO, HI, Tune_P>::
01411     truncated(const Scalar_T& limit) const -> const multivector_t
01412     {
01413       using traits_t = numeric_traits<Scalar_T>;
01414
01415       if (this->isnan() || this->isinf())
01416         return *this;
01417       const auto truncation = traits_t::abs(limit);
01418       const auto top = max_abs();
01419       auto result = multivector_t();
01420       if (top != Scalar_T(0))
01421         for (auto& this_term : *this)
01422           if (traits_t::abs(this_term.second) > top * truncation)
01423             result.insert(this_term);
01424       return result;
01425     }
01426
01427     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01428     auto
01429     framed_multi<Scalar_T, LO, HI, Tune_P>::
01430     fold(const index_set_t frm) const -> multivector_t
01431     {
01432       if (frm.is_contiguous())
01433         return *this;
01434       else
01435       {
01436         auto result = multivector_t();
01437         for (auto& this_term : *this)
01438           result.insert(term_t(this_term.first.fold(frm), this_term.second));
01439       }
01440       return result;
01441     }
01442
01443     }
01444
01445     }
01446

```



```

01447
01449 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01450 auto
01451 framed_multi<Scalar_T,LO,HI,Tune_P>::
01452 unfold(const index_set_t frm) const -> multivector_t
01453 {
01454     if (frm.is_contiguous())
01455         return *this;
01456     else
01457     {
01458         auto result = multivector_t();
01459         for (auto& this_term : *this)
01460             result.insert(term_t(this_term.first.unfold(frm), this_term.second));
01461         return result;
01462     }
01463 }
01464
01466 // Reference: [L] 16.4 Periodicity of 8, p216
01467 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01468 auto
01469 framed_multi<Scalar_T,LO,HI,Tune_P>::
01470 centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&
01471 {
01472     // We add 4 to q by subtracting 4 from p
01473     if (q+4 > -LO)
01474         throw error_t("centre_pm4_qp4(p,q): LO is too high to represent this value");
01475     if (this->frame().max() > p-4)
01476     {
01477         using index_pair_t = typename index_set_t::index_pair_t;
01478         const auto pm3210 = index_set_t(index_pair_t(p-3,p), true);
01479         const auto qm4321 = index_set_t(index_pair_t(-q-4,-q-1), true);
01480         const auto& tqm4321 = term_t(qm4321, Scalar_T(1));
01481         auto result = multivector_t();
01482         for (auto& this_term : *this)
01483         {
01484             const auto ist = this_term.first;
01485             if (ist.max() > p-4)
01486             {
01487                 auto var_term = var_term_t();
01488                 for (auto
01489                     n = index_t(0);
01490                     n != index_t(4);
01491                     ++n)
01492                     if (ist[n+p-3])
01493                         var_term *= term_t(index_set_t(n-q-4), Scalar_T(1)) * tqm4321;
01494                 // Mask out {p-3}..{p}
01495                 result.insert(term_t(ist & ~pm3210, this_term.second) *
01496                             term_t(var_term.first, var_term.second));
01497             }
01498             else
01499                 result.insert(this_term);
01500         }
01501         *this = result;
01502     }
01503     p -=4; q += 4;
01504     return *this;
01505 }
01506
01508 // Reference: [L] 16.4 Periodicity of 8, p216
01509 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01510 auto
01511 framed_multi<Scalar_T,LO,HI,Tune_P>::
01512 centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&
01513 {
01514     // We add 4 to p by subtracting 4 from q
01515     if (p+4 > HI)
01516         throw error_t("centre_pp4_qm4(p,q): HI is too low to represent this value");
01517     if (this->frame().min() < -q+4)
01518     {
01519         using index_pair_t = typename index_set_t::index_pair_t;
01520         const auto qp0123 = index_set_t(index_pair_t(-q,-q+3), true);
01521         const auto pp1234 = index_set_t(index_pair_t(p+1,p+4), true);
01522         const auto& tpp1234 = term_t(pp1234, Scalar_T(1));
01523         auto result = multivector_t();
01524         for (auto& this_term : *this)
01525         {
01526             index_set_t ist = this_term.first;
01527             if (ist.min() < -q+4)
01528             {
01529                 auto var_term = var_term_t();
01530                 for (auto
01531                     n = index_t(0);
01532                     n != index_t(4);
01533                     ++n)
01534                     if (ist[n-q])
01535                         var_term *= term_t(index_set_t(n+p+1), Scalar_T(1)) * tpp1234;
01536                 // Mask out {-q}..{-q+3}

```

```

01537         result.insert(term_t(var_term.first, var_term.second) *
01538                        term_t(ist & ~qp0123, this_term.second));
01539     }
01540     else
01541         result.insert(this_term);
01542     }
01543     *this = result;
01544 }
01545 p +=4; q -= 4;
01546 return *this;
01547 }
01548
01549 // Reference: [P] Proposition 15.20, p 131
01550 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01551 auto
01552 framed_multi<Scalar_T,LO,HI,Tune_P>::
01553 centre_qp1_pml(index_t& p, index_t& q) -> multivector_t&
01554 {
01555     {
01556         if (q+1 > HI)
01557             throw error_t("centre_qp1_pml(p,q): HI is too low to represent this value");
01558         if (p-1 > -LO)
01559             throw error_t("centre_qp1_pml(p,q): LO is too high to represent this value");
01560         const auto qp1 = index_set_t(q+1);
01561         const auto& tqp1 = term_t(qp1, Scalar_T(1));
01562         auto result = multivector_t();
01563         for (auto& this_term : *this)
01564         {
01565             const auto ist = this_term.first;
01566             auto var_term = var_term_t(index_set_t(), this_term.second);
01567             for (auto
01568                 n = -q;
01569                 n != p;
01570                 ++n)
01571                 if (n != 0 && ist[n])
01572                     var_term *= term_t(index_set_t(-n) | qp1, Scalar_T(1));
01573             if (p != 0 && ist[p])
01574                 var_term *= tqp1;
01575             result.insert(term_t(var_term.first, var_term.second));
01576         }
01577         index_t orig_p = p;
01578         p = q+1;
01579         q = orig_p-1;
01580         return *this = result;
01581     }
01582 }
01583
01584 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01585 auto
01586 framed_multi<Scalar_T,LO,HI,Tune_P>::
01587 divide(const index_set_t ist) const -> const framed_pair_t
01588 {
01589     auto quo = multivector_t();
01590     auto rem = multivector_t();
01591     for (auto& this_term : *this)
01592         if ((this_term.first | ist) == this_term.first)
01593             quo.insert(term_t(this_term.first ^ ist, this_term.second));
01594         else
01595             rem.insert(this_term);
01596     return framed_pair_t(quo, rem);
01597 }
01598
01599 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01600 auto
01601 framed_multi<Scalar_T,LO,HI,Tune_P>::
01602 fast(const index_t level, const bool odd) const -> const matrix_t
01603 {
01604     {
01605         // Assume val is already folded and centred
01606         if (this->empty())
01607         {
01608             using matrix_index_t = typename matrix_multi_t::matrix_index_t;
01609             const auto dim = matrix_index_t(1) << level;
01610             auto result = matrix_t(dim, dim);
01611             result.clear();
01612             return result;
01613         }
01614         if (level == 0)
01615             return matrix::unit<matrix_t>(1) * this->scalar();
01616
01617         using basis_matrix_t = typename matrix_multi_t::basis_matrix_t;
01618         using basis_scalar_t = typename basis_matrix_t::value_type;
01619
01620         const auto& I = matrix::unit<basis_matrix_t>(2);
01621         auto J = basis_matrix_t(2,2,2);
01622         J.clear();
01623         J(0,1) = basis_scalar_t(-1);
01624         J(1,0) = basis_scalar_t( 1);
01625         auto K = J;
01626         K(0,1) = basis_scalar_t( 1);

```

```

01627     auto JK = I;
01628     JK(0,0) = basis_scalar_t(-1);
01629
01630     const auto ist_mn = index_set_t(-level);
01631     const auto ist_pn = index_set_t(level);
01632     if (level == 1)
01633     {
01634         if (odd)
01635             return matrix_t(J) * (*this)[ist_mn] + matrix_t(K) * (*this)[ist_pn];
01636         else
01637             return matrix_t(I) * this->scalar() + matrix_t(JK) * (*this)[ist_mn ^ ist_pn];
01638     }
01639     else
01640     {
01641         const auto& pair_mn = this->divide(ist_mn);
01642         const auto& quo_mn = pair_mn.first;
01643         const auto& rem_mn = pair_mn.second;
01644         const auto& pair_quo_mnpn = quo_mn.divide(ist_pn);
01645         const auto& val_mnpn = pair_quo_mnpn.first;
01646         const auto& val_mn = pair_quo_mnpn.second;
01647         const auto& pair_rem_mnpn = rem_mn.divide(ist_pn);
01648         const auto& val_pn = pair_rem_mnpn.first;
01649         const auto& val_l = pair_rem_mnpn.second;
01650         using matrix::kron;
01651         if (odd)
01652             return - kron(JK, val_l.fast (level-1, 1))
01653                   + kron(I, val_mnpn.fast (level-1, 1))
01654                   + kron(J, val_mn.fast (level-1, 0))
01655                   + kron(K, val_pn.fast (level-1, 0));
01656         else
01657             return kron(I, val_l.fast (level-1, 0))
01658                   + kron(JK, val_mnpn.fast (level-1, 0))
01659                   + kron(K, val_mn.fast (level-1, 1))
01660                   - kron(J, val_pn.fast (level-1, 1));
01661     }
01662 }
01663
01664 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01665 template< typename Other_Scalar_T >
01666 auto
01667 framed_multi<Scalar_T,LO,HI,Tune_P>::
01668 fast_matrix_multi(const index_set_t frm) const -> const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>
01669 {
01670     // Fold val
01671     auto val = this->fold(frm);
01672     auto p = frm.count_pos();
01673     auto q = frm.count_neg();
01674     const auto bott_offset = gen::offset_to_super[pos_mod(p - q, 8)];
01675     p += std::max(bott_offset, index_t(0));
01676     q -= std::min(bott_offset, index_t(0));
01677     if (p > HI)
01678         throw error_t("fast_matrix_multi(frm): HI is too low to represent this value");
01679     if (q > -LO)
01680         throw error_t("fast_matrix_multi(frm): LO is too high to represent this value");
01681     // Centre val
01682     while (p - q > 4)
01683         val.centre_pm4_qp4(p, q);
01684     while (p - q < -3)
01685         val.centre_pp4_qm4(p, q);
01686     if (p - q > 1)
01687         val.centre_qp1_pm1(p, q);
01688     const index_t level = (p + q)/2;
01689
01690     // Do the fast transform
01691     const auto& ev_val = val.even();
01692     const auto& od_val = val.odd();
01693     return matrix_multi<Other_Scalar_T,LO,HI,Tune_P>(ev_val.fast(level, 0) + od_val.fast(level, 1),
01694 frm);
01695 }
01696
01697 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01698 inline
01699 auto
01700 framed_multi<Scalar_T,LO,HI,Tune_P>::
01701 fast_framed_multi() const -> const multivector_t
01702 { return *this; }
01703
01704 template< typename Scalar_T, const index_t LO, const index_t HI >
01705 inline
01706 static
01707 auto
01708 crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01709             const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T
01710 { return lhs.first.sign_of_mult(rhs.first) * lhs.second * rhs.second; }
01711
01712 template< typename Scalar_T, const index_t LO, const index_t HI >
01713 inline

```

```

01716     auto
01717     operator* (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01718               const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const
index_set<LO,HI>, Scalar_T>
01719     {
01720         using term_t = std::pair<const index_set<LO,HI>, Scalar_T>;
01721         return term_t(lhs.first ^ rhs.first, crd_of_mult(lhs, rhs));
01722     }
01723
01725     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01726     auto
01727     sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01728     {
01729         using traits_t = numeric_traits<Scalar_T>;
01730         if (val.isnan())
01731             return traits_t::NaN();
01732
01733         check_complex(val, i, prechecked);
01734
01735         const auto realval = val.scalar();
01736         if (val == realval)
01737         {
01738             if (realval < Scalar_T(0))
01739                 return i * traits_t::sqrt(-realval);
01740             else
01741                 return traits_t::sqrt(realval);
01742         }
01743         using matrix_multi_t = typename framed_multi<Scalar_T,LO,HI,Tune_P>::matrix_multi_t;
01744         return sqrt(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01745     }
01746
01748     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01749     auto
01750     exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01751     {
01752         using traits_t = numeric_traits<Scalar_T>;
01753         if (val.isnan())
01754             return traits_t::NaN();
01755
01756         const auto s = scalar(val);
01757         if (val == s)
01758             return traits_t::exp(s);
01759
01760         const double size = val.size();
01761         const auto frm_count = val.frame().count();
01762         const auto algebra_dim = set_value_t(1) << frm_count;
01763
01764         if ( (size * size <= double(algebra_dim)) || (frm_count < Tune_P::mult_matrix_threshold) )
01765         {
01766             switch (Tune_P::function_precision)
01767             {
01768                 case precision_demoted:
01769                 {
01770                     using demoted_scalar_t = typename traits_t::demoted::type;
01771                     using demoted_multivector_t = framed_multi<demoted_scalar_t,LO,HI,Tune_P>;
01772
01773                     const auto& demoted_val = demoted_multivector_t(val);
01774                     return clifford_exp(demoted_val);
01775                 }
01776                 break;
01777                 case precision_promoted:
01778                 {
01779                     using promoted_scalar_t = typename traits_t::promoted::type;
01780                     using promoted_multivector_t = framed_multi<promoted_scalar_t,LO,HI,Tune_P>;
01781
01782                     const auto& promoted_val = promoted_multivector_t(val);
01783                     return clifford_exp(promoted_val);
01784                 }
01785                 break;
01786                 default:
01787                     return clifford_exp(val);
01788             }
01789         }
01790         else
01791         {
01792             using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01793             return exp(matrix_multi_t(val));
01794         }
01795     }
01796
01798     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01799     auto
01800     log(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01801     {
01802         using traits_t = numeric_traits<Scalar_T>;

```

```

01803     if (val == Scalar_T(0) || val.isnan())
01804         return traits_t::NaN();
01805
01806     check_complex(val, i, prechecked);
01807
01808     const auto realval = val.scalar();
01809     if (val == realval)
01810     {
01811         if (realval < Scalar_T(0))
01812             return i * traits_t::pi() + traits_t::log(-realval);
01813         else
01814             return traits_t::log(realval);
01815     }
01816     using matrix_multi_t = typename framed_multi<Scalar_T, LO, HI, Tune_P>::matrix_multi_t;
01817     return log(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01818 }
01819 }
01820 #endif // _GLUCAT_FRAMED_MULTI_IMP_H

```

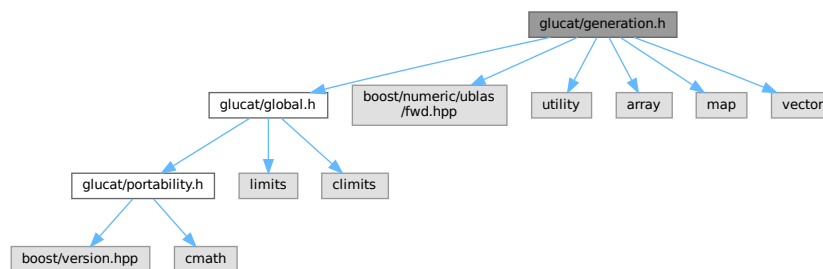
## 7.13 glucat/generation.h File Reference

```

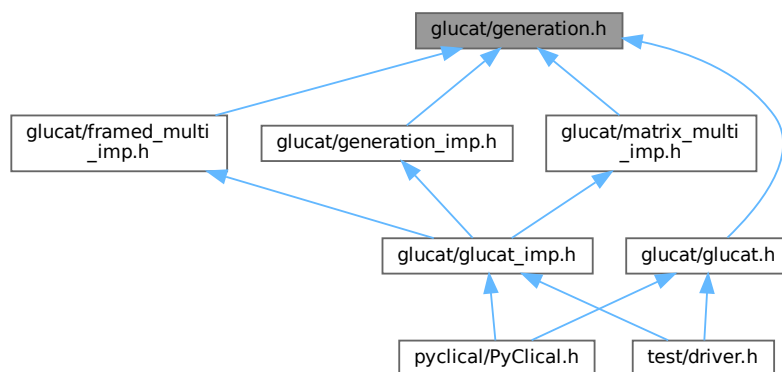
#include "glucat/global.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <utility>
#include <array>
#include <map>
#include <vector>

```

Include dependency graph for generation.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::gen::generator_table< Matrix_T >`  
*Table of generators for specific signatures.*

## Namespaces

- namespace `glucat`
- namespace `glucat::gen`

## Typedefs

- using `glucat::gen::signature_t = std::pair<index_t, index_t>`  
*A signature is a pair of indices,  $p$ ,  $q$ , with  $p == \text{frame.max}()$ ,  $q == -\text{frame.min}()$*

## Variables

- static const `std::array< index_t, 8 > glucat::gen::offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1}`  
*Offsets between the current signature and that of the real superalgebra.*

## 7.14 generation.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_GENERATION_H
00002 #define _GLUCAT_GENERATION_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     generation.h : Declare functions for generation of the matrix representation
00006     -----
00007     begin                               : Wed Jan 23 2002
00008     copyright                           : (C) 2002-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035
00036 #include <boost/numeric/ublas/fwd.hpp>
00037
00038 #include <utility>
00039 #include <array>
00040 #include <map>
00041 #include <vector>
00042
```

```

00043 namespace glucat { namespace gen
00044 {
00045     namespace ublas = boost::numeric::ublas;
00046
00048     using signature_t = std::pair<index_t, index_t>;
00049
00051     template< class Matrix_T >
00052     class generator_table :
00053     private std::map< signature_t, std::vector<Matrix_T> >
00054     {
00055     public:
00057         auto operator() (const index_t p, const index_t q) -> const Matrix_T*;
00059         static auto generator() -> generator_table<Matrix_T>&;
00060     private:
00062         auto gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&;
00064         void gen_from_pml_qml(const std::vector<Matrix_T>& old, const signature_t sig);
00066         void gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig);
00068         void gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig);
00070         void gen_from_qp1_pml(const std::vector<Matrix_T>& old, const signature_t sig);
00071
00075         friend class friend_for_private_destructor;
00076         // Enforce singleton
00077         // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00078         generator_table() = default;
00079         ~generator_table() = default;
00080     public:
00081         generator_table(const generator_table&) = delete;
00082         auto operator= (const generator_table&) -> generator_table& = delete;
00083     };
00084
00086     static const std::array<index_t, 8> offset_to_super = {0, -1, 0, -1, -2, 3, 2, 1};
00087 } }
00089 #endif // _GLUCAT_GENERATION_H

```

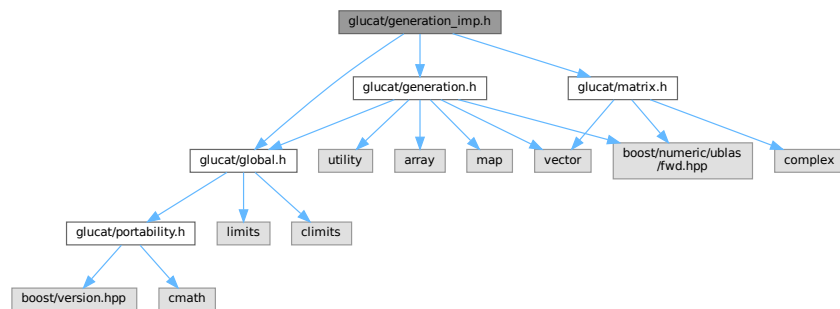
## 7.15 glucat/generation\_imp.h File Reference

```

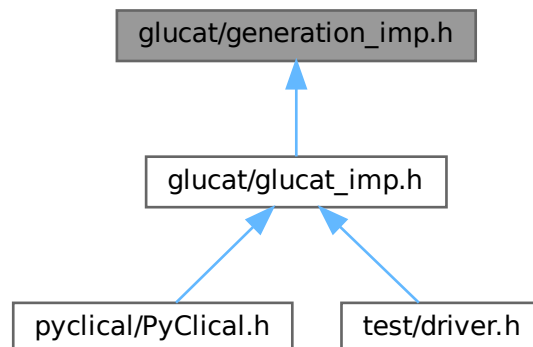
#include "glucat/global.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"

```

Include dependency graph for generation\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`
- namespace `glucat::gen`

## 7.16 generation\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GENERATION_IMP_H
00002 #define _GLUCAT_GENERATION_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     generation_imp.h : Implement functions for generation of the matrix representation
00006     -----
00007     begin                : Wed Jan 23 2002
00008     copyright            : (C) 2002-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/generation.h"
00036 #include "glucat/matrix.h"
00037

```



```

00038 namespace glucat { namespace gen
00039 {
00040     // References for algorithms:
00041     // [M]: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.
00042     // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00043     // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00044
00046     // Reference: [M] Item 47
00047     template< class Matrix_T >
00048     auto
00049     generator_table<Matrix_T>::
00050     generator() -> generator_table<Matrix_T>&
00051     { static generator_table<Matrix_T> g; return g;}
00052
00054     // Reference: [P] Table 15.27, p 133
00055     template< class Matrix_T >
00056     inline
00057     auto
00058     generator_table<Matrix_T>::
00059     operator() (const index_t p, const index_t q) -> const Matrix_T*
00060     {
00061         const auto bott = pos_mod(p-q, 8);
00062         switch(bott)
00063         {
00064             case 0:
00065             case 2:
00066                 // Construct generators
00067                 return &(gen_vector(p, q)[q]);
00068             default:
00069                 // Select generators from the vector for a larger frame
00070                 const auto super_p = p + std::max(offset_to_super[bott], index_t(0));
00071                 const auto super_q = q - std::min(offset_to_super[bott], index_t(0));
00072                 return &(gen_vector(super_p, super_q)[super_q]);
00073         }
00074     }
00075
00077     template< class Matrix_T >
00078     auto
00079     generator_table<Matrix_T>::
00080     gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&
00081     {
00082         using result_t = std::vector<Matrix_T>;
00083         const auto card = p + q;
00084         const auto bias = p - q;
00085         const auto bott = pos_mod(bias, 8);
00086         const auto sig = signature_t(p, q);
00087         if (this->find(sig) == this->end())
00088             switch(bott)
00089             {
00090                 case 0:
00091                     if (bias < 0)
00092                         // Construct generators for p,q given generators for p+4,q-4
00093                         gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00094                     else if (bias > 0)
00095                         // Construct generators for p,q given generators for p-4,q+4
00096                         gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00097                     else if (card == 0)
00098                     { // Base case. Save a generator vector containing one matrix, size 1.
00099                         auto result = result_t(1, matrix::unit<Matrix_T>(1));
00100                         this->insert(make_pair(sig, result));
00101                     }
00102                     else
00103                         // Construct generators for p,q given generators for p-1,q-1
00104                         gen_from_pml_qml(gen_vector(p-1, q-1), sig);
00105                     break;
00106                 case 2:
00107                     if (bias < 2)
00108                         // Construct generators for p,q given generators for p+4,q-4
00109                         gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00110                     else if (bias > 2)
00111                         // Construct generators for p,q given generators for p-4,q+4
00112                         gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00113                     else
00114                         // Construct generators for p,q given generators for q+1,p-1
00115                         gen_from_qp1_pml(gen_vector(q+1, p-1), sig);
00116                     break;
00117             default:
00118                 break;
00119             }
00120         return (*this)[sig];
00121     }
00122
00124     // Reference: [P] Proposition 15.17, p 131
00125     template< class Matrix_T >
00126     void
00127     generator_table<Matrix_T>::
00128     gen_from_pml_qml(const std::vector<Matrix_T>& old, const signature_t sig)

```

```

00129 {
00130     const auto new_size = old.size() + 2;
00131     using size_t = decltype(new_size);
00132     using result_t = std::vector<Matrix_T>;
00133     auto result = result_t(new_size);
00134
00135     const auto old_dim = old[0].size1();
00136     const auto& eye = matrix::unit<Matrix_T>(old_dim);
00137
00138     auto neg = Matrix_T(2,2,2);
00139     neg(0,1) = -1;
00140     neg(1,0) = 1;
00141
00142     auto pos = neg;
00143     pos(0,1) = 1;
00144
00145     auto dup = Matrix_T(2,2,2);
00146     dup(0,0) = 1;
00147     dup(1,1) = -1;
00148
00149     result[0] = matrix::mono_kron(neg, eye);
00150     for (auto
00151         k = size_t(1);
00152         k != new_size-1;
00153         ++k)
00154         result[k] = matrix::mono_kron(dup, old[k-1]);
00155     result[new_size-1] = matrix::mono_kron(pos, eye);
00156
00157     // Save the resulting generator array.
00158     this->insert(make_pair(sig, result));
00159 }
00160
00162 // Reference: [L] 16.4 Periodicity of 8, p216
00163 template< class Matrix_T >
00164 void
00165 generator_table<Matrix_T>::
00166 gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig)
00167 {
00168     const auto old_size = old.size();
00169     using size_t = decltype(old_size);
00170     using result_t = std::vector<Matrix_T>;
00171     auto result = result_t(old_size);
00172
00173     auto h = old[0];
00174     for (auto
00175         k = size_t(1);
00176         k != size_t(4);
00177         ++k)
00178         h = matrix::mono_prod(old[k], h);
00179
00180     for (auto
00181         k = size_t(0);
00182         k != old_size-4;
00183         ++k)
00184         result[k] = old[k+4];
00185     for (auto
00186         k = old_size-4;
00187         k != old_size;
00188         ++k)
00189         result[k] = matrix::mono_prod(old[k+4-old_size], h);
00190     // Save the resulting generator array.
00191     this->insert(make_pair(sig, result));
00192 }
00193
00195 // Reference: [L] 16.4 Periodicity of 8, p216
00196 template< class Matrix_T >
00197 void
00198 generator_table<Matrix_T>::
00199 gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig)
00200 {
00201     const auto old_size = old.size();
00202     using size_t = decltype(old_size);
00203     using result_t = std::vector<Matrix_T>;
00204     auto result = result_t(old_size);
00205
00206     auto h = old[old_size-1];
00207     for (auto
00208         k = size_t(1);
00209         k != size_t(4);
00210         ++k)
00211         h = matrix::mono_prod(old[old_size-1-k], h);
00212
00213     for (auto
00214         k = size_t(0);
00215         k != size_t(4);
00216         ++k)
00217         result[k] = matrix::mono_prod(old[k+old_size-4], h);

```

```

00218     for (auto
00219         k = size_t(4);
00220         k != old_size;
00221         ++k)
00222         result[k] = old[k-4];
00223     // Save the resulting generator array.
00224     this->insert(make_pair(sig, result));
00225 }
00226
00228 // Reference: [P] Proposition 15.20, p 131
00229 template< class Matrix_T >
00230 void
00231 generator_table<Matrix_T>::
00232 gen_from_gpl_pml(const std::vector<Matrix_T>& old, const signature_t sig)
00233 {
00234     const auto old_size = old.size();
00235     using size_t = decltype(old_size);
00236     using result_t = std::vector<Matrix_T>;
00237     auto result = result_t(old_size);
00238
00239     const auto& h = old[old_size-1];
00240     for (auto
00241         k = size_t(0);
00242         k != old_size-1;
00243         ++k)
00244         result[k] = matrix::mono_prod(old[old_size-2-k], h);
00245     result[old_size-1] = h;
00246
00247     // Save the resulting generator array.
00248     this->insert(make_pair(sig, result));
00249 }
00250
00251 } }
00252 #endif // _GLUCAT_GENERATION_IMP_H

```

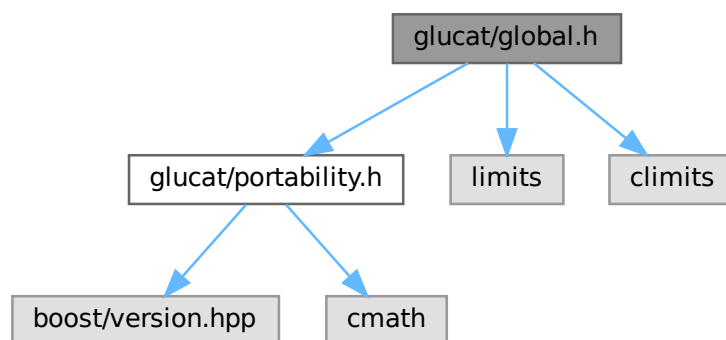
## 7.17 glucat/global.h File Reference

```

#include "glucat/portability.h"
#include <limits>
#include <climits>

```

Include dependency graph for global.h:





## Variables

- const double `glucat::MS_PER_S` = 1000.0  
*Timing constant: deprecated here - moved to [test/timing.h](#).*
- const `index_t glucat::BITS_PER_SET_VALUE` = `std::numeric_limits<set_value_t>::digits`  
*Number of bits in `set_value_t`.*
- const `index_t glucat::DEFAULT_HI` = `index_t(BITS_PER_SET_VALUE / 2)`  
*Default highest index in an index set.*

## 7.17.1 Macro Definition Documentation

## 7.17.1.1 \_GLUCAT\_CTAssert

```
#define _GLUCAT_CTAssert(
    expr,
    msg)
```

## Value:

```
namespace { struct msg { glucat::CTAssertion<(expr)> ERROR_##msg; }; }
```

Definition at line 48 of file [global.h](#).

## 7.18 global.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_GLOBAL_H
00002 #define _GLUCAT_GLOBAL_H
00003 /*****
00004  GluCat : Generic library of universal Clifford algebra templates
00005  global.h : Global declarations
00006  -----
00007  begin                : Sun 2001-12-09
00008  copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  *****/
00010
00011  This library is free software: you can redistribute it and/or modify
00012  it under the terms of the GNU Lesser General Public License as published
00013  by the Free Software Foundation, either version 3 of the License, or
00014  (at your option) any later version.
00015
00016  This library is distributed in the hope that it will be useful,
00017  but WITHOUT ANY WARRANTY; without even the implied warranty of
00018  MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019  GNU Lesser General Public License for more details.
00020
00021  You should have received a copy of the GNU Lesser General Public License
00022  along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024  *****/
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030  *****/
00031  See also Arvind Raja's original header comments and references in glucat.h
00032  *****/
00033
00034 #include "glucat/portability.h"
00035
00036 #include <limits>
00037 #include <climits>
00038
00039 namespace glucat
00040 {
00041     // References:
```

```

00042 // [AA]: A. Alexandrescu, "Modern C++ Design", Addison-Wesley, 2001.
00043
00045 // Reference: [AA], p. 25
00046 template<bool> struct CTAAssertion;
00047 template<> struct CTAAssertion<true> { };
00048 #define _GLUCAT_CTAssert(expr, msg) \
00049     namespace { struct msg { glucat::CTAssertion<(expr)> ERROR_##msg; }; }
00050
00052 // Reference: [AA], pp. 34--37
00053 template < typename LHS_T, typename RHS_T >
00054 class compare_types
00055 {
00056 public:
00057     enum { are_same = false };
00058 };
00059 template < typename T >
00060 class compare_types<T, T>
00061 {
00062 public:
00063     enum { are_same = true };
00064 };
00065
00067 // Reference: [AA], 2.4, p. 29
00068 template< bool truth_value >
00069 class bool_to_type
00070 {
00071 private:
00072     enum { value = truth_value };
00073 };
00074
00075 // Global types which determine sizes
00077 using index_t = int;
00079 using set_value_t = unsigned long;
00080
00081 // Global constants
00083 const double MS_PER_S = 1000.0;
00084
00085 // Constants which determine sizes
00086
00087 // Bits per unsigned long
00088 #if (ULONG_MAX == (4294967295UL))
00089 #define _GLUCAT_BITS_PER_ULONG 32
00090 #elif (ULONG_MAX == (18446744073709551615UL))
00091 #define _GLUCAT_BITS_PER_ULONG 64
00092 #elif defined(__WORDSIZE)
00093 #define _GLUCAT_BITS_PER_ULONG __WORDSIZE
00094 #endif
00095
00097 _GLUCAT_CTAssert(std::numeric_limits<unsigned char>::radix == 2, CannotDetermineBitsPerChar)
00098
00099
00100 const index_t BITS_PER_CHAR = std::numeric_limits<unsigned char>::digits;
00101
00103 const index_t BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits;
00104
00105 _GLUCAT_CTAssert(_GLUCAT_BITS_PER_ULONG == BITS_PER_SET_VALUE, BitsPerULongDoesNotMatchSetValueType)
00106
00107 // Constants which are determined by size
00109 const index_t DEFAULT_LO = -index_t(BITS_PER_SET_VALUE / 2);
00111 const index_t DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2);
00112
00114 template< typename LHS_T, typename RHS_T >
00115 inline
00116 auto
00117 pos_mod(LHS_T lhs, RHS_T rhs) -> LHS_T
00118 { return lhs > 0? lhs % rhs : (-lhs) % rhs == 0 ? 0 : rhs - (-lhs) % rhs; }
00119
00120 }
00121 #endif // _GLUCAT_GLOBAL_H

```

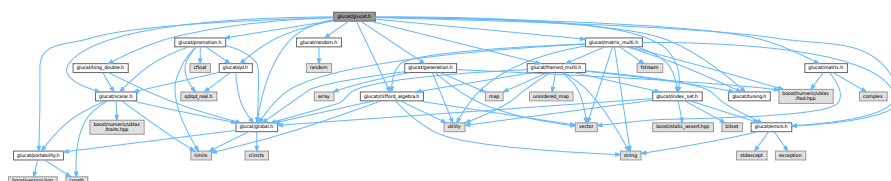
## 7.19 glucat/glucat.h File Reference

```

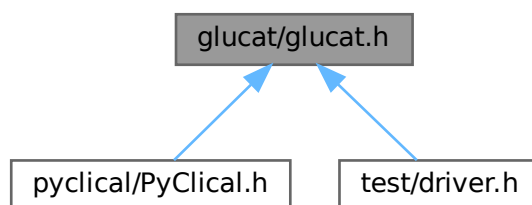
#include "glucat/portability.h"
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/scalar.h"
#include "glucat/long_double.h"

```

```
#include "glucat/qd.h"
#include "glucat/promotion.h"
#include "glucat/random.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include "glucat/framed_multi.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include "glucat/matrix_multi.h"
Include dependency graph for glucat.h:
```



This graph shows which files directly or indirectly include this file:



## 7.20 glucat.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GLUCAT_H
00002 #define _GLUCAT_GLUCAT_H
00003 /*****
00004      GluCat : Generic library of universal Clifford algebra templates
00005      glucat.h : Organize GluCat header files for applications
00006      -----
00007      begin                : Sun 2001-12-09
00008      copyright            : (C) 2001-2021 by Paul C. Leopardi
00009      *****/
00010
00011      This library is free software: you can redistribute it and/or modify
00012      it under the terms of the GNU Lesser General Public License as published
00013      by the Free Software Foundation, either version 3 of the License, or
00014      (at your option) any later version.
00015
00016      This library is distributed in the hope that it will be useful,
00017      but WITHOUT ANY WARRANTY; without even the implied warranty of
00018      MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019      GNU Lesser General Public License for more details.
00020
00021      You should have received a copy of the GNU Lesser General Public License
00022      along with this library.  If not, see <http://www.gnu.org/licenses/>.

```

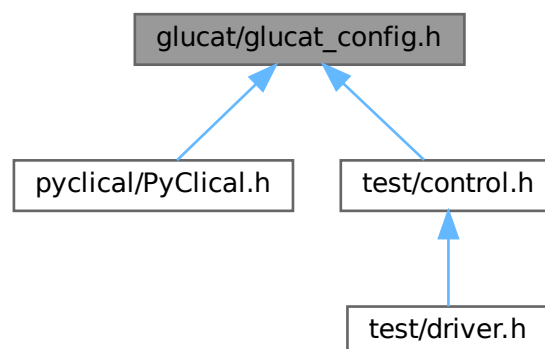
```

00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031 Arvind Raja's original header comments and references follow.
00032 *****
00033 // clifford algebra package, Arvind.Raja@hut.fi
00034 // ref: Press et.al. "Numerical Recipes in C", 2nd ed., C.U.P., 1992.
00035 // ref: LEDA, v 3.0, Stefan N\aher, Max-Planck-Institut f\ur Informatik
00036 // ref: Stroustrup B., "The C++ Programming Language", 2nd ed.,
00037 // Addison-Wesley, 1991.
00038 // ref: R. Sedgewick, "Algorithms in C++", Addison-Wesley, 1992.
00039 // ref: S. Meyers, "Effective C++ ", Addison-Wesley, 1992.
00040 *****/
00041
00042 #include "glucat/portability.h"
00043
00044 #include "glucat/global.h"
00045
00046 #include "glucat/errors.h"
00047
00048 #include "glucat/index_set.h"
00049
00050 #include "glucat/scalar.h"
00051
00052 #include "glucat/long_double.h"
00053
00054 #include "glucat/qd.h"
00055
00056 #include "glucat/promotion.h"
00057
00058 #include "glucat/random.h"
00059
00060 #include "glucat/clifford_algebra.h"
00061
00062 #include "glucat/tuning.h"
00063
00064 #include "glucat/framed_multi.h"
00065
00066 #include "glucat/generation.h"
00067
00068 #include "glucat/matrix.h"
00069
00070 #include "glucat/matrix_multi.h"
00071
00072 #endif // _GLUCAT_GLUCAT_H

```

## 7.21 glucat/glucat\_config.h File Reference

This graph shows which files directly or indirectly include this file:





## Macros

- `#define GLUCAT_HAVE_CXX11 1`
- `#define GLUCAT_HAVE_INTTYPES_H 1`
- `#define GLUCAT_HAVE_STDINT_H 1`
- `#define GLUCAT_HAVE_STDIO_H 1`
- `#define GLUCAT_HAVE_STDLIB_H 1`
- `#define GLUCAT_HAVE_STRINGS_H 1`
- `#define GLUCAT_HAVE_STRING_H 1`
- `#define GLUCAT_HAVE_SYS_STAT_H 1`
- `#define GLUCAT_HAVE_SYS_TYPES_H 1`
- `#define GLUCAT_HAVE_UNISTD_H 1`
- `#define GLUCAT_PACKAGE "glucat"`
- `#define GLUCAT_PACKAGE_BUGREPORT ""`
- `#define GLUCAT_PACKAGE_NAME "glucat"`
- `#define GLUCAT_PACKAGE_STRING "glucat 0.12.0"`
- `#define GLUCAT_PACKAGE_TARNAME "glucat"`
- `#define GLUCAT_PACKAGE_URL ""`
- `#define GLUCAT_PACKAGE_VERSION "0.12.0"`
- `#define GLUCAT_STDC_HEADERS 1`
- `#define GLUCAT_VERSION "0.12.0"`

## 7.21.1 Macro Definition Documentation

### 7.21.1.1 GLUCAT\_HAVE\_CXX11

```
#define GLUCAT_HAVE_CXX11 1
```

Definition at line 20 of file [glucat\\_config.h](#).

### 7.21.1.2 GLUCAT\_HAVE\_INTTYPES\_H

```
#define GLUCAT_HAVE_INTTYPES_H 1
```

Definition at line 28 of file [glucat\\_config.h](#).

### 7.21.1.3 GLUCAT\_HAVE\_STDINT\_H

```
#define GLUCAT_HAVE_STDINT_H 1
```

Definition at line 39 of file [glucat\\_config.h](#).

### 7.21.1.4 GLUCAT\_HAVE\_STDIO\_H

```
#define GLUCAT_HAVE_STDIO_H 1
```

Definition at line 44 of file [glucat\\_config.h](#).

#### 7.21.1.5 GLUCAT\_HAVE\_STDLIB\_H

```
#define GLUCAT_HAVE_STDLIB_H 1
```

Definition at line 49 of file [glucat\\_config.h](#).

#### 7.21.1.6 GLUCAT\_HAVE\_STRING\_H

```
#define GLUCAT_HAVE_STRING_H 1
```

Definition at line 59 of file [glucat\\_config.h](#).

#### 7.21.1.7 GLUCAT\_HAVE\_STRINGS\_H

```
#define GLUCAT_HAVE_STRINGS_H 1
```

Definition at line 54 of file [glucat\\_config.h](#).

#### 7.21.1.8 GLUCAT\_HAVE\_SYS\_STAT\_H

```
#define GLUCAT_HAVE_SYS_STAT_H 1
```

Definition at line 64 of file [glucat\\_config.h](#).

#### 7.21.1.9 GLUCAT\_HAVE\_SYS\_TYPES\_H

```
#define GLUCAT_HAVE_SYS_TYPES_H 1
```

Definition at line 69 of file [glucat\\_config.h](#).

#### 7.21.1.10 GLUCAT\_HAVE\_UNISTD\_H

```
#define GLUCAT_HAVE_UNISTD_H 1
```

Definition at line 74 of file [glucat\\_config.h](#).

#### 7.21.1.11 GLUCAT\_PACKAGE

```
#define GLUCAT_PACKAGE "glucat"
```

Definition at line 79 of file [glucat\\_config.h](#).

#### 7.21.1.12 GLUCAT\_PACKAGE\_BUGREPORT

```
#define GLUCAT_PACKAGE_BUGREPORT ""
```

Definition at line 84 of file [glucat\\_config.h](#).

#### 7.21.1.13 GLUCAT\_PACKAGE\_NAME

```
#define GLUCAT_PACKAGE_NAME "glucat"
```

Definition at line 89 of file [glucat\\_config.h](#).

Referenced by [glucat::control\\_t::control\\_t\(\)](#).

#### 7.21.1.14 GLUCAT\_PACKAGE\_STRING

```
#define GLUCAT_PACKAGE_STRING "glucat 0.12.0"
```

Definition at line 94 of file [glucat\\_config.h](#).

#### 7.21.1.15 GLUCAT\_PACKAGE\_TARNAME

```
#define GLUCAT_PACKAGE_TARNAME "glucat"
```

Definition at line 99 of file [glucat\\_config.h](#).

#### 7.21.1.16 GLUCAT\_PACKAGE\_URL

```
#define GLUCAT_PACKAGE_URL ""
```

Definition at line 104 of file [glucat\\_config.h](#).

#### 7.21.1.17 GLUCAT\_PACKAGE\_VERSION

```
#define GLUCAT_PACKAGE_VERSION "0.12.0"
```

Definition at line 109 of file [glucat\\_config.h](#).

#### 7.21.1.18 GLUCAT\_STDC\_HEADERS

```
#define GLUCAT_STDC_HEADERS 1
```

Definition at line 116 of file [glucat\\_config.h](#).

#### 7.21.1.19 GLUCAT\_VERSION

```
#define GLUCAT_VERSION "0.12.0"
```

Definition at line 121 of file [glucat\\_config.h](#).

Referenced by [glucat::control\\_t::control\\_t\(\)](#).

## 7.22 glucat\_config.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GLUCAT_CONFIG_H
00002 #define _GLUCAT_GLUCAT_CONFIG_H 1
00003
00004 /* glucat/glucat_config.h. Generated automatically at end of configure. */
00005 /* config.h. Generated from config.h.in by configure. */
00006 /* config.h.in. Generated from configure.ac by autoheader. */
00007
00008 /* Define to dummy `main' function (if any) required to link to the Fortran
00009    libraries. */
00010 /* #undef F77_DUMMY_MAIN */
00011
00012 /* Define if F77 and FC dummy `main' functions are identical. */
00013 /* #undef FC_DUMMY_MAIN_EQ_F77 */
00014
00015 /* Define if you have a BLAS library. */
00016 /* #undef HAVE_BLAS */
00017
00018 /* define if the compiler supports basic C++11 syntax */
00019 #ifndef GLUCAT_HAVE_CXX11
00020 #define GLUCAT_HAVE_CXX11 1
00021 #endif
00022
00023 /* define if the compiler supports basic C++14 syntax */
00024 /* #undef HAVE_CXX14 */
00025
00026 /* Define to 1 if you have the <inttypes.h> header file. */
00027 #ifndef GLUCAT_HAVE_INTTYPES_H
00028 #define GLUCAT_HAVE_INTTYPES_H 1
00029 #endif
00030
00031 /* Define if you have LAPACK library. */
00032 /* #undef HAVE_LAPACK */
00033
00034 /* Define to 1 if you have the `lmf' library (-lmf). */
00035 /* #undef HAVE_LIBIMF */
00036
00037 /* Define to 1 if you have the <stdint.h> header file. */
00038 #ifndef GLUCAT_HAVE_STDINT_H
00039 #define GLUCAT_HAVE_STDINT_H 1
00040 #endif
00041
00042 /* Define to 1 if you have the <stdio.h> header file. */
00043 #ifndef GLUCAT_HAVE_STDIO_H
00044 #define GLUCAT_HAVE_STDIO_H 1
00045 #endif
00046
00047 /* Define to 1 if you have the <stdlib.h> header file. */
00048 #ifndef GLUCAT_HAVE_STDLIB_H
00049 #define GLUCAT_HAVE_STDLIB_H 1
00050 #endif
00051
00052 /* Define to 1 if you have the <strings.h> header file. */
00053 #ifndef GLUCAT_HAVE_STRINGS_H
00054 #define GLUCAT_HAVE_STRINGS_H 1
00055 #endif
00056
00057 /* Define to 1 if you have the <string.h> header file. */
00058 #ifndef GLUCAT_HAVE_STRING_H
00059 #define GLUCAT_HAVE_STRING_H 1
00060 #endif
00061
00062 /* Define to 1 if you have the <sys/stat.h> header file. */
00063 #ifndef GLUCAT_HAVE_SYS_STAT_H
00064 #define GLUCAT_HAVE_SYS_STAT_H 1
00065 #endif
00066
00067 /* Define to 1 if you have the <sys/types.h> header file. */
00068 #ifndef GLUCAT_HAVE_SYS_TYPES_H
00069 #define GLUCAT_HAVE_SYS_TYPES_H 1
00070 #endif
00071
00072 /* Define to 1 if you have the <unistd.h> header file. */
00073 #ifndef GLUCAT_HAVE_UNISTD_H
00074 #define GLUCAT_HAVE_UNISTD_H 1
00075 #endif
00076
00077 /* Name of package */
00078 #ifndef GLUCAT_PACKAGE
00079 #define GLUCAT_PACKAGE "glucat"
00080 #endif
00081
00082 /* Define to the address where bug reports for this package should be sent. */

```

```

00083 #ifndef GLUCAT_PACKAGE_BUGREPORT
00084 #define GLUCAT_PACKAGE_BUGREPORT ""
00085 #endif
00086
00087 /* Define to the full name of this package. */
00088 #ifndef GLUCAT_PACKAGE_NAME
00089 #define GLUCAT_PACKAGE_NAME "glucat"
00090 #endif
00091
00092 /* Define to the full name and version of this package. */
00093 #ifndef GLUCAT_PACKAGE_STRING
00094 #define GLUCAT_PACKAGE_STRING "glucat 0.12.0"
00095 #endif
00096
00097 /* Define to the one symbol short name of this package. */
00098 #ifndef GLUCAT_PACKAGE_TARNAME
00099 #define GLUCAT_PACKAGE_TARNAME "glucat"
00100 #endif
00101
00102 /* Define to the home page for this package. */
00103 #ifndef GLUCAT_PACKAGE_URL
00104 #define GLUCAT_PACKAGE_URL ""
00105 #endif
00106
00107 /* Define to the version of this package. */
00108 #ifndef GLUCAT_PACKAGE_VERSION
00109 #define GLUCAT_PACKAGE_VERSION "0.12.0"
00110 #endif
00111
00112 /* Define to 1 if all of the C90 standard headers exist (not just the ones
00113    required in a freestanding environment). This macro is provided for
00114    backward compatibility; new code need not use it. */
00115 #ifndef GLUCAT_STDC_HEADERS
00116 #define GLUCAT_STDC_HEADERS 1
00117 #endif
00118
00119 /* Version number of package */
00120 #ifndef GLUCAT_VERSION
00121 #define GLUCAT_VERSION "0.12.0"
00122 #endif
00123
00124 /* once: _GLUCAT_GLUCAT_CONFIG_H */
00125 #endif

```

## 7.23 glucat/glucat\_imp.h File Reference

```

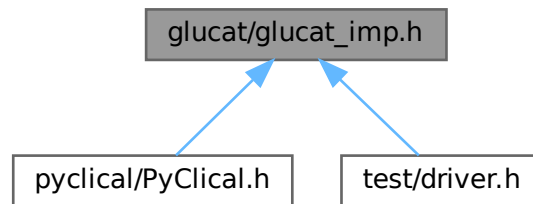
#include "glucat/errors_imp.h"
#include "glucat/index_set_imp.h"
#include "glucat/scalar_imp.h"
#include "glucat/clifford_algebra_imp.h"
#include "glucat/random.h"
#include "glucat/framed_multi_imp.h"
#include "glucat/matrix_imp.h"
#include "glucat/generation_imp.h"
#include "glucat/matrix_multi_imp.h"

```

Include dependency graph for glucat\_imp.h:



This graph shows which files directly or indirectly include this file:



## 7.24 glucat\_imp.h

[Go to the documentation of this file.](#)

```

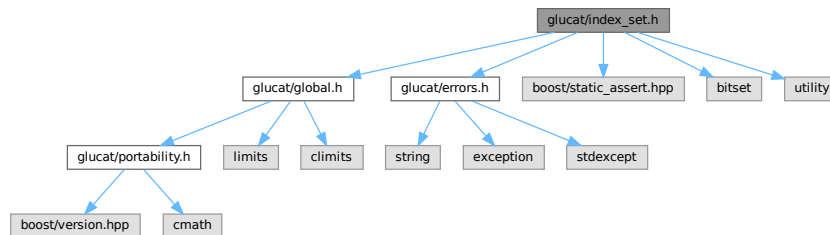
00001 #ifndef _GLUCAT_GLUCAT_IMP_H
00002 #define _GLUCAT_GLUCAT_IMP_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   glucat_imp.h : Organize GluCat template definitions which cannot be compiled separately
00006   -----
00007   begin                : Sun 2001-12-25
00008   copyright             : (C) 2001-2012 by Paul C. Leopardi
00009   *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030   *****/
00031   For Arvind Raja's original header comments, see glucat.h
00032   *****/
00033
00034   // Template definitions which cannot be compiled separately
00035
00036   #include "glucat/errors_imp.h"
00037
00038   #include "glucat/index_set_imp.h"
00039
00040   #include "glucat/scalar_imp.h"
00041
00042   #include "glucat/clifford_algebra_imp.h"
00043
00044   #include "glucat/random.h"
00045
00046   #include "glucat/framed_multi_imp.h"
00047
00048   #include "glucat/matrix_imp.h"
00049
00050   #include "glucat/generation_imp.h"
00051
00052   #include "glucat/matrix_multi_imp.h"
00053
00054 #endif // _GLUCAT_GLUCAT_IMP_H

```

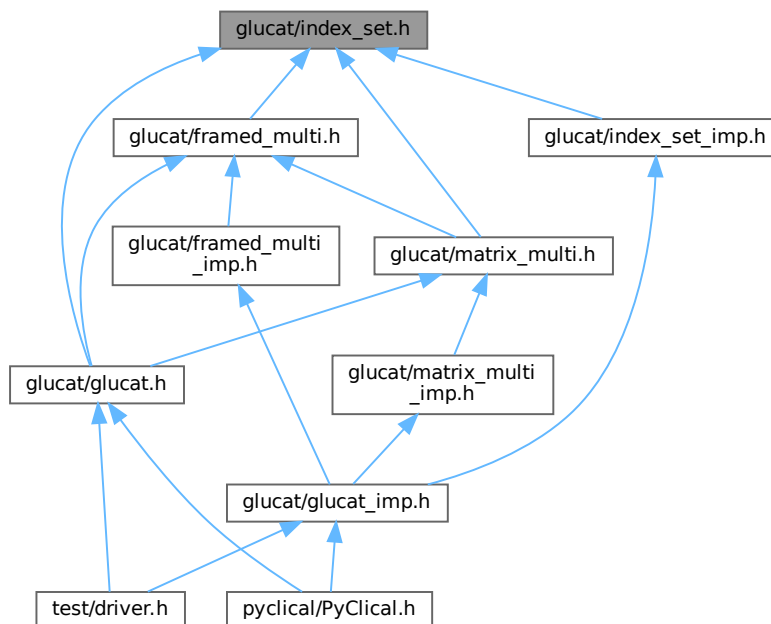
## 7.25 glucat/index\_set.h File Reference

```
#include "glucat/global.h"
#include "glucat/errors.h"
#include <boost/static_assert.hpp>
#include <bitset>
#include <utility>
```

Include dependency graph for index\_set.h:



This graph shows which files directly or indirectly include this file:



### Classes

- class `glucat::index_set< LO, HI >`  
Index set class based on `std::bitset<>` in Gnu standard C++ library.
- class `glucat::index_set< LO, HI >::reference`  
Index set member reference.

## Namespaces

- namespace [glucat](#)

## Functions

- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator^ (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Symmetric set difference: exclusive or.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator& (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Set intersection: and.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator| (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Set union: or.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::compare (const index\_set< LO, HI > &a, const index\_set< LO, HI > &b) -> int`  
*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*
- `glucat::GLUCAT\_CTAssert (sizeof(set\_value\_t) >=sizeof(std::bitset< DEFAULT\_HI-DEFAULT\_LO >),`  
`Default_index_set_too_big_for_value) template< const index\_t LO`  
*Size of [set\\_value\\_t](#) should be enough to contain `bitset<DEFAULT_HI-DEFAULT_LO>`*
- `const index\_t HI auto glucat::operator<< (std::ostream &os, const index\_set< LO, HI > &ist) -> std::ostream &`
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator>> (std::istream &s, index\_set< LO, HI > &ist) -> std::istream &`  
*Read in index set.*
- `auto glucat::sign\_of\_square (index\_t j) -> int`  
*Square of generator {j}.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::min\_neg (const index\_set< LO, HI > &ist) -> index\_t`  
*Minimum negative index, or 0 if none.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::max\_pos (const index\_set< LO, HI > &ist) -> index\_t`  
*Maximum positive index, or 0 if none.*

## 7.26 [index\\_set.h](#)

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_INDEX_SET_H
00002 #define _GLUCAT_INDEX_SET_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     index_set.h : Declare a class for a set of non-zero integer indices
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
```



```

00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author.  See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036
00037 #include <boost/static_assert.hpp>
00038
00039 #include <bitset>
00040 #include <utility>
00041
00042 namespace glucat
00043 {
00044     template<const index_t LO, const index_t HI>
00045     class index_set; // forward
00046
00047     template<const index_t LO, const index_t HI>
00048     auto
00049     operator^ (const index_set<LO,HI>& lhs,
00050               const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00051
00052     template<const index_t LO, const index_t HI>
00053     auto
00054     operator& (const index_set<LO,HI>& lhs,
00055               const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00056
00057     template<const index_t LO, const index_t HI>
00058     auto
00059     operator| (const index_set<LO,HI>& lhs,
00060               const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00061
00062     // -1 if a<b, +1 if a>b, 0 if a==b
00063     template<const index_t LO, const index_t HI>
00064     auto
00065     compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int;
00066
00067     template<const index_t LO, const index_t HI>
00068     class index_set :
00069     private std::bitset<HI-LO>
00070     {
00071     private:
00072         BOOST_STATIC_ASSERT((LO <= 0) && (0 <= HI) && (LO < HI) && \
00073                             (~LO < _GLUCAT_BITS_PER_ULONG) && \
00074                             (HI < _GLUCAT_BITS_PER_ULONG) && \
00075                             (HI-LO <= _GLUCAT_BITS_PER_ULONG));
00076     public:
00077         using bitset_t = std::bitset<HI - LO>;
00078         using error_t = error<index_set>;
00079
00080         using index_set_t = index_set;
00081         using index_pair_t = std::pair<index_t, index_t>;
00082
00083         static const index_t v_lo = LO;
00084         static const index_t v_hi = HI;
00085
00086         static auto classname() -> const std::string;
00087         index_set() = default;
00088         index_set(const bitset_t bst);
00089         index_set(const index_t idx);
00090         index_set(const set_value_t folded_val, const index_set_t frm, const bool prechecked = false);
00091         index_set(const index_pair_t& range, const bool prechecked = false);
00092         index_set(const std::string& str);
00093
00094         auto operator== (const index_set_t rhs) const -> bool;
00095         auto operator!= (const index_set_t rhs) const -> bool;
00096         auto operator~ () const -> index_set_t;
00097         auto operator^= (const index_set_t rhs) -> index_set_t&;
00098         auto operator&= (const index_set_t rhs) -> index_set_t&;
00099         auto operator|= (const index_set_t rhs) -> index_set_t&;
00100         auto operator[] (const index_t idx) const -> bool;
00101         auto test(const index_t idx) const -> bool;
00102         auto set() -> index_set_t&;
00103         auto set(const index_t idx) -> index_set_t&;
00104         auto set(const index_t idx, const int val) -> index_set_t&;

```

```

00127     auto reset() -> index_set_t&;
00129     auto reset(const index_t idx) -> index_set_t&;
00131     auto flip() -> index_set_t&;
00133     auto flip(const index_t idx) -> index_set_t&;
00135     auto count() const -> index_t;
00137     auto count_neg() const -> index_t;
00139     auto count_pos() const -> index_t;
00141     auto min() const -> index_t;
00143     auto max() const -> index_t;
00144
00145     // Functions which support Clifford algebra operations
00147     auto operator< (const index_set_t rhs) const -> bool;
00149     auto is_contiguous () const -> bool;
00151     auto fold () const -> const index_set_t;
00153     auto fold (const index_set_t frm, const bool prechecked = false) const -> const
index_set_t;
00155     auto unfold (const index_set_t frm, const bool prechecked = false) const -> const
index_set_t;
00157     auto value_of_fold (const index_set_t frm) const -> set_value_t;
00159     auto sign_of_mult (const index_set_t ist) const -> int;
00161     auto sign_of_square () const -> int;
00162
00164     auto hash_fn () const -> size_t;
00165
00166     // Friends
00167     friend auto operator^<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00168     friend auto operator&<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00169     friend auto operator|<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00170     friend auto compare<> (const index_set_t& lhs, const index_set_t& rhs) -> int;
00171
00172     // Member reference:
00173     class reference;
00174     friend class reference;
00175
00177     class reference {
00178     friend class index_set;
00179
00180     public:
00182     reference() = delete;
00183     reference (index_set_t& ist, index_t idx);
00184     ~reference () = default;
00186     auto operator== (const reference& c_j) const -> bool;
00188     auto operator= (const bool x) -> reference&;
00190     auto operator= (const reference& c_j) -> reference&;
00192     auto operator~ () const -> bool;
00194     operator bool () const;
00196     auto flip() -> reference&;
00197
00198     private:
00199     index_set_t* m_pst;
00200     index_t m_idx;
00201     };
00203     auto operator[](index_t idx) -> reference;
00204 private:
00206     auto lex_less_than (const index_set_t rhs) const -> bool;
00207     };
00208
00210     _GLUCAT_CTAssert(sizeof(set_value_t) >= sizeof(std::bitset<DEFAULT_HI-DEFAULT_LO>),
00211         Default_index_set_too_big_for_value)
00212
00213     // non-members
00214
00216     template<const index_t LO, const index_t HI>
00217     auto
00218     operator<< (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&;
00219
00221     template<const index_t LO, const index_t HI>
00222     auto
00223     operator>> (std::istream& s, index_set<LO,HI>& ist) -> std::istream&;
00224
00225     // Functions which support Clifford algebra operations
00227     auto sign_of_square(index_t j) -> int;
00228
00230     template<const index_t LO, const index_t HI>
00231     auto
00232     min_neg(const index_set<LO,HI>& ist) -> index_t;
00233
00235     template<const index_t LO, const index_t HI>
00236     auto
00237     max_pos(const index_set<LO,HI>& ist) -> index_t;
00238 }
00239 #endif // _GLUCAT_INDEX_SET_H

```

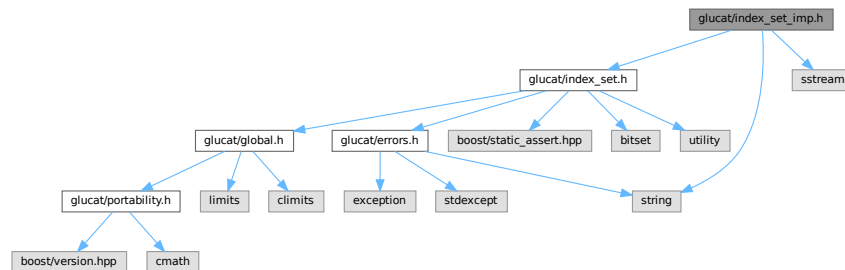
## 7.27 glucat/index\_set\_imp.h File Reference

```
#include "glucat/index_set.h"
```

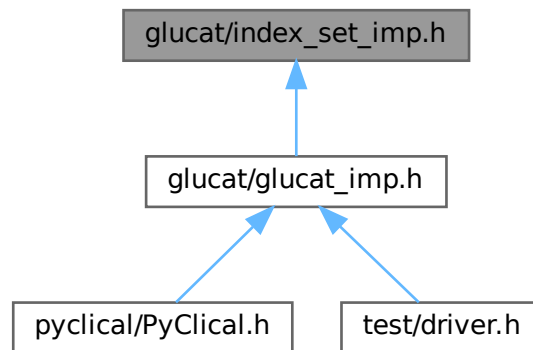
```
#include <string>
```

```
#include <sstream>
```

Include dependency graph for index\_set\_imp.h:



This graph shows which files directly or indirectly include this file:



### Namespaces

- namespace [glucat](#)

### Functions

- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator^ (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Symmetric set difference: exclusive or.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator& (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`

*Set intersection: and.*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator| (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >`

*Set union: or.*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::compare (const index_set< LO, HI > &a, const index_set< LO, HI > &b) -> int`  
*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator<< (std::ostream &os, const index_set< LO, HI > &ist) -> std::ostream &`

*Write out index set.*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator>> (std::istream &s, index_set< LO, HI > &ist) -> std::istream &`

*Read in index set.*

- `static auto glucat::inverse_reversed_gray (unsigned long x) -> unsigned long`

*Inverse reversed Gray code.*

- `static auto glucat::inverse_gray (unsigned long x) -> unsigned long`

*Inverse Gray code.*

- `auto glucat::sign_of_square (index_t j) -> int`

*Square of generator [j].*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::min_neg (const index_set< LO, HI > &ist) -> index_t`

*Minimum negative index, or 0 if none.*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::max_pos (const index_set< LO, HI > &ist) -> index_t`

*Maximum positive index, or 0 if none.*

## 7.28 index\_set\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_INDEX_SET_IMP_H
00002 #define _GLUCAT_INDEX_SET_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     index_set_imp.h : Implement a class for a set of non-zero integer indices
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright             : (C) 2001-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/index_set.h"

```

```

00035
00036 #include <string>
00037 #include <sstream>
00038
00039 namespace glucat
00040 {
00041     // References for algorithms:
00042     // [JA]: Joerg Arndt, "Algorithms for programmers", http://www.jjj.de/fxt/fxtbook.pdf
00043     //       Chapter 1, Bit wizardry, http://www.jjj.de/bitwizardry/bitwizardrypage.html
00044     // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00045
00046     template<const index_t LO, const index_t HI>
00047     inline
00048     auto
00049     index_set<LO,HI>::
00050     classname() -> const std::string
00051     { return "index_set"; }
00052
00053     template<const index_t LO, const index_t HI>
00054     index_set<LO,HI>::
00055     index_set(const index_t idx)
00056     { this->set(idx); }
00057
00058     template<const index_t LO, const index_t HI>
00059     index_set<LO,HI>::
00060     index_set(const bitset_t bst):
00061     bitset_t(bst)
00062     { }
00063
00064     template<const index_t LO, const index_t HI>
00065     index_set<LO,HI>::
00066     index_set(const set_value_t folded_val, const index_set_t frm, const bool prechecked)
00067     {
00068         if (!prechecked && folded_val >= (set_value_t(1) << frm.count()))
00069             throw error_t("index_set(val,frm): cannot create: value gives an index set outside of frame");
00070         const index_set_t folded_frame = frm.fold();
00071         const index_t min_index = folded_frame.min();
00072         const index_t skip = min_index > 0 ? 1 : 0;
00073         const index_set_t folded_set = index_set_t(bitset_t(folded_val) << (min_index - skip - LO));
00074         *this = folded_set.unfold(frm);
00075     }
00076
00077     template<const index_t LO, const index_t HI>
00078     index_set<LO,HI>::
00079     index_set(const index_pair_t& range, const bool prechecked)
00080     {
00081         if (!prechecked && (range.first < LO || range.second > HI))
00082             throw error_t("index_set(range): cannot create: range is too large");
00083         const index_t begin_bit = (range.first < 0)
00084             ? range.first-LO
00085             : range.first-LO-1;
00086         const index_t end_bit = (range.second < 0)
00087             ? range.second-LO+1
00088             : range.second-LO;
00089         unsigned long mask = ( (end_bit == _GLUCAT_BITS_PER_ULONGLONG)
00090             ? -1UL
00091             : (1UL << end_bit)-1UL)
00092             & ~((1UL << begin_bit)-1UL);
00093         *this = bitset_t(mask);
00094     }
00095
00096     template<const index_t LO, const index_t HI>
00097     index_set<LO,HI>::
00098     index_set(const std::string& str)
00099     {
00100         std::istringstream ss(str);
00101         ss >> *this;
00102         if (!ss)
00103             throw error_t("index_set_t(str): could not parse string");
00104         // Peek to see if the end of the string has been reached.
00105         ss.peek();
00106         if (!ss.eof())
00107             throw error_t("index_set_t(str): could not parse entire string");
00108     }
00109
00110     template<const index_t LO, const index_t HI>
00111     inline
00112     auto
00113     index_set<LO,HI>::
00114     operator== (const index_set_t rhs) const -> bool
00115     {
00116         const auto* pthis = static_cast<const bitset_t*>(this);
00117         return *pthis == static_cast<bitset_t>(rhs);
00118     }
00119
00120     template<const index_t LO, const index_t HI>
00121     inline

```

```

00129     auto
00130     index_set<LO,HI>::
00131     operator!= (const index_set_t rhs) const -> bool
00132     {
00133         const auto* pthis = static_cast<const bitset_t*>(this);
00134         return *pthis != static_cast<bitset_t>(rhs);
00135     }
00136
00137     template<const index_t LO, const index_t HI>
00138     inline
00139     auto
00140     index_set<LO,HI>::
00141     operator~ () const -> index_set_t
00142     { return bitset_t::operator~(); }
00143
00144     template<const index_t LO, const index_t HI>
00145     inline
00146     auto
00147     index_set<LO,HI>::
00148     operator^= (const index_set_t rhs) -> index_set_t&
00149     {
00150         bitset_t* pthis = this;
00151         *pthis ^= static_cast<bitset_t>(rhs);
00152         return *this;
00153     }
00154
00155     template<const index_t LO, const index_t HI>
00156     inline
00157     auto
00158     operator^ (const index_set<LO,HI>& lhs,
00159                const index_set<LO,HI>& rhs) -> const
00160     index_set<LO,HI>
00161     {
00162         using index_set_t = index_set<LO, HI>;
00163         using bitset_t = typename index_set_t::bitset_t;
00164         return static_cast<bitset_t>(lhs) ^ static_cast<bitset_t>(rhs);
00165     }
00166
00167     template<const index_t LO, const index_t HI>
00168     inline
00169     auto
00170     index_set<LO,HI>::
00171     operator&= (const index_set_t rhs) -> index_set_t&
00172     {
00173         bitset_t* pthis = this;
00174         *pthis &= static_cast<bitset_t>(rhs);
00175         return *this;
00176     }
00177
00178     template<const index_t LO, const index_t HI>
00179     inline
00180     auto
00181     operator& (const index_set<LO,HI>& lhs,
00182                const index_set<LO,HI>& rhs) -> const
00183     index_set<LO,HI>
00184     {
00185         using index_set_t = index_set<LO, HI>;
00186         using bitset_t = typename index_set_t::bitset_t;
00187         return static_cast<bitset_t>(lhs) & static_cast<bitset_t>(rhs);
00188     }
00189
00190     template<const index_t LO, const index_t HI>
00191     inline
00192     auto
00193     index_set<LO,HI>::
00194     operator|= (const index_set_t rhs) -> index_set_t&
00195     {
00196         bitset_t* pthis = this;
00197         *pthis |= static_cast<bitset_t>(rhs);
00198         return *this;
00199     }
00200
00201     template<const index_t LO, const index_t HI>
00202     inline
00203     auto
00204     operator| (const index_set<LO,HI>& lhs,
00205                const index_set<LO,HI>& rhs) -> const
00206     index_set<LO,HI>
00207     {
00208         using index_set_t = index_set<LO, HI>;
00209         using bitset_t = typename index_set_t::bitset_t;
00210         return static_cast<bitset_t>(lhs) | static_cast<bitset_t>(rhs);
00211     }
00212
00213     template<const index_t LO, const index_t HI>
00214     inline
00215     auto

```

```

00224 index_set<LO,HI>::
00225 operator[] (const index_t idx) -> reference
00226 { return reference(*this, idx); }
00227
00229 template<const index_t LO, const index_t HI>
00230 inline
00231 auto
00232 index_set<LO,HI>::
00233 operator[] (const index_t idx) const -> bool
00234 { return this->test(idx); }
00235
00237 template<const index_t LO, const index_t HI>
00238 inline
00239 auto
00240 index_set<LO,HI>::
00241 test(const index_t idx) const -> bool
00242 {
00243     // Reference: [JA], 1.2.1
00244     return (idx < 0)
00245         ? bool(bitset_t::to_ulong() & (1UL << (idx - LO)))
00246         : (idx > 0)
00247         ? bool(bitset_t::to_ulong() & (1UL << (idx - LO - 1)))
00248         : false;
00249 }
00250
00252 template<const index_t LO, const index_t HI>
00253 inline
00254 auto
00255 index_set<LO,HI>::
00256 set() -> index_set_t&
00257 {
00258     bitset_t::set();
00259     return *this;
00260 }
00261
00263 template<const index_t LO, const index_t HI>
00264 inline
00265 auto
00266 index_set<LO,HI>::
00267 set(index_t idx) -> index_set_t&
00268 {
00269     if (idx > 0)
00270         bitset_t::set(idx-LO-1);
00271     else if (idx < 0)
00272         bitset_t::set(idx-LO);
00273     return *this;
00274 }
00275
00277 template<const index_t LO, const index_t HI>
00278 inline
00279 auto
00280 index_set<LO,HI>::
00281 set(const index_t idx, const int val) -> index_set_t&
00282 {
00283     if (idx > 0)
00284         bitset_t::set(idx-LO-1, val);
00285     else if (idx < 0)
00286         bitset_t::set(idx-LO, val);
00287     return *this;
00288 }
00289
00291 template<const index_t LO, const index_t HI>
00292 inline
00293 auto
00294 index_set<LO,HI>::
00295 reset() -> index_set_t&
00296 {
00297     bitset_t::reset();
00298     return *this;
00299 }
00300
00302 template<const index_t LO, const index_t HI>
00303 inline
00304 auto
00305 index_set<LO,HI>::
00306 reset(const index_t idx) -> index_set_t&
00307 {
00308     if (idx > 0)
00309         bitset_t::reset(idx-LO-1);
00310     else if (idx < 0)
00311         bitset_t::reset(idx-LO);
00312     return *this;
00313 }
00314
00316 template<const index_t LO, const index_t HI>
00317 inline
00318 auto

```

```

00319 index_set<LO,HI>::
00320 flip() -> index_set<LO,HI>&
00321 {
00322     bitset_t::flip();
00323     return *this;
00324 }
00325
00326 template<const index_t LO, const index_t HI>
00327 inline
00328 auto
00329 index_set<LO,HI>::
00330 flip(const index_t idx) -> index_set_t&
00331 {
00332     {
00333         if (idx > 0)
00334             bitset_t::flip(idx-LO-1);
00335         else if (idx < 0)
00336             bitset_t::flip(idx-LO);
00337         return *this;
00338     }
00339
00340 template<const index_t LO, const index_t HI>
00341 inline
00342 auto
00343 index_set<LO,HI>::
00344 count() const -> index_t
00345 {
00346     {
00347         unsigned long val = bitset_t::to_ulong();
00348         // Reference: [JA], 1.3
00349         if (val == 0)
00350             return 0;
00351         else
00352         {
00353             index_t result = 1;
00354             while (val &= val-1)
00355                 ++result;
00356             return result;
00357         }
00358     }
00359
00360 template<const index_t LO, const index_t HI>
00361 inline
00362 auto
00363 index_set<LO,HI>::
00364 count_neg() const -> index_t
00365 {
00366     {
00367         static const index_set_t lo_mask = bitset_t((1UL << -LO) - 1UL);
00368         const index_set_t neg_part = *this & lo_mask;
00369         return neg_part.count();
00370     }
00371
00372 template<const index_t LO, const index_t HI>
00373 inline
00374 auto
00375 index_set<LO,HI>::
00376 count_pos() const -> index_t
00377 {
00378     {
00379         const auto* pthis = static_cast<const bitset_t*>(this);
00380         const index_set_t pos_part = *pthis >> -LO;
00381         return pos_part.count();
00382     }
00383
00384 #if (_GLUCAT_BITS_PER_ULONG == 64)
00385 template<const index_t LO, const index_t HI>
00386 inline
00387 auto
00388 index_set<LO,HI>::
00389 min() const -> index_t
00390 {
00391     {
00392         // Reference: [JA], 1.3
00393         unsigned long val = bitset_t::to_ulong();
00394         if (val == 0)
00395             return 0;
00396         else
00397         {
00398             val -= val & (val-1); // isolate lowest bit
00399
00400             index_t idx = 0;
00401             const index_t nbits = HI - LO;
00402
00403             if (nbits > 8)
00404             {
00405                 if (val & 0xffffffff00000000ul)
00406                     idx += 32;
00407                 if (val & 0xffff0000ffff0000ul)
00408                     idx += 16;
00409                 if (val & 0xff00ff00ff00ff00ul)
00410                     idx += 8;

```



```

00411     }
00412     if (val & 0xf0f0f0f0f0f0f0ul)
00413         idx += 4;
00414     if (val & 0xccccccccccccccul)
00415         idx += 2;
00416     if (val & 0aaaaaaaaaaaaaul)
00417         idx += 1;
00418
00419     return idx + ((idx < -LO) ? LO : LO+1);
00420 }
00421 }
00422 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00423 template<const index_t LO, const index_t HI>
00424 inline
00425 index_t
00426 index_set<LO,HI>::
00427 min() const
00428 {
00429     // Reference: [JA], 1.3
00430     unsigned long val = bitset_t::to_ulong();
00431     if (val == 0)
00432         return 0;
00433     else
00434     {
00435         val -= val & (val-1); // isolate lowest bit
00436
00437         index_t idx = 0;
00438         const index_t nbits = HI - LO;
00439         if (nbits > 8)
00440         {
00441             if (val & 0xffff0000ul)
00442                 idx += 16;
00443             if (val & 0xff00ff00ul)
00444                 idx += 8;
00445         }
00446         if (val & 0xf0f0f0f0ul)
00447             idx += 4;
00448         if (val & 0xccccccccul)
00449             idx += 2;
00450         if (val & 0aaaaaaaaul)
00451             idx += 1;
00452
00453         return idx + ((idx < -LO) ? LO : LO+1);
00454     }
00455 }
00456 }
00457 #else
00458 template<const index_t LO, const index_t HI>
00459 auto
00460 index_set<LO,HI>::
00461 min() const -> index_t
00462 {
00463     for (auto
00464         idx = LO;
00465         idx != 0;
00466         ++idx)
00467         if (this->test(idx))
00468             return idx;
00469     for (auto
00470         idx = index_t(1);
00471         idx <= HI;
00472         ++idx)
00473         if (this->test(idx))
00474             return idx;
00475     return 0;
00476 }
00477 }
00478 #endif
00479
00480 #if (_GLUCAT_BITS_PER_ULONG == 64)
00481 template<const index_t LO, const index_t HI>
00482 inline
00483 auto
00484 index_set<LO,HI>::
00485 max() const -> index_t
00486 {
00487     // Reference: [JA], 1.6
00488     auto val = bitset_t::to_ulong();
00489     if (val == 0)
00490         return 0;
00491     else
00492     {
00493         auto idx = index_t(0);
00494         const auto nbits = HI - LO;
00495         if (nbits > 8)
00496         {
00497             if (val & 0xffffffff00000000ul)
00498                 { val >>= 32; idx += 32; }
00499             if (val & 0x00000000ffff0000ul)

```

```

00501         { val >>= 16; idx += 16; }
00502         if (val & 0x000000000000ff00ul)
00503         { val >>= 8; idx += 8; }
00504     }
00505     if (val & 0x00000000000000f0ul)
00506     { val >>= 4; idx += 4; }
00507     if (val & 0x000000000000000cul)
00508     { val >>= 2; idx += 2; }
00509     if (val & 0x0000000000000002ul)
00510     { idx += 1; }
00511     return idx + ((idx < -LO) ? LO : LO+1);
00512 }
00513 }
00514 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00515 template<const index_t LO, const index_t HI>
00516 inline
00517 auto
00518 index_set<LO,HI>::
00519 max() const -> index_t
00520 {
00521     // Reference: [JA], 1.6
00522     auto val = bitset_t::to_ulong();
00523     if (val == 0)
00524         return 0;
00525     else
00526     {
00527         auto idx = index_t(0);
00528         const auto nbits = HI - LO;
00529         if (nbits > 8)
00530         {
00531             if (val & 0xffff0000ul)
00532             { val >>= 16; idx += 16; }
00533             if (val & 0x0000ff00ul)
00534             { val >>= 8; idx += 8; }
00535         }
00536         if (val & 0x000000f0ul)
00537         { val >>= 4; idx += 4; }
00538         if (val & 0x0000000cul)
00539         { val >>= 2; idx += 2; }
00540         if (val & 0x00000002ul)
00541         { idx += 1; }
00542         return idx + ((idx < -LO) ? LO : LO+1);
00543     }
00544 }
00545 }
00546 #else
00547 template<const index_t LO, const index_t HI>
00548 auto
00549 index_set<LO,HI>::
00550 max() const -> index_t
00551 {
00552     for (auto
00553         idx = HI;
00554         idx != 0;
00555         --idx)
00556         if (this->test(idx))
00557             return idx;
00558     for (auto
00559         idx = index_t(-1);
00560         idx >= LO;
00561         --idx)
00562         if (this->test(idx))
00563             return idx;
00564     return 0;
00565 }
00566 #endif
00567 // eg. {3,4,5} is less than {3,7,8}
00568 template<const index_t LO, const index_t HI>
00569 inline
00570 auto
00571 compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int
00572 {
00573     return (a == b)
00574         ? 0
00575         : a.lex_less_than(b)
00576         ? -1
00577         : 1;
00578 }
00579 // eg. {3,4,5} is less than {3,7,8}
00580 template<const index_t LO, const index_t HI>
00581 inline
00582 auto
00583 index_set<LO,HI>::
00584 lex_less_than(const index_set_t rhs) const -> bool
00585 { return bitset_t::to_ulong() < rhs.bitset_t::to_ulong(); }
00586
00587
00588
00589
00590
00591

```

```

00593 // Order by count, then order lexicographically within the equivalence class of count.
00594 template<const index_t LO, const index_t HI>
00595 inline
00596 auto
00597 index_set<LO,HI>::
00598 operator< (const index_set_t rhs) const -> bool
00599 {
00600     const auto this_grade = this->count();
00601     const auto rhs_grade = rhs.count();
00602     return (this_grade < rhs_grade)
00603         ? true
00604         : (this_grade > rhs_grade)
00605         ? false
00606         : this->lex_less_than(rhs);
00607 }
00608
00609 template<const index_t LO, const index_t HI>
00610 auto
00611 operator<< (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&
00612 {
00613     {
00614         index_t i;
00615         os << '{';
00616         for (i = LO;
00617              (i <= HI) && !(ist[i]);
00618              ++i)
00619             { }
00620         if (i <= HI)
00621             os << i;
00622         for (++i;
00623              i <= HI;
00624              ++i)
00625             if (ist[i])
00626                 os << ',' << i;
00627         os << '}';
00628         return os;
00629     }
00630
00631 template<const index_t LO, const index_t HI>
00632 auto
00633 operator>> (std::istream& s, index_set<LO,HI>& ist) -> std::istream&
00634 {
00635     // Parsing variables.
00636     auto i = index_t(0);
00637     using index_set_t = index_set<LO,HI>;
00638     auto local_ist = index_set_t();
00639     // Parsing control variables.
00640     auto parse_index_list = true;
00641     auto expect_closing_brace = false;
00642     auto expect_index = false;
00643     // Parse an optional opening brace.
00644     auto c = s.peek();
00645     // If there is a failure or end of file, this ends parsing.
00646     if (!s.good())
00647         parse_index_list = false;
00648     else
00649     { // Check for an opening brace.
00650         expect_closing_brace = (c == int('{'));
00651         if (expect_closing_brace)
00652         { // Consume the opening brace.
00653             s.get();
00654             // The next character may be a closing brace,
00655             // indicating the empty index set.
00656             c = s.peek();
00657             if (s.good() && (c == int('}')))
00658             { // A closing brace has been parsed and is no longer expected.
00659                 expect_closing_brace = false;
00660                 // Consume the closing brace.
00661                 s.get();
00662                 // This ends parsing.
00663                 parse_index_list = false;
00664             }
00665         }
00666     }
00667     if (s.good() && parse_index_list)
00668     { // Parse an optional index list.
00669         // The index list starts with a first index.
00670         for (s >> i;
00671              !s.fail();
00672              s >> i)
00673         { // An index has been parsed. Check to see if it is in range.
00674             if ((i < LO) || (i > HI))
00675             { // An index out of range is a failure.
00676                 s.clear(std::istream::failbit);
00677                 break;
00678             }
00679             // Add the index to the index set local_ist.
00680             local_ist.set(i);
00681         }

```

```

00682         // Immediately after parsing an index, an index is no longer expected.
00683         expect_index = false;
00684         // Reading the index may have resulted in an end of file condition.
00685         // If so, this ends the index list.
00686         if (s.eof())
00687             break;
00688         // The index list continues with a comma, and
00689         // may be ended by a closing brace, if it was begun with an opening brace.
00690         // Parse a possible comma or closing brace.
00691         c = s.peek();
00692         if (!s.good())
00693             break;
00694         // First, test for a closing brace, if expected.
00695         if (expect_closing_brace && (c == int('}')))
00696         { // Consume the closing brace.
00697             s.get();
00698             // Immediately after parsing the closing brace, it is no longer expected.
00699             expect_closing_brace = false;
00700             // A closing brace ends the index list.
00701             break;
00702         }
00703         // Now test for a comma.
00704         if (c == int(','))
00705         { // Consume the comma.
00706             s.get();
00707             // A index is expected after the comma.
00708             expect_index = true;
00709         }
00710         else
00711         { // Any other character here is a failure.
00712             s.clear(std::istream::failbit);
00713             break;
00714         }
00715     }
00716 }
00717 // If an index or a closing brace is still expected, this is a failure.
00718 if (expect_index || expect_closing_brace)
00719     s.clear(std::istream::failbit);
00720 // End of file is not a failure.
00721 if (s)
00722 { // The index set has been successfully parsed.
00723     ist = local_ist;
00724 }
00725 return s;
00726 }
00727
00728 template<const index_t LO, const index_t HI>
00729 inline
00730 auto
00731 index_set<LO,HI>::
00732 is_contiguous () const -> bool
00733 {
00734     {
00735         const auto min_index = this->min();
00736         const auto max_index = this->max();
00737         return (min_index < 0 && max_index > 0)
00738             ? max_index - min_index == this->count()
00739             : (min_index == 1 || max_index == -1) &&
00740               (max_index - min_index == this->count() - 1);
00741     }
00742 }
00743
00744 template<const index_t LO, const index_t HI>
00745 inline
00746 auto
00747 index_set<LO,HI>::
00748 fold() const -> const
00749 index_set<LO,HI>
00750 { return this->fold(*this, true); }
00751
00752 template<const index_t LO, const index_t HI>
00753 auto
00754 index_set<LO,HI>::
00755 fold(const index_set_t frm, const bool prechecked) const -> const
00756 index_set<LO,HI>
00757 {
00758     {
00759         if (!prechecked && ((*this | frm) != frm))
00760             throw error_t("fold(frm): cannot fold from outside of frame");
00761         const auto frm_min = frm.min();
00762         const auto frm_max = frm.max();
00763         auto result = index_set_t();
00764         auto fold_idx = index_t(-1);
00765         for (auto
00766             unfold_idx = fold_idx;
00767             unfold_idx >= frm_min;
00768             --unfold_idx)
00769             if (frm.test(unfold_idx))
00770                 // result.set(fold_idx--, this->test(unfold_idx));
00771         {

```

```

00772         if (this->test(unfold_idx))
00773             result.set(fold_idx);
00774         --fold_idx;
00775     }
00776     fold_idx = index_t(1);
00777     for (auto
00778         unfold_idx = fold_idx;
00779         unfold_idx <= frm_max;
00780         ++unfold_idx)
00781         if (frm.test(unfold_idx))
00782             // result.set(fold_idx++, this->test(unfold_idx));
00783         {
00784             if (this->test(unfold_idx))
00785                 result.set(fold_idx);
00786             ++fold_idx;
00787         }
00788     return result;
00789 }
00790
00791 template<const index_t LO, const index_t HI>
00792 auto
00793 index_set<LO,HI>::
00794 unfold(const index_set_t frm, const bool prechecked) const -> const index_set_t
00795 {
00796     {
00797         const char* msg =
00798             "unfold(frm): cannot unfold into a smaller frame";
00799         const auto frm_min = frm.min();
00800         const auto frm_max = frm.max();
00801         auto result = index_set_t();
00802         auto fold_idx = index_t(-1);
00803         for (auto
00804             unfold_idx = fold_idx;
00805             unfold_idx >= frm_min;
00806             --unfold_idx)
00807             if (frm.test(unfold_idx))
00808                 if (this->test(fold_idx--))
00809                     result.set(unfold_idx);
00810         if (!prechecked && ((fold_idx+1) > this->min()))
00811             throw error_t(msg);
00812         fold_idx = index_t(1);
00813         for (auto
00814             unfold_idx = fold_idx;
00815             unfold_idx <= frm_max;
00816             ++unfold_idx)
00817             if (frm.test(unfold_idx))
00818                 if (this->test(fold_idx++))
00819                     result.set(unfold_idx);
00820         if (!prechecked && ((fold_idx-1) < this->max()))
00821             throw error_t(msg);
00822         return result;
00823     }
00824
00825 template<const index_t LO, const index_t HI>
00826 inline
00827 auto
00828 index_set<LO,HI>::
00829 value_of_fold(const index_set_t frm) const -> set_value_t
00830 {
00831     {
00832         const auto min_index = frm.fold().min();
00833         if (min_index == 0)
00834             return 0;
00835         else
00836         {
00837             const auto folded_set = this->fold(frm);
00838             const auto skip = min_index > 0 ? index_t(1) : index_t(0);
00839             return folded_set.bitset_t::to_ulong() >> (min_index-LO-skip);
00840         }
00841     }
00842
00843 inline
00844 static
00845 auto inverse_reversed_gray(unsigned long x) -> unsigned long
00846 {
00847     {
00848         // Reference: [JA]
00849 #if (_GLUCAT_BITS_PER_ULONG >= 64)
00850         x ^= x << 32; // for 64-bit words
00851 #endif
00852         x ^= x << 16; // reversed_gray ** 16
00853         x ^= x << 8;  // reversed_gray ** 8
00854         x ^= x << 4;  // reversed_gray ** 4
00855         x ^= x << 2;  // reversed_gray ** 2
00856         x ^= x << 1;  // reversed_gray ** 1
00857         return x;
00858     }
00859
00860 inline
00861 static

```

```

00863 auto inverse_gray(unsigned long x) -> unsigned long
00864 {
00865     // Reference: [JA]
00866     #if (_GLUCAT_BITS_PER_ULONG >= 64)
00867         x ^= x » 32; // for 64-bit words
00868     #endif
00869     x ^= x » 16; // gray ** 16
00870     x ^= x » 8; // gray ** 8
00871     x ^= x » 4; // gray ** 4
00872     x ^= x » 2; // gray ** 2
00873     x ^= x » 1; // gray ** 1
00874     return x;
00875 }
00876
00877 template<const index_t LO, const index_t HI>
00878 auto
00880 index_set<LO,HI>::
00881 sign_of_mult(const index_set_t rhs) const -> int
00882 {
00883     // Implemented using Walsh functions and Gray codes.
00884     // Reference: [L] Chapter 21, 21.3
00885     // Reference: [JA]
00886     const auto uthis = this->bitset_t::to_ulong();
00887     const auto urhs = rhs.bitset_t::to_ulong();
00888     const auto nbits = HI - LO;
00889     auto negative = 0UL;
00890     if (nbits > 8)
00891     {
00892         // Set h to be the inverse reversed Gray code of rhs.
00893         // This sets each bit of h to be the cumulative ^ of
00894         // the same and lower bits of rhs.
00895         const auto h = inverse_reversed_gray(urhs);
00896         // Set k to be the inverse Gray code of *this & h.
00897         // This sets the low bit of k to be parity(*this & h).
00898         const auto k = inverse_gray(uthis & h);
00899         // Set q to be the inverse Gray code of the positive part of *this & rhs.
00900         const auto q = inverse_gray((uthis & urhs) » -LO);
00901         negative = k ^ q;
00902     }
00903     else
00904     {
00905         auto h = 0UL;
00906         for (auto
00907             j = index_t(0);
00908             j < -LO;
00909             ++j)
00910         {
00911             h ^= urhs » j;
00912             negative ^= h & (uthis » j);
00913         }
00914         for (auto
00915             j = index_t(-LO);
00916             j < nbits;
00917             ++j)
00918         {
00919             negative ^= h & (uthis » j);
00920             h ^= urhs » j;
00921         }
00922     }
00923     return 1 - int((negative & 1) « 1);
00924 }
00925
00927 template<const index_t LO, const index_t HI>
00928 inline
00929 auto
00930 index_set<LO,HI>::
00931 sign_of_square() const -> int
00932 {
00933     auto result = 1 - int((this->count_neg() % 2) « 1);
00934     switch (this->count() % 4)
00935     {
00936     case 2:
00937     case 3:
00938         result *= -1;
00939         break;
00940     default:
00941         break;
00942     }
00943     return result;
00944 }
00945
00947 template<const index_t LO, const index_t HI>
00948 inline
00949 auto
00950 index_set<LO,HI>::
00951 hash_fn() const -> size_t
00952 {

```

```

00953     static const auto lo_mask = (1UL « -LO) - 1UL;
00954     const auto uthis = bitset_t::to_ulong();
00955     const auto neg_part = uthis & lo_mask;
00956     const auto pos_part = uthis » -LO;
00957     return size_t(neg_part ^ pos_part);
00958 }
00959
00961     inline
00962     auto
00963     sign_of_square(index_t j) -> int
00964     { return (j < 0) ? -1 : 1; }
00965
00967     template<const index_t LO, const index_t HI>
00968     inline
00969     auto
00970     min_neg(const index_set<LO,HI>& ist) -> index_t
00971     { return std::min(ist.min(), 0); }
00972
00974     template<const index_t LO, const index_t HI>
00975     inline
00976     auto
00977     max_pos(const index_set<LO,HI>& ist) -> index_t
00978     { return std::max(ist.max(), 0); }
00979
00980 // index_set reference
00981
00983     template<const index_t LO, const index_t HI>
00984     inline
00985     index_set<LO,HI>::reference::
00986     reference( index_set_t& ist, index_t idx ) :
00987         m_pst(&ist),
00988         m_idx(idx)
00989     { }
00990
00992     template<const index_t LO, const index_t HI>
00993     inline
00994     auto
00995     index_set<LO,HI>::reference::
00996     operator== (const reference& c_j) const -> bool
00997     { return m_pst == c_j.m_pst && m_idx == c_j.m_idx; }
00998
01000     template<const index_t LO, const index_t HI>
01001     inline
01002     auto
01003     index_set<LO,HI>::reference::
01004     operator= (bool x) -> reference&
01005     {
01006         if ( x )
01007             m_pst->set(m_idx);
01008         else
01009             m_pst->reset(m_idx);
01010         return *this;
01011     }
01012
01014     template<const index_t LO, const index_t HI>
01015     inline
01016     auto
01017     index_set<LO,HI>::reference::
01018     operator= (const reference& c_j) -> reference&
01019     {
01020         if (&c_j != this && c_j != *this)
01021         {
01022             if ( (c_j.m_pst)[c_j.m_idx] )
01023                 m_pst->set(m_idx);
01024             else
01025                 m_pst->reset(m_idx);
01026         }
01027         return *this;
01028     }
01029
01031     template<const index_t LO, const index_t HI>
01032     inline
01033     auto
01034     index_set<LO,HI>::reference::
01035     operator~ () const -> bool
01036     { return !(m_pst->test(m_idx)); }
01037
01039     template<const index_t LO, const index_t HI>
01040     inline
01041     index_set<LO,HI>::reference::
01042     operator bool () const
01043     { return m_pst->test(m_idx); }
01044
01046     template<const index_t LO, const index_t HI>
01047     inline
01048     auto
01049     index_set<LO,HI>::reference::

```

```

01050     flip() -> reference&
01051     {
01052         m_pst->flip(m_idx);
01053         return *this;
01054     }
01055 }
01056 #endif // _GLUCAT_INDEX_SET_IMP_H

```

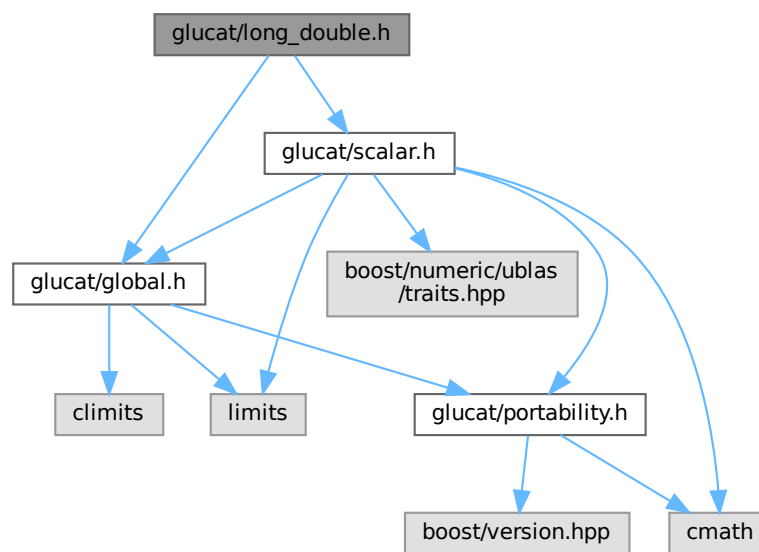
## 7.29 glucat/long\_double.h File Reference

```

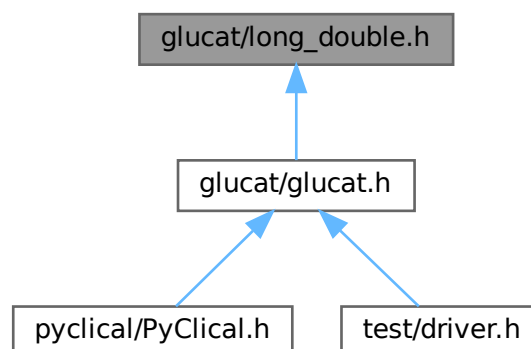
#include "glucat/global.h"
#include "glucat/scalar.h"

```

Include dependency graph for long\_double.h:



This graph shows which files directly or indirectly include this file:





## Namespaces

- namespace `glucat`

## Variables

- static const long double `glucat::l_pi` = 3.1415926535897932384626433832795029L
- static const long double `glucat::l_ln2` = 0.6931471805599453094172321214581766L

## 7.30 long\_double.h

[Go to the documentation of this file.](#)

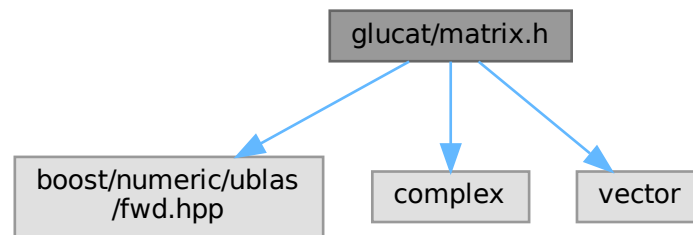
```

00001 #ifndef _GLUCAT_LONG_DOUBLE_H
00002 #define _GLUCAT_LONG_DOUBLE_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     long_double.h : Define std functions for long double
00006     -----
00007     begin                : 2001-12-18
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036
00037 namespace glucat
00038 {
00039 #if defined(__USE_GNU)
00040     static const long double l_pi    = M_PI1;
00041     static const long double l_ln2   = M_LN21;
00042 #else
00043     static const long double l_pi    = 3.1415926535897932384626433832795029L;
00044     static const long double l_ln2   = 0.6931471805599453094172321214581766L;
00045 #endif
00046
00047     template<>
00048     inline
00049     auto
00050     numeric_traits<long double>::
00051     pi() -> long double
00052     { return l_pi; }
00053
00054     template<>
00055     inline
00056     auto
00057     numeric_traits<long double>::
00058     ln_2() -> long double
00059     { return l_ln2; }
00060 }
00061 #endif // _GLUCAT_LONG_DOUBLE_H

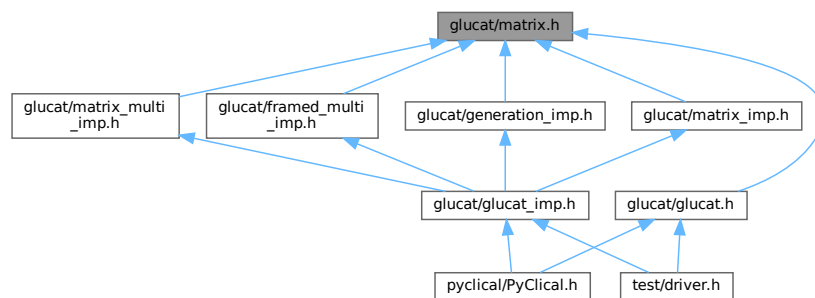
```

## 7.31 glucat/matrix.h File Reference

```
#include <boost/numeric/ublas/fwd.hpp>
#include <complex>
#include <vector>
Include dependency graph for matrix.h:
```



This graph shows which files directly or indirectly include this file:



### Classes

- struct [glucat::matrix::eig\\_genus< Matrix\\_T >](#)  
*Structure containing classification of eigenvalues.*

### Namespaces

- namespace [glucat](#)
- namespace [glucat::matrix](#)

### Typedefs

- using [glucat::matrix::eig\\_case\\_t](#)  
*Classification of eigenvalues of a matrix.*

## Functions

- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Kronecker tensor product of matrices - as per Matlab kron.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::mono\\_kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Sparse Kronecker tensor product of monomial matrices.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::nork](#) (const LHS\_T &lhs, const RHS\_T &rhs, const bool mono=true) -> const RHS\_T  
*Left inverse of Kronecker product.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::signed\\_perm\\_nork](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::nnz](#) (const Matrix\_T &m) -> typename Matrix\_T::size\_type  
*Number of non-zeros.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::isinf](#) (const Matrix\_T &m) -> bool  
*Infinite.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::isnan](#) (const Matrix\_T &m) -> bool  
*Not a Number.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::unit](#) (const typename Matrix\_T::size\_type n) -> const Matrix\_T  
*Unit matrix - as per Matlab eye.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::mono\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↵  
 expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of monomial matrices.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::sparse\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↵  
 expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of sparse matrices.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression<  
 RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of matrices.*
- template<typename Scalar\_T, typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::inner](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> Scalar\_T  
*Inner product:  $\sum(x(i,j)*y(i,j))/x.nrows()$*
- template<typename Matrix\_T>  
 auto [glucat::matrix::norm\\_frob2](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Square of Frobenius norm.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::trace](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Matrix trace.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::eigenvalues](#) (const Matrix\_T &val) -> std::vector< std::complex< double > >  
*Eigenvalues of a matrix.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::classify\\_eigenvalues](#) (const Matrix\_T &val) -> [eig\\_genus](#)< Matrix\_T >  
*Classify the eigenvalues of a matrix.*

## 7.32 matrix.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_H
00002 #define _GLUCAT_MATRIX_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix.h : Declare common matrix functions
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009                        : uBLAS interface contributed by Joerg Walter
00010     *****/
00011
00012     This library is free software: you can redistribute it and/or modify
00013     it under the terms of the GNU Lesser General Public License as published
00014     by the Free Software Foundation, either version 3 of the License, or
00015     (at your option) any later version.
00016
00017     This library is distributed in the hope that it will be useful,
00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
00023     along with this library. If not, see <http://www.gnu.org/licenses/>.
00024
00025     *****/
00026     This library is based on a prototype written by Arvind Raja and was
00027     licensed under the LGPL with permission of the author. See Arvind Raja,
00028     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029     in Ablamowicz, Lounesto and Parra (eds.)
00030     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031     *****/
00032     See also Arvind Raja's original header comments in glucat.h
00033     *****/
00034
00035 #include <boost/numeric/ublas/fwd.hpp>
00036
00037 #include <complex>
00038 #include <vector>
00039
00040 namespace glucat
00041 {
00042     namespace ublas = boost::numeric::ublas;
00043
00044     namespace matrix
00045     {
00046         template< typename LHS_T, typename RHS_T >
00047         auto
00048         kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00049         RHS_T;
00050
00051         template< typename LHS_T, typename RHS_T >
00052         auto
00053         mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00054         RHS_T;
00055
00056         template< typename LHS_T, typename RHS_T >
00057         auto
00058         nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono = true) -> const
00059         RHS_T;
00060
00061         template< typename LHS_T, typename RHS_T >
00062         auto
00063         signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00064         RHS_T;
00065
00066         template< typename Matrix_T >
00067         auto
00068         nnz(const Matrix_T& m) -> typename Matrix_T::size_type;
00069
00070         template< typename Matrix_T >
00071         auto
00072         isinf(const Matrix_T& m) -> bool;
00073
00074         template< typename Matrix_T >
00075         auto
00076         isnan(const Matrix_T& m) -> bool;
00077
00078         template< typename Matrix_T >
00079         auto
00080         unit(const typename Matrix_T::size_type n) -> const
00081         Matrix_T;
00082
00083     }
00084
00085 }

```

```

00092     template< typename LHS_T, typename RHS_T >
00093     auto
00094     mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00095              const ublas::matrix_expression<RHS_T>& rhs) -> const
00096     typename RHS_T::expression_type;
00097
00099     template< typename LHS_T, typename RHS_T >
00100     auto
00101     sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00102               const ublas::matrix_expression<RHS_T>& rhs) -> const
00103     typename RHS_T::expression_type;
00104
00106     template< typename LHS_T, typename RHS_T >
00107     auto
00108     prod(const ublas::matrix_expression<LHS_T>& lhs,
00109          const ublas::matrix_expression<RHS_T>& rhs) -> const
00110     typename RHS_T::expression_type;
00111
00113     template< typename Scalar_T, typename LHS_T, typename RHS_T >
00114     auto
00115     inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T;
00116
00118     template< typename Matrix_T >
00119     auto
00120     norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type;
00121
00123     template< typename Matrix_T >
00124     auto
00125     trace(const Matrix_T& val) -> typename Matrix_T::value_type;
00126
00128     template< typename Matrix_T >
00129     auto
00130     eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >;
00131
00133     using eig_case_t = enum {
00134         safe_eigs,
00135         neg_real_eigs,
00136         both_eigs};
00137
00139     template< typename Matrix_T >
00140     struct eig_genus
00141     {
00142         using Scalar_T = typename Matrix_T::value_type;
00144         bool m_is_singular = false;
00146         eig_case_t m_eig_case = safe_eigs;
00148         Scalar_T m_safe_arg = Scalar_T(0);
00149     };
00150
00152     template< typename Matrix_T >
00153     auto
00154     classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>;
00155 }
00156 }
00157
00158 #endif // _GLUCAT_MATRIX_H

```

## 7.33 glucat/matrix\_imp.h File Reference

```

#include "glucat/errors.h"
#include "glucat/scalar.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/vector.hpp>
#include <boost/numeric/ublas/vector_proxy.hpp>
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <boost/numeric/bindings/lapack/driver/gees.hpp>
#include <boost/numeric/bindings/ublas.hpp>
#include <set>

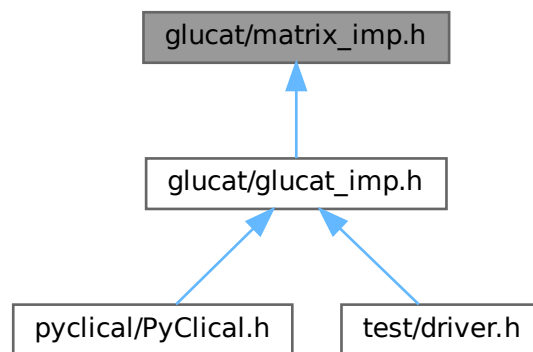
```

```
#include <vector>
```

Include dependency graph for `matrix_imp.h`:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`
- namespace `glucat::matrix`

## Functions

- `template<typename LHS_T, typename RHS_T>`  
`auto glucat::matrix::kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Kronecker tensor product of matrices - as per Matlab kron.*
- `template<typename LHS_T, typename RHS_T>`  
`auto glucat::matrix::mono_kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Sparse Kronecker tensor product of monomial matrices.*
- `template<typename LHS_T, typename RHS_T>`  
`void glucat::matrix::nork_range (RHS_T &result, const typename LHS_T::const_iterator2 lhs_it2, const RHS_T &rhs, const typename RHS_T::size_type res_s1, const typename RHS_T::size_type res_s2)`  
*Utility routine for nork: calculate result for a range of indices.*
- `template<typename LHS_T, typename RHS_T>`  
`auto glucat::matrix::nork (const LHS_T &lhs, const RHS_T &rhs, const bool mono=true) -> const RHS_T`  
*Left inverse of Kronecker product.*
- `template<typename LHS_T, typename RHS_T>`  
`auto glucat::matrix::signed_perm_nork (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*

- template<typename Matrix\_T>  
auto `glucat::matrix::nnz` (const Matrix\_T &m) -> typename Matrix\_T::size\_type  
*Number of non-zeros.*
- template<typename Matrix\_T>  
auto `glucat::matrix::isinf` (const Matrix\_T &m) -> bool  
*Infinite.*
- template<typename Matrix\_T>  
auto `glucat::matrix::isnan` (const Matrix\_T &m) -> bool  
*Not a Number.*
- template<typename Matrix\_T>  
auto `glucat::matrix::unit` (const typename Matrix\_T::size\_type n) -> const Matrix\_T  
*Unit matrix - as per Matlab eye.*
- template<typename LHS\_T, typename RHS\_T>  
auto `glucat::matrix::mono_prod` (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↵  
expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of monomial matrices.*
- template<typename LHS\_T, typename RHS\_T>  
auto `glucat::matrix::sparse_prod` (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↵  
expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of sparse matrices.*
- template<typename LHS\_T, typename RHS\_T>  
auto `glucat::matrix::prod` (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression<  
RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of matrices.*
- template<typename Scalar\_T, typename LHS\_T, typename RHS\_T>  
auto `glucat::matrix::inner` (const LHS\_T &lhs, const RHS\_T &rhs) -> Scalar\_T  
*Inner product:  $\sum(x(i,j)*y(i,j))/x.rows()$*
- template<typename Matrix\_T>  
auto `glucat::matrix::norm_frob2` (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Square of Frobenius norm.*
- template<typename Matrix\_T>  
auto `glucat::matrix::trace` (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Matrix trace.*
- template<typename Matrix\_T>  
static auto `glucat::matrix::to_lapack` (const Matrix\_T &val) -> ublas::matrix< double, ublas::column\_major >  
*Convert matrix to LAPACK format.*
- template<typename Matrix\_T>  
auto `glucat::matrix::eigenvalues` (const Matrix\_T &val) -> std::vector< std::complex< double > >  
*Eigenvalues of a matrix.*
- template<typename Matrix\_T>  
auto `glucat::matrix::classify_eigenvalues` (const Matrix\_T &val) -> `eig_genus`< Matrix\_T >  
*Classify the eigenvalues of a matrix.*

## 7.34 matrix\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_IMP_H
00002 #define _GLUCAT_MATRIX_IMP_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   matrix_imp.h : Implement common matrix functions
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright            : (C) 2001-2012 by Paul C. Leopardi

```

```

00009             : uBLAS interface contributed by Joerg Walter
00010 *****
00011
00012     This library is free software: you can redistribute it and/or modify
00013     it under the terms of the GNU Lesser General Public License as published
00014     by the Free Software Foundation, either version 3 of the License, or
00015     (at your option) any later version.
00016
00017     This library is distributed in the hope that it will be useful,
00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
00023     along with this library. If not, see <http://www.gnu.org/licenses/>.
00024
00025 *****
00026     This library is based on a prototype written by Arvind Raja and was
00027     licensed under the LGPL with permission of the author. See Arvind Raja,
00028     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029     in Ablamowicz, Lounesto and Parra (eds.)
00030     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031 *****
00032     See also Arvind Raja's original header comments in glucat.h
00033 *****/
00034
00035 #include "glucat/errors.h"
00036 #include "glucat/scalar.h"
00037 #include "glucat/matrix.h"
00038
00039 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00040 #   pragma GCC diagnostic push
00041 #   pragma GCC diagnostic ignored "-Wunused-local-typedefs"
00042 # endif
00043 # if defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00044 #   include <boost/serialization/array_wrapper.hpp>
00045 # endif
00046 #include <boost/numeric/ublas/vector.hpp>
00047 #include <boost/numeric/ublas/vector_proxy.hpp>
00048 #include <boost/numeric/ublas/matrix.hpp>
00049 #include <boost/numeric/ublas/matrix_expression.hpp>
00050 #include <boost/numeric/ublas/matrix_proxy.hpp>
00051 #include <boost/numeric/ublas/matrix_sparse.hpp>
00052 #include <boost/numeric/ublas/operation.hpp>
00053 #include <boost/numeric/ublas/operation_sparse.hpp>
00054
00055 #if defined(_GLUCAT_USE_BINDINGS)
00056 # include <boost/numeric/bindings/lapack/driver/gees.hpp>
00057 # include <boost/numeric/bindings/ublas.hpp>
00058 #endif
00059
00060 #if defined(_GLUCAT_USE_BLAZE)
00061 #include <blaze/Math.h>
00062 #include <blaze/math/DynamicMatrix.h>
00063 #include <blaze/math/DynamicVector.h>
00064 #endif
00065
00066 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00067 #   pragma GCC diagnostic pop
00068 # endif
00069
00070 #include <set>
00071 #include <vector>
00072
00073 namespace glucat { namespace matrix
00074 {
00075     // References for algorithms:
00076     // [v]: C. F. van Loan and N. Pitsianis, "Approximation with Kronecker products",
00077     // in Linear Algebra for Large Scale and Real-Time Applications, Marc S. Moonen,
00078     // Gene H. Golub, and Bart L. R. Moor (eds.), 1993, pp. 293--314.
00079
00080     template< typename LHS_T, typename RHS_T >
00081     auto
00082     kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00083     RHS_T
00084     {
00085         {
00086             const auto rhs_s1 = rhs.size1();
00087             const auto rhs_s2 = rhs.size2();
00088             auto result = RHS_T(lhs.size1()*rhs_s1, lhs.size2()*rhs_s2);
00089             result.clear();
00090
00091             for (auto
00092                 lhs_it1 = lhs.begin1();
00093                 lhs_it1 != lhs.end1();
00094                 ++lhs_it1)
00095                 for (auto
00096                     lhs_it2 = lhs_it1.begin();

```



```

00097         lhs_it2 != lhs_it1.end();
00098         ++lhs_it2)
00099     {
00100         const auto start1 = rhs_s1 * lhs_it2.index1();
00101         const auto start2 = rhs_s2 * lhs_it2.index2();
00102         const auto& lhs_val = *lhs_it2;
00103         for (auto
00104             rhs_it1 = rhs.begin1();
00105             rhs_it1 != rhs.end1();
00106             ++rhs_it1)
00107             for (auto
00108                 rhs_it2 = rhs_it1.begin();
00109                 rhs_it2 != rhs_it1.end();
00110                 ++rhs_it2)
00111                 result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00112     }
00113     return result;
00114 }
00115
00116 template< typename LHS_T, typename RHS_T >
00117 auto
00118 mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00119 RHS_T
00120 {
00121     {
00122         const auto rhs_s1 = rhs.size1();
00123         const auto rhs_s2 = rhs.size2();
00124         const auto dim = lhs.size1()*rhs_s1;
00125         auto result = RHS_T(dim, dim, dim);
00126         result.clear();
00127
00128         for (auto
00129             lhs_it1 = lhs.begin1();
00130             lhs_it1 != lhs.end1();
00131             ++lhs_it1)
00132         {
00133             const auto lhs_it2 = lhs_it1.begin();
00134             const auto start1 = rhs_s1 * lhs_it2.index1();
00135             const auto start2 = rhs_s2 * lhs_it2.index2();
00136             const auto& lhs_val = *lhs_it2;
00137             for (auto
00138                 rhs_it1 = rhs.begin1();
00139                 rhs_it1 != rhs.end1();
00140                 ++rhs_it1)
00141             {
00142                 const auto rhs_it2 = rhs_it1.begin();
00143                 result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00144             }
00145         }
00146         return result;
00147     }
00148
00149 template< typename LHS_T, typename RHS_T >
00150 void
00151 nork_range(RHS_T& result,
00152            const typename LHS_T::const_iterator2 lhs_it2,
00153            const RHS_T& rhs,
00154            const typename RHS_T::size_type res_s1,
00155            const typename RHS_T::size_type res_s2)
00156 {
00157     // Definition matches [v] Section 4, Theorem 4.1.
00158     const auto start1 = res_s1 * lhs_it2.index1();
00159     const auto start2 = res_s2 * lhs_it2.index2();
00160     using ublas::range;
00161     const auto& range1 = range(start1, start1 + res_s1);
00162     const auto& range2 = range(start2, start2 + res_s2);
00163     using matrix_range_t = ublas::matrix_range<const RHS_T>;
00164     const auto& rhs_range = matrix_range_t(rhs, range1, range2);
00165     using Scalar_T = typename RHS_T::value_type;
00166     const auto lhs_val = numeric_traits<Scalar_T>::to_scalar_t(*lhs_it2);
00167     for (auto
00168         rhs_it1 = rhs_range.begin1();
00169         rhs_it1 != rhs_range.end1();
00170         ++rhs_it1)
00171         for (auto
00172             rhs_it2 = rhs_it1.begin();
00173             rhs_it2 != rhs_it1.end();
00174             ++rhs_it2)
00175             result(rhs_it2.index1(), rhs_it2.index2()) += lhs_val * *rhs_it2;
00176 }
00177
00178 template< typename LHS_T, typename RHS_T >
00179 auto
00180 nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono) -> const
00181 RHS_T
00182 {
00183     // nork(A, kron(A, B)) is close to B
00184     // Definition matches [v] Section 4, Theorem 4.1.

```

```

00187     const auto lhs_s1 = lhs.size1();
00188     const auto lhs_s2 = lhs.size2();
00189     const auto rhs_s1 = rhs.size1();
00190     const auto rhs_s2 = rhs.size2();
00191     const auto res_s1 = rhs_s1 / lhs_s1;
00192     const auto res_s2 = rhs_s2 / lhs_s2;
00193     using Scalar_T = typename RHS_T::value_type;
00194     const auto norm_frob2_lhs = norm_frob2(lhs);
00195     if (!mono)
00196     {
00197         using error_t = error<RHS_T>;
00198         if (rhs_s1 == 0)
00199             throw error_t("matrix", "nork: number of rows must not be 0");
00200         if (rhs_s2 == 0)
00201             throw error_t("matrix", "nork: number of cols must not be 0");
00202         if (res_s1 * lhs_s1 != rhs_s1)
00203             throw error_t("matrix", "nork: incompatible numbers of rows");
00204         if (res_s2 * lhs_s2 != rhs_s2)
00205             throw error_t("matrix", "nork: incompatible numbers of cols");
00206         if (norm_frob2_lhs == Scalar_T(0))
00207             throw error_t("matrix", "nork: LHS must not be 0");
00208     }
00209     auto result = RHS_T(res_s1, res_s2);
00210     result.clear();
00211     for (auto
00212         lhs_it1 = lhs.begin1();
00213         lhs_it1 != lhs.end1();
00214         ++lhs_it1)
00215         for (auto
00216             lhs_it2 = lhs_it1.begin();
00217             lhs_it2 != lhs_it1.end();
00218             ++lhs_it2)
00219             if (*lhs_it2 != Scalar_T(0))
00220                 nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00221     result /= norm_frob2_lhs;
00222     return result;
00223 }
00224
00226 template< typename LHS_T, typename RHS_T >
00227 auto
00228 signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00229 RHS_T
00230 {
00231     // signed_perm_nork(A, kron(A, B)) is close to B
00232     // Definition matches [v] Section 4, Theorem 4.1.
00233     const auto lhs_s1 = lhs.size1();
00234     const auto lhs_s2 = lhs.size2();
00235     const auto rhs_s1 = rhs.size1();
00236     const auto rhs_s2 = rhs.size2();
00237     const auto res_s1 = rhs_s1 / lhs_s1;
00238     const auto res_s2 = rhs_s2 / lhs_s2;
00239     using Scalar_T = typename RHS_T::value_type;
00240     const auto norm_frob2_lhs = Scalar_T( double(lhs_s1) );
00241     auto result = RHS_T(res_s1, res_s2);
00242     result.clear();
00243     for (auto
00244         lhs_it1 = lhs.begin1();
00245         lhs_it1 != lhs.end1();
00246         ++lhs_it1)
00247     {
00248         const auto lhs_it2 = lhs_it1.begin();
00249         nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00250     }
00251     result /= norm_frob2_lhs;
00252     return result;
00253 }
00254
00256 template< typename Matrix_T >
00257 auto
00258 nnz(const Matrix_T& m) -> typename Matrix_T::size_type
00259 {
00260     using size_t = typename Matrix_T::size_type;
00261     auto result = size_t(0);
00262     for (auto
00263         it1 = m.begin1();
00264         it1 != m.end1();
00265         ++it1)
00266         for (auto& entry : it1)
00267             if (entry != 0)
00268                 ++result;
00269     return result;
00270 }
00271
00273 template< typename Matrix_T >
00274 auto
00275 isinf(const Matrix_T& m) -> bool
00276 {

```

```

00277     using Scalar_T = typename Matrix_T::value_type;
00278     for (auto
00279         it1 = m.begin1();
00280         it1 != m.end1();
00281         ++it1)
00282         for (auto& entry : it1)
00283             if (numeric_traits<Scalar_T>::isInf(entry))
00284                 return true;
00285
00286     return false;
00287 }
00288
00290 template< typename Matrix_T >
00291 auto
00292 isnan(const Matrix_T& m) -> bool
00293 {
00294     using Scalar_T = typename Matrix_T::value_type;
00295     for (auto
00296         it1 = m.begin1();
00297         it1 != m.end1();
00298         ++it1)
00299         for (auto& entry : it1)
00300             if (numeric_traits<Scalar_T>::isNaN(entry))
00301                 return true;
00302
00303     return false;
00304 }
00305
00307 template< typename Matrix_T >
00308 inline
00309 auto
00310 unit(const typename Matrix_T::size_type dim) -> const
00311 Matrix_T
00312 {
00313     using Scalar_T = typename Matrix_T::value_type;
00314     return ublas::identity_matrix<Scalar_T>(dim);
00315 }
00316
00318 template< typename LHS_T, typename RHS_T >
00319 auto
00320 mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00321           const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00322 {
00323     using rhs_expression_t = const RHS_T;
00324     using matrix_row_t = typename ublas::matrix_row<rhs_expression_t>;
00325
00326     const auto dim = lhs().size1();
00327     // The following assumes that RHS_T is a sparse matrix type.
00328     auto result = RHS_T(dim, dim, dim);
00329     for (auto
00330         lhs_row = lhs().begin1();
00331         lhs_row != lhs().end1();
00332         ++lhs_row)
00333     {
00334         const auto& lhs_it = lhs_row.begin();
00335         if (lhs_it != lhs_row.end())
00336         {
00337             const auto& rhs_row = matrix_row_t(rhs(), lhs_it.index2());
00338             const auto& rhs_it = rhs_row.begin();
00339             if (rhs_it != rhs_row.end())
00340                 result(lhs_it.index1(), rhs_it.index()) = (*lhs_it) * (*rhs_it);
00341         }
00342     }
00343     return result;
00344 }
00345
00347 template< typename LHS_T, typename RHS_T >
00348 inline
00349 auto
00350 sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00351            const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00352 {
00353     using expression_t = typename RHS_T::expression_type;
00354     return ublas::sparse_prod<expression_t>(lhs(), rhs());
00355 }
00356
00358 template< typename LHS_T, typename RHS_T >
00359 inline
00360 auto
00361 prod(const ublas::matrix_expression<LHS_T>& lhs,
00362      const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00363 {
00364     const auto dim = lhs().size1();
00365     RHS_T result(dim, dim);
00366     ublas::axpy_prod(lhs, rhs, result, true);
00367     return result;
00368 }

```

```

00369
00371 template< typename Scalar_T, typename LHS_T, typename RHS_T >
00372 auto
00373 inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T
00374 {
00375     auto result = Scalar_T(0);
00376     for (auto
00377         lhs_it1 = lhs.begin1();
00378         lhs_it1 != lhs.end1();
00379         ++lhs_it1)
00380         for (auto
00381             lhs_it2 = lhs_it1.begin();
00382             lhs_it2 != lhs_it1.end();
00383             ++lhs_it2)
00384             {
00385                 const auto& rhs_val = rhs(lhs_it2.index1(), lhs_it2.index2());
00386                 if (rhs_val != Scalar_T(0))
00387                     result += (*lhs_it2) * rhs_val;
00388             }
00389     return result / lhs.size1();
00390 }
00391
00393 template< typename Matrix_T >
00394 auto
00395 norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type
00396 {
00397     using Scalar_T = typename Matrix_T::value_type;
00398
00399     auto result = Scalar_T(0);
00400     for (auto
00401         val_it1 = val.begin1();
00402         val_it1 != val.end1();
00403         ++val_it1)
00404         for (auto& val_entry : val_it1)
00405             {
00406                 if (numeric_traits<Scalar_T>::isNaN(val_entry))
00407                     return numeric_traits<Scalar_T>::NaN();
00408                 result += val_entry * val_entry;
00409             }
00410     return result;
00411 }
00412
00414 template< typename Matrix_T >
00415 auto
00416 trace(const Matrix_T& val) -> typename Matrix_T::value_type
00417 {
00418     using Scalar_T = typename Matrix_T::value_type;
00419
00420     auto result = Scalar_T(0);
00421     auto dim = val.size1();
00422     for (auto
00423         ndx = decltype(dim)(0);
00424         ndx != dim;
00425         ++ndx)
00426         {
00427             const Scalar_T crd = val(ndx, ndx);
00428             if (numeric_traits<Scalar_T>::isNaN(crd))
00429                 return numeric_traits<Scalar_T>::NaN();
00430             result += crd;
00431         }
00432     return result;
00433 }
00434
00435 #if defined(_GLUCAT_USE_BINDINGS)
00437 template< typename Matrix_T >
00438 static
00439 auto
00440 to_lapack(const Matrix_T& val) -> ublas::matrix<double, ublas::column_major>
00441 {
00442     const auto s1 = val.size1();
00443     const auto s2 = val.size2();
00444
00445     using lapack_matrix_t = typename ublas::matrix<double, ublas::column_major>;
00446     auto result = lapack_matrix_t(s1, s2);
00447     result.clear();
00448
00449     using Scalar_T = typename Matrix_T::value_type;
00450     using traits_t = numeric_traits<Scalar_T>;
00451
00452     for (auto
00453         val_it1 = val.begin1();
00454         val_it1 != val.end1();
00455         ++val_it1)
00456         for (auto
00457             val_it2 = val_it1.begin();
00458             val_it2 != val_it1.end();
00459             ++val_it2)

```

```

00460         result(val_it2.index1(), val_it2.index2()) = traits_t::to_double(*val_it2);
00461     }
00462     return result;
00463 }
00464 #endif
00465
00466 #if defined(_GLUCAT_USE_BLAZE)
00467 template< typename Matrix_T >
00468 static
00469 auto
00470 to_blaze(const Matrix_T& val) -> blaze::DynamicMatrix<double, blaze::rowMajor>
00471 {
00472     {
00473         const auto s1 = val.size1();
00474         const auto s2 = val.size2();
00475
00476         using blaze_matrix_t = typename blaze::DynamicMatrix<double, blaze::rowMajor>;
00477         auto result = blaze_matrix_t(s1, s2);
00478
00479         using Scalar_T = typename Matrix_T::value_type;
00480         using traits_t = numeric_traits<Scalar_T>;
00481
00482         for (auto
00483             val_it1 = val.begin1();
00484             val_it1 != val.end1();
00485             ++val_it1)
00486             for (auto
00487                 val_it2 = val_it1.begin();
00488                 val_it2 != val_it1.end();
00489                 ++val_it2)
00490                 result(val_it2.index1(), val_it2.index2()) = traits_t::to_double(*val_it2);
00491
00492         return result;
00493     }
00494 }
00495 #endif
00496
00497 template< typename Matrix_T >
00498 auto
00499 eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >
00500 {
00501     {
00502         using complex_t = std::complex<double>;
00503         using complex_vector_t = typename std::vector<complex_t>;
00504
00505         const auto dim = val.size1();
00506         auto lambda = complex_vector_t(dim);
00507
00508         #if defined(_GLUCAT_USE_BINDINGS)
00509         namespace lapack = boost::numeric::bindings::lapack;
00510         using lapack_matrix_t = typename ublas::matrix<double, ublas::column_major>;
00511
00512         auto T = to_lapack(val);
00513         auto V = T;
00514         using vector_t = typename ublas::vector<double>;
00515         auto real_lambda = vector_t(dim);
00516         auto imag_lambda = vector_t(dim);
00517         fortran_int_t sdim = 0;
00518
00519         lapack::gees('N', 'N', nullptr, T, sdim, real_lambda, imag_lambda, V);
00520
00521         for (auto
00522             k = decltype(dim)(0);
00523             k != dim;
00524             ++k)
00525             lambda[k] = complex_t(real_lambda[k], imag_lambda[k]);
00526         #endif
00527         #if defined(_GLUCAT_USE_BLAZE)
00528         using blaze_matrix_t = typename blaze::DynamicMatrix<double, blaze::rowMajor>;
00529         using complex_t = std::complex<double>;
00530         using blaze_complex_vector_t = blaze::DynamicVector<complex_t, blaze::columnVector>;
00531
00532         auto blaze_val = to_blaze(val);
00533         auto blaze_lambda = blaze_complex_vector_t(dim);
00534         blaze::geev(blaze_val, blaze_lambda);
00535
00536         for (auto
00537             k = decltype(dim)(0);
00538             k != dim;
00539             ++k)
00540             lambda[k] = blaze_lambda[k];
00541         #endif
00542         return lambda;
00543     }
00544 }
00545
00546 template< typename Matrix_T >
00547 auto
00548 classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>
00549 {

```

```

00550     using Scalar_T = typename Matrix_T::value_type;
00551     eig_genus<Matrix_T> result;
00552
00553     using complex_t = std::complex<double>;
00554     using complex_vector_t = typename std::vector<complex_t>;
00555     auto lambda = eigenvalues(val);
00556
00557     std::set<double> arg_set;
00558
00559     using vector_index_t = typename complex_vector_t::size_type;
00560     const auto dim = lambda.size();
00561     static const auto epsilon =
00562         std::max(std::numeric_limits<double>::epsilon(),
00563             numeric_traits<Scalar_T>::to_double(std::numeric_limits<Scalar_T>::epsilon()));
00564     static const auto zero_eig_tol = 4096.0*epsilon;
00565
00566     bool neg_real_eig_found = false;
00567     bool imag_eig_found = false;
00568     bool zero_eig_found = false;
00569
00570     for (auto
00571         k = decltype(dim)(0);
00572         k != dim;
00573         ++k)
00574     {
00575         const auto lambda_k = lambda[k];
00576         arg_set.insert(std::arg(lambda_k));
00577
00578         const auto real_lambda_k = std::real(lambda_k);
00579         const auto imag_lambda_k = std::imag(lambda_k);
00580         const auto norm_tol = 4096.0*epsilon*std::norm(lambda_k);
00581
00582         if (!neg_real_eig_found &&
00583             real_lambda_k < -epsilon &&
00584             (imag_lambda_k == 0.0 ||
00585              imag_lambda_k * imag_lambda_k < norm_tol))
00586             neg_real_eig_found = true;
00587         if (!imag_eig_found &&
00588             imag_lambda_k > epsilon &&
00589             (real_lambda_k == 0.0 ||
00590              real_lambda_k * real_lambda_k < norm_tol))
00591             imag_eig_found = true;
00592         if (!zero_eig_found &&
00593             std::norm(lambda_k) < zero_eig_tol)
00594             zero_eig_found = true;
00595     }
00596
00597     if (zero_eig_found)
00598         result.m_is_singular = true;
00599
00600     static const auto pi = numeric_traits<double>::pi();
00601     if (neg_real_eig_found)
00602     {
00603         if (imag_eig_found)
00604             result.m_eig_case = both_eigs;
00605         else
00606         {
00607             result.m_eig_case = neg_real_eigs;
00608             result.m_safe_arg = Scalar_T(-pi / 2.0);
00609         }
00610     }
00611
00612     if (result.m_eig_case == both_eigs)
00613     {
00614         auto arg_it = arg_set.begin();
00615         auto first_arg = *arg_it;
00616         auto best_arg = first_arg;
00617         auto best_diff = 0.0;
00618         auto previous_arg = first_arg;
00619         for (++arg_it;
00620             arg_it != arg_set.end();
00621             ++arg_it)
00622         {
00623             const auto arg_diff = *arg_it - previous_arg;
00624             if (arg_diff > best_diff)
00625             {
00626                 best_diff = arg_diff;
00627                 best_arg = previous_arg;
00628             }
00629             previous_arg = *arg_it;
00630         }
00631         const auto arg_diff = first_arg + 2.0*pi - previous_arg;
00632         if (arg_diff > best_diff)
00633         {
00634             best_diff = arg_diff;
00635             best_arg = previous_arg;
00636         }
00637     }

```

```

00637     result.m_safe_arg = Scalar_T(pi - (best_arg + best_diff / 2.0));
00638     }
00639     return result;
00640 }
00641 } }
00642
00643 #endif // _GLUCAT_MATRIX_IMP_H

```

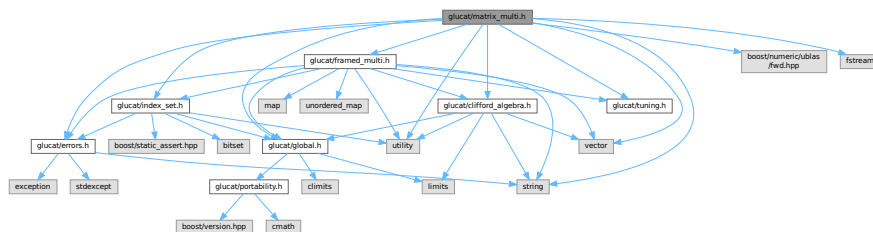
## 7.35 glucat/matrix\_multi.h File Reference

```

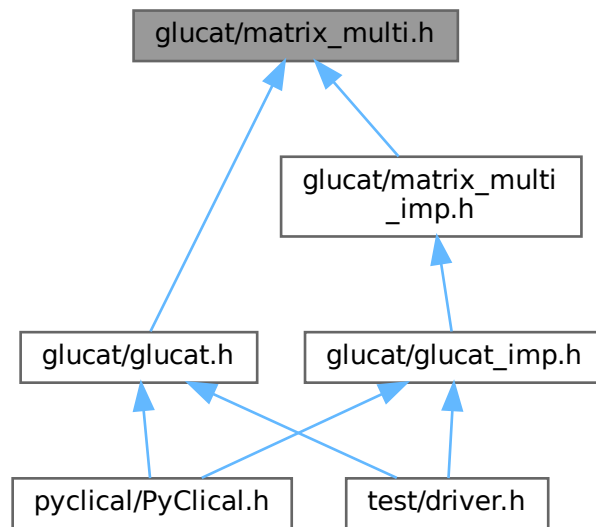
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include "glucat/framed_multi.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <fstream>
#include <string>
#include <utility>
#include <vector>

```

Include dependency graph for matrix\_multi.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*A `matrix_multi<Scalar_T,LO,HI,Tune_P>` is a matrix approximation to a multivector.*
- struct `std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >`  
*Numeric limits for `matrix_multi` inherit limits for the corresponding scalar type.*

## Namespaces

- namespace `glucat`
- namespace `std`

## Functions

- template<typename `Scalar_T`, const `index_t` `LO`, const `index_t` `HI`, typename `Tune_P`>  
`auto glucat::operator*` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric product.*
- template<typename `Scalar_T`, const `index_t` `LO`, const `index_t` `HI`, typename `Tune_P`>  
`auto glucat::operator^` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Outer product.*
- template<typename `Scalar_T`, const `index_t` `LO`, const `index_t` `HI`, typename `Tune_P`>  
`auto glucat::operator&` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Inner product.*



- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator% (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Left contraction.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::star (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`  
*Hestenes scalar product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator/ (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator| (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator>> (std::istream &s, matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`  
*Read multivector from input.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator<< (std::ostream &os, const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &`  
*Write multivector to output.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::reframe (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs, matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs_reframed, matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs_reframed) -> const index_set< LO, HI >`  
*Find a common frame for operands of a binary operator.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::matrix_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::matrix_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::exp (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Exponential of multivector.*

## 7.36 matrix\_multi.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_MULTI_H
00002 #define _GLUCAT_MATRIX_MULTI_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   matrix_multi.h : Declare a class for the matrix representation of a multivector
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright            : (C) 2001-2021 by Paul C. Leopardi
00009   *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030   *****/
00031   See also Arvind Raja's original header comments in glucat.h
00032   *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039 #include "glucat/framed_multi.h"
00040
00041 #include <boost/numeric/ublas/fwd.hpp>
00042
00043 #include <fstream>
00044 #include <string>
00045 #include <utility>
00046 #include <vector>
00047
00048 namespace glucat
00049 {
00050     namespace ublas = boost::numeric::ublas;
00051
00052     // Forward declarations for friends
00053
00054     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00055     class framed_multi; // forward
00056
00057     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00058     class matrix_multi; // forward
00059
00060     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00061     auto
00062     operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00064
00065     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00066     auto
00067     operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00069
00070     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00071     auto
00072     operator& (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00074
00075     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00076     auto
00077     operator% (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00079
00080     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00081     auto
00082     star(const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs)
-> Scalar_T;
00084
00085     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00086     auto
00087     operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const

```

```

    matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00089
00091     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092     auto
00093     operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00094
00096     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00097     auto
00098     operator» (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00099
00101     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00102     auto
00103     operator« (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00104
00106     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00107     auto
00108     reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs,
matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs_reframed,
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs_reframed) -> const index_set<LO,HI>;
00109
00110     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00111     auto
00112     sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00113
00115     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00116     auto
00117     matrix_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00118
00119     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00120     auto
00121     log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00122
00124     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00125     auto
00126     log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00127
00129     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00130     auto
00131     matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00132
00133     template< typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<> >
00134     class matrix_multi {
00135     public:
00136         clifford_algebra< Scalar_T, index_set<LO,HI>, matrix_multi<Scalar_T,LO,HI,Tune_P> >
00137         {
00138         public:
00139             using multivector_t = matrix_multi;
00140             using matrix_multi_t = multivector_t;
00141             using scalar_t = Scalar_T;
00142             using tune_p = Tune_P;
00143             using index_set_t = index_set<LO, HI>;
00144             using term_t = std::pair<const index_set_t, Scalar_T>;
00145             using vector_t = std::vector<Scalar_T>;
00146             using error_t = error<multivector_t>;
00147             using framed_multi_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00148             template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00149             friend class framed_multi;
00150             template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00151             friend class matrix_multi;
00152
00153     private:
00154         using orientation_t = ublas::row_major;
00155         using basis_matrix_t = ublas::compressed_matrix<int, orientation_t>;
00156         using matrix_t = ublas::matrix<Scalar_T, orientation_t>;
00157         using matrix_index_t = typename matrix_t::size_type;
00158
00159     public:
00160         static auto classname() -> const std::string;
00161         ~matrix_multi() override = default;
00162         matrix_multi();
00163         template< typename Other_Scalar_T >
00164         matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00165         template< typename Other_Scalar_T >
00166         matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
const index_set_t frm, const bool prechecked = false);
00167         matrix_multi(const multivector_t& val,
const index_set_t frm, const bool prechecked = false);
00168         matrix_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00169         matrix_multi(const index_set_t ist, const Scalar_T& crd,
const index_set_t frm, const bool prechecked = false);
00170         matrix_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());

```

```

00186     matrix_multi(const int scr, const index_set_t frm = index_set_t());
00188     matrix_multi(const vector_t& vec,
00189                 const index_set_t frm, const bool prechecked = false);
00191     matrix_multi(const std::string& str);
00193     matrix_multi(const std::string& str,
00194                 const index_set_t frm, const bool prechecked = false);
00196     matrix_multi(const char* str)
00197     { *this = matrix_multi(std::string(str)); };
00199     matrix_multi(const char* str,
00200                 const index_set_t frm, const bool prechecked = false)
00201     { *this = matrix_multi(std::string(str), frm, prechecked); };
00203     template< typename Other_Scalar_T >
00204     matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00206     template< typename Other_Scalar_T >
00207     matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00208                 const index_set_t frm, const bool prechecked = false);
00210     auto fast_matrix_multi(const index_set_t frm) const -> const matrix_multi_t;
00212     template< typename Other_Scalar_T >
00213     auto fast_framed_multi() const -> const framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00214
00215 private:
00217     template< typename Matrix_T >
00218     matrix_multi(const Matrix_T& mtx, const index_set_t frm);
00220     matrix_multi(const matrix_t& mtx, const index_set_t frm);
00222     auto basis_element(const index_set<LO,HI>& ist) const -> const basis_matrix_t;
00223
00224 public:
00225     _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00226
00228     auto operator= (const multivector_t& rhs) -> multivector_t&;
00229
00231     static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const matrix_multi_t;
00232
00233     // Friend declarations
00234
00235     friend auto
00236     operator* <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00237     friend auto
00238     operator^ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00239     friend auto
00240     operator& <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00241     friend auto
00242     operator% <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00243     friend auto
00244     star <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> Scalar_T;
00245     friend auto
00246     operator/ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00247     friend auto
00248     operator| <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00249
00250     friend auto
00251     operator» <>(std::istream& s, multivector_t& val) -> std::istream&;
00252     friend auto
00253     operator« <>(std::ostream& os, const multivector_t& val) -> std::ostream&;
00254     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00255     friend auto
00256     reframe (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs, const
matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs,
00257             matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs_reframed,
matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs_reframed) -> const
index_set<Other_LO,Other_HI>;
00258     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00259     friend auto
00260     matrix_sqrt (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
00261                 const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00262                 const index_t level)
00263     -> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00264     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00265     friend auto
00266     matrix_log (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
00267                const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00268                const index_t level)
00269     -> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00270
00272     auto operator+= (const term_t& rhs) -> multivector_t&;
00273
00274 private:
00275     // Data members
00276
00278     index_set_t      m_frame;
00280     matrix_t         m_matrix;
00281 };
00282
00283 // Non-members

```

```

00284
00286     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00287     auto
00288     exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00289
00290 }
00291
00292 namespace std
00293 {
00295     template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00296     struct numeric_limits< glucat::matrix_multi<Scalar_T,LO,HI,Tune_P> > :
00297     public numeric_limits<Scalar_T>
00298     { };
00299 }
00300 #endif // _GLUCAT_MATRIX_MULTI_H

```

## 7.37 glucat/matrix\_multi\_imp.h File Reference

```

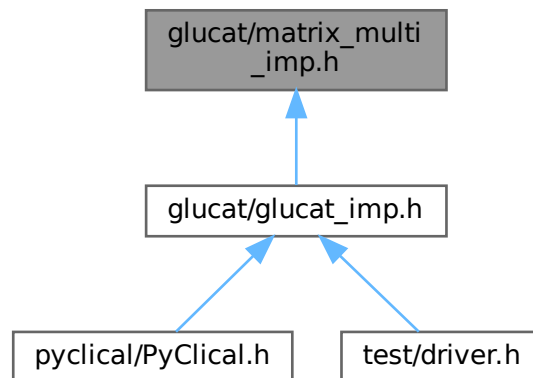
#include "glucat/matrix_multi.h"
#include "glucat/scalar.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <boost/numeric/ublas/triangular.hpp>
#include <boost/numeric/ublas/lu.hpp>
#include <boost/numeric/ublas/io.hpp>
#include <fstream>
#include <iomanip>
#include <array>
#include <iostream>

```

Include dependency graph for matrix\_multi\_imp.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class [glucat::basis\\_table< Scalar\\_T, LO, HI, Matrix\\_T >](#)  
*Table of basis elements used as a cache by basis\_element()*
- struct [pade::pade\\_sqrt\\_numer< Scalar\\_T >](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*
- struct [pade::pade\\_sqrt\\_denom< Scalar\\_T >](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*
- struct [pade::pade\\_sqrt\\_numer< float >](#)
- struct [pade::pade\\_sqrt\\_denom< float >](#)
- struct [pade::pade\\_sqrt\\_numer< long double >](#)
- struct [pade::pade\\_sqrt\\_denom< long double >](#)
- struct [pade::pade\\_sqrt\\_numer< dd\\_real >](#)
- struct [pade::pade\\_sqrt\\_denom< dd\\_real >](#)
- struct [pade::pade\\_sqrt\\_numer< qd\\_real >](#)
- struct [pade::pade\\_sqrt\\_denom< qd\\_real >](#)
- struct [pade::pade\\_log\\_numer< Scalar\\_T >](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*
- struct [pade::pade\\_log\\_denom< Scalar\\_T >](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*
- struct [pade::pade\\_log\\_numer< float >](#)
- struct [pade::pade\\_log\\_denom< float >](#)
- struct [pade::pade\\_log\\_numer< long double >](#)
- struct [pade::pade\\_log\\_denom< long double >](#)
- struct [pade::pade\\_log\\_numer< dd\\_real >](#)
- struct [pade::pade\\_log\\_denom< dd\\_real >](#)
- struct [pade::pade\\_log\\_numer< qd\\_real >](#)
- struct [pade::pade\\_log\\_denom< qd\\_real >](#)

## Namespaces

- namespace [glucat](#)
- namespace [pade](#)

## Functions

- auto [glucat::offset\\_level](#) (const [index\\_t](#) p, const [index\\_t](#) q) -> [index\\_t](#)  
*Determine the log2 dim corresponding to signature p, q.*
- template<typename Matrix\_Index\_T, const [index\\_t](#) LO, const [index\\_t](#) HI>  
static auto [glucat::folded\\_dim](#) (const [index\\_set](#)< LO, HI > &sub) -> Matrix\_Index\_T  
*Determine the matrix dimension of the fold of a subalgebra.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::reframe](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs\_reframed, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs\_reframed) -> const [index\\_set](#)< LO, HI >  
*Find a common frame for operands of a binary operator.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator\\*](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Geometric product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator^](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Outer product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator&](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Inner product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator%](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Left contraction.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::star](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T  
*Hestenes scalar product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator/](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Geometric quotient.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator|](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Transformation via twisted adjoint action.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator<<](#) (std::ostream &os, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::ostream &  
*Write multivector to output.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator>>](#) (std::istream &s, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::istream &  
*Read multivector from input.*
- template<typename Multivector\_T, typename Matrix\_T, typename Basis\_Matrix\_T>  
static auto [glucat::fast](#) (const Matrix\_T &X, [index\\_t](#) level) -> Multivector\_T  
*Inverse generalized Fast Fourier Transform.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P, const size\_t Size>  
static auto [glucat::pade\\_approx](#) (const std::array< Scalar\_T, Size > &numer, const std::array< Scalar\_T, Size > &denom, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &X) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static void [glucat::db\\_step](#) ([matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &M, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &Y)

*Single step of product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [glucat::db\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 4)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [glucat::cr\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_Y\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 1)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Cyclic reduction square root iteration.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::matrix\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector with specified complexifier.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, bool prechecked) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector with specified complexifier.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [glucat::pade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation of log.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [glucat::cascade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Incomplete square root cascade and Pade' approximation of log.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::matrix\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector with specified complexifier.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, bool prechecked) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector with specified complexifier.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::exp](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Exponential of multivector.*

## Variables

- template<typename Scalar\_T>  
const [pade\\_sqrt\\_numer](#)< Scalar\_T >::array [pade::pade\\_sqrt\\_numer](#)< Scalar\_T >::numer
- template<typename Scalar\_T>  
const [pade\\_sqrt\\_denom](#)< Scalar\_T >::array [pade::pade\\_sqrt\\_denom](#)< Scalar\_T >::denom
- const [pade\\_sqrt\\_numer](#)< float >::array [pade::pade\\_sqrt\\_numer](#)< float >::numer
- const [pade\\_sqrt\\_denom](#)< float >::array [pade::pade\\_sqrt\\_denom](#)< float >::denom
- const [pade\\_sqrt\\_numer](#)< longdouble >::array [pade::pade\\_sqrt\\_numer](#)< longdouble >::numer
- const [pade\\_sqrt\\_denom](#)< longdouble >::array [pade::pade\\_sqrt\\_denom](#)< longdouble >::denom
- const [pade\\_sqrt\\_numer](#)< dd\_real >::array [pade::pade\\_sqrt\\_numer](#)< dd\_real >::numer
- const [pade\\_sqrt\\_denom](#)< dd\_real >::array [pade::pade\\_sqrt\\_denom](#)< dd\_real >::denom



- const pade\_sqrt\_numer< qd\_real >::array pade::pade\_sqrt\_numer< qd\_real >::numer
- const pade\_sqrt\_denom< qd\_real >::array pade::pade\_sqrt\_denom< qd\_real >::denom
- template<typename Scalar\_T>  
const pade\_log\_numer< Scalar\_T >::array pade::pade\_log\_numer< Scalar\_T >::numer
- template<typename Scalar\_T>  
const pade\_log\_denom< Scalar\_T >::array pade::pade\_log\_denom< Scalar\_T >::denom
- const pade\_log\_numer< float >::array pade::pade\_log\_numer< float >::numer
- const pade\_log\_denom< float >::array pade::pade\_log\_denom< float >::denom
- const pade\_log\_numer< longdouble >::array pade::pade\_log\_numer< longdouble >::numer
- const pade\_log\_denom< longdouble >::array pade::pade\_log\_denom< longdouble >::denom
- const pade\_log\_numer< dd\_real >::array pade::pade\_log\_numer< dd\_real >::numer
- const pade\_log\_denom< dd\_real >::array pade::pade\_log\_denom< dd\_real >::denom
- const pade\_log\_numer< qd\_real >::array pade::pade\_log\_numer< qd\_real >::numer
- const pade\_log\_denom< qd\_real >::array pade::pade\_log\_denom< qd\_real >::denom

## 7.38 matrix\_multi\_imp.h

Go to the documentation of this file.

```

00001 #ifndef _GLUCAT_MATRIX_MULTI_IMP_H
00002 #define _GLUCAT_MATRIX_MULTI_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix_multi_imp.h : Implement the matrix representation of a multivector
00006
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/matrix_multi.h"
00035
00036 #include "glucat/scalar.h"
00037 #include "glucat/generation.h"
00038 #include "glucat/matrix.h"
00039
00040 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00041 # pragma GCC diagnostic push
00042 # pragma GCC diagnostic ignored "-Wunused-local-typedefs"
00043 # endif
00044 # if defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00045 # include <boost/serialization/array_wrapper.hpp>
00046 # endif
00047 #include <boost/numeric/ublas/matrix.hpp>
00048 #include <boost/numeric/ublas/matrix_expression.hpp>
00049 #include <boost/numeric/ublas/matrix_proxy.hpp>
00050 #include <boost/numeric/ublas/matrix_sparse.hpp>
00051 #include <boost/numeric/ublas/operation.hpp>
00052 #include <boost/numeric/ublas/operation_sparse.hpp>
00053 #include <boost/numeric/ublas/triangular.hpp>
00054 #include <boost/numeric/ublas/lu.hpp>
00055 #include <boost/numeric/ublas/io.hpp>

```

```

00056 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00057 #   pragma GCC diagnostic pop
00058 # endif
00059
00060 #include <fstream>
00061 #include <iomanip>
00062 #include <array>
00063 #include <iostream>
00064
00065 namespace glucat
00066 {
00067     // References for algorithms:
00068     // [CHKL]:
00069     // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00070     // [MB]: Beatrice Meini, "The Matrix Square Root From a New Functional Perspective:
00071     // Theoretical Results and Computational Issues", SIAM Journal on
00072     // Matrix Analysis and Applications 26(2):362-376, 2004.
00073     // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00074
00075     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00076     auto
00077     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00078     classname() -> const std::string
00079     { return "matrix_multi"; }
00080
00081
00082     // Reference: [P] Table 15.27, p 133
00083     inline
00084     auto
00085     offset_level(const index_t p, const index_t q) -> index_t
00086     {
00087         // Offsets between the log2 of the matrix dimension for the current signature
00088         // and that of the real superalgebra
00089         static const std::array<int, 8> offset_log2_dim = {0, 1, 0, 1, 1, 2, 1, 1};
00090         const index_t bott = pos_mod(p-q, 8);
00091         return (p+q)/2 + offset_log2_dim[bott];
00092     }
00093
00094
00095     // Reference: [P] Table 15.27, p 133
00096     template< typename Matrix_Index_T, const index_t LO, const index_t HI >
00097     inline
00098     static
00099     auto
00100     folded_dim( const index_set<LO,HI>& sub ) -> Matrix_Index_T
00101     { return 1 « offset_level(sub.count_pos(), sub.count_neg()); }
00102
00103
00104     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00105     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00106     matrix_multi()
00107     : m_frame( index_set_t() ),
00108       m_matrix( matrix_t( 1, 1 ) )
00109     { this->m_matrix.clear(); }
00110
00111
00112     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00113     template< typename Other_Scalar_T >
00114     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00115     matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00116     : m_frame( val.m_frame ), m_matrix( val.m_matrix.size1(), val.m_matrix.size2() )
00117     {
00118         this->m_matrix.clear();
00119         for (auto
00120             val_it1 = val.m_matrix.begin1();
00121             val_it1 != val.m_matrix.end1();
00122             ++val_it1)
00123             for (auto
00124                 val_it2 = val_it1.begin();
00125                 val_it2 != val_it1.end();
00126                 ++val_it2)
00127                 this->m_matrix(val_it2.index1(), val_it2.index2()) =
00128                 numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00129     }
00130
00131
00132     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00133     template< typename Other_Scalar_T >
00134     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00135     matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val, const index_set_t frm, const bool
00136     prechecked)
00137     : m_frame( frm )
00138     {
00139         if (frm != val.m_frame)
00140             *this = multivector_t(framed_multi_t(val), frm);
00141         else
00142         {
00143             const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00144             this->m_matrix.resize(dim, dim, false);
00145             this->m_matrix.clear();
00146             for (auto
00147                 val_it1 = val.m_matrix.begin1();

```

```

00147         val_it1 != val.m_matrix.end1();
00148         ++val_it1)
00149     for (auto
00150         val_it2 = val_it1.begin();
00151         val_it2 != val_it1.end();
00152         ++val_it2)
00153         this->m_matrix(val_it2.index1(), val_it2.index2()) =
numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00154     }
00155 }
00156
00158 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00159 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00160 matrix_multi(const multivector_t& val, const index_set_t frm, const bool prechecked)
00161 : m_frame( frm )
00162 {
00163     if (frm != val.m_frame)
00164         *this = multivector_t(framed_multi_t(val), frm);
00165     else
00166         this->m_matrix = val.m_matrix;
00167 }
00168
00170 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00171 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00172 matrix_multi(const index_set_t ist, const Scalar_T& crd)
00173 : m_frame( ist )
00174 {
00175     const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00176     this->m_matrix.resize(dim, dim, false);
00177     this->m_matrix.clear();
00178     *this += term_t(ist, crd);
00179 }
00180
00182 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00183 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00184 matrix_multi(const index_set_t ist, const Scalar_T& crd, const index_set_t frm, const bool
prechecked)
00185 : m_frame( frm )
00186 {
00187     if (!prechecked && (ist | frm) != frm)
00188         throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00189     const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00190     this->m_matrix.resize(dim, dim, false);
00191     this->m_matrix.clear();
00192     *this += term_t(ist, crd);
00193 }
00194
00196 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00197 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00198 matrix_multi(const Scalar_T& scr, const index_set_t frm)
00199 : m_frame( frm )
00200 {
00201     const auto dim = folded_dim<matrix_index_t>(frm);
00202     this->m_matrix.resize(dim, dim, false);
00203     this->m_matrix.clear();
00204     *this += term_t(index_set_t(), scr);
00205 }
00206
00208 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00209 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00210 matrix_multi(const int scr, const index_set_t frm)
00211 { *this = multivector_t(Scalar_T(scr), frm); }
00212
00214 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00215 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00216 matrix_multi(const vector_t& vec,
00217             const index_set_t frm, const bool prechecked)
00218 : m_frame( frm )
00219 {
00220     if (!prechecked && index_t(vec.size()) != frm.count())
00221         throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00222     const auto dim = folded_dim<matrix_index_t>(frm);
00223     this->m_matrix.resize(dim, dim, false);
00224     this->m_matrix.clear();
00225     auto idx = frm.min();
00226     const auto frm_end = frm.max()+1;
00227     for (auto& crd : vec)
00228     {
00229         *this += term_t(index_set_t(idx), crd);
00230         for (
00231             ++idx;
00232             idx != frm_end && !frm[idx];
00233             ++idx)
00234         ;
00235     }
00236 }
00237

```

```

00239     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00240     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00241     matrix_multi(const std::string& str)
00242     { *this = framed_multi_t(str); }
00243
00244     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00245     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00246     matrix_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00247     { *this = multivector_t(framed_multi_t(str), frm, prechecked); }
00248
00249     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00250     template< typename Other_Scalar_T >
00251     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00252     matrix_multi(const framed_multi_t<Other_Scalar_T,LO,HI,Tune_P>& val)
00253     : m_frame( val.frame() )
00254     {
00255         if (val.size() >= Tune_P::fast_size_threshold)
00256             try
00257             {
00258                 *this = val.template fast_matrix_multi<Scalar_T>(this->m_frame);
00259                 return;
00260             }
00261             catch (const glucat_error& e)
00262             { }
00263         const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00264         this->m_matrix.resize(dim, dim, false);
00265         this->m_matrix.clear();
00266
00267         using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00268         for (auto& val_term : val)
00269             *this += val_term;
00270     }
00271
00272     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00273     template< typename Other_Scalar_T >
00274     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00275     matrix_multi(const framed_multi_t<Other_Scalar_T,LO,HI,Tune_P>& framed_val, const index_set_t frm,
00276     const bool prechecked)
00277     {
00278         const auto val = framed_val.truncated();
00279         const auto our_frame = val.frame() | frm;
00280         if (val.size() >= Tune_P::fast_size_threshold)
00281             try
00282             {
00283                 *this = val.template fast_matrix_multi<Scalar_T>(our_frame);
00284                 return;
00285             }
00286             catch (const glucat_error& e)
00287             { }
00288         this->m_frame = our_frame;
00289         const auto dim = folded_dim<matrix_index_t>(our_frame);
00290         this->m_matrix.resize(dim, dim, false);
00291         this->m_matrix.clear();
00292
00293         using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00294         for (auto& val_term : val)
00295             *this += val_term;
00296     }
00297
00298     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00299     template< typename Matrix_T >
00300     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00301     matrix_multi(const Matrix_T& mtx, const index_set_t frm)
00302     : m_frame( frm ), m_matrix( mtx.size1(), mtx.size2() )
00303     {
00304         this->m_matrix.clear();
00305
00306         for (auto
00307             mtx_it1 = mtx.begin1();
00308             mtx_it1 != mtx.end1();
00309             ++mtx_it1)
00310             for (auto
00311                 mtx_it2 = mtx_it1.begin();
00312                 mtx_it2 != mtx_it1.end();
00313                 ++mtx_it2)
00314                 this->m_matrix(mtx_it2.index1(), mtx_it2.index2()) =
00315                 numeric_traits<Scalar_T>::to_scalar_t(*mtx_it2);
00316     }
00317
00318     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00319     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00320     matrix_multi(const matrix_t& mtx, const index_set_t frm)
00321     : m_frame( frm ), m_matrix( mtx )
00322     { }
00323
00324     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00325     auto

```

```

00330 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00331 operator= (const multivector_t& rhs) -> multivector_t&
00332 {
00333     // Check for assignment to self
00334     if (this == &rhs)
00335         return *this;
00336     this->m_frame = rhs.m_frame;
00337     this->m_matrix = rhs.m_matrix;
00338     return *this;
00339 }
00340
00341 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00342 inline
00343 auto
00344 reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00345 matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs,
00346 matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs_reframed,
00347 matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs_reframed) -> const index_set<LO,HI>
00348 {
00349     using index_set_t = index_set<LO, HI>;
00350     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00351     using framed_multi_t = typename multivector_t::framed_multi_t;
00352     // Determine the initial common frame
00353     index_set_t our_frame = lhs.m_frame | rhs.m_frame;
00354     framed_multi_t framed_lhs;
00355     framed_multi_t framed_rhs;
00356     if ((lhs.m_frame != our_frame) || (rhs.m_frame != our_frame))
00357     {
00358         // The common frame may expand as a result of the transform to framed_multi_t
00359         framed_lhs = framed_multi_t(lhs);
00360         framed_rhs = framed_multi_t(rhs);
00361         our_frame |= framed_lhs.frame() | framed_rhs.frame();
00362     }
00363     // Do the reframing only where necessary
00364     if (lhs.m_frame != our_frame)
00365         lhs_reframed = multivector_t(framed_lhs, our_frame, true);
00366     if (rhs.m_frame != our_frame)
00367         rhs_reframed = multivector_t(framed_rhs, our_frame, true);
00368     return our_frame;
00369 }
00370
00371 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00372 auto
00373 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00374 operator== (const multivector_t& rhs) const -> bool
00375 {
00376     // Ensure that there is no aliasing
00377     if (this == &rhs)
00378         return true;
00379
00380     // Operate only within a common frame
00381     multivector_t lhs_reframed;
00382     multivector_t rhs_reframed;
00383     const index_set_t our_frame = reframe(*this, rhs, lhs_reframed, rhs_reframed);
00384     const multivector_t& lhs_ref = (this->m_frame == our_frame)
00385         ? *this
00386         : lhs_reframed;
00387     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00388         ? rhs
00389         : rhs_reframed;
00390
00391     return ublas::norm_inf(lhs_ref.m_matrix - rhs_ref.m_matrix) == 0;
00392 }
00393
00394 // Test for equality of multivector and scalar
00395 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00396 inline
00397 auto
00398 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00399 operator== (const Scalar_T& scr) const -> bool
00400 {
00401     if (scr != Scalar_T(0))
00402         return *this == multivector_t(framed_multi_t(scr), this->m_frame, true);
00403     else if (ublas::norm_inf(this->m_matrix) != 0)
00404         return false;
00405     else
00406     {
00407         const matrix_index_t dim = this->m_matrix.size();
00408         return !(dim == 1 && this->isnan());
00409     }
00410 }
00411
00412 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00413 inline
00414 auto
00415 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00416 operator+= (const Scalar_T& scr) -> multivector_t&

```

```

00418 { return *this += term_t(index_set_t(), scr); }
00419
00421 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00422 inline
00423 auto
00424 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00425 operator+= (const multivector_t& rhs) -> multivector_t&
00426 {
00427     // Ensure that there is no aliasing
00428     if (this == &rhs)
00429         return *this *= Scalar_T(2);
00430
00431     // Operate only within a common frame
00432     multivector_t rhs_reframed;
00433     const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00434     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00435         ? rhs
00436         : rhs_reframed;
00437
00438     noalias(this->m_matrix) += rhs_ref.m_matrix;
00439     return *this;
00440 }
00441
00443 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00444 inline
00445 auto
00446 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00447 operator-= (const Scalar_T& scr) -> multivector_t&
00448 { return *this += term_t(index_set_t(), -scr); }
00449
00451 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00452 inline
00453 auto
00454 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00455 operator-= (const multivector_t& rhs) -> multivector_t&
00456 {
00457     // Ensure that there is no aliasing
00458     if (this == &rhs)
00459         return *this = Scalar_T(0);
00460
00461     // Operate only within a common frame
00462     multivector_t rhs_reframed;
00463     const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00464     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00465         ? rhs
00466         : rhs_reframed;
00467
00468     noalias(this->m_matrix) -= rhs_ref.m_matrix;
00469     return *this;
00470 }
00471
00473 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00474 inline
00475 auto
00476 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00477 operator- () const -> const multivector_t
00478 { return multivector_t(-(this->m_matrix), this->m_frame); }
00479
00481 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00482 inline
00483 auto
00484 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00485 operator*= (const Scalar_T& scr) -> multivector_t&
00486 { // multiply coordinates of all terms by scalar
00487
00488     using traits_t = numeric_traits<Scalar_T>;
00489     if (traits_t::isNaN_or_isInf(scr) || this->isnan())
00490         return *this = traits_t::NaN();
00491     if (scr == Scalar_T(0))
00492         *this = Scalar_T(0);
00493     else
00494         this->m_matrix *= scr;
00495     return *this;
00496 }
00497
00499 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00500 inline
00501 auto
00502 operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00503 {
00504     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00505     using index_set_t = typename multivector_t::index_set_t;
00506
00507     if (lhs.isnan() || rhs.isnan())
00508         return numeric_traits<Scalar_T>::NaN();
00509

```

```

00510     // Operate only within a common frame
00511     multivector_t lhs_reframed;
00512     multivector_t rhs_reframed;
00513     const index_set_t our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00514     const multivector_t& lhs_ref = (lhs.m_frame == our_frame)
00515         ? lhs
00516         : lhs_reframed;
00517     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00518         ? rhs
00519         : rhs_reframed;
00520
00521     using matrix_t = typename multivector_t::matrix_t;
00522     using matrix_index_t = typename matrix_t::size_type;
00523
00524     const matrix_index_t dim = lhs_ref.m_matrix.size1();
00525     multivector_t result = multivector_t(matrix_t(dim, dim), our_frame);
00526     result.m_matrix.clear();
00527     ublas::axpy_prod(lhs_ref.m_matrix, rhs_ref.m_matrix, result.m_matrix, true);
00528     return result;
00529 }
00530
00531 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00532 inline
00533 auto
00534 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00535 operator*= (const multivector_t& rhs) -> multivector_t&
00536 { return *this = *this * rhs; }
00537
00538 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00539 inline
00540 auto
00541 operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00542 matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00543 {
00544     {
00545         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00546         using framed_multi_t = typename multivector_t::framed_multi_t;
00547         return framed_multi_t(lhs) ^ framed_multi_t(rhs);
00548     }
00549 }
00550
00551 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00552 inline
00553 auto
00554 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00555 operator^= (const multivector_t& rhs) -> multivector_t&
00556 { return *this = *this ^ rhs; }
00557
00558 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00559 inline
00560 auto
00561 operator& (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00562 matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00563 {
00564     {
00565         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00566         using framed_multi_t = typename multivector_t::framed_multi_t;
00567         return framed_multi_t(lhs) & framed_multi_t(rhs);
00568     }
00569 }
00570
00571 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00572 inline
00573 auto
00574 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00575 operator&= (const multivector_t& rhs) -> multivector_t&
00576 { return *this = *this & rhs; }
00577
00578 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00579 inline
00580 auto
00581 operator% (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00582 matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00583 {
00584     {
00585         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00586         using framed_multi_t = typename multivector_t::framed_multi_t;
00587         return framed_multi_t(lhs) % framed_multi_t(rhs);
00588     }
00589 }
00590
00591 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00592 inline
00593 auto
00594 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00595 operator%= (const multivector_t& rhs) -> multivector_t&
00596 { return *this = *this % rhs; }
00597
00598 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00599 inline
00600 auto
00601 star(const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs)
00602 -> Scalar_T

```

```

00601 { return (lhs * rhs).scalar(); }
00602
00603 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00604 inline
00605 auto
00606 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00607 operator/=( const Scalar_T& scr) -> multivector_t&
00608 { return *this *= Scalar_T(1)/scr; }
00609
00610 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00611 auto
00612 operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00613 matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00614 {
00615     {
00616         using traits_t = numeric_traits<Scalar_T>;
00617
00618         if (lhs.isnan() || rhs.isnan())
00619             return traits_t::NaN();
00620
00621         if (rhs == Scalar_T(0))
00622             return traits_t::NaN();
00623
00624         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00625         using index_set_t = typename multivector_t::index_set_t;
00626
00627         // Operate only within a common frame
00628         multivector_t lhs_reframed;
00629         multivector_t rhs_reframed;
00630         const auto our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00631         const auto& lhs_ref = (lhs.m_frame == our_frame)
00632             ? lhs
00633             : lhs_reframed;
00634         const auto& rhs_ref = (rhs.m_frame == our_frame)
00635             ? rhs
00636             : rhs_reframed;
00637
00638         // Solve result == lhs_ref/rhs_ref <=> result*rhs_ref == lhs_ref
00639         // We now solve X == B/A
00640         // (where X == result, B == lhs_ref.m_matrix and A == rhs_ref.m_matrix)
00641         // X == B/A <=> X*A == B <=> AT*XT == BT
00642         // So, we solve AT*XT == BT
00643
00644         using matrix_t = typename multivector_t::matrix_t;
00645         using matrix_index_t = typename matrix_t::size_type;
00646
00647         const auto& AT = matrix_t(ublas::trans(rhs_ref.m_matrix));
00648         auto LU = AT;
00649
00650         using permutation_t = ublas::permutation_matrix<matrix_index_t>;
00651
00652         auto pvector = permutation_t(AT.size());
00653         if (! ublas::lu_factorize(LU, pvector))
00654         {
00655             const auto& BT = matrix_t(ublas::trans(lhs_ref.m_matrix));
00656             auto XT = BT;
00657             ublas::lu_substitute(LU, pvector, XT);
00658             if (matrix::isnan(XT))
00659                 return traits_t::NaN();
00660
00661             // Iterative refinement.
00662             // Reference: Nicholas J. Higham, "Accuracy and Stability of Numerical Algorithms",
00663             // SIAM, 1996, ISBN 0-89871-355-2, Chapter 11
00664             if (Tune_P::div_max_steps > 0)
00665             {
00666                 // matrix_t R = ublas::prod(AT, XT) - BT;
00667                 auto R = matrix_t(-BT);
00668                 ublas::axpy_prod(AT, XT, R, false);
00669                 if (matrix::isnan(R))
00670                     return traits_t::NaN();
00671
00672                 auto nr = Scalar_T(ublas::norm_inf(R));
00673                 if ( nr != Scalar_T(0) && !traits_t::isNaN_or_isInf(nr) )
00674                 {
00675                     auto XTnew = XT;
00676                     auto nrold = nr + Scalar_T(1);
00677                     for (auto
00678                         step = 0;
00679                         step != Tune_P::div_max_steps &&
00680                         nr < nrold &&
00681                         nr != Scalar_T(0) &&
00682                         nr == nr;
00683                         ++step)
00684                     {
00685                         nrold = nr;
00686                         if (step != 0)
00687                             XT = XTnew;
00688                         auto& D = R;

```



```

00689         ublas::lu_substitute(LU, pvector, D);
00690         XTnew -= D;
00691         // noalias(R) = ublas::prod(AT, XTnew) - BT;
00692         R = -BT;
00693         ublas::axpy_prod(AT, XTnew, R, false);
00694         nr = ublas::norm_inf(R);
00695     }
00696 }
00697 }
00698     return multivector_t(ublas::trans(XT), our_frame);
00699 }
00700     else
00701         // AT is singular. Return NaN
00702         return traits_t::NaN();
00703 }
00704
00705 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00706 inline
00707 auto
00708 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00709 operator/=(const multivector_t& rhs) -> multivector_t&
00710 { return *this = *this / rhs; }
00711
00712 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00713 inline
00714 auto
00715 operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00716 matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00717 { return rhs * lhs / rhs.involute(); }
00718
00719 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00720 inline
00721 auto
00722 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00723 operator|=(const multivector_t& rhs) -> multivector_t&
00724 { return *this = rhs * *this / rhs.involute(); }
00725
00726 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00727 inline
00728 auto
00729 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00730 inv() const -> const multivector_t
00731 { return multivector_t(Scalar_T(1), this->m_frame) / *this; }
00732
00733 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00734 inline
00735 auto
00736 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00737 pow(int m) const -> const multivector_t
00738 { return glucat::pow(*this, m); }
00739
00740 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00741 auto
00742 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00743 outer_pow(int m) const -> const multivector_t
00744 {
00745     if (m < 0)
00746         throw error_t("outer_pow(m): negative exponent");
00747     framed_multi_t a = *this;
00748     return a.outer_pow(m);
00749 }
00750
00751 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00752 inline
00753 auto
00754 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00755 grade() const -> index_t
00756 { return framed_multi_t(*this).grade(); }
00757
00758 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00759 inline
00760 auto
00761 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00762 frame() const -> const index_set_t
00763 { return this->m_frame; }
00764
00765 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00766 inline
00767 auto
00768 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00769 operator[] (const index_set_t ist) const -> Scalar_T
00770 {
00771     // Use matrix inner product only if ist is in frame
00772     if ( (ist | this->m_frame) == this->m_frame)
00773         return matrix::inner<Scalar_T>(this->basis_element(ist), this->m_matrix);
00774     else
00775         return Scalar_T(0);
00776 }

```

```

00784     }
00785
00786     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00787     inline
00788     auto
00789     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00790     operator() (index_t grade) const -> const multivector_t
00791     {
00792     {
00793         if ((grade < 0) || (grade > HI-LO))
00794             return 0;
00795         else
00796             return (framed_multi_t(*this))(grade);
00797     }
00798
00799     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00800     inline
00801     auto
00802     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00803     scalar() const -> Scalar_T
00804     {
00805     {
00806         const matrix_index_t dim = this->m_matrix.size1();
00807         return matrix::trace(this->m_matrix) / Scalar_T( double(dim) );
00808     }
00809
00810     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00811     inline
00812     auto
00813     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00814     pure() const -> const multivector_t
00815     { return *this - this->scalar(); }
00816
00817     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00818     inline
00819     auto
00820     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00821     even() const -> const multivector_t
00822     { return framed_multi_t(*this).even(); }
00823
00824     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00825     inline
00826     auto
00827     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00828     odd() const -> const multivector_t
00829     { return framed_multi_t(*this).odd(); }
00830
00831     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00832     auto
00833     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00834     vector_part() const -> const vector_t
00835     { return this->vector_part(this->frame(), true); }
00836
00837     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00838     auto
00839     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00840     vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00841     {
00842     {
00843         if (!prechecked && (this->frame() | frm) != frm)
00844             throw error_t("vector_part(frm): value is outside of requested frame");
00845         vector_t result;
00846         // If we need to enlarge the frame we may as well use a framed_multi_t
00847         if (this->frame() != frm)
00848             return framed_multi_t(*this).vector_part(frm, true);
00849
00850         const auto begin_index = frm.min();
00851         const auto end_index = frm.max()+1;
00852         for (auto
00853             idx = begin_index;
00854             idx != end_index;
00855             ++idx)
00856             if (frm[idx])
00857                 // Frame may contain indices which do not correspond to a grade 1 term but
00858                 // frame cannot omit any index corresponding to a grade 1 term
00859                 result.push_back(
00860                     matrix::inner<Scalar_T>(this->basis_element(index_set_t(idx)),
00861                         this->m_matrix));
00862         return result;
00863     }
00864
00865     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00866     inline
00867     auto
00868     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00869     involute() const -> const multivector_t
00870     { return framed_multi_t(*this).involute(); }
00871
00872     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00873     inline

```

```

00880     auto
00881     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00882     reverse() const -> const multivector_t
00883     { return framed_multi_t(*this).reverse(); }
00884
00885     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00886     inline
00887     auto
00888     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00889     conj() const -> const multivector_t
00890     { return framed_multi_t(*this).conj(); }
00891
00892     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00893     inline
00894     auto
00895     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00896     quad() const -> Scalar_T
00897     { // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
00898       // Arvind Raja ref: "old clical: quadfunction(p:pter):pterm in file compmod.pas"
00899       return framed_multi_t(*this).quad();
00900     }
00901
00902     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00903     inline
00904     auto
00905     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00906     norm() const -> Scalar_T
00907     {
00908       const matrix_index_t dim = this->m_matrix.size1();
00909       return matrix::norm_frob2(this->m_matrix) / Scalar_T( double(dim) );
00910     }
00911
00912     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00913     inline
00914     auto
00915     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00916     max_abs() const -> Scalar_T
00917     { return framed_multi_t(*this).max_abs(); }
00918
00919     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00920     auto
00921     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00922     random(const index_set<LO,HI> frm, Scalar_T fill) -> const multivector_t
00923     {
00924       return framed_multi<Scalar_T,LO,HI,Tune_P>::random(frm, fill);
00925     }
00926
00927     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00928     inline
00929     void
00930     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00931     write(const std::string& msg) const
00932     { framed_multi_t(*this).write(msg); }
00933
00934     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00935     inline
00936     void
00937     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00938     write(std::ofstream& ofile, const std::string& msg) const
00939     {
00940       if (!ofile)
00941         throw error_t("write(ofile,msg): cannot write to output file");
00942       framed_multi_t(*this).write(ofile, msg);
00943     }
00944
00945     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00946     inline
00947     auto
00948     operator<< (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
00949     {
00950       os << typename matrix_multi<Scalar_T,LO,HI,Tune_P>::framed_multi_t(val);
00951       return os;
00952     }
00953
00954     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00955     inline
00956     auto
00957     operator>> (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&
00958     { // Input looks like 1.0-2.0{1,2}+3.2{3,4}
00959       framed_multi<Scalar_T,LO,HI,Tune_P> local;
00960       s >> local;
00961       // If s.bad() then we have a corrupt input
00962       // otherwise we are fine and can copy the resulting matrix_multi
00963       if (!s.bad())
00964         val = local;
00965       return s;
00966     }

```

```

00976
00977 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00978 inline
00979 auto
00980 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00981 isinf() const -> bool
00982 {
00983     if (std::numeric_limits<Scalar_T>::has_infinity)
00984         return matrix::isinf(this->m_matrix);
00985     else
00986         return false;
00987 }
00988
00989 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00990 inline
00991 auto
00992 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00993 isnan() const -> bool
00994 {
00995     if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
00996         return matrix::isnan(this->m_matrix);
00997     else
00998         return false;
00999 }
01000
01001
01002 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01003 inline
01004 auto
01005 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01006 truncated(const Scalar_T& limit) const -> const multivector_t
01007 { return framed_multi_t(*this).truncated(limit); }
01008
01009 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01010 inline
01011 auto
01012 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01013 operator+= (const term_t& term) -> multivector_t&
01014 {
01015     if (term.second != Scalar_T(0))
01016         this->m_matrix.plus_assign(matrix_t(this->basis_element(term.first)) * term.second);
01017     return *this;
01018 }
01019
01020 template< typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T >
01021 static
01022 auto
01023 fast(const Matrix_T& X, index_t level) -> Multivector_T
01024 {
01025     using framed_multi_t = Multivector_T;
01026
01027     using index_set_t = typename framed_multi_t::index_set_t;
01028     using Scalar_T = typename framed_multi_t::scalar_t;
01029     using matrix_t = Matrix_T;
01030     using basis_matrix_t = Basis_Matrix_T;
01031     using basis_scalar_t = typename basis_matrix_t::value_type;
01032     using traits_t = numeric_traits<Scalar_T>;
01033
01034     if (level == 0)
01035         return framed_multi_t(traits_t::to_scalar_t(X(0,0)));
01036
01037     if (ublas::norm_inf(X) == 0)
01038         return Scalar_T(0);
01039
01040     const basis_matrix_t& I = matrix::unit<basis_matrix_t>(2);
01041     basis_matrix_t J(2,2,2);
01042     J.clear();
01043     J(0,1) = basis_scalar_t(-1);
01044     J(1,0) = basis_scalar_t( 1);
01045     basis_matrix_t K = J;
01046     K(0,1) = basis_scalar_t( 1);
01047     basis_matrix_t JK = I;
01048     JK(0,0) = basis_scalar_t(-1);
01049
01050     using matrix::signed_perm_nork;
01051     const index_set_t ist_mn = index_set_t(-level);
01052     const index_set_t ist_pn = index_set_t(level);
01053     const index_set_t ist_mnpn = ist_mn | ist_pn;
01054     if (level == 1)
01055     {
01056         using term_t = typename framed_multi_t::term_t;
01057         const Scalar_T i_x = traits_t::to_scalar_t(signed_perm_nork(I, X)(0, 0));
01058         const Scalar_T j_x = traits_t::to_scalar_t(signed_perm_nork(J, X)(0, 0));
01059         const Scalar_T k_x = traits_t::to_scalar_t(signed_perm_nork(K, X)(0, 0));
01060         const Scalar_T jk_x = traits_t::to_scalar_t(signed_perm_nork(JK,X)(0, 0));
01061         framed_multi_t
01062             result = i_x;
01063         result += term_t(ist_mn, j_x); // j_x * mn;
01064     }

```

```

01068         result += term_t(ist_pn, k_x); // k_x * pn;
01069         return result += term_t(ist_mnpn, jk_x); // jk_x * mnpn;
01070     }
01071     else
01072     {
01073         const framed_multi_t& mn = framed_multi_t(ist_mn);
01074         const framed_multi_t& pn = framed_multi_t(ist_pn);
01075         const framed_multi_t& mnpn = framed_multi_t(ist_mnpn);
01076         const framed_multi_t i_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01077             (signed_perm_nork(I, X), level-1);
01078         const framed_multi_t j_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01079             (signed_perm_nork(J, X), level-1);
01080         const framed_multi_t k_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01081             (signed_perm_nork(K, X), level-1);
01082         const framed_multi_t jk_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01083             (signed_perm_nork(JK, X), level-1);
01084         framed_multi_t
01085             result = i_x.even() - jk_x.odd();
01086             result += (j_x.even() - k_x.odd()) * mn;
01087             result += (k_x.even() - j_x.odd()) * pn;
01088         return result += (jk_x.even() - i_x.odd()) * mnpn;
01089     }
01090 }
01091
01092 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01093 inline
01094 auto
01095 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01096 fast_matrix_multi(const index_set_t frm) const -> const multivector_t
01097 {
01098     if (this->m_frame == frm)
01099         return *this;
01100     else
01101         return (this->template fast_framed_multi<Scalar_T>()).template fast_matrix_multi<Scalar_T>(frm);
01102 }
01103
01104 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01105 template <typename Other_Scalar_T>
01106 auto
01107 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01108 fast_framed_multi() const -> const framed_multi<Other_Scalar_T,LO,HI,Tune_P>
01109 {
01110     // Determine the amount of off-centering needed
01111     index_t p = this->m_frame.count_pos();
01112     index_t q = this->m_frame.count_neg();
01113
01114     const index_t bott = pos_mod(p-q, 8);
01115     p += std::max(gen::offset_to_super[bott],index_t(0));
01116     q -= std::min(gen::offset_to_super[bott],index_t(0));
01117
01118     const index_t orig_p = p;
01119     const index_t orig_q = q;
01120     while (p-q > 4)
01121     { p -= 4; q += 4; }
01122     while (p-q < -3)
01123     { p += 4; q -= 4; }
01124     if (p-q > 1)
01125     {
01126         index_t old_p = p;
01127         p = q+1;
01128         q = old_p-1;
01129     }
01130     const index_t level = (p+q)/2;
01131
01132     // Do the inverse fast transform
01133     using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
01134     framed_multi_t val = fast<framed_multi_t, matrix_t, basis_matrix_t>(this->m_matrix, level);
01135
01136     // Off-centre val
01137     switch (pos_mod(orig_p-orig_q, 8))
01138     {
01139     case 2:
01140     case 3:
01141     case 4:
01142         val.centred_qpl_pml(p, q);
01143         break;
01144     default:
01145         break;
01146     }
01147     if (orig_p-orig_q > 4)
01148     while (p != orig_p)
01149         val.centred_pp4_qm4(p, q);
01150     if (orig_p-orig_q < -3)
01151     while (p != orig_p)
01152         val.centred_pm4_qp4(p, q);
01153
01154     // Return unfolded val

```

```

01157     return val.unfold(this->m_frame);
01158 }
01159
01161 template< typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T >
01162 class basis_table :
01163 public std::map< std::pair< const index_set<LO,HI>, const index_set<LO,HI> >,
01164               Matrix_T* >
01165 {
01166 public:
01168     static auto basis() -> basis_table& { static basis_table b; return b;}
01169 private:
01173     friend class friend_for_private_destructor;
01174     // Enforce singleton
01175     // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
01176     basis_table() = default;
01177     ~basis_table() = default;
01178 public:
01179     basis_table(const basis_table&) = delete;
01180     auto operator= (const basis_table&) -> basis_table& = delete;
01181 };
01182
01184 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01185 auto
01186 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01187 basis_element(const index_set_t& ist) const -> const basis_matrix_t
01188 {
01189     using index_set_pair_t = std::pair<const index_set_t, const index_set_t>;
01190     const auto& unfolded_pair = index_set_pair_t(ist, this->m_frame);
01191
01192     using basis_table_t = basis_table<Scalar_T, LO, HI, basis_matrix_t>;
01193     auto& basis_cache = basis_table_t::basis();
01194
01195     const auto frame_count = this->m_frame.count();
01196     const auto use_cache = frame_count <= index_t(Tune_P::basis_max_count);
01197
01198     if (use_cache)
01199     {
01200         const auto basis_it = basis_cache.find(unfolded_pair);
01201         if (basis_it != basis_cache.end())
01202             return *(basis_it->second);
01203     }
01204     const auto folded_set = ist.fold(this->m_frame);
01205     const auto folded_frame = this->m_frame.fold();
01206     const auto& folded_pair = index_set_pair_t(folded_set, folded_frame);
01207     using basis_pair_t = std::pair<const index_set_pair_t, basis_matrix_t *>;
01208     if (use_cache)
01209     {
01210         const auto basis_it = basis_cache.find(folded_pair);
01211         if (basis_it != basis_cache.end())
01212         {
01213             auto* result_ptr = basis_it->second;
01214             basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01215             return *result_ptr;
01216         }
01217     }
01218     const auto folded_max = folded_frame.max();
01219     const auto folded_min = folded_frame.min();
01220     const auto p = std::max(folded_max, index_t(0));
01221     const auto q = std::max(index_t(-folded_min), index_t(0));
01222     const auto* e = (gen::generator_table<basis_matrix_t>::generator())(p, q);
01223     const auto dim = matrix_index_t(1) << offset_level(p, q);
01224     auto result = matrix::unit<basis_matrix_t>(dim);
01225     for (auto
01226         k = folded_min;
01227         k <= folded_max;
01228         ++k)
01229         if (folded_set[k])
01230             result = matrix::mono_prod(result, e[k]);
01231     if (use_cache)
01232     {
01233         auto* result_ptr = new basis_matrix_t(result);
01234         basis_cache.insert(basis_pair_t(folded_pair, result_ptr));
01235         basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01236     }
01237     return result;
01238 }
01239
01241 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P, const size_t Size
01242 >
01243 static
01244 auto
01245 pade_approx(
01246     const std::array<Scalar_T, Size>& numer,
01247     const std::array<Scalar_T, Size>& denom,
01248     const matrix_multi<Scalar_T,LO,HI,Tune_P>& X) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01249 {

```

```

01250 // Pade' approximation
01251 // Reference: [GW], Section 4.3, pp318-322
01252 // Reference: [GL], Section 11.3, p572-576.
01253
01254 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01255 using traits_t = numeric_traits<Scalar_T>;
01256
01257 if (X.isnan())
01258     return traits_t::NaN();
01259
01260 // Array size is assumed to be even
01261 const auto nbr_even_powers = Size/2 - 1;
01262
01263 // Create an array of even powers
01264 auto XX = std::vector<multivector_t>(nbr_even_powers);
01265 XX[0] = X * X;
01266 XX[1] = XX[0] * XX[0];
01267 for (auto
01268     k = size_t(2);
01269     k != nbr_even_powers;
01270     ++k)
01271     XX[k] = XX[k-2] * XX[1];
01272
01273 // Calculate numerator N and denominator D
01274 auto N = multivector_t(Scalar_T(1));
01275 for (auto
01276     k = size_t(0);
01277     k != nbr_even_powers;
01278     ++k)
01279     N += XX[k] * numer[2*k + 3];
01280 N *= X;
01281 N += numer[0];
01282 for (auto
01283     k = size_t(0);
01284     k != nbr_even_powers;
01285     ++k)
01286     N += XX[k] * numer[2*k + 2];
01287 auto D = multivector_t(Scalar_T(1));
01288 for (auto
01289     k = size_t(0);
01290     k != nbr_even_powers;
01291     ++k)
01292     D += XX[k] * denom[2*k + 3];
01293 D *= X;
01294 D += denom[0];
01295 for (auto
01296     k = size_t(0);
01297     k != nbr_even_powers;
01298     ++k)
01299     D += XX[k] * denom[2*k + 2];
01300 return N / D;
01301 }
01302
01303 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01304 inline
01305 static
01306 void
01307 db_step(matrix_multi<Scalar_T,LO,HI,Tune_P>& M, matrix_multi<Scalar_T,LO,HI,Tune_P>& Y)
01308 {
01309     // Reference: [CHKL]
01310     const auto& invM = inv(M);
01311     M = ((M + invM)/Scalar_T(2) + Scalar_T(1)) / Scalar_T(2);
01312     Y *= (invM + Scalar_T(1)) / Scalar_T(2);
01313 }
01314
01315 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01316 static
01317 auto
01318 db_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01319         Scalar_T norm_tol=std::pow(std::numeric_limits<Scalar_T>::epsilon(), 4)) -> const
01320 matrix_multi<Scalar_T,LO,HI,Tune_P>
01321 {
01322     // Reference: [CHKL]
01323     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01324
01325     if (val == Scalar_T(0))
01326         return val;
01327
01328     static const auto sqrt_max_steps = Tune_P::db_sqrt_max_steps;
01329     auto M = val;
01330     auto Y = val;
01331
01332     for (auto
01333         step = 0;
01334         step != sqrt_max_steps && norm(M - Scalar_T(1)) > norm_tol;
01335         ++step)
01336     {

```

```

01338         if (Y.isnan())
01339             return numeric_traits<Scalar_T>::NaN();
01340         db_step(M, Y);
01341     }
01342     return Y;
01343 }
01344
01345 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01346 static
01347 auto
01348 cr_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01349         Scalar_T norm_Y_tol=std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1)) -> const
01350 matrix_multi<Scalar_T,LO,HI,Tune_P>
01351 {
01352     // Reference: [MB]
01353     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01354
01355     if (val == Scalar_T(0))
01356         return val;
01357
01358     static const auto sqrt_max_steps = Tune_P::cr_sqrt_max_steps;
01359     auto Z = Scalar_T(2) * (Scalar_T(1) + val);
01360     auto Y = Scalar_T(1) - val;
01361     using traits_t = numeric_traits<Scalar_T>;
01362     auto norm_Y = norm(Y);
01363     for (auto
01364         step = 0;
01365         step != sqrt_max_steps && norm_Y > norm_Y_tol;
01366         ++step)
01367     {
01368         const auto old_norm_Y = norm_Y;
01369         Y = (-Y / Z) * Y;
01370         norm_Y = norm(Y);
01371         if (Y.isnan() || (norm_Y > old_norm_Y * Scalar_T(2)))
01372             return numeric_traits<Scalar_T>::NaN();
01373
01374         Z += Y * Scalar_T(2);
01375     }
01376     return Z / Scalar_T(4);
01377 }
01378 }
01379
01380 namespace pade {
01381     // Reference: [Z], Padel
01382     template< typename Scalar_T >
01383     struct pade_sqrt_number
01384     {
01385         using array = std::array<Scalar_T, 14>;
01386         static const array number;
01387     };
01388     template< typename Scalar_T >
01389     const typename pade_sqrt_number<Scalar_T>::array pade_sqrt_number<Scalar_T>::number =
01390     {
01391         {
01392             1.0,                27.0/4.0,            81.0/4.0,            2277.0/64.0,
01393             10395.0/256.0,       32319.0/1024.0,       8721.0/512.0,       26163.0/4096.0,
01394             53703.0/32768.0,     36465.0/131072.0,    3861.0/131072.0,    7371.0/4194304.0,
01395             819.0/16777216.0,    27.0/67108864.0
01396         };
01397     };
01398     // Reference: [Z], Padel
01399     template< typename Scalar_T >
01400     struct pade_sqrt_denom
01401     {
01402         using array = std::array<Scalar_T, 14>;
01403         static const array denom;
01404     };
01405     template< typename Scalar_T >
01406     const typename pade_sqrt_denom<Scalar_T>::array pade_sqrt_denom<Scalar_T>::denom =
01407     {
01408         {
01409             1.0,                25.0/4.0,            69.0/4.0,            1771.0/64.0,
01410             7315.0/256.0,       20349.0/1024.0,       4845.0/512.0,       12597.0/4096.0,
01411             21879.0/32768.0,    12155.0/131072.0,    1001.0/131072.0,    1365.0/4194304.0,
01412             91.0/16777216.0,    1.0/67108864.0
01413         };
01414     };
01415     template< >
01416     struct pade_sqrt_number<float>
01417     {
01418         using array = std::array<float, 10>;
01419         static const array number;
01420     };
01421     const typename pade_sqrt_number<float>::array pade_sqrt_number<float>::number =
01422     {
01423         {
01424             1.0,                19.0/4.0,            19.0/2.0,            665.0/64.0,
01425             1729.0/256.0,       2717.0/1024.0,       627.0/1024.0,       627.0/8192.0,
01426             285.0/65536.0,     19.0/262144.0
01427         };
01428     };

```



```

01427     template< >
01428     struct pade_sqrt_denom<float>
01429     {
01430         using array = std::array<float, 10>;
01431         static const array denom;
01432     };
01433     const typename pade_sqrt_denom<float>::array pade_sqrt_denom<float>::denom =
01434     {
01435         1.0,          17.0/4.0,          15.0/2.0,          455.0/64.0,
01436         1001.0/256.0,  1287.0/1024.0,    231.0/1024.0,    165.0/8192.0,
01437         45.0/65536,    1.0/262144.0
01438     };
01439
01440     template< >
01441     struct pade_sqrt_number<long double>
01442     {
01443         using array = std::array<long double, 18>;
01444         static const array number;
01445     };
01446     const typename pade_sqrt_number<long double>::array pade_sqrt_number<long double>::number =
01447     {
01448         1.0L,          35.0L/4.0L,          35.0L,          5425.0L/64.0L,
01449         35525.0L/256.0L,  166257.0L/1024.0L,  143325.0L/1024.0L,  740025.0L/8192.0L,
01450         2877875.0L/65536.0L,  4206125.0L/262144.0L,  572033.0L/131072.0L,  1820105.0L/2097152.0L,
01451         1028755.0L/8388608.0L,  395675.0L/33554432.0L,  24225.0L/33554432.0L,  6783.0L/268435456.0L,
01452         1785.0L/4294967296.0L,  35.0L/17179869184.0L
01453     };
01454
01455     template< >
01456     struct pade_sqrt_denom<long double>
01457     {
01458         using array = std::array<long double, 18>;
01459         static const array denom;
01460     };
01461     const typename pade_sqrt_denom<long double>::array pade_sqrt_denom<long double>::denom =
01462     {
01463         1.0L,          33.0L/4.0L,          31.0L,          4495.0L/64.0L,
01464         27405.0L/256.0L,  118755.0L/1024.0L,  94185.0L/1024.0L,  444015.0L/8192.0L,
01465         1562275.0L/65536.0L,  2042975.0L/262144.0L,  245157.0L/131072.0L,  676039.0L/2097152.0L,
01466         323323.0L/8388608.0L,  101745.0L/33554432.0L,  4845.0L/33554432.0L,  969.0L/268435456.0L,
01467         153.0L/4294967296.0L,  1.0L/17179869184.0L
01468     };
01469 #if defined(_GLUCAT_USE_QD)
01470     template< >
01471     struct pade_sqrt_number<dd_real>
01472     {
01473         using array = std::array<dd_real, 22>;
01474         static const array number;
01475     };
01476     const typename pade_sqrt_number<dd_real>::array pade_sqrt_number<dd_real>::number =
01477     {
01478         dd_real("1"),          dd_real("43")/dd_real("4"),
01479         dd_real("215")/dd_real("4"),          dd_real("10621")/dd_real("64"),
01480         dd_real("90687")/dd_real("256"),          dd_real("567987")/dd_real("1024"),
01481         dd_real("168861")/dd_real("256"),          dd_real("1246355")/dd_real("2048"),
01482         dd_real("7228859")/dd_real("16384"),          dd_real("16583853")/dd_real("65536"),
01483         dd_real("7538115")/dd_real("65536"),          dd_real("173376645")/dd_real("4194304"),
01484         dd_real("195747825")/dd_real("16777216"),          dd_real("171655785")/dd_real("67108864"),
01485         dd_real("14375115")/dd_real("33554432"),          dd_real("14375115")/dd_real("268435456"),
01486         dd_real("20764055")/dd_real("4294967296"),          dd_real("5167525")/dd_real("17179869184"),
01487         dd_real("206701")/dd_real("17179869184"),          dd_real("76153")/dd_real("274877906944"),
01488         dd_real("3311")/dd_real("1099511627776"),          dd_real("43")/dd_real("4398046511104")
01489     };
01490
01491     template< >
01492     struct pade_sqrt_denom<dd_real>
01493     {
01494         using array = std::array<dd_real, 22>;
01495         static const array denom;
01496     };
01497     const typename pade_sqrt_denom<dd_real>::array pade_sqrt_denom<dd_real>::denom =
01498     {
01499         dd_real("1"),          dd_real("41")/dd_real("4"),
01500         dd_real("195")/dd_real("4"),          dd_real("9139")/dd_real("64"),
01501         dd_real("73815")/dd_real("256"),          dd_real("435897")/dd_real("1024"),
01502         dd_real("121737")/dd_real("256"),          dd_real("840565")/dd_real("2048"),
01503         dd_real("4539051")/dd_real("16384"),          dd_real("9641775")/dd_real("65536"),
01504         dd_real("4032015")/dd_real("65536"),          dd_real("84672315")/dd_real("4194304"),
01505         dd_real("86493225")/dd_real("16777216"),          dd_real("67863915")/dd_real("67108864"),
01506         dd_real("5014575")/dd_real("33554432"),          dd_real("4345965")/dd_real("268435456"),
01507         dd_real("5311735")/dd_real("4294967296"),          dd_real("1081575")/dd_real("17179869184"),
01508         dd_real("33649")/dd_real("17179869184"),          dd_real("8855")/dd_real("274877906944"),
01509         dd_real("231")/dd_real("1099511627776"),          dd_real("1")/dd_real("4398046511104")
01510     };
01511
01512     template< >
01513     struct pade_sqrt_number<qd_real>
01514     {

```

```

01514     using array = std::array<qd_real, 34>;
01515     static const array number;
01516 };
01517 const typename pade_sqrt_numer<qd_real>::array pade_sqrt_numer<qd_real>::number =
01518 {
01519     qd_real("1"),
01520     qd_real("134"),
01521     qd_real("633485")/qd_real("256"),
01522     qd_real("15246721")/qd_real("1024"),
01523     qd_real("2518145487")/qd_real("65536"),
01524     qd_real("12301285425")/qd_real("262144"),
01525     qd_real("6344873535")/qd_real("131072"),
01526     qd_real("89075432355")/qd_real("2097152"),
01527     qd_real("267226297065")/qd_real("8388608"),
01528     qd_real("687479618945")/qd_real("33554432"),
01529     qd_real("379874182975")/qd_real("33554432"),
01530     qd_real("1443521895305")/qd_real("268435456"),
01531     qd_real("9425348845815")/qd_real("4294967296"),
01532     qd_real("13195488384141")/qd_real("17179869184"),
01533     qd_real("987417498133")/qd_real("4294967296"),
01534     qd_real("8055248011085")/qd_real("137438953472"),
01535     qd_real("6958363175533")/qd_real("549755813888"),
01536     qd_real("5056698705201")/qd_real("2199023255552"),
01537     qd_real("766166470485")/qd_real("2199023255552"),
01538     qd_real("766166470485")/qd_real("17592186044416"),
01539     qd_real("623623871325")/qd_real("140737488355328"),
01540     qd_real("203123203803")/qd_real("562949953421312"),
01541     qd_real("6478601247")/qd_real("281474976710656"),
01542     qd_real("5038912081")/qd_real("4503599627370496"),
01543     qd_real("719844583")/qd_real("18014398509481984"),
01544     qd_real("71853815")/qd_real("72057594037927936"),
01545     qd_real("1165197")/qd_real("72057594037927936"),
01546     qd_real("87703")/qd_real("576460752303423488"),
01547     qd_real("12529")/qd_real("18446744073709551616"),
01548     qd_real("67")/qd_real("73786976294838206464")
01549 };
01550 template< >
01551 struct pade_sqrt_denom<qd_real>
01552 {
01553     using array = std::array<qd_real, 34>;
01554     static const array denom;
01555 };
01556 const typename pade_sqrt_denom<qd_real>::array pade_sqrt_denom<qd_real>::denom =
01557 {
01558     qd_real("1"),
01559     qd_real("126"),
01560     qd_real("557845")/qd_real("256"),
01561     qd_real("12515965")/qd_real("1024"),
01562     qd_real("1916797311")/qd_real("65536"),
01563     qd_real("8996462475")/qd_real("262144"),
01564     qd_real("4450881435")/qd_real("131072"),
01565     qd_real("59826782925")/qd_real("2097152"),
01566     qd_real("171503444385")/qd_real("8388608"),
01567     qd_real("420696483235")/qd_real("33554432"),
01568     qd_real("221120793075")/qd_real("33554432"),
01569     qd_real("797168807855")/qd_real("268435456"),
01570     qd_real("4923689695575")/qd_real("4294967296"),
01571     qd_real("6499270398159")/qd_real("17179869184"),
01572     qd_real("456864812569")/qd_real("4294967296"),
01573     qd_real("3486599885395")/qd_real("137438953472"),
01574     qd_real("2804116503573")/qd_real("549755813888"),
01575     qd_real("1886827875075")/qd_real("2199023255552"),
01576     qd_real("263012370465")/qd_real("2199023255552"),
01577     qd_real("240141729555")/qd_real("17592186044416"),
01578     qd_real("176848560525")/qd_real("140737488355328"),
01579     qd_real("51538723353")/qd_real("562949953421312"),
01580     qd_real("1450433115")/qd_real("281474976710656"),
01581     qd_real("977699359")/qd_real("4503599627370496"),
01582     qd_real("118183439")/qd_real("18014398509481984"),
01583     qd_real("9652005")/qd_real("72057594037927936"),
01584     qd_real("121737")/qd_real("72057594037927936"),
01585     qd_real("6545")/qd_real("576460752303423488"),
01586     qd_real("561")/qd_real("18446744073709551616"),
01587     qd_real("1")/qd_real("73786976294838206464")
01588 };
01589 #endif
01590 }
01591 namespace glucat
01592 {
01593     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01594     auto
01595     matrix_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01596                 const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01597                 const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01598     {
01599         // Reference: [GW], Section 4.3, pp318-322
01600     }
01601 }

```

```

01576 // Reference: [GL], Section 11.3, p572-576
01577 // Reference: [Z], Padel
01578
01579 using traits_t = numeric_traits<Scalar_T>;
01580
01581 if (val.isnan())
01582     return traits_t::NaN();
01583
01584 const auto scr_val = val.scalar();
01585 if (val == scr_val)
01586 {
01587     if (scr_val < Scalar_T(0))
01588         return i * traits_t::sqrt(-scr_val);
01589     else
01590         return traits_t::sqrt(scr_val);
01591 }
01592
01593 // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01594 const auto scale =
01595     (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < Scalar_T(1))
01596     ? scr_val
01597     : (scr_val < Scalar_T(0))
01598       ? -abs(val)
01599       : abs(val);
01600 const auto sqrt_scale = traits_t::sqrt(traits_t::abs(scale));
01601 if (traits_t::isNaN_or_isInf(sqrt_scale))
01602     return traits_t::NaN();
01603
01604 using multivector_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
01605 auto rescale = multivector_t(sqrt_scale);
01606 if (scale < Scalar_T(0))
01607     rescale = i * sqrt_scale;
01608
01609 const auto& unitval = val / scale;
01610 static const auto max_norm = Scalar_T(1.0/4.0);
01611 auto use_approx_sqrt = true;
01612 auto use_cr_sqrt = false;
01613 auto scaled_result = multivector_t();
01614 #if defined(_GLUCAT_USE_EIGENVALUES)
01615 static const auto sqrt_2 = traits_t::sqrt(Scalar_T(2));
01616 if (level == 0)
01617 {
01618     using matrix_t = typename multivector_t::matrix_t;
01619
01620     // What kind of eigenvalues does the matrix contain?
01621     const auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
01622     const index_t next_level =
01623         (genus.m_is_singular)
01624         ? level
01625         : level + 1;
01626     switch (genus.m_eig_case)
01627     {
01628     case matrix::neg_real_eigs:
01629         scaled_result = matrix_sqrt(-i * unitval, i, next_level) * (i + Scalar_T(1)) / sqrt_2;
01630         use_approx_sqrt = false;
01631         break;
01632     case matrix::both_eigs:
01633     {
01634         const auto safe_arg = genus.m_safe_arg;
01635         scaled_result = matrix_sqrt(exp(i*safe_arg) * unitval, i, next_level) * exp(-i*safe_arg /
01636         Scalar_T(2));
01637     }
01638     case matrix::neg_real_eigs:
01639         scaled_result = matrix_sqrt(-i * unitval, i, next_level) * (i + Scalar_T(1)) / sqrt_2;
01640         use_approx_sqrt = false;
01641         break;
01642     default:
01643         break;
01644     }
01645     use_cr_sqrt = genus.m_is_singular;
01646 }
01647 #endif
01648 if (use_approx_sqrt)
01649 {
01650     scaled_result =
01651         (norm(unitval - Scalar_T(1)) < max_norm)
01652         // Pade' approximation of square root
01653         ? pade_approx(pade::pade_sqrt_numer<Scalar_T>::numer,
01654                       pade::pade_sqrt_denom<Scalar_T>::denom,
01655                       unitval - Scalar_T(1))
01656         // Product form of Denman-Beavers square root iteration
01657         : (use_cr_sqrt)
01658           ? cr_sqrt(unitval)
01659           : db_sqrt(unitval);
01660 }
01661 return (scaled_result.isnan() ||
01662         !approx_equal(pow(scaled_result, 2), unitval))
01663     ? traits_t::NaN()
01664     : scaled_result * rescale;

```

```

01662     }
01663
01664     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01665     auto
01666     sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01667     bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01668     {
01669         // Reference: [GW], Section 4.3, pp318-322
01670         // Reference: [GL], Section 11.3, p572-576
01671         // Reference: [Z], Padel
01672
01673         using traits_t = numeric_traits<Scalar_T>;
01674
01675         if (val.isnan())
01676             return traits_t::NaN();
01677
01678         check_complex(val, i, prechecked);
01679
01680         switch (Tune_P::function_precision)
01681         {
01682             case precision_demoted:
01683             {
01684                 using demoted_scalar_t = typename traits_t::demoted::type;
01685                 using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
01686
01687                 const auto& demoted_val = demoted_multivector_t(val);
01688                 const auto& demoted_i = demoted_multivector_t(i);
01689
01690                 return matrix_sqrt(demoted_val, demoted_i, 0);
01691             }
01692             break;
01693             case precision_promoted:
01694             {
01695                 using promoted_scalar_t = typename traits_t::promoted::type;
01696                 using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
01697
01698                 const auto& promoted_val = promoted_multivector_t(val);
01699                 const auto& promoted_i = promoted_multivector_t(i);
01700
01701                 return matrix_sqrt(promoted_val, promoted_i, 0);
01702             }
01703             break;
01704             default:
01705                 return matrix_sqrt(val, i, 0);
01706         }
01707     }
01708 }
01709
01710 namespace pade {
01711     // Reference: [Z], Padel
01712     template< typename Scalar_T >
01713     struct pade_log_number
01714     {
01715     {
01716         using array = std::array<Scalar_T, 14>;
01717         static const array number;
01718     };
01719     template< typename Scalar_T >
01720     const typename pade_log_number<Scalar_T>::array pade_log_number<Scalar_T>::number =
01721     {
01722         0.0, 1.0, 6.0, 4741.0/300.0,
01723         1441.0/60.0, 107091.0/4600.0, 8638.0/575.0, 263111.0/40250.0,
01724         153081.0/80500.0, 395243.0/1101240.0, 28549.0/688275.0, 605453.0/228813200.0,
01725         785633.0/10296594000.0, 1145993.0/1873980108000.0
01726     };
01727
01728     // Reference: [Z], Padel
01729     template< typename Scalar_T >
01730     struct pade_log_denom
01731     {
01732     {
01733         using array = std::array<Scalar_T, 14>;
01734         static const array denom;
01735     };
01736     template< typename Scalar_T >
01737     const typename pade_log_denom<Scalar_T>::array pade_log_denom<Scalar_T>::denom =
01738     {
01739         1.0, 13.0/2.0, 468.0/25.0, 1573.0/50.0,
01740         1573.0/46.0, 11583.0/460.0, 10296.0/805.0, 2574.0/575.0,
01741         11583.0/10925.0, 143.0/874.0, 572.0/37145.0, 117.0/148580.0,
01742         13.0/742900.0, 1.0/10400600.0
01743     };
01744
01745     template< >
01746     struct pade_log_number<float>
01747     {
01748         using array = std::array<float, 10>;
01749         static const array number;
01750     };

```

```

01751 const typename pade_log_numer<float>::array pade_log_numer<float>::number =
01752 {
01753     0.0,          1.0,          4.0,          1337.0/204.0,
01754     385.0/68.0,    1879.0/680.0,    193.0/255.0,    197.0/1820.0,
01755     419.0/61880.0, 7129.0/61261200.0
01756 };
01757 template< >
01758 struct pade_log_denom<float>
01759 {
01760     using array = std::array<float, 10>;
01761     static const array denom;
01762 };
01763 const typename pade_log_denom<float>::array pade_log_denom<float>::denom =
01764 {
01765     1.0,          9.0/2.0,          144.0/17.0,    147.0/17.0,
01766     441.0/85.0,    63.0/34.0,          84.0/221.0,    9.0/221.0,
01767     9.0/4862.0,    1.0/48620.0
01768 };
01769
01770 template< >
01771 struct pade_log_numer<long double>
01772 {
01773     using array = std::array<long double, 18>;
01774     static const array number;
01775 };
01776 const typename pade_log_numer<long double>::array pade_log_numer<long double>::number =
01777 {
01778     0.0L,          1.0L,          8.0L,
01779     3835.0L/132.0L,    8365.0L/132.0L,    11363807.0L/122760.0L,    162981.0L/1705.0L,
01780     9036157.0L/125860.0L,    18009875.0L/453096.0L,    44211925.0L/2718576.0L,    4149566.0L/849555.0L,
01781     16973929.0L/16020180.0L,    172459.0L/1068012.0L,    116317061.0L/7025382936.0L,    19679783.0L/18441630207.0L,
01782     23763863.0L/614721006900.0L,    50747.0L/79318839600.0L,    42142223.0L/14295951736466400.0L
01783 };
01784 template< >
01785 struct pade_log_denom<long double>
01786 {
01787     using array = std::array<long double, 18>;
01788     static const array denom;
01789 };
01790 const typename pade_log_denom<long double>::array pade_log_denom<long double>::denom =
01791 {
01792     1.0L,          17.0L/2.0L,          1088.0L/33.0L,
01793     850.0L/11.0L,    41650.0L/341.0L,    140777.0L/1023.0L,    1126216.0L/9889.0L,
01794     63206.0L/899.0L,    790075.0L/24273.0L,    60775.0L/5394.0L,    38896.0L/13485.0L,
01795     21658.0L/40455.0L,    21658.0L/310155.0L,    4165.0L/682341.0L,    680.0L/2047023.0L,
01796     34.0L/3411705.0L,    17.0L/129644790.0L,    1.0L/2333606220
01797 };
01798 #if defined(_GLUCAT_USE_QD)
01799 template< >
01800 struct pade_log_numer<dd_real>
01801 {
01802     using array = std::array<dd_real, 22>;
01803     static const array number;
01804 };
01805 const typename pade_log_numer<dd_real>::array pade_log_numer<dd_real>::number =
01806 {
01807     dd_real("0"),          dd_real("1"),
01808     dd_real("10"),          dd_real("22781")/dd_real("492"),
01809     dd_real("21603")/dd_real("164"),    dd_real("5492649")/dd_real("21320"),
01810     dd_real("978724")/dd_real("2665"),    dd_real("4191605")/dd_real("10619"),
01811     dd_real("12874933")/dd_real("39442"),    dd_real("11473457")/dd_real("54612"),
01812     dd_real("2406734")/dd_real("22755"),    dd_real("166770367")/dd_real("4004880"),
01813     dd_real("30653165")/dd_real("2402928"),    dd_real("647746389")/dd_real("215195552"),
01814     dd_real("25346331")/dd_real("47074027"),    dd_real("278270613")/dd_real("3900419380"),
01815     dd_real("105689791")/dd_real("15601677520"),    dd_real("606046475")/dd_real("1379188292768"),
01816     dd_real("969715")/dd_real("53502994116"),    dd_real("11098301")/dd_real("26204577562592"),
01817     dd_real("118999")/dd_real("26204577562592"),    dd_real("18858053")/dd_real("1392249205900512960")
01818 };
01819 template< >
01820 struct pade_log_denom<dd_real>
01821 {
01822     using array = std::array<dd_real, 22>;
01823     static const array denom;
01824 };
01825 const typename pade_log_denom<dd_real>::array pade_log_denom<dd_real>::denom =
01826 {
01827     dd_real("1"),          dd_real("21")/dd_real("2"),
01828     dd_real("2100")/dd_real("41"),    dd_real("12635")/dd_real("82"),
01829     dd_real("341145")/dd_real("1066"),    dd_real("1037799")/dd_real("2132"),

```

```

01830     dd_real("11069856")/dd_real("19721"),          dd_real("9883800")/dd_real("19721"),
01831     dd_real("6918660")/dd_real("19721"),          dd_real("293930")/dd_real("1517"),
01832     dd_real("1410864")/dd_real("16687"),          dd_real("88179")/dd_real("3034"),
01833     dd_real("734825")/dd_real("94054"),            dd_real("305235")/dd_real("188108"),
01834     dd_real("348840")/dd_real("1363783"),          dd_real("40698")/dd_real("1363783"),
01835     dd_real("6783")/dd_real("2727566"),            dd_real("9975")/dd_real("70916716"),
01836     dd_real("266")/dd_real("53187537"),            dd_real("7")/dd_real("70916716"),
01837     dd_real("7")/dd_real("8155422340"),            dd_real("1")/dd_real("538257874440")
01838 };
01839
01840 template< >
01841 struct pade_log_numer<qd_real>
01842 {
01843     using array = std::array<qd_real, 34>;
01844     static const array numer;
01845 };
01846 const typename pade_log_numer<qd_real>::array pade_log_numer<qd_real>::numer =
01847 {
01848     qd_real("0"),                                     qd_real("1"),
01849     qd_real("16"),
01850     qd_real("95201")/qd_real("780"),
01851     qd_real("30721")/qd_real("52"),
01852     qd_real("7416257")/qd_real("3640"),
01853     qd_real("1039099")/qd_real("195"),
01854     qd_real("6097772319")/qd_real("555100"),
01855     qd_real("1564058073")/qd_real("85400"),
01856     qd_real("30404640205")/qd_real("1209264"),
01857     qd_real("725351278")/qd_real("25193"),
01858     qd_real("4092322670789")/qd_real("147429436"),
01859     qd_real("4559713849589")/qd_real("201040140"),
01860     qd_real("5049361751189")/qd_real("320023080"),
01861     qd_real("74979677195")/qd_real("8000577"),
01862     qd_real("16569850691873")/qd_real("3481514244"),
01863     qd_real("1065906022369")/qd_real("515779888"),
01864     qd_real("335956770855841")/qd_real("438412904800"),
01865     qd_real("1462444287585964")/qd_real("6041877844275"),
01866     qd_real("397242326339851")/qd_real("6122436215532"),
01867     qd_real("64211291334131")/qd_real("4373168725380"),
01868     qd_real("142322343550859")/qd_real("51080680851480"),
01869     qd_real("154355972958659")/qd_real("351179680853925"),
01870     qd_real("167483568676259")/qd_real("2937139148960100"),
01871     qd_real("4230788929433")/qd_real("704913395750424"),
01872     qd_real("197968763176019")/qd_real("392923948371995600"),
01873     qd_real("10537522306718")/qd_real("319250708052246425"),
01874     qd_real("236648286272519")/qd_real("144249197475035425500"),
01875     qd_real("260715545088119")/qd_real("4375558990076074573500"),
01876     qd_real("289596255666839")/qd_real("192874640282553367199880"),
01877     qd_real("8802625510547")/qd_real("361639950529787563499775"),
01878     qd_real("373831661521439")/qd_real("1659204093030665341336967700"),
01879     qd_real("446033437968239")/qd_real("464577146048586295574350956000"),
01880     qd_real("53676090078349")/qd_real("47386868896955802148583797512000")
01881 };
01882 template< >
01883 struct pade_log_denom<qd_real>
01884 {
01885     using array = std::array<qd_real, 34>;
01886     static const array denom;
01887 };
01888 const typename pade_log_denom<qd_real>::array pade_log_denom<qd_real>::denom =
01889 {
01890     qd_real("1"),
01891     qd_real("33")/qd_real("2"),
01892     qd_real("8448")/qd_real("65"),
01893     qd_real("42284")/qd_real("65"),
01894     qd_real("211420")/qd_real("91"),
01895     qd_real("573562")/qd_real("91"),
01896     qd_real("32119472")/qd_real("2379"),
01897     qd_real("92917044")/qd_real("3965"),
01898     qd_real("603960786")/qd_real("17995"),
01899     qd_real("144626625")/qd_real("3599"),
01900     qd_real("2776831200")/qd_real("68381"),
01901     qd_real("16692542100")/qd_real("478667"),
01902     qd_real("12241197540")/qd_real("478667"),
01903     qd_real("1098569010")/qd_real("68381"),
01904     qd_real("31387686000")/qd_real("3624193"),
01905     qd_real("9939433900")/qd_real("2479711"),
01906     qd_real("67091178825")/qd_real("42155087"),
01907     qd_real("2683647153")/qd_real("4959422"),
01908     qd_real("19083713088")/qd_real("121505839"),
01909     qd_real("4708152900")/qd_real("121505839"),
01910     qd_real("941630580")/qd_real("116546417"),
01911     qd_real("88704330")/qd_real("62755763"),
01912     qd_real("12902448")/qd_real("62755763"),
01913     qd_real("1542684")/qd_real("62755763"),
01914     qd_real("6427850")/qd_real("2698497809"),
01915     qd_real("3471039")/qd_real("18889484663"),
01916     qd_real("8544096")/qd_real("774468871183"),

```

```

    qd_real("39556")/qd_real("79027435835"),
01888    qd_real("118668")/qd_real("7191496660985"),
    qd_real("10230")/qd_real("27327687311743"),
01889    qd_real("5456")/qd_real("1011124430534491"),
    qd_real("44")/qd_real("1011124430534491"),
01890    qd_real("11")/qd_real("70778710137414370"),
    qd_real("1")/qd_real("7219428434016265740")
01891 };
01892 #endif
01893 }
01894
01895 namespace glucat{
01897     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01898     static
01899     auto
01900     pade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
matrix_multi<Scalar_T,LO,HI,Tune_P>
01901     {
01902         // Reference: [GW], Section 4.3, pp318-322
01903         // Reference: [CHKL]
01904         // Reference: [GL], Section 11.3, p572-576
01905         // Reference: [Z], Padel
01906
01907         using traits_t = numeric_traits<Scalar_T>;
01908         if (val == Scalar_T(0) || val.isnan())
01909             return traits_t::NaN();
01910         else
01911             return pade_approx(pade::pade_log_numer<Scalar_T>::numer,
01912                                pade::pade_log_denom<Scalar_T>::denom,
01913                                val - Scalar_T(1));
01914     }
01915
01917     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01918     static
01919     auto
01920     cascade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
matrix_multi<Scalar_T,LO,HI,Tune_P>
01921     {
01922         // Reference: [CHKL]
01923         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01924         using traits_t = numeric_traits<Scalar_T>;
01925         if (val == Scalar_T(0) || val.isnan())
01926             return traits_t::NaN();
01927
01928         using limits_t = std::numeric_limits<Scalar_T>;
01929         static const auto epsilon = limits_t::epsilon();
01930         static const auto max_inner_norm = traits_t::pow(epsilon, 2);
01931         static const auto max_outer_norm = Scalar_T(6.0/limits_t::digits);
01932         auto Y = val;
01933         auto E = multivector_t(Scalar_T(0));
01934         Scalar_T norm_Y_1;
01935         auto pow_2_outer_step = Scalar_T(1);
01936         auto pow_4_outer_step = Scalar_T(1);
01937         int outer_step;
01938         for (outer_step = 0, norm_Y_1 = norm(Y - Scalar_T(1));
01939             outer_step != Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm;
01940             ++outer_step, norm_Y_1 = norm(Y - Scalar_T(1)))
01941         {
01942             if (Y == Scalar_T(0) || Y.isnan())
01943                 return traits_t::NaN();
01944
01945             // Incomplete product form of Denman-Beavers square root iteration
01946             auto M = Y;
01947             for (auto
01948                 inner_step = 0;
01949                 inner_step != Tune_P::log_max_inner_steps &&
01950                 norm(M - Scalar_T(1)) * pow_4_outer_step > max_inner_norm;
01951                 ++inner_step)
01952                 db_step(M, Y);
01953
01954             E += (M - Scalar_T(1)) * pow_2_outer_step;
01955             pow_2_outer_step *= Scalar_T(2);
01956             pow_4_outer_step *= Scalar_T(4);
01957         }
01958         if (outer_step == Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm)
01959             return traits_t::NaN();
01960         else
01961             return pade_log(Y) * pow_2_outer_step - E;
01962     }
01963
01965     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01966     auto
01967     matrix_log( const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01968                const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01969                const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01970     {
01971         // Scaled incomplete square root cascade and scaled Pade' approximation of log

```

```

01972 // Reference: [CHKL]
01973
01974 using traits_t = numeric_traits<Scalar_T>;
01975 if (val == Scalar_T(0) || val.isnan())
01976     return traits_t::NaN();
01977
01978 static const auto pi = traits_t::pi();
01979 const auto scr_val = val.scalar();
01980 if (val == scr_val)
01981 {
01982     if (scr_val < Scalar_T(0))
01983         return i * pi + traits_t::log(-scr_val);
01984     else
01985         return traits_t::log(scr_val);
01986 }
01987
01988 // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01989 const auto max_norm = Scalar_T(1.0/9.0);
01990 const auto scale =
01991     (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < max_norm)
01992     ? scr_val
01993     : (scr_val < Scalar_T(0))
01994       ? -abs(val)
01995       : abs(val);
01996 if (scale == Scalar_T(0))
01997     return traits_t::NaN();
01998
01999 using multivector_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
02000 const auto log_scale = traits_t::log(traits_t::abs(scale));
02001 auto rescale = multivector_t(log_scale);
02002 if (scale < Scalar_T(0))
02003     rescale = i * pi + log_scale;
02004 const auto unitval = val/scale;
02005 if (inv(unitval).isnan())
02006     return traits_t::NaN();
02007
02008 #if defined(_GLUCAT_USE_EIGENVALUES)
02009 auto scaled_result = multivector_t();
02010 if (level == 0)
02011 {
02012     using matrix_t = typename multivector_t::matrix_t;
02013
02014     // What kind of eigenvalues does the matrix contain?
02015     auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
02016     switch (genus.m_eig_case)
02017     {
02018     case matrix::neg_real_eigs:
02019         scaled_result = matrix_log(-i * unitval, i, level + 1) + i * pi/Scalar_T(2);
02020         break;
02021     case matrix::both_eigs:
02022     {
02023         const Scalar_T safe_arg = genus.m_safe_arg;
02024         scaled_result = matrix_log(exp(i*safe_arg) * unitval, i, level + 1) - i * safe_arg;
02025     }
02026     break;
02027     default:
02028         scaled_result = cascade_log(unitval);
02029         break;
02030     }
02031 }
02032 else
02033     scaled_result = cascade_log(unitval);
02034 #else
02035 auto scaled_result = cascade_log(unitval);
02036 #endif
02037 return (scaled_result.isnan())
02038     ? traits_t::NaN()
02039     : scaled_result + rescale;
02040 }
02041
02042 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02043 auto
02044 log(const matrix_multi<Scalar_T, LO, HI, Tune_P>& val, const matrix_multi<Scalar_T, LO, HI, Tune_P>& i,
02045 bool prechecked) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>
02046 {
02047     using traits_t = numeric_traits<Scalar_T>;
02048
02049     if (val == Scalar_T(0) || val.isnan())
02050         return traits_t::NaN();
02051
02052     check_complex(val, i, prechecked);
02053
02054     switch (Tune_P::function_precision)
02055     {
02056     case precision_demoted:
02057     {
02058         using demoted_scalar_t = typename traits_t::demoted::type;

```



```

02059         using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
02060
02061         const auto& demoted_val = demoted_multivector_t(val);
02062         const auto& demoted_i = demoted_multivector_t(i);
02063
02064         return matrix_log(demoted_val, demoted_i, 0);
02065     }
02066     break;
02067 case precision_promoted:
02068     {
02069         using promoted_scalar_t = typename traits_t::promoted::type;
02070         using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
02071
02072         const auto& promoted_val = promoted_multivector_t(val);
02073         const auto& promoted_i = promoted_multivector_t(i);
02074
02075         return matrix_log(promoted_val, promoted_i, 0);
02076     }
02077     break;
02078 default:
02079     return matrix_log(val, i, 0);
02080 }
02081 }
02082
02083 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02084 auto
02085 exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
02086 {
02087     {
02088         using traits_t = numeric_traits<Scalar_T>;
02089         if (val.isnan())
02090             return traits_t::NaN();
02091
02092         const auto scr_val = val.scalar();
02093         if (val == scr_val)
02094             return traits_t::exp(scr_val);
02095
02096         switch (Tune_P::function_precision)
02097         {
02098         case precision_demoted:
02099             {
02100                 using demoted_scalar_t = typename traits_t::demoted::type;
02101                 using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
02102
02103                 const auto& demoted_val = demoted_multivector_t(val);
02104                 return clifford_exp(demoted_val);
02105             }
02106             break;
02107         case precision_promoted:
02108             {
02109                 using promoted_scalar_t = typename traits_t::promoted::type;
02110                 using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
02111
02112                 const auto& promoted_val = promoted_multivector_t(val);
02113                 return clifford_exp(promoted_val);
02114             }
02115             break;
02116         default:
02117             return clifford_exp(val);
02118         }
02119     }
02120 }
02121 #endif // _GLUCAT_MATRIX_MULTI_IMP_H

```

## 7.39 glucat/portability.h File Reference

```

#include <boost/version.hpp>
#include <cmath>

```



## 7.39.1.2 \_GLUCAT\_ISNAN

```
#define _GLUCAT_ISNAN(
    x)
```

**Value:**

```
(x != x)
```

Definition at line 42 of file [portability.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::isNaN\(\)](#).

## 7.39.1.3 UBLAS\_ABS

```
#define UBLAS_ABS abs
```

Definition at line 51 of file [portability.h](#).

## 7.39.1.4 UBLAS\_SQRT

```
#define UBLAS_SQRT sqrt
```

Definition at line 52 of file [portability.h](#).

## 7.40 portability.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_PORTABILITY_H
00002 #define _GLUCAT_PORTABILITY_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     portability.h : Work around non-standard compilers and libraries
00006     -----
00007     begin                : Sun 2001-08-18
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include <boost/version.hpp>
00035 #include <cmath>
00036
00037 // Workaround for isnan and isinf
```

```

00038 #if __cplusplus > 199711L
00039 # define _GLUCAT_ISNAN(x) (std::isnan(x))
00040 # define _GLUCAT_ISINF(x) (std::isinf(x))
00041 #else
00042 # define _GLUCAT_ISNAN(x) (x != x)
00043 # define _GLUCAT_ISINF(x) (!_GLUCAT_ISNAN(x) && _GLUCAT_ISNAN(x-x))
00044 #endif
00045
00046 // Workaround for abs and sqrt
00047 #if BOOST_VERSION >= 103400
00048 # define UBLAS_ABS type_abs
00049 # define UBLAS_SQRT type_sqrt
00050 #else
00051 # define UBLAS_ABS abs
00052 # define UBLAS_SQRT sqrt
00053 #endif
00054
00055 // Use with Cygwin gcc to obtain __WORDSIZE
00056 #if defined(HAVE_BITS_WORDSIZE_H)
00057 # include <bits/wordsize.h>
00058 #endif
00059
00060 #endif // _GLUCAT_PORTABILITY_H

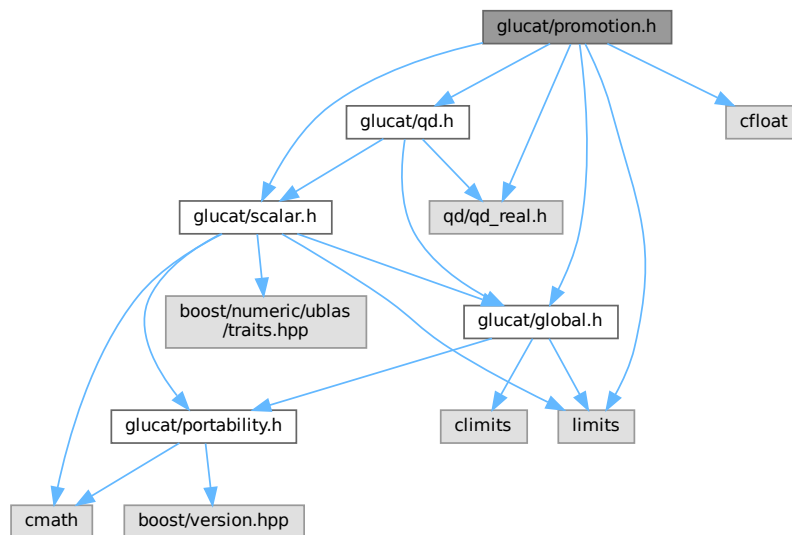
```

## 7.41 glucat/promotion.h File Reference

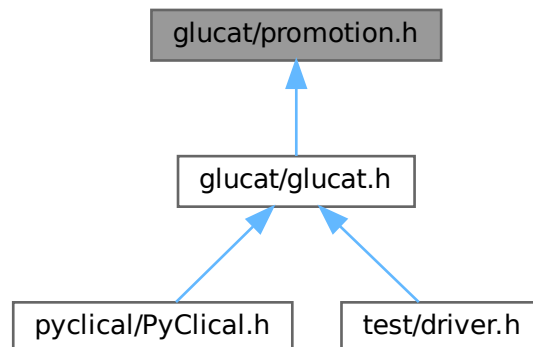
```

#include "glucat/global.h"
#include "glucat/scalar.h"
#include "glucat/qd.h"
#include <cfloat>
#include <limits>
#include <qd/qd_real.h>
Include dependency graph for promotion.h:

```



This graph shows which files directly or indirectly include this file:



## Classes

- struct `glucat::numeric_traits< Scalar_T >::promoted`  
*Extra traits which extend numeric limits.*
- struct `glucat::numeric_traits< Scalar_T >::demoted`  
*Demoted type for long double.*

## Namespaces

- namespace `glucat`

## 7.42 promotion.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_PROMOTION_H
00002 #define _GLUCAT_PROMOTION_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   promotion.h : Define promotion and demotion for specific scalar types
00006   -----
00007   begin                : 2021-11-13
00008   copyright             : (C) 2021 by Paul C. Leopardi
00009 *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,

```

```

00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036 #include "glucat/qd.h"
00037
00038 #include <cfloat>
00039 #include <limits>
00040
00041 #if defined(_GLUCAT_USE_QD)
00042 # include <qd/qd_real.h>
00043 #endif
00044
00045 namespace glucat
00046 {
00047     // Reference: [AA], 2.4, p. 30-31
00048
00049     #if !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00050
00051     # if DBL_MANT_DIG < LDBL_MANT_DIG
00052
00053         template<>
00054         struct
00055         numeric_traits<double>::
00056         promoted {using type = long double;};
00057
00058         template<>
00059         struct
00060         numeric_traits<long double>::
00061         demoted {using type = double;};
00062
00063     # else
00064
00065         template<>
00066         struct
00067         numeric_traits<double>::
00068         promoted {using type = double;};
00069
00070         template<>
00071         struct
00072         numeric_traits<long double>::
00073         demoted {using type = float;};
00074
00075     # endif // DBL_MANT_DIG < LDBL_MANT_DIG
00076
00077     template<>
00078     struct
00079     numeric_traits<long double>::
00080     promoted {using type = long double;};
00081
00082 #else
00083
00084 # if (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00085
00086         template<>
00087         struct
00088         numeric_traits<double>::
00089         promoted {using type = long double;};
00090
00091         template<>
00092         struct
00093         numeric_traits<long double>::
00094         demoted {using type = double;};
00095
00096         template<>
00097         struct
00098         numeric_traits<long double>::
00099         promoted {using type = dd_real;};
00100
00101         template<>
00102         struct
00103         numeric_traits<dd_real>::
00104         demoted {using type = long double;};
00105
00106         template<>
00107         struct
00108         numeric_traits<dd_real>::
00109         promoted {using type = qd_real;};
00110
00111         template<>
00112         struct
00113         numeric_traits<qd_real>::

```

```

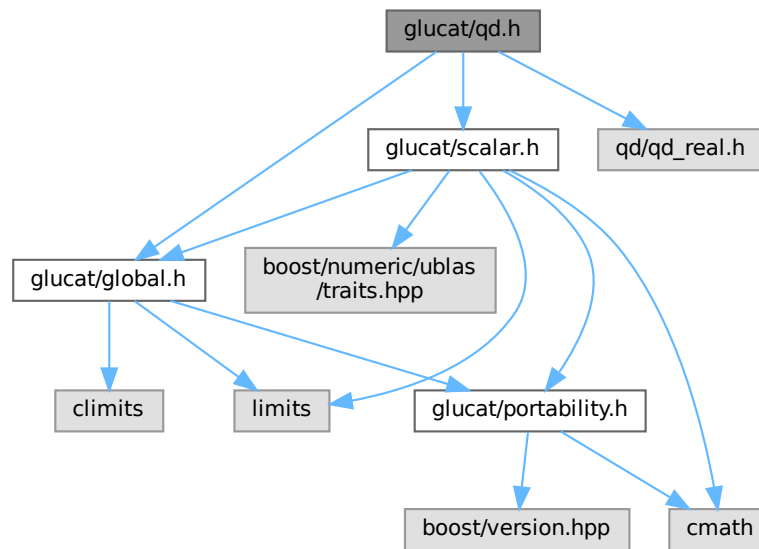
00126     demoted {using type = dd_real;};
00127
00128 # elif (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00129
00131     template<>
00132     struct
00133     numeric_traits<double>::
00134     promoted {using type = dd_real;};
00135
00137     template<>
00138     struct
00139     numeric_traits<long double>::
00140     demoted {using type = float;};
00141
00143     template<>
00144     struct
00145     numeric_traits<long double>::
00146     promoted {using type = dd_real;};
00147
00149     template<>
00150     struct
00151     numeric_traits<dd_real>::
00152     demoted {using type = double;};
00153
00155     template<>
00156     struct
00157     numeric_traits<dd_real>::
00158     promoted {using type = qd_real;};
00159
00161     template<>
00162     struct
00163     numeric_traits<qd_real>::
00164     demoted {using type = dd_real;};
00165
00166 # else
00167
00169     template<>
00170     struct
00171     numeric_traits<double>::
00172     promoted {using type = dd_real;};
00173
00175     template<>
00176     struct
00177     numeric_traits<dd_real>::
00178     demoted {using type = double;};
00179
00181     template<>
00182     struct
00183     numeric_traits<dd_real>::
00184     promoted {using type = long double;};
00185
00187     template<>
00188     struct
00189     numeric_traits<long double>::
00190     demoted {using type = dd_real;};
00191
00193     template<>
00194     struct
00195     numeric_traits<long double>::
00196     promoted {using type = qd_real;};
00197
00199     template<>
00200     struct
00201     numeric_traits<qd_real>::
00202     demoted {using type = long double;};
00203
00204 # endif // (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00205
00207     template<>
00208     struct
00209     numeric_traits<qd_real>::
00210     promoted {using type = qd_real;};
00211
00212 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00213
00214 } // namespace glucat
00215
00216 #endif // _GLUCAT_PROMOTION_H

```

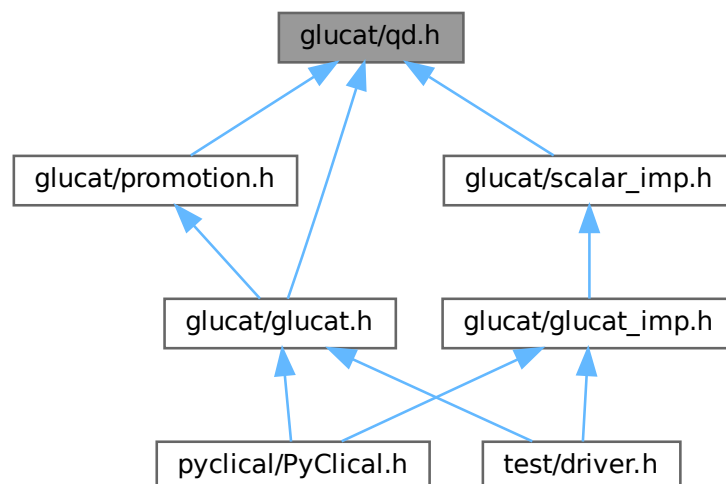
## 7.43 glucat/qd.h File Reference

```
#include "glucat/global.h"
#include "glucat/scalar.h"
#include <qd/qd_real.h>
```

Include dependency graph for qd.h:



This graph shows which files directly or indirectly include this file:





## Namespaces

- namespace `glucat`

## 7.44 qd.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_QD_H
00002 #define _GLUCAT_QD_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     qd.h : Define functions for dd_real and qd_real as scalar_t
00006
00007     begin                : 2010-03-23
00008     copyright            : (C) 2010-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036
00037 #if defined(_GLUCAT_USE_QD)
00038 # include <qd/qd_real.h>
00039 #endif
00040
00041 namespace glucat
00042 {
00043     // Reference: [AA], 2.4, p. 30-31
00044
00045     #if defined(_GLUCAT_USE_QD) && defined(QD_API)
00046     # define _GLUCAT_QD_F(_T, _F) \
00047     template<> \
00048     inline \
00049     auto \
00050     numeric_traits<_T>:: \
00051     _F(const _T& val) -> _T \
00052     { return ::_F(val); }
00053
00054     template<>
00055     inline
00056     auto
00057     numeric_traits<dd_real>::
00058     isNaN(const dd_real& val) -> bool
00059     { return val.isnan(); }
00060
00061     template<>
00062     inline
00063     auto
00064     numeric_traits<dd_real>::
00065     isInf(const dd_real& val) -> bool
00066     { return val.isinf(); }
00067
00068     template<>
00069     inline
00070     auto
00071     numeric_traits<dd_real>::
00072     isNaN_or_isInf(const dd_real& val) -> bool

```

```

00079 { return val.isnan() || val.isinf(); }
00080
00082 template<>
00083 inline
00084 auto
00085 numeric_traits<dd_real>::
00086 to_int(const dd_real& val) -> int
00087 { return ::to_int(val); }
00088
00090 template<>
00091 inline
00092 auto
00093 numeric_traits<dd_real>::
00094 to_double(const dd_real& val) -> double
00095 { return ::to_double(val); }
00096
00098 template<>
00099 inline
00100 auto
00101 numeric_traits<dd_real>::
00102 fmod(const dd_real& lhs, const dd_real& rhs) -> dd_real
00103 { return ::fmod(lhs, rhs); }
00104
00106 template<>
00107 inline
00108 auto
00109 numeric_traits<dd_real>::
00110 pow(const dd_real& val, int n) -> dd_real
00111 {
00112     if (val == dd_real(0))
00113     {
00114         return
00115             (n < 0)
00116             ? NaN()
00117             : (n == 0)
00118             ? dd_real(1)
00119             : dd_real(0);
00120     }
00121     auto result = dd_real(1);
00122     auto power =
00123         (n < 0)
00124         ? dd_real(1)/val
00125         : val;
00126     for (auto
00127         k = std::abs(n);
00128         k != 0;
00129         k /= 2)
00130     {
00131         if (k % 2)
00132             result *= power;
00133         power *= power;
00134     }
00135     return result;
00136 }
00137
00139 template<>
00140 inline
00141 auto
00142 numeric_traits<dd_real>::
00143 pi() -> dd_real
00144 { return dd_real::_pi; }
00145
00147 template<>
00148 inline
00149 auto
00150 numeric_traits<dd_real>::
00151 ln_2() -> dd_real
00152 { return dd_real::_log2; }
00153
00155 _GLUCAT_QD_F(dd_real, exp)
00156
00157 _GLUCAT_QD_F(dd_real, log)
00158
00159 _GLUCAT_QD_F(dd_real, cos)
00160
00161 _GLUCAT_QD_F(dd_real, acos)
00162
00163 _GLUCAT_QD_F(dd_real, cosh)
00164
00165 _GLUCAT_QD_F(dd_real, sinh)
00166
00167 _GLUCAT_QD_F(dd_real, cosh)
00168
00169 _GLUCAT_QD_F(dd_real, sinh)
00170
00171 _GLUCAT_QD_F(dd_real, sin)
00172

```

```

00173  _GLUCAT_QD_F(dd_real, asin)
00174
00175
00176  _GLUCAT_QD_F(dd_real, sinh)
00177
00178
00179  _GLUCAT_QD_F(dd_real, tan)
00180
00181
00182  _GLUCAT_QD_F(dd_real, atan)
00183
00184
00185  _GLUCAT_QD_F(dd_real, tanh)
00186
00187
00188  template<>
00189  inline
00190  auto
00191  numeric_traits<qd_real>::
00192  isNaN(const qd_real& val) -> bool
00193  { return val.isnan(); }
00194
00196  template<>
00197  inline
00198  auto
00199  numeric_traits<qd_real>::
00200  isInf(const qd_real& val) -> bool
00201  { return val.isinf(); }
00202
00204  template<>
00205  inline
00206  auto
00207  numeric_traits<qd_real>::
00208  isNaN_or_isInf(const qd_real& val) -> bool
00209  { return val.isnan() || val.isinf(); }
00210
00212  template<>
00213  inline
00214  auto
00215  numeric_traits<qd_real>::
00216  to_int(const qd_real& val) -> int
00217  { return ::to_int(val); }
00218
00220  template<>
00221  inline
00222  auto
00223  numeric_traits<qd_real>::
00224  to_double(const qd_real& val) -> double
00225  { return ::to_double(val); }
00226
00228  template<>
00229  inline
00230  auto
00231  numeric_traits<qd_real>::
00232  fmod(const qd_real& lhs, const qd_real& rhs) -> qd_real
00233  { return ::fmod(lhs, rhs); }
00234
00236  template<>
00237  inline
00238  auto
00239  numeric_traits<qd_real>::
00240  pow(const qd_real& val, int n) -> qd_real
00241  {
00242      if (val == qd_real(0))
00243      {
00244          return
00245              (n < 0)
00246              ? NaN()
00247              : (n == 0)
00248              ? qd_real(1)
00249              : qd_real(0);
00250      }
00251      auto result = qd_real(1);
00252      auto power =
00253          (n < 0)
00254          ? qd_real(1)/val
00255          : val;
00256      for (auto
00257          k = std::abs(n);
00258          k != 0;
00259          k /= 2)
00260      {
00261          if (k % 2)
00262              result *= power;
00263          power *= power;
00264      }
00265      return result;

```

```

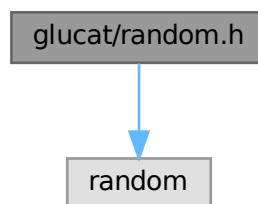
00266     }
00267
00269     template<>
00270     inline
00271     auto
00272     numeric_traits<qd_real>::
00273     pi() -> qd_real
00274     { return qd_real::_pi; }
00275
00277     template<>
00278     inline
00279     auto
00280     numeric_traits<qd_real>::
00281     ln_2() -> qd_real
00282     { return qd_real::_log2; }
00283
00285     _GLUCAT_QD_F(qd_real, exp)
00286
00287
00288     _GLUCAT_QD_F(qd_real, log)
00289
00290
00291     _GLUCAT_QD_F(qd_real, cos)
00292
00293
00294     _GLUCAT_QD_F(qd_real, acos)
00295
00296
00297     _GLUCAT_QD_F(qd_real, cosh)
00298
00299
00300     _GLUCAT_QD_F(qd_real, sin)
00301
00302
00303     _GLUCAT_QD_F(qd_real, asin)
00304
00305
00306     _GLUCAT_QD_F(qd_real, sinh)
00307
00308
00309     _GLUCAT_QD_F(qd_real, tan)
00310
00311
00312     _GLUCAT_QD_F(qd_real, atan)
00313
00314
00315     _GLUCAT_QD_F(qd_real, tanh)
00316
00317 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00318
00319 } // namespace glucat
00320
00321 #endif // _GLUCAT_QD_H

```

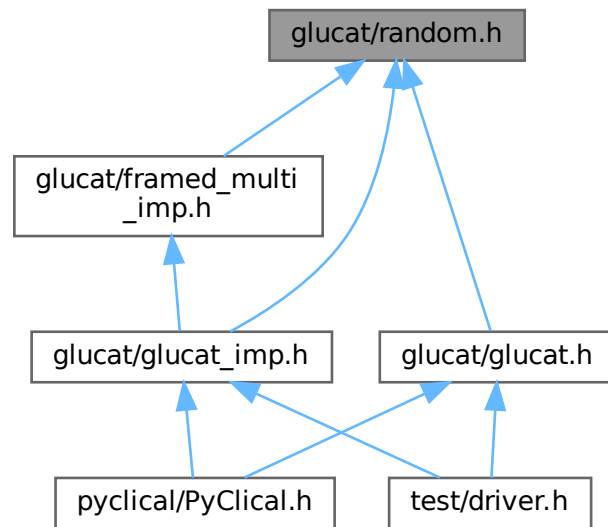
## 7.45 glucat/random.h File Reference

#include <random>

Include dependency graph for random.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::random_generator< Scalar_T >`  
Random number generator with single instance per `Scalar_T`.

## Namespaces

- namespace `glucat`

## 7.46 random.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_RANDOM_H
00002 #define _GLUCAT_RANDOM_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     random.h : Random number generator with single instance per Scalar_T
00006     -----
00007     begin                : 2010-03-28
00008     copyright             : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License

```

```

00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include <random>
00035
00036 namespace glucat
00037 {
00038     // Enforce singleton
00039     // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00040     template< typename Scalar_T >
00041     class random_generator
00042     {
00043     private:
00044         friend class friend_for_private_destructor;
00045     public:
00051         static auto generator() -> random_generator& { static random_generator g; return g;}
00052         random_generator(const random_generator&) = delete;
00053         auto operator= (const random_generator&) -> random_generator& = delete;
00054     private:
00055         static const unsigned long seed = 19590921UL;
00056
00057         std::mt19937 uint_gen;
00058         std::uniform_real_distribution<double> uniform_dist;
00059         std::normal_distribution<double> normal_dist;
00060
00061         random_generator() :
00062             uint_gen(), uniform_dist(0.0, 1.0), normal_dist(0.0, 1.0)
00063         { this->uint_gen.seed(seed); }
00064
00065         ~random_generator() = default;
00066
00067     public:
00068         auto uniform() -> Scalar_T
00069         { return Scalar_T(this->uniform_dist(this->uint_gen)); }
00070         auto normal() -> Scalar_T
00071         { return Scalar_T(this->normal_dist(this->uint_gen)); }
00072     };
00073 }
00074
00075 #endif // _GLUCAT_RANDOM_H

```

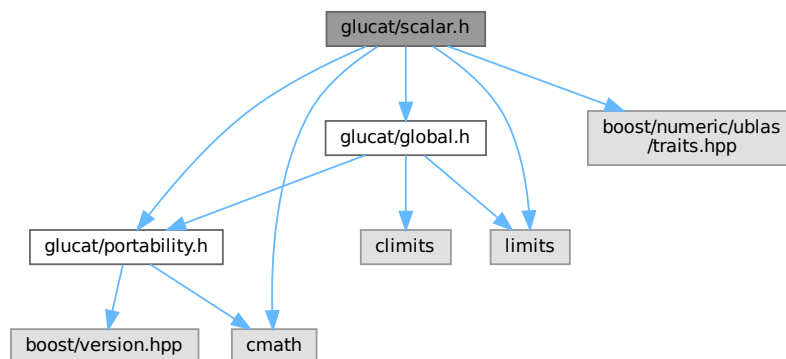
## 7.47 glucat/scalar.h File Reference

```

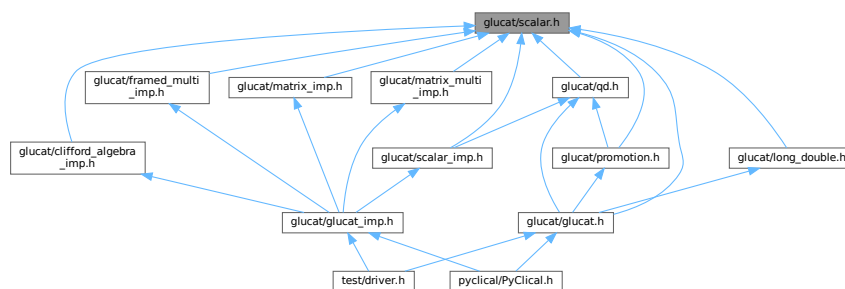
#include "glucat/portability.h"
#include "glucat/global.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>

```

Include dependency graph for scalar.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class [glucat::numeric\\_traits< Scalar\\_T >](#)  
*Extra traits which extend numeric limits.*
- struct [glucat::numeric\\_traits< Scalar\\_T >::promoted](#)  
*Extra traits which extend numeric limits.*
- struct [glucat::numeric\\_traits< Scalar\\_T >::demoted](#)  
*Demoted type for long double.*

## Namespaces

- namespace [glucat](#)

## Functions

- template<typename Scalar\_T>  
auto [glucat::log2](#) (const Scalar\_T &x) -> Scalar\_T  
*Log base 2 of scalar.*

## 7.48 scalar.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_SCALAR_H
00002 #define _GLUCAT_SCALAR_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     scalar.h : Define functions for scalar_t
00006     -----
00007     begin                : 2001-12-20
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009 *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****/
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/portability.h"
00035 #include "glucat/global.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
00041
00042 namespace glucat
00043 {
00044     // Reference: [AA], 2.4, p. 30-31
00045     template< typename Scalar_T >
00046     class numeric_traits
00047     {
00048     private:
00049         inline
00050         static
00051         auto
00052         isInf(const Scalar_T& val, bool_to_type<false>) -> bool
00053         { return false; }
00054
00055         inline
00056         static
00057         auto
00058         isInf(const Scalar_T& val, bool_to_type<true>) -> bool
00059         { return _GLUCAT_ISINF(val); }
00060
00061         inline
00062         static
00063         auto
00064         isNaN(const Scalar_T& val, bool_to_type<false>) -> bool
00065         { return false; }
00066
00067         inline
00068         static
00069         auto
00070         isNaN(const Scalar_T& val, bool_to_type<true>) -> bool
00071         { return _GLUCAT_ISNAN(val); }
00072
00073     public:
00074         inline
00075         static
00076         auto
00077         isInf(const Scalar_T& val) -> bool
00078         {
00079             return isInf(val,
00080                 bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >() );
00081         }
00082     }
00083
00084 }

```



```

00090     inline
00091     static
00092     auto
00093     isNaN(const Scalar_T& val) -> bool
00094     {
00095         return isNaN(val,
00096             bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >() );
00097     }
00098
00100     inline
00101     static
00102     auto
00103     isNaN_or_isInf(const Scalar_T& val) -> bool
00104     {
00105         return isNaN(val,
00106             bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >() )
00107             || isInf(val,
00108                 bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >() );
00109     }
00110
00112     inline
00113     static
00114     auto
00115     NaN() -> Scalar_T
00116     {
00117         return std::numeric_limits<Scalar_T>::has_quiet_NaN
00118             ? std::numeric_limits<Scalar_T>::quiet_NaN()
00119             : Scalar_T(std::log(0.0));
00120     }
00121
00123     inline
00124     static
00125     auto
00126     to_int(const Scalar_T& val) -> int
00127     { return static_cast<int>(val); }
00128
00130     inline
00131     static
00132     auto
00133     to_double(const Scalar_T& val) -> double
00134     { return static_cast<double>(val); }
00135
00137     template <typename Other_Scalar_T >
00138     inline
00139     static
00140     auto
00141     to_scalar_t(const Other_Scalar_T& val) -> Scalar_T
00142     { return static_cast<Scalar_T>(val); }
00143
00145     struct promoted {using type = double;};
00146
00148     struct demoted {using type = float;};
00149
00151     inline
00152     static
00153     auto
00154     fmod(const Scalar_T& lhs, const Scalar_T& rhs) -> Scalar_T
00155     { return std::fmod(lhs, rhs); }
00156
00158     inline
00159     static
00160     auto
00161     conj(const Scalar_T& val) -> Scalar_T
00162     { return val; }
00163
00165     inline
00166     static
00167     auto
00168     real(const Scalar_T& val) -> Scalar_T
00169     { return val; }
00170
00172     inline
00173     static
00174     auto
00175     imag(const Scalar_T& val) -> Scalar_T
00176     { return Scalar_T(0); }
00177
00179     inline
00180     static
00181     auto
00182     abs(const Scalar_T& val) -> Scalar_T
00183     { return boost::numeric::ublas::type_traits<Scalar_T>::UBLAS_ABS(val); }
00184
00186     inline
00187     static
00188     auto
00189     pi() -> Scalar_T

```

```

00190     { return Scalar_T(3.14159265358979323); }
00191
00192     inline
00193     static
00194     auto
00195     ln_2() -> Scalar_T
00196     { return Scalar_T(0.693147180559945309); }
00197
00198     inline
00199     static
00200     auto
00201     pow(const Scalar_T& val, int n) -> Scalar_T
00202     { return std::pow(val, n); }
00203
00204     inline
00205     static
00206     auto
00207     sqrt(const Scalar_T& val) -> Scalar_T
00208     { return boost::numeric::ublas::type_traits<Scalar_T>::UBLAS_SQRT(val); }
00209
00210     inline
00211     static
00212     auto
00213     exp(const Scalar_T& val) -> Scalar_T
00214     { return std::exp(val); }
00215
00216     inline
00217     static
00218     auto
00219     log(const Scalar_T& val) -> Scalar_T
00220     { return std::log(val); }
00221
00222     inline
00223     static
00224     auto
00225     log2(const Scalar_T& val) -> Scalar_T
00226     { return log(val)/ln_2(); }
00227
00228     inline
00229     static
00230     auto
00231     cos(const Scalar_T& val) -> Scalar_T
00232     { return std::cos(val); }
00233
00234     inline
00235     static
00236     auto
00237     acos(const Scalar_T& val) -> Scalar_T
00238     { return std::acos(val); }
00239
00240     inline
00241     static
00242     auto
00243     cosh(const Scalar_T& val) -> Scalar_T
00244     { return std::cosh(val); }
00245
00246     inline
00247     static
00248     auto
00249     sinh(const Scalar_T& val) -> Scalar_T
00250     { return std::sinh(val); }
00251
00252     inline
00253     static
00254     auto
00255     sin(const Scalar_T& val) -> Scalar_T
00256     { return std::sin(val); }
00257
00258     inline
00259     static
00260     auto
00261     asin(const Scalar_T& val) -> Scalar_T
00262     { return std::asin(val); }
00263
00264     inline
00265     static
00266     auto
00267     tan(const Scalar_T& val) -> Scalar_T
00268     { return std::tan(val); }
00269
00270     inline
00271     static
00272     auto
00273     atan(const Scalar_T& val) -> Scalar_T
00274     { return std::atan(val); }
00275
00276     inline
00277     static
00278     auto
00279     atan2(const Scalar_T& val1, const Scalar_T& val2) -> Scalar_T
00280     { return std::atan2(val1, val2); }
00281
00282     inline
00283     static
00284     auto
00285     erf(const Scalar_T& val) -> Scalar_T
00286     { return std::erf(val); }
00287
00288     inline
00289     static
00290     auto
00291     erfc(const Scalar_T& val) -> Scalar_T
00292     { return std::erfc(val); }

```

```

00292     static
00293     auto
00294     tanh(const Scalar_T& val) -> Scalar_T
00295     { return std::tanh(val); }
00296
00297 };
00298
00300 template< typename Scalar_T >
00301 inline
00302 auto
00303 log2(const Scalar_T& x) -> Scalar_T
00304 { return numeric_traits<Scalar_T>::log2(x); }
00305 }
00306
00307 #endif // _GLUCAT_SCALAR_H

```

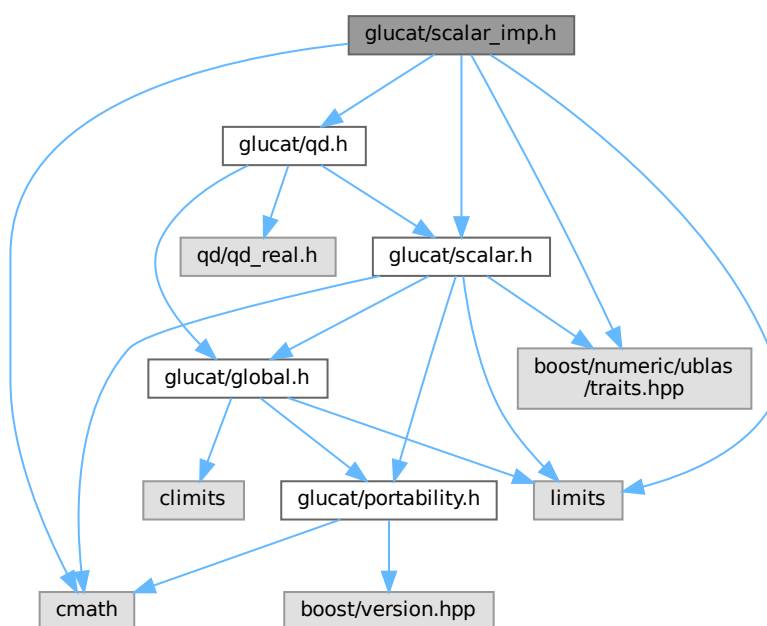
## 7.49 glucat/scalar\_imp.h File Reference

```

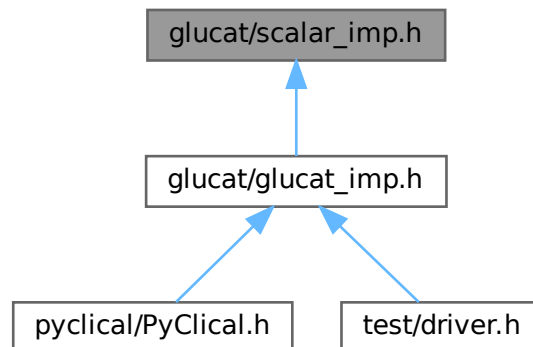
#include "glucat/scalar.h"
#include "glucat/qd.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>

```

Include dependency graph for scalar\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace [glucat](#)

## Functions

- `template<typename Scalar_T>`  
`auto glucat::to\_promote (const Scalar_T &val) -> typename numeric\_traits< Scalar_T >::promoted::type`  
*Cast to promote.*
- `template<typename Scalar_T>`  
`auto glucat::to\_demote (const Scalar_T &val) -> typename numeric\_traits< Scalar_T >::demoted::type`  
*Cast to demote.*

## 7.50 scalar\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_SCALAR_IMP_H
00002 #define _GLUCAT_SCALAR_IMP_H
00003 /*****
00004  GluCat : Generic library of universal Clifford algebra templates
00005  scalar_imp.h : Define functions for scalar_t
00006  -----
00007  begin                : 2001-12-20
00008  copyright            : (C) 2001-2014 by Paul C. Leopardi
00009  *****/
00010
00011  This library is free software: you can redistribute it and/or modify
00012  it under the terms of the GNU Lesser General Public License as published
00013  by the Free Software Foundation, either version 3 of the License, or
00014  (at your option) any later version.
00015
00016  This library is distributed in the hope that it will be useful,
00017  but WITHOUT ANY WARRANTY; without even the implied warranty of
00018  MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019  GNU Lesser General Public License for more details.
00020
00021  You should have received a copy of the GNU Lesser General Public License
00022  along with this library. If not, see <http://www.gnu.org/licenses/>.
00023

```

```

00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/scalar.h"
00035 #include "glucat/qd.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
00041
00042 namespace glucat
00043 {
00044     // Reference: [AA], 2.4, p. 30-31
00045
00046     template< >
00047     template< typename Other_Scalar_T >
00048     inline
00049     auto
00050     numeric_traits<float>::
00051     to_scalar_t(const Other_Scalar_T& val) -> float
00052     { return static_cast<float>(numeric_traits<Other_Scalar_T>::to_double(val)); }
00053
00054     template< >
00055     template< typename Other_Scalar_T >
00056     inline
00057     auto
00058     numeric_traits<double>::
00059     to_scalar_t(const Other_Scalar_T& val) -> double
00060     { return numeric_traits<Other_Scalar_T>::to_double(val); }
00061
00062     #if defined(_GLUCAT_USE_QD)
00063     template< >
00064     template< >
00065     inline
00066     auto
00067     numeric_traits<long double>::
00068     to_scalar_t(const dd_real& val) -> long double
00069     { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00070
00071     template< >
00072     template< >
00073     inline
00074     auto
00075     numeric_traits<long double>::
00076     to_scalar_t(const qd_real& val) -> long double
00077     { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00078
00079     template< >
00080     template< >
00081     inline
00082     auto
00083     numeric_traits<dd_real>::
00084     to_scalar_t(const long double& val) -> dd_real
00085     { return {double(val), double(val - static_cast<long double>(double(val)))}; }
00086
00087     template< >
00088     template< >
00089     inline
00090     auto
00091     numeric_traits<dd_real>::
00092     to_scalar_t(const qd_real& val) -> dd_real
00093     { return {val.x[0], val.x[1]}; }
00094
00095     template< >
00096     template< >
00097     inline
00098     auto
00099     numeric_traits<qd_real>::
00100     to_scalar_t(const long double& val) -> qd_real
00101     { return {double(val), double(val - static_cast<long double>(double(val))), 0.0, 0.0}; }
00102
00103     template< >
00104     template< >
00105     inline
00106     auto
00107     numeric_traits<qd_real>::
00108     to_scalar_t(const dd_real& val) -> qd_real
00109     { return {val.x[0], val.x[1], 0.0, 0.0}; }
00110
00111     #endif
00112 }

```

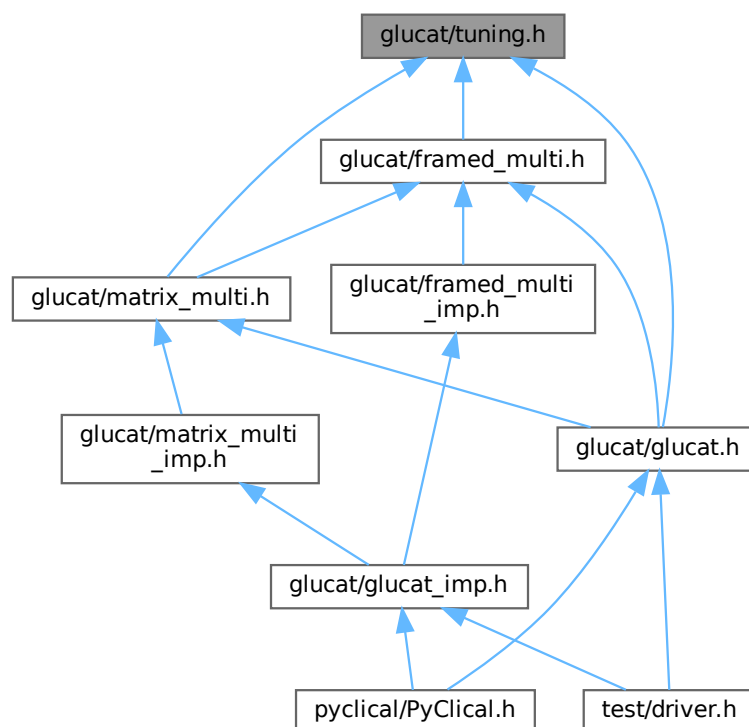
```

00120
00122 template< typename Scalar_T >
00123 inline
00124 auto
00125 to_promote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::promoted::type
00126 {
00127     using promoted_scalar_t = typename numeric_traits<Scalar_T>::promoted::type;
00128     return numeric_traits<promoted_scalar_t>::to_scalar_t(val);
00129 }
00130
00132 template< typename Scalar_T >
00133 inline
00134 auto
00135 to_demote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::demoted::type
00136 {
00137     using demoted_scalar_t = typename numeric_traits<Scalar_T>::demoted::type;
00138     return numeric_traits<demoted_scalar_t>::to_scalar_t(val);
00139 }
00140 }
00141
00142 #endif // _GLUCAT_SCALAR_IMP_H

```

## 7.51 glucat/tuning.h File Reference

This graph shows which files directly or indirectly include this file:



### Functions

- [\\_GLUCAT\\_CTAssert](#) (std::numeric\_limits< unsigned int >::radix==2, CannotSetThresholds) namespace glucat

## 7.51.1 Function Documentation

### 7.51.1.1 `_GLUCAT_CTAssert()`

```
_GLUCAT_CTAssert (
    std::numeric_limits< unsigned int >::radix == 2,
    CannotSetThresholds )
```

Base class for policies

Precision policy

Tuning policy

Minimum index count needed to invoke matrix multiplication algorithm

Maximum steps of iterative refinement in division algorithm

Maximum number of steps in cyclic reduction square root iteration

Maximum number of steps in Denman-Beavers square root iteration

Maximum number of incomplete square roots in cascade log algorithm

Maximum number of steps in incomplete square root within cascade log algorithm

Maximum index count of folded frames in basis cache

Minimum map size needed to invoke generalized FFT

Minimum matrix dimension needed to invoke inverse generalized FFT

Minimum size needed for to invoke faster products algorithms

Denominator of proportion of different bits allowed in approximate equality

Extra number of different bits allowed in approximate equality

Precision used for exp, log and sqrt functions

Definition at line 35 of file [tuning.h](#).

## 7.52 tuning.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TUNING_H
00002 #define GLUCAT_TUNING_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     tuning.h : Policy classes to control tuning
00006
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 // If radix of int is not 2, we can't easily set thresholds
00035 _GLUCAT_CTAssert(std::numeric_limits<unsigned int>::radix == 2, CannotSetThresholds)
00036
00037 namespace glucat
00038 {
00039     struct policy{};
00040
00041     enum precision_t
00042     {
00043         precision_demoted,
00044         precision_same,
00045         precision_promoted
00046     };
00047
00048 // Tuning policy default constants
00049 const unsigned int Tuning_Default_Mult_Matrix_Threshold = 8;
00050 const unsigned int Tuning_Default_Div_Max_Steps = 4;
00051 const unsigned int Tuning_Default_CR_Sqrt_Max_Steps = 256;
00052 const unsigned int Tuning_Default_DB_Sqrt_Max_Steps = 256;
00053 const unsigned int Tuning_Default_Log_Max_Outer_Steps = 256;
00054 const unsigned int Tuning_Default_Log_Max_Inner_Steps = 32;
00055 const unsigned int Tuning_Default_Basis_Max_Count = 12;
00056 const unsigned int Tuning_Default_Fast_Size_Threshold = 1 << 6;
00057 const unsigned int Tuning_Default_Inv_Fast_Dim_Threshold = 1 << 3;
00058 const unsigned int Tuning_Default_Products_Size_Threshold = 1 << 22;
00059 const unsigned int Tuning_Default_Denom_Different_Bits = 8;
00060 const unsigned int Tuning_Default_Extra_Different_Bits = 8;
00061 const precision_t Tuning_Default_Function_Precision = precision_same;
00062
00063 template
00064 <
00065     unsigned int Mult_Matrix_Threshold = Tuning_Default_Mult_Matrix_Threshold,
00066     unsigned int Div_Max_Steps = Tuning_Default_Div_Max_Steps,
00067     unsigned int CR_Sqrt_Max_Steps = Tuning_Default_CR_Sqrt_Max_Steps,
00068     unsigned int DB_Sqrt_Max_Steps = Tuning_Default_DB_Sqrt_Max_Steps,
00069     unsigned int Log_Max_Outer_Steps = Tuning_Default_Log_Max_Outer_Steps,
00070     unsigned int Log_Max_Inner_Steps = Tuning_Default_Log_Max_Inner_Steps,
00071     unsigned int Basis_Max_Count = Tuning_Default_Basis_Max_Count,
00072     unsigned int Fast_Size_Threshold = Tuning_Default_Fast_Size_Threshold,
00073     unsigned int Inv_Fast_Dim_Threshold = Tuning_Default_Inv_Fast_Dim_Threshold,
00074     unsigned int Products_Size_Threshold = Tuning_Default_Products_Size_Threshold,
00075     unsigned int Denom_Different_Bits = Tuning_Default_Denom_Different_Bits,
00076     unsigned int Extra_Different_Bits = Tuning_Default_Extra_Different_Bits,
00077     precision_t Function_Precision = Tuning_Default_Function_Precision
00078 >
00079 struct tuning : policy
00080 {
00081     using tune_p = tuning
00082 <

```



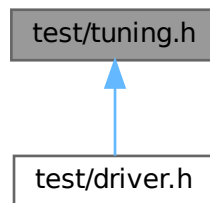
```

00086     Mult_Matrix_Threshold,
00087     Div_Max_Steps,
00088     CR_Sqrt_Max_Steps,
00089     DB_Sqrt_Max_Steps,
00090     Log_Max_Outer_Steps,
00091     Log_Max_Inner_Steps,
00092     Basis_Max_Count,
00093     Fast_Size_Threshold,
00094     Inv_Fast_Dim_Threshold,
00095     Products_Size_Threshold,
00096     Denom_Different_Bits,
00097     Extra_Different_Bits,
00098     Function_Precision
00099 >;
00100 // Tuning for multiplication
00102     enum { mult_matrix_threshold = Mult_Matrix_Threshold };
00103 // Tuning for division
00105     enum { div_max_steps = Div_Max_Steps };
00106 // Tuning for sqrt
00108     enum { cr_sqrt_max_steps = CR_Sqrt_Max_Steps };
00110     enum { db_sqrt_max_steps = DB_Sqrt_Max_Steps };
00111 // Tuning for log
00113     enum { log_max_outer_steps = Log_Max_Outer_Steps };
00115     enum { log_max_inner_steps = Log_Max_Inner_Steps };
00116 // Tuning for basis cache
00118     enum { basis_max_count = Basis_Max_Count };
00119 // Tuning for FFT
00121     enum { fast_size_threshold = Fast_Size_Threshold };
00123     enum { inv_fast_dim_threshold = Inv_Fast_Dim_Threshold };
00124 // Tuning for products (other than geometric product)
00126     enum { products_size_threshold = Products_Size_Threshold };
00127 // Tuning for precision of exp, log and sqrt functions
00129     enum { denom_different_bits = Denom_Different_Bits };
00131     enum { extra_different_bits = Extra_Different_Bits };
00133     static const precision_t function_precision = Function_Precision;
00134 };
00135
00136 using tuning_demoted = tuning
00137 <
00138     Tuning_Default_Mult_Matrix_Threshold,
00139     Tuning_Default_Div_Max_Steps,
00140     Tuning_Default_CR_Sqrt_Max_Steps,
00141     Tuning_Default_DB_Sqrt_Max_Steps,
00142     Tuning_Default_Log_Max_Outer_Steps,
00143     Tuning_Default_Log_Max_Inner_Steps,
00144     Tuning_Default_Basis_Max_Count,
00145     Tuning_Default_Fast_Size_Threshold,
00146     Tuning_Default_Inv_Fast_Dim_Threshold,
00147     Tuning_Default_Products_Size_Threshold,
00148     Tuning_Default_Denom_Different_Bits,
00149     Tuning_Default_Extra_Different_Bits,
00150     precision_demoted
00151 >;
00152
00153 using tuning_promoted = tuning
00154 <
00155     Tuning_Default_Mult_Matrix_Threshold,
00156     Tuning_Default_Div_Max_Steps,
00157     Tuning_Default_CR_Sqrt_Max_Steps,
00158     Tuning_Default_DB_Sqrt_Max_Steps,
00159     Tuning_Default_Log_Max_Outer_Steps,
00160     Tuning_Default_Log_Max_Inner_Steps,
00161     Tuning_Default_Basis_Max_Count,
00162     Tuning_Default_Fast_Size_Threshold,
00163     Tuning_Default_Inv_Fast_Dim_Threshold,
00164     Tuning_Default_Products_Size_Threshold,
00165     Tuning_Default_Denom_Different_Bits,
00166     Tuning_Default_Extra_Different_Bits,
00167     precision_promoted
00168 >;
00169 }
00170
00171 #endif // GLUCAT_TUNING_H

```

## 7.53 test/tuning.h File Reference

This graph shows which files directly or indirectly include this file:



### Namespaces

- namespace [glucat](#)

### Typedefs

- using [glucat::tuning\\_slow](#)
- using [glucat::tuning\\_naive](#)
- using [glucat::tuning\\_fast](#)

### Variables

- const unsigned int [glucat::Tuning\\_Int\\_Digits](#) = std::numeric\_limits<int>::digits
- const unsigned int [glucat::Tuning\\_Max\\_Threshold](#) = 1 << [Tuning\\_Int\\_Digits](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Mult\\_Matrix\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Basis\\_Max\\_Count](#) = 0
- const unsigned int [glucat::Tuning\\_Slow\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Products\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Naive\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Naive\\_Basis\\_Max\\_Count](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Naive\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Naive\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Fast\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Div\\_Max\\_Steps](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_CR\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [glucat::Tuning\\_Fast\\_DB\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [glucat::Tuning\\_Fast\\_Log\\_Max\\_Outer\\_Steps](#) = 16
- const unsigned int [glucat::Tuning\\_Fast\\_Log\\_Max\\_Inner\\_Steps](#) = 8
- const unsigned int [glucat::Tuning\\_Fast\\_Basis\\_Max\\_Count](#) = 1
- const unsigned int [glucat::Tuning\\_Fast\\_Fast\\_Size\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Inv\\_Fast\\_Dim\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Products\\_Size\\_Threshold](#) = 0

## 7.54 tuning.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TEST_TUNING_H
00002 #define GLUCAT_TEST_TUNING_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     tuning.h : Class definitions to control test tuning
00006
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 namespace glucat
00035 {
00036     const unsigned int Tuning_Int_Digits = std::numeric_limits<int>::digits;
00037     const unsigned int Tuning_Max_Threshold = 1 << Tuning_Int_Digits;
00038
00039     // Specific tuning policy constants and tuning policies
00040
00041     const unsigned int Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold;
00042     const unsigned int Tuning_Slow_Basis_Max_Count = 0;
00043     const unsigned int Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold;
00044     const unsigned int Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold;
00045     const unsigned int Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold;
00046
00047     using tuning_slow = tuning
00048     <
00049         Tuning_Slow_Mult_Matrix_Threshold,
00050         Tuning_Default_Div_Max_Steps,
00051         Tuning_Default_CR_Sqrt_Max_Steps,
00052         Tuning_Default_DB_Sqrt_Max_Steps,
00053         Tuning_Default_Log_Max_Outer_Steps,
00054         Tuning_Default_Log_Max_Inner_Steps,
00055         Tuning_Slow_Basis_Max_Count,
00056         Tuning_Slow_Fast_Size_Threshold,
00057         Tuning_Slow_Inv_Fast_Dim_Threshold,
00058         Tuning_Slow_Products_Size_Threshold,
00059         Tuning_Default_Denom_Different_Bits,
00060         Tuning_Default_Extra_Different_Bits,
00061         Tuning_Default_Function_Precision
00062     >;
00063
00064     const unsigned int Tuning_Naive_Mult_Matrix_Threshold = 0;
00065     const unsigned int Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold;
00066     const unsigned int Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold;
00067     const unsigned int Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold;
00068
00069     using tuning_naive = tuning
00070     <
00071         Tuning_Naive_Mult_Matrix_Threshold,
00072         Tuning_Default_Div_Max_Steps,
00073         Tuning_Default_CR_Sqrt_Max_Steps,
00074         Tuning_Default_DB_Sqrt_Max_Steps,
00075         Tuning_Default_Log_Max_Outer_Steps,
00076         Tuning_Default_Log_Max_Inner_Steps,
00077         Tuning_Naive_Basis_Max_Count,
00078         Tuning_Naive_Fast_Size_Threshold,
00079         Tuning_Naive_Inv_Fast_Dim_Threshold,
00080         Tuning_Default_Products_Size_Threshold,
00081         Tuning_Default_Denom_Different_Bits,
00082         Tuning_Default_Extra_Different_Bits,

```

```

00083     Tuning_Default_Function_Precision
00084     >;
00085
00086     const unsigned int Tuning_Fast_Mult_Matrix_Threshold = 0;
00087     const unsigned int Tuning_Fast_Div_Max_Steps = 0;
00088     const unsigned int Tuning_Fast_CR_Sqrt_Max_Steps = 256;
00089     const unsigned int Tuning_Fast_DB_Sqrt_Max_Steps = 256;
00090     const unsigned int Tuning_Fast_Log_Max_Outer_Steps = 16;
00091     const unsigned int Tuning_Fast_Log_Max_Inner_Steps = 8;
00092     const unsigned int Tuning_Fast_Basis_Max_Count = 1;
00093     const unsigned int Tuning_Fast_Fast_Size_Threshold = 0;
00094     const unsigned int Tuning_Fast_Inv_Fast_Dim_Threshold = 0;
00095     const unsigned int Tuning_Fast_Products_Size_Threshold = 0;
00096
00097     using tuning_fast = tuning
00098     <
00099         Tuning_Fast_Mult_Matrix_Threshold,
00100         Tuning_Fast_Div_Max_Steps,
00101         Tuning_Fast_CR_Sqrt_Max_Steps,
00102         Tuning_Fast_DB_Sqrt_Max_Steps,
00103         Tuning_Fast_Log_Max_Outer_Steps,
00104         Tuning_Fast_Log_Max_Inner_Steps,
00105         Tuning_Fast_Basis_Max_Count,
00106         Tuning_Fast_Fast_Size_Threshold,
00107         Tuning_Fast_Inv_Fast_Dim_Threshold,
00108         Tuning_Fast_Products_Size_Threshold,
00109         Tuning_Default_Denom_Different_Bits,
00110         Tuning_Default_Extra_Different_Bits,
00111         Tuning_Default_Function_Precision
00112     >;
00113 }
00114 #endif // GLUCAT_TEST_TUNING_H

```

## 7.55 pyclical/glucat.pxd File Reference

### Namespaces

- namespace [glucat](#)

## 7.56 glucat.pxd

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClical: Python interface to GluCat:
00005 #     Generic library of universal Clifford algebra templates
00006 #
00007 # glucat.pxd: Basic Cython definitions
00008 #     corresponding to C++ definitions from PyClical.h.
00009 # Kept as a separate module from PyClical.pxd to avoid namespace clashes.
00010 #
00011 #     copyright           : (C) 2008-2012 by Paul C. Leopardi
00012 #
00013 #     This library is free software: you can redistribute it and/or modify
00014 #     it under the terms of the GNU Lesser General Public License as published
00015 #     by the Free Software Foundation, either version 3 of the License, or
00016 #     (at your option) any later version.
00017 #
00018 #     This library is distributed in the hope that it will be useful,
00019 #     but WITHOUT ANY WARRANTY; without even the implied warranty of
00020 #     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00021 #     GNU Lesser General Public License for more details.
00022 #
00023 #     You should have received a copy of the GNU Lesser General Public License
00024 #     along with this library. If not, see <http://www.gnu.org/licenses/>.
00025
00026 from libcpp.vector cimport vector
00027
00028 cdef extern from "PyClical.h":
00029
00030     cdef cppclass String:
00031         char* c_str()
00032

```

```

00033     cdef cppclass IndexSet:
00034         IndexSet ()
00035         IndexSet (IndexSet Ist) except+
00036         IndexSet (int idx) except+
00037         IndexSet (char* str) except+
00038         inline bint operator==(IndexSet Rhs)
00039         inline bint operator!=(IndexSet Rhs)
00040         inline bint operator<(IndexSet Rhs)
00041         inline IndexSet invert "operator~"()
00042         inline bint getitem "operator[]"(int idx)
00043         inline IndexSet set()
00044         inline IndexSet set(int idx) except+
00045         inline IndexSet set(int idx, int val) except+
00046         inline IndexSet reset()
00047         inline IndexSet reset(int idx) except+
00048         int count()
00049         int count_pos()
00050         int count_neg()
00051         int min()
00052         int max()
00053         int sign_of_mult(IndexSet Rhs)
00054         int sign_of_square()
00055         int hash_fn()
00056
00057     int compare(IndexSet Lhs, IndexSet Rhs)
00058     int min_neg(IndexSet Ist)
00059     int max_pos(IndexSet Ist)
00060
00061     ctypedef double scalar_t
00062
00063     cdef cppclass Clifford:
00064         Clifford ()
00065         Clifford (Clifford Clf) except+
00066         Clifford (Clifford Clf, IndexSet ist) except+
00067         Clifford (scalar_t scr) except+
00068         Clifford (char* str) except+
00069         Clifford (IndexSet ist, scalar_t scr) except+
00070         Clifford (vector[scalar_t] vec, IndexSet ist) except+
00071         bint operator==(Clifford Rhs)
00072         bint operator!=(Clifford Rhs)
00073         Clifford neg "operator-"()
00074         scalar_t getitem "operator[]"(IndexSet Ist)
00075         Clifford call "operator()"(int grade)
00076         scalar_t scalar()
00077         Clifford pure()
00078         Clifford even()
00079         Clifford odd()
00080         vector[scalar_t] vector_part()
00081         vector[scalar_t] vector_part(IndexSet frm) except+
00082         Clifford involute()
00083         Clifford reverse()
00084         Clifford conj()
00085         Clifford random(IndexSet Ist, scalar_t fill)
00086         scalar_t norm()
00087         scalar_t quad()
00088         IndexSet frame()
00089         scalar_t max_abs()
00090         Clifford inv()
00091         Clifford pow(int m)
00092         Clifford outer_pow(int m)
00093         Clifford truncated(scalar_t limit)
00094         bint isinf()
00095         bint isnan()
00096         void write(char* msg)
00097
00098         scalar_t error_squared_tol(Clifford Clf)
00099         scalar_t error_squared(Clifford Lhs, Clifford Rhs, scalar_t threshold)
00100         bint approx_equal(Clifford Lhs, Clifford Rhs, scalar_t threshold, scalar_t tol)
00101         scalar_t scalar(Clifford Clf)
00102         scalar_t real(Clifford Clf)
00103         scalar_t imag(Clifford Clf)
00104         Clifford pure(Clifford Clf)
00105         Clifford even(Clifford Clf)
00106         Clifford odd(Clifford Clf)
00107         Clifford involute(Clifford Clf)
00108         Clifford reverse(Clifford Clf)
00109         Clifford conj(Clifford Clf)
00110         scalar_t norm(Clifford Clf)
00111         scalar_t abs(Clifford Clf)
00112         scalar_t max_abs(Clifford Clf)
00113         scalar_t quad(Clifford Clf)
00114         Clifford inv(Clifford Clf)
00115         Clifford pow(Clifford Clf, int m)
00116         Clifford outer_pow(Clifford Clf, int m)
00117
00118         Clifford complexifier(Clifford Clf)
00119         Clifford sqrt(Clifford Clf, Clifford I) except+

```

```

00120     Clifford sqrt(Clifford Clf)
00121     Clifford exp(Clifford Clf)
00122     Clifford log(Clifford Clf, Clifford I) except+
00123     Clifford log(Clifford Clf)
00124     Clifford cos(Clifford Clf, Clifford I) except+
00125     Clifford cos(Clifford Clf)
00126     Clifford acos(Clifford Clf, Clifford I) except+
00127     Clifford acos(Clifford Clf)
00128     Clifford cosh(Clifford Clf)
00129     Clifford acosh(Clifford Clf, Clifford I) except+
00130     Clifford acosh(Clifford Clf)
00131     Clifford sin(Clifford Clf, Clifford I) except+
00132     Clifford sin(Clifford Clf)
00133     Clifford asin(Clifford Clf, Clifford I) except+
00134     Clifford asin(Clifford Clf)
00135     Clifford sinh(Clifford Clf)
00136     Clifford asinh(Clifford Clf, Clifford I) except+
00137     Clifford asinh(Clifford Clf)
00138     Clifford tan(Clifford Clf, Clifford I) except+
00139     Clifford tan(Clifford Clf)
00140     Clifford atan(Clifford Clf, Clifford I) except+
00141     Clifford atan(Clifford Clf)
00142     Clifford tanh(Clifford Clf)
00143     Clifford atanh(Clifford Clf, Clifford I) except+
00144     Clifford atanh(Clifford Clf)
00145
00146 cdef extern from "PyClical.h" namespace "cga3":
00147     Clifford agc3(Clifford Clf)
00148     Clifford cga3(Clifford Clf)
00149     Clifford cga3std(Clifford Clf)

```

## 7.57 pyclical/PyClical.h File Reference

```

#include "glucat/glucat_config.h"
#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include <iostream>
#include <sstream>
#include <iomanip>
#include <limits>

```

Include dependency graph for PyClical.h:



### Namespaces

- namespace `cga3`  
*Definitions for 3D Conformal Geometric Algebra [DL].*

### Typedefs

- using `String` = `std::string`
- using `IndexSet` = `index_set<lo_ndx, hi_ndx>`
- using `scalar_t` = `double`
- using `Clifford` = `matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>`

## Functions

- `template<typename Scalar_T>`  
`PyObject * PyFloat_FromDouble (Scalar_T v)`
- `template<typename Index_Set_T>`  
`String index_set_to_repr (const Index_Set_T &ist)`  
*The "official" string representation of Index\_Set\_T ist.*
- `template<typename Index_Set_T>`  
`String index_set_to_str (const Index_Set_T &ist)`  
*The "informal" string representation of Index\_Set\_T ist.*
- `template<typename Multivector_T>`  
`String clifford_to_repr (const Multivector_T &mv)`  
*The "official" string representation of Multivector\_T mv.*
- `template<typename Multivector_T>`  
`String clifford_to_str (const Multivector_T &mv)`  
*The "informal" string representation of Multivector\_T mv.*
- `template<typename Multivector_T>`  
`Multivector_T cga3::cga3 (const Multivector_T &x)`  
*Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].*
- `template<typename Multivector_T>`  
`Multivector_T cga3::cga3std (const Multivector_T &X)`  
*Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].*
- `template<typename Multivector_T>`  
`Multivector_T cga3::agc3 (const Multivector_T &X)`  
*Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].*

## Variables

- `String glucat_package_version = GLUCAT_PACKAGE_VERSION`
- `const index_t lo_ndx = DEFAULT_LO`
- `const index_t hi_ndx = DEFAULT_HI`
- `const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon()`

## 7.57.1 Typedef Documentation

### 7.57.1.1 Clifford

```
using Clifford = matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>
```

Definition at line 148 of file [PyClical.h](#).

### 7.57.1.2 IndexSet

```
using IndexSet = index_set<lo_ndx, hi_ndx>
```

Definition at line 145 of file [PyClical.h](#).

### 7.57.1.3 scalar\_t

```
using scalar_t = double
```

Definition at line 147 of file [PyClical.h](#).

### 7.57.1.4 String

```
using String = std::string
```

Definition at line 51 of file [PyClical.h](#).

## 7.57.2 Function Documentation

### 7.57.2.1 clifford\_to\_repr()

```
template<typename Multivector_T>  
String clifford_to_repr (  
    const Multivector_T & mv) [inline]
```

The “official” string representation of Multivector\_T mv.

Definition at line 75 of file [PyClical.h](#).

Referenced by [PyClical.clifford::\\_\\_repr\\_\\_\(\)](#).

### 7.57.2.2 clifford\_to\_str()

```
template<typename Multivector_T>  
String clifford_to_str (  
    const Multivector_T & mv) [inline]
```

The “informal” string representation of Multivector\_T mv.

Definition at line 86 of file [PyClical.h](#).

References [glucat::abs\(\)](#).

Referenced by [PyClical.clifford::\\_\\_str\\_\\_\(\)](#).

### 7.57.2.3 index\_set\_to\_repr()

```
template<typename Index_Set_T>  
String index_set_to_repr (  
    const Index_Set_T & ist) [inline]
```

The “official” string representation of Index\_Set\_T ist.

Definition at line 57 of file [PyClical.h](#).

Referenced by [PyClical.index\\_set::\\_\\_repr\\_\\_\(\)](#).



#### 7.57.2.4 index\_set\_to\_str()

```
template<typename Index_Set_T>
String index_set_to_str (
    const Index_Set_T & ist) [inline]
```

The "informal" string representation of Index\_Set\_T ist.

Definition at line 66 of file [PyClical.h](#).

Referenced by [PyClical.index\\_set::\\_\\_str\\_\\_\(\)](#).

#### 7.57.2.5 PyFloat\_FromDouble()

```
template<typename Scalar_T>
PyObject * PyFloat_FromDouble (
    Scalar_T v) [inline]
```

Create a PyFloatObject object from Scalar\_T v. Needed because Scalar\_T might not be the same as double.

Definition at line 45 of file [PyClical.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_double\(\)](#).

### 7.57.3 Variable Documentation

#### 7.57.3.1 epsilon

```
const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon()
```

Definition at line 150 of file [PyClical.h](#).

Referenced by [glucat::cascade\\_log\(\)](#), and [glucat::matrix::classify\\_eigenvalues\(\)](#).

#### 7.57.3.2 glucat\_package\_version

```
String glucat_package_version = GLUCAT_PACKAGE_VERSION
```

Definition at line 53 of file [PyClical.h](#).

#### 7.57.3.3 hi\_ndx

```
const index_t hi_ndx = DEFAULT_HI
```

Definition at line 144 of file [PyClical.h](#).

### 7.57.3.4 lo\_ndx

```
const index_t lo_ndx = DEFAULT_LO
```

Definition at line 143 of file [PyClicl.h](#).

## 7.58 PyClicl.h

[Go to the documentation of this file.](#)

```
00001 /*****
00002     GluCat : Generic library of universal Clifford algebra templates
00003     PyClicl.h : C++ definitions needed by PyClicl
00004     -----
00005     copyright          : (C) 2008-2021 by Paul C. Leopardi
00006     *****/
00007
00008     This library is free software: you can redistribute it and/or modify
00009     it under the terms of the GNU Lesser General Public License as published
00010     by the Free Software Foundation, either version 3 of the License, or
00011     (at your option) any later version.
00012
00013     This library is distributed in the hope that it will be useful,
00014     but WITHOUT ANY WARRANTY; without even the implied warranty of
00015     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00016     GNU Lesser General Public License for more details.
00017
00018     You should have received a copy of the GNU Lesser General Public License
00019     along with this library. If not, see <http://www.gnu.org/licenses/>.
00020
00021     *****/
00022     This library is based on a prototype written by Arvind Raja and was
00023     licensed under the LGPL with permission of the author. See Arvind Raja,
00024     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00025     in Ablamowicz, Lounesto and Parra (eds.)
00026     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00027     *****/
00028     See also Arvind Raja's original header comments in glucat/glucat.h
00029     *****/
00030 // References for algorithms:
00031 // [DL]:
00032 // C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00033
00034 #include "glucat/glucat_config.h"
00035 #include "glucat/glucat.h"
00036 #include "glucat/glucat_imp.h"
00037 #include <iostream>
00038 #include <sstream>
00039 #include <iomanip>
00040 #include <limits>
00041
00042 template<typename Scalar_T>
00043 inline PyObject* PyFloat_FromDouble(Scalar_T v)
00044 { return ::PyFloat_FromDouble(glucat::numeric_traits<Scalar_T>::to_double(v)); }
00045
00046 // String representations for use by PyClicl Python classes.
00047
00048 using String = std::string;
00049
00050 String glucat_package_version = GLUCAT_PACKAGE_VERSION;
00051
00052 template<typename Index_Set_T>
00053 inline String index_set_to_repr(const Index_Set_T& ist)
00054 {
00055     std::ostringstream os;
00056     os << "index_set(" << ist << ")";
00057     return os.str();
00058 }
00059
00060 template<typename Index_Set_T>
00061 inline String index_set_to_str(const Index_Set_T& ist)
00062 {
00063     std::ostringstream os;
00064     os << ist;
00065     return os.str();
00066 }
00067
00068 template<typename Multivector_T>
```

```

00075 inline String clifford_to_repr(const Multivector_T& mv)
00076 {
00077     using scalar_t = typename Multivector_T::scalar_t;
00078     std::ostringstream os;
00079     os << std::setprecision(std::numeric_limits<scalar_t>::digits10 + 1);
00080     os << "clifford(\"" << mv << "\")";
00081     return os.str();
00082 }
00083
00085 template<typename Multivector_T>
00086 inline String clifford_to_str(const Multivector_T& mv)
00087 {
00088     using scalar_t = typename Multivector_T::scalar_t;
00089     std::ostringstream os;
00090     if (abs(mv) < std::numeric_limits<scalar_t>::epsilon())
00091         os << 0.0;
00092     else
00093         os << std::setprecision(4) << mv.truncated(scalar_t(1.0e-4));
00094     return os.str();
00095 }
00096
00097 namespace cga3
00098 {
00099     template<typename Multivector_T>
00100     inline Multivector_T cga3(const Multivector_T& x)
00101     {
00102         using cl = Multivector_T;
00103         using ist = typename cl::index_set_t;
00104         static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00105
00106         return (cl(ist(4)) - x) * ninf3 * (x - cl(ist(4)));
00107     }
00108
00109     template<typename Multivector_T>
00110     inline Multivector_T cga3std(const Multivector_T& X)
00111     {
00112         using cl = Multivector_T;
00113         using ist = typename cl::index_set_t;
00114         using scalar_t = typename cl::scalar_t;
00115         static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00116
00117         return scalar_t(-2.0) * X / (X & ninf3);
00118     }
00119
00120     template<typename Multivector_T>
00121     inline Multivector_T agc3(const Multivector_T& X)
00122     {
00123         using cl = Multivector_T;
00124         using ist = typename cl::index_set_t;
00125         using scalar_t = typename cl::scalar_t;
00126
00127         const cl& cga3stdX = cga3std(X);
00128         return (cl(ist(1))*cga3stdX[ist(1)] +
00129                 cl(ist(2))*cga3stdX[ist(2)] +
00130                 cl(ist(3))*cga3stdX[ist(3)]) / scalar_t(2.0);
00131     }
00132 }
00133
00134 // Specifications of the IndexSet and Clifford C++ classes for use with PyClical.
00135 using namespace glucat;
00136 const index_t lo_ndx = DEFAULT_LO;
00137 const index_t hi_ndx = DEFAULT_HI;
00138 using IndexSet = index_set<lo_ndx, hi_ndx>;
00139
00140 using scalar_t = double;
00141 using Clifford = matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>;
00142
00143 const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon();
00144
00145 // Do not warn about unused values. This affects clang++ as well as g++.
00146 #pragma GCC diagnostic ignored "-Wunused-value"
00147
00148 #if defined(__clang__)
00149 // Do not warn about unused functions. The affects clang++ only.
00150 #pragma clang diagnostic ignored "-Wunused-function"
00151
00152 // Do not warn about unneeded internal declarations. The affects clang++ only.
00153 #pragma clang diagnostic ignored "-Wunused-declaration"
00154 #endif

```

## 7.59 pyclical/PyClical.pxd File Reference

### Namespaces

- namespace [PyClical](#)

## 7.60 PyClical.pxd

[Go to the documentation of this file.](#)

```
00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClical: Python interface to GluCat:
00005 #         Generic library of universal Clifford algebra templates
00006 #
00007 # PyClical.pxd: Basic Cython definitions for PyClical
00008 #         corresponding to C++ definitions from PyClical.h.
00009 #
00010 #         copyright          : (C) 2008-2021 by Paul C. Leopardi
00011 #
00012 #         This library is free software: you can redistribute it and/or modify
00013 #         it under the terms of the GNU Lesser General Public License as published
00014 #         by the Free Software Foundation, either version 3 of the License, or
00015 #         (at your option) any later version.
00016 #
00017 #         This library is distributed in the hope that it will be useful,
00018 #         but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 #         MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 #         GNU Lesser General Public License for more details.
00021 #
00022 #         You should have received a copy of the GNU Lesser General Public License
00023 #         along with this library. If not, see <http://www.gnu.org/licenses/>.
00024 #
00025 cimport glucat
00026 from glucat cimport IndexSet, String, Clifford, scalar_t, vector
00027 from libcpp.string cimport string
00028 #
00029 cdef extern from "PyClical.h":
00030     string glucat_package_version
00031 #
00032     IndexSet operator&(IndexSet Lhs, IndexSet Rhs)
00033     IndexSet operator|(IndexSet Lhs, IndexSet Rhs)
00034     IndexSet operator^(IndexSet Lhs, IndexSet Rhs)
00035 #
00036     string index_set_to_repr(IndexSet& Ist)
00037     string index_set_to_str(IndexSet& Ist)
00038 #
00039     Clifford operator+(Clifford Lhs, Clifford Rhs)
00040     Clifford operator-(Clifford Lhs, Clifford Rhs)
00041     Clifford operator*(Clifford Lhs, Clifford Rhs)
00042     Clifford operator&(Clifford Lhs, Clifford Rhs)
00043     Clifford operator%(Clifford Lhs, Clifford Rhs)
00044     Clifford operator^(Clifford Lhs, Clifford Rhs)
00045     Clifford operator/(Clifford Lhs, Clifford Rhs)
00046     Clifford operator|(Clifford Lhs, Clifford Rhs)
00047 #
00048     string clifford_to_repr(Clifford& Clf)
00049     string clifford_to_str(Clifford& Clf)
00050 #
00051     const scalar_t epsilon
```

## 7.61 pyclical/PyClical.pyx File Reference

### Classes

- class [PyClical.index\\_set](#)
- class [PyClical.clifford](#)

## Namespaces

- namespace [PyClical](#)

## Functions

- [PyClical.index\\_set\\_hidden\\_doctests](#) ()
- [PyClical.clifford\\_hidden\\_doctests](#) ()
- [PyClical.e](#) (obj)
- [PyClical.istpq](#) (p, q)
- [PyClical.\\_test](#) ()

## Variables

- [PyClical.\\_\\_version\\_\\_](#) = str([glucat\\_package\\_version](#), 'utf-8')
- [PyClical.lhs](#)
- [PyClical.rhs](#)
- [PyClical.threshold](#) = error\_squared\_tol([rhs](#)) if threshold is [None](#) else threshold
- [PyClical.None](#)
- [PyClical.tol](#) = error\_squared\_tol([rhs](#)) if tol is [None](#) else tol
- [PyClical.obj](#)
- [PyClical.i](#)
- [PyClical.ixt](#)
- [PyClical.fill](#)
- [PyClical.scalar\\_epsilon](#) = [epsilon](#)
- float [PyClical.pi](#) = atan([clifford](#)(1.0)) \* 4.0
- float [PyClical.tau](#) = atan([clifford](#)(1.0)) \* 8.0
- [PyClical.cl](#) = [clifford](#)
- [PyClical.ist](#) = [index\\_set](#)
- [PyClical.ninf3](#) = [e](#)(4) + [e](#)(-1)
- [PyClical.nbar3](#) = [e](#)(4) - [e](#)(-1)

## 7.62 PyClical.pyx

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 # distutils: language = c++
00004 #
00005 # PyClical: Python interface to GluCat:
00006 #         Generic library of universal Clifford algebra templates
00007 #
00008 # PyClical.pyx: Cython definitions visible from Python.
00009 #
00010 #     copyright          : (C) 2008-2021 by Paul C. Leopardi
00011 #
00012 #     This library is free software: you can redistribute it and/or modify
00013 #     it under the terms of the GNU Lesser General Public License as published
00014 #     by the Free Software Foundation, either version 3 of the License, or
00015 #     (at your option) any later version.
00016 #
00017 #     This library is distributed in the hope that it will be useful,
00018 #     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 #     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 #     GNU Lesser General Public License for more details.
00021 #
00022 #     You should have received a copy of the GNU Lesser General Public License
00023 #     along with this library. If not, see <http://www.gnu.org/licenses/>.
00024 #
00025 # References for definitions:

```

```

00026 # [DL]:
00027 # C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00028
00029 import math
00030 import numbers
00031 import collections
00032
00033 from PyClical cimport *
00034
00035 __version__ = str(glucat_package_version, 'utf-8')
00036
00037 # Forward reference
00038 cdef class index_set
00039
00040 cdef inline IndexSet toIndexSet(obj):
00041     """
00042     Return the C++ IndexSet instance wrapped by index_set(obj).
00043     """
00044     return index_set(obj).instance[0]
00045
00046 cdef class index_set:
00047     """
00048     Python class index_set wraps C++ class IndexSet.
00049     """
00050     cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
00051
00052     cdef inline wrap(index_set self, IndexSet other):
00053         """
00054         Wrap an instance of the C++ class IndexSet.
00055         """
00056         self.instance[0] = other
00057         return self
00058
00059     cdef inline IndexSet unwrap(index_set self):
00060         """
00061         Return the wrapped C++ IndexSet instance.
00062         """
00063         return self.instance[0]
00064
00065     cpdef copy(index_set self):
00066         """
00067         Copy this index_set object.
00068
00069         >> s=index_set(1); t=s.copy(); print(t)
00070         {1}
00071         """
00072         return index_set(self)
00073
00074     def __cinit__(self, other = 0):
00075         """
00076         Construct an object of type index_set.
00077
00078         >> print(index_set(1))
00079         {1}
00080         >> print(index_set({1,2}))
00081         {1,2}
00082         >> print(index_set(index_set({1,2})))
00083         {1,2}
00084         >> print(index_set({1,2}))
00085         {1,2}
00086         >> print(index_set({1,2,1}))
00087         {1,2}
00088         >> print(index_set("{1,2,1}"))
00089         {1,2}
00090         >> print(index_set(""))
00091         {}
00092         """
00093         error_msg_prefix = "Cannot initialize index_set object from"
00094         if isinstance(other, index_set):
00095             self.instance = new IndexSet((<index_set>other).unwrap())
00096         elif isinstance(other, numbers.Integral):
00097             self.instance = new IndexSet(<int>other)
00098         elif isinstance(other, (set, frozenset)):
00099             try:
00100                 self.instance = new IndexSet()
00101                 for idx in other:
00102                     self[idx] = True
00103             except IndexError:
00104                 raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
00105             except (RuntimeError, TypeError):
00106                 raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
00107         elif isinstance(other, str):
00108             try:
00109                 bother = other.encode("UTF-8")
00110                 self.instance = new IndexSet(<char *>bother)
00111             except RuntimeError:
00112                 raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")

```

```

00113         else:
00114             raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00115
00116     def __dealloc__(self):
00117         """
00118         Clean up by deallocating the instance of C++ class IndexSet.
00119         """
00120         del self.instance
00121
00122     def __richcmp__(lhs, rhs, int op):
00123         """
00124         Compare two objects of class index_set.
00125
00126         >> index_set(1) == index_set({1})
00127         True
00128         >> index_set({1}) != index_set({1})
00129         False
00130         >> index_set({1}) != index_set({2})
00131         True
00132         >> index_set({1}) == index_set({2})
00133         False
00134         >> index_set({1}) < index_set({2})
00135         True
00136         >> index_set({1}) <= index_set({2})
00137         True
00138         >> index_set({1}) > index_set({2})
00139         False
00140         >> index_set({1}) >= index_set({2})
00141         False
00142         """
00143         if (lhs is None) or (rhs is None):
00144             eq = bool(lhs is rhs)
00145             if op == 2: # ==
00146                 return eq
00147             elif op == 3: # !=
00148                 return not eq
00149             else:
00150                 if op == 0: # <
00151                     return False
00152                 elif op == 1: # <=
00153                     return eq
00154                 elif op == 4: # >
00155                     return False
00156                 elif op == 5: # >=
00157                     return eq
00158                 else:
00159                     return NotImplemented
00160         else:
00161             eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
00162             if op == 2: # ==
00163                 return eq
00164             elif op == 3: # !=
00165                 return not eq
00166             else:
00167                 lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
00168                 if op == 0: # <
00169                     return lt
00170                 elif op == 1: # <=
00171                     return lt or eq
00172                 elif op == 4: # >
00173                     return not (lt or eq)
00174                 elif op == 5: # >=
00175                     return not lt
00176                 else:
00177                     return NotImplemented
00178
00179     def __setitem__(self, idx, val):
00180         """
00181         Set the value of an index_set object at index idx to value val.
00182
00183         >> s=index_set({1}); s[2] = True; print(s)
00184         {1,2}
00185         >> s=index_set({1,2}); s[1] = False; print(s)
00186         {2}
00187         """
00188         self.instance.set(idx, val)
00189         return
00190
00191     def __getitem__(self, idx):
00192         """
00193         Get the value of an index_set object at an index.
00194
00195         >> index_set({1})[1]
00196         True
00197         >> index_set({1})[2]
00198         False
00199         >> index_set({2})[-1]

```

```

00200         False
00201         >> index_set({2})[1]
00202         False
00203         >> index_set({2})[2]
00204         True
00205         >> index_set({2})[33]
00206         False
00207         """
00208         return self.instance.getitem(idx)
00209
00210     def __contains__(self, idx):
00211         """
00212         Check that an index_set object contains the index idx: idx in self.
00213
00214         >> 1 in index_set({1})
00215         True
00216         >> 2 in index_set({1})
00217         False
00218         >> -1 in index_set({2})
00219         False
00220         >> 1 in index_set({2})
00221         False
00222         >> 2 in index_set({2})
00223         True
00224         >> 33 in index_set({2})
00225         False
00226         """
00227         return self.instance.getitem(idx)
00228
00229     def __iter__(self):
00230         """
00231         Iterate over the indices of an index_set.
00232
00233         >> for i in index_set({-3,4,7}):print(i, end=",")
00234         -3,4,7,
00235         """
00236         for idx in range(self.min(), self.max()+1):
00237             if idx in self:
00238                 yield idx
00239
00240     def __invert__(self):
00241         """
00242         Set complement: not.
00243
00244         >>
00245         print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))
00246         {-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32}
00247         """
00248         return index_set().wrap( self.instance.invert() )
00249
00250     def __xor__(lhs, rhs):
00251         """
00252         Symmetric set difference: exclusive or.
00253
00254         >> print(index_set({1}) ^ index_set({2}))
00255         {1,2}
00256         >> print(index_set({1,2}) ^ index_set({2}))
00257         {1}
00258         """
00259         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
00260
00261     def __ixor__(self, rhs):
00262         """
00263         Symmetric set difference: exclusive or.
00264
00265         >> x = index_set({1}); x ^= index_set({2}); print(x)
00266         {1,2}
00267         >> x = index_set({1,2}); x ^= index_set({2}); print(x)
00268         {1}
00269         """
00270         return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
00271
00272     def __and__(lhs, rhs):
00273         """
00274         Set intersection: and.
00275
00276         >> print(index_set({1}) & index_set({2}))
00277         {}
00278         >> print(index_set({1,2}) & index_set({2}))
00279         {2}
00280         """
00281         return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
00282
00283     def __iand__(self, rhs):
00284         """
00285         Set intersection: and.

```



```

00285
00286     >> x = index_set({1}); x &= index_set({2}); print(x)
00287     {}
00288     >> x = index_set({1,2}); x &= index_set({2}); print(x)
00289     {2}
00290     """
00291     return self.wrap( self.unwrap() & toIndexSet(rhs) )
00292
00293 def __or__(lhs, rhs):
00294     """
00295     Set union: or.
00296
00297     >> print(index_set({1}) | index_set({2}))
00298     {1,2}
00299     >> print(index_set({1,2}) | index_set({2}))
00300     {1,2}
00301     """
00302     return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
00303
00304 def __ior__(self, rhs):
00305     """
00306     Set union: or.
00307
00308     >> x = index_set({1}); x |= index_set({2}); print(x)
00309     {1,2}
00310     >> x = index_set({1,2}); x |= index_set({2}); print(x)
00311     {1,2}
00312     """
00313     return self.wrap( self.unwrap() | toIndexSet(rhs) )
00314
00315 def count(self):
00316     """
00317     Cardinality: Number of indices included in set.
00318
00319     >> index_set({-1,1,2}).count()
00320     3
00321     """
00322     return self.instance.count()
00323
00324 def count_neg(self):
00325     """
00326     Number of negative indices included in set.
00327
00328     >> index_set({-1,1,2}).count_neg()
00329     1
00330     """
00331     return self.instance.count_neg()
00332
00333 def count_pos(self):
00334     """
00335     Number of positive indices included in set.
00336
00337     >> index_set({-1,1,2}).count_pos()
00338     2
00339     """
00340     return self.instance.count_pos()
00341
00342 def min(self):
00343     """
00344     Minimum member.
00345
00346     >> index_set({-1,1,2}).min()
00347     -1
00348     """
00349     return self.instance.min()
00350
00351 def max(self):
00352     """
00353     Maximum member.
00354
00355     >> index_set({-1,1,2}).max()
00356     2
00357     """
00358     return self.instance.max()
00359
00360 def hash_fn(self):
00361     """
00362     Hash function.
00363     """
00364     return self.instance.hash_fn()
00365
00366 def sign_of_mult(self, rhs):
00367     """
00368     Sign of geometric product of two Clifford basis elements.
00369
00370     >> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)
00371     1

```

```

00372         """
00373         return self.instance.sign_of_mult(toIndexSet(rhs))
00374
00375     def sign_of_square(self):
00376         """
00377         Sign of geometric square of a Clifford basis element.
00378
00379         >> s = index_set({1,2}); s.sign_of_square()
00380         -1
00381         """
00382         return self.instance.sign_of_square()
00383
00384     def __repr__(self):
00385         """
00386         The "official" string representation of self.
00387
00388         >> index_set({1,2}).__repr__()
00389         'index_set({1,2})'
00390         >> repr(index_set({1,2}))
00391         'index_set({1,2})'
00392         """
00393         return index_set_to_repr( self.unwrap() ).decode()
00394
00395     def __str__(self):
00396         """
00397         The "informal" string representation of self.
00398
00399         >> index_set({1,2}).__str__()
00400         '{1,2}'
00401         >> str(index_set({1,2}))
00402         '{1,2}'
00403         """
00404         return index_set_to_str( self.unwrap() ).decode()
00405
00406     def index_set_hidden_doctests():
00407         """
00408         Tests for functions that Doctest cannot see.
00409
00410         For index_set.__cinit__: Construct index_set.
00411
00412         >> print(index_set(1))
00413         {1}
00414         >> print(index_set({1,2}))
00415         {1,2}
00416         >> print(index_set(index_set({1,2})))
00417         {1,2}
00418         >> print(index_set({1,2}))
00419         {1,2}
00420         >> print(index_set({1,2,1}))
00421         {1,2}
00422         >> print(index_set({1,2,1}))
00423         {1,2}
00424         >> print(index_set(""))
00425         {}
00426         >> print(index_set("{}"))
00427         Traceback (most recent call last):
00428         ...
00429         ValueError: Cannot initialize index_set object from invalid string '{}'.
00430         >> print(index_set("{1}"))
00431         Traceback (most recent call last):
00432         ...
00433         ValueError: Cannot initialize index_set object from invalid string '{1}'.
00434         >> print(index_set("{1,2,100}"))
00435         Traceback (most recent call last):
00436         ...
00437         ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
00438         >> print(index_set({1,2,100}))
00439         Traceback (most recent call last):
00440         ...
00441         IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.
00442         >> print(index_set([1,2]))
00443         Traceback (most recent call last):
00444         ...
00445         TypeError: Cannot initialize index_set object from <class 'list'>.
00446
00447         For index_set.__richcmp__: Compare two objects of class index_set.
00448
00449         >> index_set(1) == index_set({1})
00450         True
00451         >> index_set({1}) != index_set({1})
00452         False
00453         >> index_set({1}) != index_set({2})
00454         True
00455         >> index_set({1}) == index_set({2})
00456         False
00457         >> index_set({1}) < index_set({2})
00458         True

```

```

00459     >> index_set({1}) <= index_set({2})
00460     True
00461     >> index_set({1}) > index_set({2})
00462     False
00463     >> index_set({1}) >= index_set({2})
00464     False
00465     >> None == index_set({1,2})
00466     False
00467     >> None != index_set({1,2})
00468     True
00469     >> None < index_set({1,2})
00470     False
00471     >> None <= index_set({1,2})
00472     False
00473     >> None > index_set({1,2})
00474     False
00475     >> None >= index_set({1,2})
00476     False
00477     >> index_set({1,2}) == None
00478     False
00479     >> index_set({1,2}) != None
00480     True
00481     >> index_set({1,2}) < None
00482     False
00483     >> index_set({1,2}) <= None
00484     False
00485     >> index_set({1,2}) > None
00486     False
00487     >> index_set({1,2}) >= None
00488     False
00489     """
00490     return
00491
00492 cdef inline compare(lhs,rhs):
00493     """
00494     "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
00495     -1 if a<b, +1 if a>b, 0 if a==b.
00496
00497     >> compare(index_set({1,2}),index_set({-1,3}))
00498     -1
00499     >> compare(index_set({-1,4}),index_set({-1,3}))
00500     1
00501     """
00502     return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
00503
00504 cdef inline min_neg(obj):
00505     """
00506     Minimum negative index, or 0 if none.
00507
00508     >> min_neg(index_set({1,2}))
00509     0
00510     """
00511     return glucat.min_neg( toIndexSet(obj) )
00512
00513 cdef inline max_pos(obj):
00514     """
00515     Maximum positive index, or 0 if none.
00516
00517     >> max_pos(index_set({1,2}))
00518     2
00519     """
00520     return glucat.max_pos( toIndexSet(obj) )
00521
00522 cdef inline vector[scalar_t] list_to_vector(lst):
00523     """
00524     Create a C++ std::vector[scalar_t] from an iterable Python object.
00525     """
00526     cdef vector[scalar_t] v
00527     for s in lst:
00528         v.push_back(<scalar_t>s)
00529     return v
00530
00531 # Forward reference.
00532 cdef class clifford
00533
00534 cdef inline Clifford toClifford(obj):
00535     return clifford(obj).instance[0]
00536
00537 cdef class clifford:
00538     """
00539     Python class clifford wraps C++ class Clifford.
00540     """
00541     cdef Clifford *instance # Wrapped instance of C++ class Clifford.
00542
00543     cdef inline wrap(clifford self, Clifford other):
00544         """
00545         Wrap an instance of the C++ class Clifford.

```

```

00546         """
00547         self.instance[0] = other
00548         return self
00549
00550     cdef inline Clifford unwrap(clifford self):
00551         """
00552         Return the wrapped C++ Clifford instance.
00553         """
00554         return self.instance[0]
00555
00556     cpdef copy(clifford self):
00557         """
00558         Copy this clifford object.
00559
00560         >> x=clifford("1{2}"); y=x.copy(); print(y)
00561         {2}
00562         """
00563         return clifford(self)
00564
00565     def __cinit__(self, other = 0, ixt = None):
00566         """
00567         Construct an object of type clifford.
00568
00569         >> print(clifford(2))
00570         2
00571         >> print(clifford(2.0))
00572         2
00573         >> print(clifford(1.0e-1))
00574         0.1
00575         >> print(clifford("2"))
00576         2
00577         >> print(clifford("2{1,2,3}"))
00578         2{1,2,3}
00579         >> print(clifford(clifford("2{1,2,3}")))
00580         2{1,2,3}
00581         >> print(clifford("-{1}"))
00582         -{1}
00583         >> print(clifford(2, index_set({1,2})))
00584         2{1,2}
00585         >> print(clifford([2,3], index_set({1,2})))
00586         2{1}+3{2}
00587         """
00588         error_msg_prefix = "Cannot initialize clifford object from"
00589         if ixt is None:
00590             try:
00591                 if isinstance(other, Clifford):
00592                     self.instance = new Clifford((<clifford>other).unwrap())
00593                 elif isinstance(other, index_set):
00594                     self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
00595                 elif isinstance(other, numbers.Real):
00596                     self.instance = new Clifford(<scalar_t>other)
00597                 elif isinstance(other, str):
00598                     try:
00599                         bother = other.encode("UTF-8")
00600                         self.instance = new Clifford(<char *>bother)
00601                     except RuntimeError:
00602                         raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
00603                 else:
00604                     raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00605             except RuntimeError as err:
00606                 raise ValueError(error_msg_prefix + " " + str(type(other))
00607                                + " value " + repr(other) + ":"
00608                                + "\n\t" + str(err))
00609         elif isinstance(ixt, index_set):
00610             if isinstance(other, numbers.Real):
00611                 self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
00612             elif isinstance(other, collections.abc.Sequence):
00613                 self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
00614             else:
00615                 raise TypeError(error_msg_prefix + " (" + str(type(other))
00616                                + ", " + repr(ixt) + ").")
00617         else:
00618             raise TypeError(error_msg_prefix + " (" + str(type(other))
00619                            + ", " + str(type(ixt)) + ").")
00620
00621     def __dealloc__(self):
00622         """
00623         Clean up by deallocating the instance of C++ class Clifford.
00624         """
00625         del self.instance
00626
00627     def __contains__(self, x):
00628         """
00629         Not applicable.
00630
00631         >> x=clifford(index_set({-3,4,7})); -3 in x
00632         Traceback (most recent call last):

```

```

00633         ...
00634         TypeError: Not applicable.
00635         """
00636         raise TypeError("Not applicable.")
00637
00638     def __iter__(self):
00639         """
00640         Not applicable.
00641
00642         >> for a in clifford(index_set({-3,4,7})):print(a, end=",")
00643         Traceback (most recent call last):
00644             ...
00645         TypeError: Not applicable.
00646         """
00647         raise TypeError("Not applicable.")
00648
00649     def reframe(self, ixt):
00650         """
00651         Put self into a larger frame, containing the union of self.frame() and index set ixt.
00652         This can be used to make multiplication faster, by multiplying within a common frame.
00653
00654         >> clifford("2+3{1}").reframe(index_set({1,2,3}))
00655         clifford("2+3{1}")
00656         >> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() ==
(s|t);
00657         True
00658         """
00659         error_msg_prefix = "Cannot reframe"
00660         if isinstance(ixt, index_set):
00661             try:
00662                 result = clifford()
00663                 result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
00664             except RuntimeError as err:
00665                 raise ValueError(error_msg_prefix + " from " + str(self) + " to frame " +
00666                                 + str(ixt) + ":" +
00667                                 + "\n\t" + str(err))
00668         else:
00669             raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
00670         return result
00671
00672     def __richcmp__(lhs, rhs, int op):
00673         """
00674         Compare objects of type clifford.
00675
00676         >> clifford("{1}") == clifford("1{1}")
00677         True
00678         >> clifford("{1}") != clifford("1.0{1}")
00679         False
00680         >> clifford("{1}") != clifford("1.0")
00681         True
00682         >> clifford("{1,2}") == None
00683         False
00684         >> clifford("{1,2}") != None
00685         True
00686         >> None == clifford("{1,2}")
00687         False
00688         >> None != clifford("{1,2}")
00689         True
00690         """
00691         if op == 2: # ==
00692             if (lhs is None) or (rhs is None):
00693                 return bool(lhs is rhs)
00694             else:
00695                 return bool(toClifford(lhs) == toClifford(rhs))
00696         elif op == 3: # !=
00697             if (lhs is None) or (rhs is None):
00698                 return not bool(lhs is rhs)
00699             else:
00700                 return bool(toClifford(lhs) != toClifford(rhs))
00701         elif isinstance(lhs, Clifford) or isinstance(rhs, Clifford):
00702             raise TypeError("This comparison operator is not implemented for "
00703                             + str(type(lhs)) + ", " + str(type(rhs)) + ".")
00704         else:
00705             return NotImplemented
00706
00707     def __getitem__(self, ixt):
00708         """
00709         Subscripting: map from index set to scalar coordinate.
00710
00711         >> clifford("{1}")[index_set(1)]
00712         1.0
00713         >> clifford("{1}")[index_set({1})]
00714         1.0
00715         >> clifford("{1}")[index_set({1,2})]
00716         0.0
00717         >> clifford("2{1,2}")[index_set({1,2})]
00718         2.0

```

```

00719         """
00720         return self.instance.getitem(toIndexSet(ixt))
00721
00722     def __neg__(self):
00723         """
00724         Unary -.
00725
00726         >> print(-clifford("{1}"))
00727         -{1}
00728         """
00729         return clifford().wrap( self.instance.neg() )
00730
00731     def __pos__(self):
00732         """
00733         Unary +.
00734
00735         >> print(+clifford("{1}"))
00736         {1}
00737         """
00738         return clifford(self)
00739
00740     def __add__(lhs, rhs):
00741         """
00742         Geometric sum.
00743
00744         >> print(clifford(1) + clifford("{2}"))
00745         1+{2}
00746         >> print(clifford("{1}") + clifford("{2}"))
00747         {1}+{2}
00748         """
00749         return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
00750
00751     def __iadd__(self, rhs):
00752         """
00753         Geometric sum.
00754
00755         >> x = clifford(1); x += clifford("{2}"); print(x)
00756         1+{2}
00757         """
00758         return self.wrap( self.unwrap() + toClifford(rhs) )
00759
00760     def __sub__(lhs, rhs):
00761         """
00762         Geometric difference.
00763
00764         >> print(clifford(1) - clifford("{2}"))
00765         1-{2}
00766         >> print(clifford("{1}") - clifford("{2}"))
00767         {1}-{2}
00768         """
00769         return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
00770
00771     def __isub__(self, rhs):
00772         """
00773         Geometric difference.
00774
00775         >> x = clifford(1); x -= clifford("{2}"); print(x)
00776         1-{2}
00777         """
00778         return self.wrap( self.unwrap() - toClifford(rhs) )
00779
00780     def __mul__(lhs, rhs):
00781         """
00782         Geometric product.
00783
00784         >> print(clifford("{1}") * clifford("{2}"))
00785         {1,2}
00786         >> print(clifford(2) * clifford("{2}"))
00787         2{2}
00788         >> print(clifford("{1}") * clifford("{1,2}"))
00789         {2}
00790         """
00791         return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
00792
00793     def __imul__(self, rhs):
00794         """
00795         Geometric product.
00796
00797         >> x = clifford(2); x *= clifford("{2}"); print(x)
00798         2{2}
00799         >> x = clifford("{1}"); x *= clifford("{2}"); print(x)
00800         {1,2}
00801         >> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
00802         {2}
00803         """
00804         return self.wrap( self.unwrap() * toClifford(rhs) )
00805

```

```

00806     def __mod__(lhs, rhs):
00807         """
00808         Contraction.
00809
00810         >> print(clifford("{1}") % clifford("{2}"))
00811         0
00812         >> print(clifford(2) % clifford("{2}"))
00813         2{2}
00814         >> print(clifford("{1}") % clifford("{1}"))
00815         1
00816         >> print(clifford("{1}") % clifford("{1,2}"))
00817         {2}
00818         """
00819         return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
00820
00821     def __imod__(self, rhs):
00822         """
00823         Contraction.
00824
00825         >> x = clifford("{1}"); x %= clifford("{2}"); print(x)
00826         0
00827         >> x = clifford(2); x %= clifford("{2}"); print(x)
00828         2{2}
00829         >> x = clifford("{1}"); x %= clifford("{1}"); print(x)
00830         1
00831         >> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
00832         {2}
00833         """
00834         return self.wrap( self.unwrap() % toClifford(rhs) )
00835
00836     def __and__(lhs, rhs):
00837         """
00838         Inner product.
00839
00840         >> print(clifford("{1}") & clifford("{2}"))
00841         0
00842         >> print(clifford(2) & clifford("{2}"))
00843         0
00844         >> print(clifford("{1}") & clifford("{1}"))
00845         1
00846         >> print(clifford("{1}") & clifford("{1,2}"))
00847         {2}
00848         """
00849         return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
00850
00851     def __iand__(self, rhs):
00852         """
00853         Inner product.
00854
00855         >> x = clifford("{1}"); x &= clifford("{2}"); print(x)
00856         0
00857         >> x = clifford(2); x &= clifford("{2}"); print(x)
00858         0
00859         >> x = clifford("{1}"); x &= clifford("{1}"); print(x)
00860         1
00861         >> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
00862         {2}
00863         """
00864         return self.wrap( self.unwrap() & toClifford(rhs) )
00865
00866     def __xor__(lhs, rhs):
00867         """
00868         Outer product.
00869
00870         >> print(clifford("{1}") ^ clifford("{2}"))
00871         {1,2}
00872         >> print(clifford(2) ^ clifford("{2}"))
00873         2{2}
00874         >> print(clifford("{1}") ^ clifford("{1}"))
00875         0
00876         >> print(clifford("{1}") ^ clifford("{1,2}"))
00877         0
00878         """
00879         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
00880
00881     def __ixor__(self, rhs):
00882         """
00883         Outer product.
00884
00885         >> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
00886         {1,2}
00887         >> x = clifford(2); x ^= clifford("{2}"); print(x)
00888         2{2}
00889         >> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
00890         0
00891         >> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
00892         0

```

```

00893         """
00894         return self.wrap( self.unwrap() ^ toClifford(rhs) )
00895
00896     def __truediv__(lhs, rhs):
00897         """
00898         Geometric quotient.
00899
00900         >> print(clifford("{1}") / clifford("{2}"))
00901         {1,2}
00902         >> print(clifford(2) / clifford("{2}"))
00903         2{2}
00904         >> print(clifford("{1}") / clifford("{1}"))
00905         1
00906         >> print(clifford("{1}") / clifford("{1,2}"))
00907         -{2}
00908         """
00909         return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
00910
00911     def __idiv__(self, rhs):
00912         """
00913         Geometric quotient.
00914
00915         >> x = clifford("{1}"); x /= clifford("{2}"); print(x)
00916         {1,2}
00917         >> x = clifford(2); x /= clifford("{2}"); print(x)
00918         2{2}
00919         >> x = clifford("{1}"); x /= clifford("{1}"); print(x)
00920         1
00921         >> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
00922         -{2}
00923         """
00924         return self.wrap( self.unwrap() / toClifford(rhs) )
00925
00926     def inv(self):
00927         """
00928         Geometric multiplicative inverse.
00929
00930         >> x = clifford("{1}"); print(x.inv())
00931         {1}
00932         >> x = clifford(2); print(x.inv())
00933         0.5
00934         >> x = clifford("{1,2}"); print(x.inv())
00935         -{1,2}
00936         """
00937         return clifford().wrap( self.instance.inv() )
00938
00939     def __or__(lhs, rhs):
00940         """
00941         Transform left hand side, using right hand side as a transformation.
00942
00943         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
00944         -{1}
00945         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
00946         -{1}
00947         """
00948         return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
00949
00950     def __ior__(self, rhs):
00951         """
00952         Transform left hand side, using right hand side as a transformation.
00953
00954         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
00955         -{1}
00956         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
00957         -{1}
00958         """
00959         return self.wrap( self.unwrap() | toClifford(rhs) )
00960
00961     def __pow__(self, m, dummy):
00962         """
00963         Power: self to the m.
00964
00965         >> x=clifford("{1}"); print(x ** 2)
00966         1
00967         >> x=clifford("2"); print(x ** 2)
00968         4
00969         >> x=clifford("2+{1}"); print(x ** 0)
00970         1
00971         >> x=clifford("2+{1}"); print(x ** 1)
00972         2+{1}
00973         >> x=clifford("2+{1}"); print(x ** 2)
00974         5+4{1}
00975         >> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
00976         1
00977         """
00978         return pow(self, m)
00979

```



```

00980     def pow(self, m):
00981         """
00982         Power: self to the m.
00983
00984         >> x=clifford("{1}"); print(x.pow(2))
00985         1
00986         >> x=clifford("2"); print(x.pow(2))
00987         4
00988         >> x=clifford("2+{1}"); print(x.pow(0))
00989         1
00990         >> x=clifford("2+{1}"); print(x.pow(1))
00991         2+{1}
00992         >> x=clifford("2+{1}"); print(x.pow(2))
00993         5+4{1}
00994         >> print(clifford("1+{1}+{1,2}").pow(3))
00995         1+3{1}+3{1,2}
00996         >> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
00997         1
00998         """
00999         if isinstance(m, numbers.Integral):
01000             return clifford().wrap( self.instance.pow(m) )
01001         else:
01002             return exp(m * log(self))
01003
01004     def outer_pow(self, m):
01005         """
01006         Outer product power.
01007
01008         >> x=clifford("2+{1}"); print(x.outer_pow(0))
01009         1
01010         >> x=clifford("2+{1}"); print(x.outer_pow(1))
01011         2+{1}
01012         >> x=clifford("2+{1}"); print(x.outer_pow(2))
01013         4+4{1}
01014         >> print(clifford("1+{1}+{1,2}").outer_pow(3))
01015         1+3{1}+3{1,2}
01016
01017         """
01018         return clifford().wrap( self.instance.outer_pow(m) )
01019
01020     def __call__(self, grade):
01021         """
01022         Pure grade-vector part.
01023
01024         >> print(clifford("{1}") (1))
01025         {1}
01026         >> print(clifford("{1}") (0))
01027         0
01028         >> print(clifford("1+{1}+{1,2}") (0))
01029         1
01030         >> print(clifford("1+{1}+{1,2}") (1))
01031         {1}
01032         >> print(clifford("1+{1}+{1,2}") (2))
01033         {1,2}
01034         >> print(clifford("1+{1}+{1,2}") (3))
01035         0
01036         """
01037         return clifford().wrap( self.instance.call(grade) )
01038
01039     def scalar(self):
01040         """
01041         Scalar part.
01042
01043         >> clifford("1+{1}+{1,2}").scalar()
01044         1.0
01045         >> clifford("{1,2}").scalar()
01046         0.0
01047         """
01048         return self.instance.scalar()
01049
01050     def pure(self):
01051         """
01052         Pure part.
01053
01054         >> print(clifford("1+{1}+{1,2}").pure())
01055         {1}+{1,2}
01056         >> print(clifford("{1,2}").pure())
01057         {1,2}
01058         """
01059         return clifford().wrap( self.instance.pure() )
01060
01061     def even(self):
01062         """
01063         Even part of multivector, sum of even grade terms.
01064
01065         >> print(clifford("1+{1}+{1,2}").even())
01066         1+{1,2}

```

```

01067         """
01068         return clifford().wrap( self.instance.even() )
01069
01070     def odd(self):
01071         """
01072         Odd part of multivector, sum of odd grade terms.
01073
01074         >> print(clifford("1+{1}+{1,2}").odd())
01075         {1}
01076         """
01077         return clifford().wrap( self.instance.odd() )
01078
01079     def vector_part(self, frm = None):
01080         """
01081         Vector part of multivector, as a Python list, with respect to frm.
01082
01083         >> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
01084         [2.0, 3.0]
01085         >> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set((-1,1,2))))
01086         [0.0, 2.0, 3.0]
01087         """
01088         error_msg_prefix = "Cannot take vector part of "
01089         cdef vector[scalar_t] vec
01090         cdef int n
01091         cdef int i
01092         try:
01093             if frm is None:
01094                 vec = self.instance.vector_part()
01095             else:
01096                 vec = self.instance.vector_part((<index_set>frm).unwrap())
01097                 n = vec.size()
01098                 lst = [0.0]*n
01099                 for i in xrange(n):
01100                     lst[i] = vec[i]
01101                 return lst
01102         except RuntimeError as err:
01103             raise ValueError(error_msg_prefix + str(self) + " using invalid "
01104                             + repr(frm) + " as frame:\n\t"
01105                             + str(err))
01106
01107     def involute(self):
01108         """
01109         Main involution, each {i} is replaced by -{i} in each term,
01110         eg. clifford("{1}") -> -clifford("{1}").
01111
01112         >> print(clifford("{1}").involute())
01113         -{1}
01114         >> print((clifford("{2}") * clifford("{1}")).involute())
01115         -{1,2}
01116         >> print((clifford("{1}") * clifford("{2}")).involute())
01117         {1,2}
01118         >> print(clifford("1+{1}+{1,2}").involute())
01119         1-{1}+{1,2}
01120         """
01121         return clifford().wrap( self.instance.involute() )
01122
01123     def reverse(self):
01124         """
01125         Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").
01126
01127         >> print(clifford("{1}").reverse())
01128         {1}
01129         >> print((clifford("{2}") * clifford("{1}")).reverse())
01130         {1,2}
01131         >> print((clifford("{1}") * clifford("{2}")).reverse())
01132         -{1,2}
01133         >> print(clifford("1+{1}+{1,2}").reverse())
01134         1+{1}-{1,2}
01135         """
01136         return clifford().wrap( self.instance.reverse() )
01137
01138     def conj(self):
01139         """
01140         Conjugation, reverse o involute == involute o reverse.
01141
01142         >> print((clifford("{1}")).conj())
01143         -{1}
01144         >> print((clifford("{2}") * clifford("{1}")).conj())
01145         {1,2}
01146         >> print((clifford("{1}") * clifford("{2}")).conj())
01147         -{1,2}
01148         >> print(clifford("1+{1}+{1,2}").conj())
01149         1-{1}-{1,2}
01150         """
01151         return clifford().wrap( self.instance.conj() )
01152
01153     def quad(self):

```

```

01154         """
01155         Quadratic form == (rev(x)*x)(0).
01156
01157         >> print(clifford("1+{1}+{1,2}").quad())
01158         3.0
01159         >> print(clifford("1+{-1}+{1,2}+{1,2,3}").quad())
01160         2.0
01161         """
01162         return self.instance.quad()
01163
01164     def norm(self):
01165         """
01166         Norm == sum of squares of coordinates.
01167
01168         >> clifford("1+{1}+{1,2}").norm()
01169         3.0
01170         >> clifford("1+{-1}+{1,2}+{1,2,3}").norm()
01171         4.0
01172         """
01173         return self.instance.norm()
01174
01175     def abs(self):
01176         """
01177         Absolute value: square root of norm.
01178
01179         >> clifford("1+{-1}+{1,2}+{1,2,3}").abs()
01180         2.0
01181         """
01182         return glucat.abs( self.unwrap() )
01183
01184     def max_abs(self):
01185         """
01186         Maximum of absolute values of components of multivector: multivector infinity norm.
01187
01188         >> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()
01189         1.0
01190         >> clifford("3+2{1}+{1,2}").max_abs()
01191         3.0
01192         """
01193         return self.instance.max_abs()
01194
01195     def truncated(self, limit):
01196         """
01197         Remove all terms of self with relative size smaller than limit.
01198
01199         >> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
01200         clifford("100000000")
01201         >> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
01202         clifford("10000+{1}")
01203         """
01204         return clifford().wrap( self.instance.truncated(limit) )
01205
01206     def isinf(self):
01207         """
01208         Check if a multivector contains any infinite values.
01209
01210         >> clifford().isinf()
01211         False
01212         """
01213         return self.instance.isnan()
01214
01215     def isnan(self):
01216         """
01217         Check if a multivector contains any IEEE NaN values.
01218
01219         >> clifford().isnan()
01220         False
01221         """
01222         return self.instance.isnan()
01223
01224     def frame(self):
01225         """
01226         Subalgebra generated by all generators of terms of given multivector.
01227
01228         >> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
01229         {-2,-1,1,2,7}
01230         >> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
01231         <class 'PyClical.index_set'>
01232         """
01233         return index_set().wrap( self.instance.frame() )
01234
01235     def __repr__(self):
01236         """
01237         The "official" string representation of self.
01238
01239         >> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
01240         'clifford("1+3{-1}+2{1,2}+4{-2,7}")'

```

```

01241         """
01242         return clifford_to_repr( self.unwrap() ).decode()
01243
01244     def __str__(self):
01245         """
01246         The "informal" string representation of self.
01247
01248         >> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
01249         '1+3{-1}+2{1,2}+4{-2,7}'
01250         """
01251         return clifford_to_str( self.unwrap() ).decode()
01252
01253 def clifford_hidden_doctests():
01254     """
01255     Tests for functions that Doctest cannot see.
01256
01257     For clifford.__cinit__: Construct an object of type clifford.
01258
01259     >> print(clifford(2))
01260     2
01261     >> print(clifford(2.0))
01262     2
01263     >> print(clifford(1.0e-1))
01264     0.1
01265     >> print(clifford("2"))
01266     2
01267     >> print(clifford("2{1,2,3}"))
01268     2{1,2,3}
01269     >> print(clifford(clifford("2{1,2,3}")))
01270     2{1,2,3}
01271     >> print(clifford("-{1}"))
01272     -{1}
01273     >> print(clifford(2,index_set({1,2})))
01274     2{1,2}
01275     >> print(clifford([2,3],index_set({1,2})))
01276     2{1}+3{2}
01277     >> print(clifford([1,2]))
01278     Traceback (most recent call last):
01279     ...
01280     TypeError: Cannot initialize clifford object from <class 'list'>.
01281     >> print(clifford(None))
01282     Traceback (most recent call last):
01283     ...
01284     TypeError: Cannot initialize clifford object from <class 'NoneType'>.
01285     >> print(clifford(None,[1,2]))
01286     Traceback (most recent call last):
01287     ...
01288     TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
01289     >> print(clifford([1,2],[1,2]))
01290     Traceback (most recent call last):
01291     ...
01292     TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
01293     >> print(clifford(""))
01294     Traceback (most recent call last):
01295     ...
01296     ValueError: Cannot initialize clifford object from invalid string "".
01297     >> print(clifford("{}"))
01298     Traceback (most recent call last):
01299     ...
01300     ValueError: Cannot initialize clifford object from invalid string '{}'.
01301     >> print(clifford("{1}"))
01302     Traceback (most recent call last):
01303     ...
01304     ValueError: Cannot initialize clifford object from invalid string '{1}'.
01305     >> print(clifford("{+}"))
01306     Traceback (most recent call last):
01307     ...
01308     ValueError: Cannot initialize clifford object from invalid string '{+'.
01309     >> print(clifford("{-}"))
01310     Traceback (most recent call last):
01311     ...
01312     ValueError: Cannot initialize clifford object from invalid string '{-'.
01313     >> print(clifford("{1}+"))
01314     Traceback (most recent call last):
01315     ...
01316     ValueError: Cannot initialize clifford object from invalid string '{1}+'.
01317
01318     For clifford.__richcmp__: Compare objects of type clifford.
01319
01320     >> clifford("{1}") == clifford("1{1}")
01321     True
01322     >> clifford("{1}") != clifford("1.0{1}")
01323     False
01324     >> clifford("{1}") != clifford("1.0")
01325     True
01326     >> clifford("{1,2}") == None
01327     False

```

```

01328     >> clifford("{1,2}") != None
01329     True
01330     >> None == clifford("{1,2}")
01331     False
01332     >> None != clifford("{1,2}")
01333     True
01334     """
01335     return
01336
01337 cpdef inline error_squared_tol(obj):
01338     """
01339     Quadratic norm error tolerance relative to a specific multivector.
01340
01341     >> print(error_squared_tol(clifford("{1}")) * 3.0 - error_squared_tol(clifford("1{1}-2{2}+3{3}")))
01342     0.0
01343     """
01344     return glucat.error_squared_tol(toClifford(obj))
01345
01346 cpdef inline error_squared(lhs, rhs, threshold):
01347     """
01348     Relative or absolute error using the quadratic norm.
01349
01350     >> err2=scalar_epsilon*scalar_epsilon
01351
01352     >> print(error_squared(clifford("{1}"), clifford("1{1}"), err2))
01353     0.0
01354     >> print(error_squared(clifford("1{1}-3{2}+4{3}"), clifford("{1}"), err2))
01355     25.0
01356     """
01357     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
01358
01359 cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):
01360     """
01361     Test for approximate equality of multivectors.
01362
01363     >> err2=scalar_epsilon*scalar_epsilon
01364
01365     >> print(approx_equal(clifford("{1}"), clifford("1{1}")))
01366     True
01367     >> print(approx_equal(clifford("1{1}-3{2}+4{3}"), clifford("{1}")))
01368     False
01369     >> print(approx_equal(clifford("1{1}-3{2}+4{3}+0.001"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01370     False
01371     >> print(approx_equal(clifford("1{1}-3{2}+4{3}+1.0e-30"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01372     True
01373     """
01374     threshold = error_squared_tol(rhs) if threshold is None else threshold
01375     tol = error_squared_tol(rhs) if tol is None else tol
01376     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold, <scalar_t>tol)
01377
01378 cpdef inline inv(obj):
01379     """
01380     Geometric multiplicative inverse.
01381
01382     >> print(inv(clifford("{1}")))
01383     {1}
01384     >> print(inv(clifford("{-1}")))
01385     -{-1}
01386     >> print(inv(clifford("{-2,-1}")))
01387     -{-2,-1}
01388     >> print(inv(clifford("{-1}+{1}")))
01389     nan
01390     """
01391     return clifford(obj).inv()
01392
01393 cpdef inline scalar(obj):
01394     """
01395     Scalar part.
01396
01397     >> scalar(clifford("1+{1}+{1,2}"))
01398     1.0
01399     >> scalar(clifford("{1,2}"))
01400     0.0
01401     """
01402     return clifford(obj).scalar()
01403
01404 cpdef inline real(obj):
01405     """
01406     Real part: synonym for scalar part.
01407
01408     >> real(clifford("1+{1}+{1,2}"))
01409     1.0
01410     >> real(clifford("{1,2}"))
01411     0.0
01412     """
01413     return clifford(obj).scalar()
01414

```

```

01415 cpdef inline imag(obj):
01416     """
01417     Imaginary part: deprecated (always 0).
01418
01419     >> imag(clifford("1+{1}+{1,2}"))
01420     0.0
01421     >> imag(clifford("{1,2}"))
01422     0.0
01423     """
01424     return 0.0
01425
01426 cpdef inline pure(obj):
01427     """
01428     Pure part
01429
01430     >> print(pure(clifford("1+{1}+{1,2}")))
01431     {1}+{1,2}
01432     >> print(pure(clifford("{1,2}")))
01433     {1,2}
01434     """
01435     return clifford(obj).pure()
01436
01437 cpdef inline even(obj):
01438     """
01439     Even part of multivector, sum of even grade terms.
01440
01441     >> print(even(clifford("1+{1}+{1,2}")))
01442     1+{1,2}
01443     """
01444     return clifford(obj).even()
01445
01446 cpdef inline odd(obj):
01447     """
01448     Odd part of multivector, sum of odd grade terms.
01449
01450     >> print(odd(clifford("1+{1}+{1,2}")))
01451     {1}
01452     """
01453     return clifford(obj).odd()
01454
01455 cpdef inline involute(obj):
01456     """
01457     Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
01458
01459     >> print(involute(clifford("{1}")))
01460     -{1}
01461     >> print(involute(clifford("{2}") * clifford("{1}")))
01462     -{1,2}
01463     >> print(involute(clifford("{1}") * clifford("{2}")))
01464     {1,2}
01465     >> print(involute(clifford("1+{1}+{1,2}")))
01466     1-{1}+{1,2}
01467     """
01468     return clifford(obj).involute()
01469
01470 cpdef inline reverse(obj):
01471     """
01472     Reversion, eg. {1}*{2} -> {2}*{1}
01473
01474     >> print(reverse(clifford("{1}")))
01475     {1}
01476     >> print(reverse(clifford("{2}") * clifford("{1}")))
01477     {1,2}
01478     >> print(reverse(clifford("{1}") * clifford("{2}")))
01479     -{1,2}
01480     >> print(reverse(clifford("1+{1}+{1,2}")))
01481     1+{1}-{1,2}
01482     """
01483     return clifford(obj).reverse()
01484
01485 cpdef inline conj(obj):
01486     """
01487     Conjugation, reverse o involute == involute o reverse.
01488
01489     >> print(conj(clifford("{1}")))
01490     -{1}
01491     >> print(conj(clifford("{2}") * clifford("{1}")))
01492     {1,2}
01493     >> print(conj(clifford("{1}") * clifford("{2}")))
01494     -{1,2}
01495     >> print(conj(clifford("1+{1}+{1,2}")))
01496     1-{1}-{1,2}
01497     """
01498     return clifford(obj).conj()
01499
01500 cpdef inline quad(obj):
01501     """

```

```

01502     Quadratic form == (rev(x)*x)(0).
01503
01504     >> print(quad(clifford("1+{1}+{1,2}")))
01505     3.0
01506     >> print(quad(clifford("1+{-1}+{1,2}+{1,2,3}")))
01507     2.0
01508     """
01509     return clifford(obj).quad()
01510
01511 cpdef inline norm(obj):
01512     """
01513     norm == sum of squares of coordinates.
01514
01515     >> norm(clifford("1+{1}+{1,2}"))
01516     3.0
01517     >> norm(clifford("1+{-1}+{1,2}+{1,2,3}"))
01518     4.0
01519     """
01520     return clifford(obj).norm()
01521
01522 cpdef inline abs(obj):
01523     """
01524     Absolute value of multivector: multivector 2-norm.
01525
01526     >> abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01527     2.0
01528     """
01529     return glucat.abs(toClifford(obj))
01530
01531 cpdef inline max_abs(obj):
01532     """
01533     Maximum absolute value of coordinates multivector: multivector infinity-norm.
01534
01535     >> max_abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01536     1.0
01537     >> max_abs(clifford("3+2{1}+{1,2}"))
01538     3.0
01539     """
01540     return glucat.max_abs(toClifford(obj))
01541
01542 cpdef inline pow(obj, m):
01543     """
01544     Integer power of multivector: obj to the m.
01545
01546     >> x=clifford("{1}"); print(pow(x,2))
01547     1
01548     >> x=clifford("2"); print(pow(x,2))
01549     4
01550     >> x=clifford("2+{1}"); print(pow(x,0))
01551     1
01552     >> x=clifford("2+{1}"); print(pow(x,1))
01553     2+{1}
01554     >> x=clifford("2+{1}"); print(pow(x,2))
01555     5+4{1}
01556     >> print(pow(clifford("1+{1}+{1,2}"),3))
01557     1+3{1}+3{1,2}
01558     >> i=clifford("{1,2}"); print(exp(pi/2) * pow(i, i))
01559     1
01560     """
01561     try:
01562         math.pow(obj, m)
01563     except:
01564         return clifford(obj).pow(m)
01565
01566 cpdef inline outer_pow(obj, m):
01567     """
01568     Outer product power of multivector.
01569
01570     >> print(outer_pow(clifford("1+{1}+{1,2}"),3))
01571     1+3{1}+3{1,2}
01572     """
01573     return clifford(obj).outer_pow(m)
01574
01575 cpdef inline complexifier(obj):
01576     """
01577     Square root of -1 which commutes with all members of the frame of the given multivector.
01578
01579     >> print(complexifier(clifford(index_set({1}))))
01580     {1,2,3}
01581     >> print(complexifier(clifford(index_set({-1}))))
01582     {-1}
01583     >> print(complexifier(index_set({1})))
01584     {1,2,3}
01585     >> print(complexifier(index_set({-1})))
01586     {-1}
01587     """
01588

```

```

01589         return clifford().wrap( glucat.complexifier(toClifford(obj)) )
01590
01591 cpdef inline sqrt(obj, i = None):
01592     """
01593     Square root of multivector with optional complexifier.
01594
01595     >> print(sqrt(-1))
01596     {-1}
01597     >> print(sqrt(clifford("2{-1}")))
01598     1+{-1}
01599     >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)
01600     {1,2,3}
01601     -1
01602     >> j=sqrt(-1,"{1,2,3}"); print(j); print(j*j)
01603     {1,2,3}
01604     -1
01605     """
01606     if not (i is None):
01607         return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
01608     else:
01609         try:
01610             return math.sqrt(obj)
01611         except:
01612             return clifford().wrap( glucat.sqrt(toClifford(obj)) )
01613
01614 cpdef inline exp(obj):
01615     """
01616     Exponential of multivector.
01617
01618     >> x=clifford("{1,2}") * pi/4; print(exp(x))
01619     0.7071+0.7071{1,2}
01620     >> x=clifford("{1,2}") * pi/2; print(exp(x))
01621     {1,2}
01622     """
01623     try:
01624         return math.exp(obj)
01625     except:
01626         return clifford().wrap( glucat.exp(toClifford(obj)) )
01627
01628 cpdef inline log(obj,i = None):
01629     """
01630     Natural logarithm of multivector with optional complexifier.
01631
01632     >> x=clifford("{-1}"); print((log(x,"{-1}") * 2/pi))
01633     {-1}
01634     >> x=clifford("{1,2}"); print((log(x,"{1,2,3}") * 2/pi))
01635     {1,2}
01636     >> x=clifford("{1,2}"); print((log(x) * 2/pi))
01637     {1,2}
01638     >> x=clifford("{1,2}"); print((log(x,"{1,2}") * 2/pi))
01639     Traceback (most recent call last):
01640     ...
01641     RuntimeError: check_complex(val, i): i is not a valid complexifier for val
01642     """
01643     if not (i is None):
01644         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
01645     else:
01646         try:
01647             return math.log(obj)
01648         except:
01649             return clifford().wrap( glucat.log(toClifford(obj)) )
01650
01651 cpdef inline cos(obj,i = None):
01652     """
01653     Cosine of multivector with optional complexifier.
01654
01655     >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01656     {1,2}
01657     >> x=clifford("{1,2}"); print(cos(acos(x)))
01658     {1,2}
01659     """
01660     if not (i is None):
01661         return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
01662     else:
01663         try:
01664             return math.cos(obj)
01665         except:
01666             return clifford().wrap( glucat.cos(toClifford(obj)) )
01667
01668 cpdef inline acos(obj,i = None):
01669     """
01670     Inverse cosine of multivector with optional complexifier.
01671
01672     >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01673     {1,2}
01674     >> x=clifford("{1,2}"); print(cos(acos(x),"{-1,1,2,3,4}"))
01675     {1,2}

```



```

01676     >> print(acos(0) / pi)
01677     0.5
01678     >> x=clifford("{1,2}"); print(cos(acos(x)))
01679     {1,2}
01680     """
01681     if not (i is None):
01682         return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
01683     else:
01684         try:
01685             return math.acos(obj)
01686         except:
01687             return clifford().wrap( glucat.acos(toClifford(obj)) )
01688
01689 cpdef inline cosh(obj):
01690     """
01691     Hyperbolic cosine of multivector.
01692
01693     >> x=clifford("{1,2}") * pi; print(cosh(x))
01694     -1
01695     >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01696     {1,2,3}
01697     >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01698     {1,2}
01699     """
01700     try:
01701         return math.cosh(obj)
01702     except:
01703         return clifford().wrap( glucat.cosh(toClifford(obj)) )
01704
01705 cpdef inline acosh(obj,i = None):
01706     """
01707     Inverse hyperbolic cosine of multivector with optional complexifier.
01708
01709     >> print(acosh(0,"{-2,-1,1}"))
01710     1.571{-2,-1,1}
01711     >> x=clifford("{1,2,3}"); print(cosh(acosh(x,"{-1,1,2,3,4}")))
01712     {1,2,3}
01713     >> print(acosh(0))
01714     1.571{-1}
01715     >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01716     {1,2,3}
01717     >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01718     {1,2}
01719     """
01720     if not (i is None):
01721         return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
01722     else:
01723         try:
01724             return math.acosh(obj)
01725         except:
01726             return clifford().wrap( glucat.acosh(toClifford(obj)) )
01727
01728 cpdef inline sin(obj,i = None):
01729     """
01730     Sine of multivector with optional complexifier.
01731
01732     >> s="{1}"; x=clifford(s); print(asin(sin(x,s),s))
01733     {-1}
01734     >> s="{1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01735     {-1}
01736     >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01737     {1,2,3}
01738     """
01739     if not (i is None):
01740         return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
01741     else:
01742         try:
01743             return math.sin(obj)
01744         except:
01745             return clifford().wrap( glucat.sin(toClifford(obj)) )
01746
01747 cpdef inline asin(obj,i = None):
01748     """
01749     Inverse sine of multivector with optional complexifier.
01750
01751     >> s="{1}"; x=clifford(s); print(asin(sin(x,s),s))
01752     {-1}
01753     >> s="{1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01754     {-1}
01755     >> print(asin(1) / pi)
01756     0.5
01757     >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01758     {1,2,3}
01759     """
01760     if not (i is None):
01761         return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
01762     else:

```

```

01763         try:
01764             return math.asin(obj)
01765         except:
01766             return clifford().wrap( glucat.asin(toClifford(obj)) )
01767
01768 cpdef inline sinh(obj):
01769     """
01770     Hyperbolic sine of multivector.
01771
01772     >> x=clifford("{1,2}") * pi/2; print(sinh(x))
01773     {1,2}
01774     >> x=clifford("{1,2}") * pi/6; print(sinh(x))
01775     0.5{1,2}
01776     """
01777     try:
01778         return math.sinh(obj)
01779     except:
01780         return clifford().wrap( glucat.sinh(toClifford(obj)) )
01781
01782 cpdef inline asinh(obj,i = None):
01783     """
01784     Inverse hyperbolic sine of multivector with optional complexifier.
01785
01786     >> x=clifford("{1,2}"); print(asinh(x,"{1,2,3}") * 2/pi)
01787     {1,2}
01788     >> x=clifford("{1,2}"); print(asinh(x) * 2/pi)
01789     {1,2}
01790     >> x=clifford("{1,2}") / 2; print(asinh(x) * 6/pi)
01791     {1,2}
01792     """
01793     if not (i is None):
01794         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
01795     else:
01796         try:
01797             return math.asinh(obj)
01798         except:
01799             return clifford().wrap( glucat.asinh(toClifford(obj)) )
01800
01801 cpdef inline tan(obj,i = None):
01802     """
01803     Tangent of multivector with optional complexifier.
01804
01805     >> x=clifford("{1,2}"); print(tan(x,"{1,2,3}"))
01806     0.7616{1,2}
01807     >> x=clifford("{1,2}"); print(tan(x))
01808     0.7616{1,2}
01809     """
01810     if not (i is None):
01811         return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
01812     else:
01813         try:
01814             return math.tan(obj)
01815         except:
01816             return clifford().wrap( glucat.tan(toClifford(obj)) )
01817
01818 cpdef inline atan(obj,i = None):
01819     """
01820     Inverse tangent of multivector with optional complexifier.
01821
01822     >> s=index_set({1,2,3}); x=clifford("{1}"); print(tan(atan(x,s),s))
01823     {1}
01824     >> x=clifford("{1}"); print(tan(atan(x)))
01825     {1}
01826     """
01827     if not (i is None):
01828         return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
01829     else:
01830         try:
01831             return math.atan(obj)
01832         except:
01833             return clifford().wrap( glucat.atan(toClifford(obj)) )
01834
01835 cpdef inline tanh(obj):
01836     """
01837     Hyperbolic tangent of multivector.
01838
01839     >> x=clifford("{1,2}") * pi/4; print(tanh(x))
01840     {1,2}
01841     """
01842     try:
01843         return math.tanh(obj)
01844     except:
01845         return clifford().wrap( glucat.tanh(toClifford(obj)) )
01846
01847 cpdef inline atanh(obj,i = None):
01848     """
01849     Inverse hyperbolic tangent of multivector with optional complexifier.

```

```

01850
01851     >> s=index_set({1,2,3}); x=clifford("{1,2}"); print(tanh(atanh(x,s)))
01852     {1,2}
01853     >> x=clifford("{1,2}"); print(tanh(atanh(x)))
01854     {1,2}
01855     """
01856     if not (i is None):
01857         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
01858     else:
01859         try:
01860             return math.atanh(obj)
01861         except:
01862             return clifford().wrap( glucat.atanh(toClifford(obj)) )
01863
01864 cpdef inline random_clifford(index_set ixt, fill = 1.0):
01865     """
01866     Random multivector within a frame.
01867
01868     >> print(random_clifford(index_set({-3,-1,2})).frame())
01869     {-3,-1,2}
01870     """
01871     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
01872
01873 cpdef inline cga3(obj):
01874     """
01875     Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
01876     definition.
01877
01878     >> x=clifford("2{1}+9{2}+{3}"); print(cga3(x))
01879     87{-1}+4{1}+18{2}+2{3}+85{4}
01880     """
01881     return clifford().wrap( glucat.cga3(toClifford(obj)) )
01882
01883 cpdef inline cga3std(obj):
01884     """
01885     Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
01886
01887     >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x)))
01888     87{-1}+4{1}+18{2}+2{3}+85{4}
01889     >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x))-cga3(x))
01890     0
01891     """
01892     return clifford().wrap( glucat.cga3std(toClifford(obj)) )
01893
01894 cpdef inline agc3(obj):
01895     """
01896     Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
01897
01898     >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x)))
01899     2{1}+9{2}+{3}
01900     >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x))-x)
01901     0
01902     """
01903     return clifford().wrap( glucat.agc3(toClifford(obj)) )
01904
01905 # Some abbreviations.
01906 scalar_epsilon = epsilon
01907 pi = atan(clifford(1.0)) * 4.0
01908 tau = atan(clifford(1.0)) * 8.0
01909
01910 c1 = clifford
01911 """
01912 Abbreviation for clifford.
01913
01914 >> print(c1(2))
01915 2
01916 >> print(c1(2.0))
01917 2
01918 >> print(c1(5.0e-1))
01919 0.5
01920 >> print(c1("2"))
01921 2
01922 >> print(c1("2{1,2,3}"))
01923 2{1,2,3}
01924 >> print(c1(c1("2{1,2,3}")))
01925 2{1,2,3}
01926 """
01927
01928 ist = index_set
01929 """
01930 Abbreviation for index_set.
01931
01932 >> print(ist("{1,2,3}"))
01933 {1,2,3}
01934 """
01935

```

```

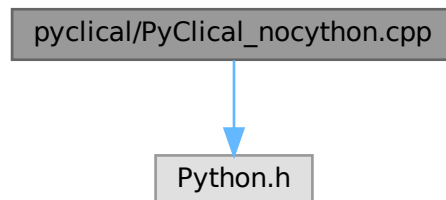
01936 def e(obj):
01937     """
01938     Abbreviation for clifford(index_set(obj)).
01939
01940     >> print(e(1))
01941     {1}
01942     >> print(e(-1))
01943     {-1}
01944     >> print(e(0))
01945     1
01946     """
01947     return clifford(index_set(obj))
01948
01949 def istpq(p, q):
01950     """
01951     Abbreviation for index_set({-q,...p}).
01952
01953     >> print(istpq(2,3))
01954     {-3,-2,-1,1,2}
01955     """
01956     return index_set(set(range(-q,p+1)))
01957
01958 ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
01959 nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
01960
01961 # Doctest interface.
01962 def _test():
01963     import PyClical, doctest
01964     return doctest.testmod(PyClical)
01965
01966 if __name__ == "__main__":
01967     _test()

```

## 7.63 pyclical/PyClical\_nocython.cpp File Reference

#include "Python.h"

Include dependency graph for PyClical\_nocython.cpp:



### Macros

- #define [PY\\_SSIZE\\_T\\_CLEAN](#)

### 7.63.1 Macro Definition Documentation

#### 7.63.1.1 PY\_SSIZE\_T\_CLEAN

```
#define PY_SSIZE_T_CLEAN
```

Definition at line 23 of file [PyClical\\_nocython.cpp](#).

## 7.64 PyClical\_nocython.cpp

[Go to the documentation of this file.](#)

```

00001 /* Generated by Cython 0.29.28 */
00002
00003 /* BEGIN: Cython Metadata
00004 {
00005     "distutils": {
00006         "depends": [
00007             "PyClical.h"
00008         ],
00009         "include_dirs": [
00010             "."
00011         ],
00012         "language": "c++",
00013         "name": "PyClical",
00014         "sources": [
00015             "PyClical.pyx"
00016         ]
00017     },
00018     "module_name": "PyClical"
00019 }
00020 END: Cython Metadata */
00021
00022 #ifndef PY_SSIZE_T_CLEAN
00023 #define PY_SSIZE_T_CLEAN
00024 #endif /* PY_SSIZE_T_CLEAN */
00025 #include "Python.h"
00026 #ifndef Py_PYTHON_H
00027     #error Python headers needed to compile C extensions, please install development version of
    Python.
00028 #elif PY_VERSION_HEX < 0x02060000 || (0x03000000 <= PY_VERSION_HEX && PY_VERSION_HEX < 0x03030000)
00029     #error Cython requires Python 2.6+ or Python 3.3+.
00030 #else
00031 #define CYTHON_ABI "0_29_28"
00032 #define CYTHON_HEX_VERSION 0x001D1CF0
00033 #define CYTHON_FUTURE_DIVISION 1
00034 #include <stddef.h>
00035 #ifndef offsetof
00036     #define offsetof(type, member) ( (size_t) & ((type*)0) -> member )
00037 #endif
00038 #if !defined(WIN32) && !defined(MS_WINDOWS)
00039     #ifndef __stdcall
00040         #define __stdcall
00041     #endif
00042     #ifndef __cdecl
00043         #define __cdecl
00044     #endif
00045     #ifndef __fastcall
00046         #define __fastcall
00047     #endif
00048 #endif
00049 #ifndef DL_IMPORT
00050     #define DL_IMPORT(t) t
00051 #endif
00052 #ifndef DL_EXPORT
00053     #define DL_EXPORT(t) t
00054 #endif
00055 #define __PYX_COMMA ,
00056 #ifndef HAVE_LONG_LONG
00057     #if PY_VERSION_HEX >= 0x02070000
00058         #define HAVE_LONG_LONG
00059     #endif
00060 #endif
00061 #ifndef PY_LONG_LONG
00062     #define PY_LONG_LONG LONG_LONG
00063 #endif
00064 #ifndef Py_HUGE_VAL
00065     #define Py_HUGE_VAL HUGE_VAL
00066 #endif
00067 #ifdef PYPY_VERSION
00068     #define CYTHON_COMPILING_IN_PYPY 1
00069     #define CYTHON_COMPILING_IN_PYSTON 0
00070     #define CYTHON_COMPILING_IN_CPYTHON 0
00071     #undef CYTHON_USE_TYPE_SLOTS
00072     #define CYTHON_USE_TYPE_SLOTS 0
00073     #undef CYTHON_USE_PYTYPE_LOOKUP
00074     #define CYTHON_USE_PYTYPE_LOOKUP 0
00075     #if PY_VERSION_HEX < 0x03050000
00076         #undef CYTHON_USE_ASYNC_SLOTS
00077         #define CYTHON_USE_ASYNC_SLOTS 0
00078     #elif !defined(CYTHON_USE_ASYNC_SLOTS)
00079         #define CYTHON_USE_ASYNC_SLOTS 1
00080     #endif
00081     #undef CYTHON_USE_PYLIST_INTERNALS

```

```

00082 #define CYTHON_USE_PYLIST_INTERNALS 0
00083 #undef CYTHON_USE_UNICODE_INTERNALS
00084 #define CYTHON_USE_UNICODE_INTERNALS 0
00085 #undef CYTHON_USE_UNICODE_WRITER
00086 #define CYTHON_USE_UNICODE_WRITER 0
00087 #undef CYTHON_USE_PYLONG_INTERNALS
00088 #define CYTHON_USE_PYLONG_INTERNALS 0
00089 #undef CYTHON_AVOID_BORROWED_REFS
00090 #define CYTHON_AVOID_BORROWED_REFS 1
00091 #undef CYTHON_ASSUME_SAFE_MACROS
00092 #define CYTHON_ASSUME_SAFE_MACROS 0
00093 #undef CYTHON_UNPACK_METHODS
00094 #define CYTHON_UNPACK_METHODS 0
00095 #undef CYTHON_FAST_THREAD_STATE
00096 #define CYTHON_FAST_THREAD_STATE 0
00097 #undef CYTHON_FAST_PYCALL
00098 #define CYTHON_FAST_PYCALL 0
00099 #undef CYTHON_PEP489_MULTI_PHASE_INIT
00100 #define CYTHON_PEP489_MULTI_PHASE_INIT 0
00101 #undef CYTHON_USE_TP_FINALIZE
00102 #define CYTHON_USE_TP_FINALIZE 0
00103 #undef CYTHON_USE_DICT_VERSIONS
00104 #define CYTHON_USE_DICT_VERSIONS 0
00105 #undef CYTHON_USE_EXC_INFO_STACK
00106 #define CYTHON_USE_EXC_INFO_STACK 0
00107 #elif defined(PYSTON_VERSION)
00108 #define CYTHON_COMPILING_IN_PYPY 0
00109 #define CYTHON_COMPILING_IN_PYSTON 1
00110 #define CYTHON_COMPILING_IN_CPYTHON 0
00111 #ifndef CYTHON_USE_TYPE_SLOTS
00112     #define CYTHON_USE_TYPE_SLOTS 1
00113 #endif
00114 #undef CYTHON_USE_PYTYPE_LOOKUP
00115 #define CYTHON_USE_PYTYPE_LOOKUP 0
00116 #undef CYTHON_USE_ASYNC_SLOTS
00117 #define CYTHON_USE_ASYNC_SLOTS 0
00118 #undef CYTHON_USE_PYLIST_INTERNALS
00119 #define CYTHON_USE_PYLIST_INTERNALS 0
00120 #ifndef CYTHON_USE_UNICODE_INTERNALS
00121     #define CYTHON_USE_UNICODE_INTERNALS 1
00122 #endif
00123 #undef CYTHON_USE_UNICODE_WRITER
00124 #define CYTHON_USE_UNICODE_WRITER 0
00125 #undef CYTHON_USE_PYLONG_INTERNALS
00126 #define CYTHON_USE_PYLONG_INTERNALS 0
00127 #ifndef CYTHON_AVOID_BORROWED_REFS
00128     #define CYTHON_AVOID_BORROWED_REFS 0
00129 #endif
00130 #ifndef CYTHON_ASSUME_SAFE_MACROS
00131     #define CYTHON_ASSUME_SAFE_MACROS 1
00132 #endif
00133 #ifndef CYTHON_UNPACK_METHODS
00134     #define CYTHON_UNPACK_METHODS 1
00135 #endif
00136 #undef CYTHON_FAST_THREAD_STATE
00137 #define CYTHON_FAST_THREAD_STATE 0
00138 #undef CYTHON_FAST_PYCALL
00139 #define CYTHON_FAST_PYCALL 0
00140 #undef CYTHON_PEP489_MULTI_PHASE_INIT
00141 #define CYTHON_PEP489_MULTI_PHASE_INIT 0
00142 #undef CYTHON_USE_TP_FINALIZE
00143 #define CYTHON_USE_TP_FINALIZE 0
00144 #undef CYTHON_USE_DICT_VERSIONS
00145 #define CYTHON_USE_DICT_VERSIONS 0
00146 #undef CYTHON_USE_EXC_INFO_STACK
00147 #define CYTHON_USE_EXC_INFO_STACK 0
00148 #else
00149 #define CYTHON_COMPILING_IN_PYPY 0
00150 #define CYTHON_COMPILING_IN_PYSTON 0
00151 #define CYTHON_COMPILING_IN_CPYTHON 1
00152 #ifndef CYTHON_USE_TYPE_SLOTS
00153     #define CYTHON_USE_TYPE_SLOTS 1
00154 #endif
00155 #if PY_VERSION_HEX < 0x02070000
00156     #undef CYTHON_USE_PYTYPE_LOOKUP
00157     #define CYTHON_USE_PYTYPE_LOOKUP 0
00158 #elif !defined(CYTHON_USE_PYTYPE_LOOKUP)
00159     #define CYTHON_USE_PYTYPE_LOOKUP 1
00160 #endif
00161 #if PY_MAJOR_VERSION < 3
00162     #undef CYTHON_USE_ASYNC_SLOTS
00163     #define CYTHON_USE_ASYNC_SLOTS 0
00164 #elif !defined(CYTHON_USE_ASYNC_SLOTS)
00165     #define CYTHON_USE_ASYNC_SLOTS 1
00166 #endif
00167 #if PY_VERSION_HEX < 0x02070000
00168     #undef CYTHON_USE_PYLONG_INTERNALS

```

```

00169     #define CYTHON_USE_PYLONG_INTERNALS 0
00170 #elif !defined(CYTHON_USE_PYLONG_INTERNALS)
00171     #define CYTHON_USE_PYLONG_INTERNALS 1
00172 #endif
00173 #ifndef CYTHON_USE_PYLIST_INTERNALS
00174     #define CYTHON_USE_PYLIST_INTERNALS 1
00175 #endif
00176 #ifndef CYTHON_USE_UNICODE_INTERNALS
00177     #define CYTHON_USE_UNICODE_INTERNALS 1
00178 #endif
00179 #if PY_VERSION_HEX < 0x030300F0 || PY_VERSION_HEX >= 0x030B00A2
00180     #undef CYTHON_USE_UNICODE_WRITER
00181     #define CYTHON_USE_UNICODE_WRITER 0
00182 #elif !defined(CYTHON_USE_UNICODE_WRITER)
00183     #define CYTHON_USE_UNICODE_WRITER 1
00184 #endif
00185 #ifndef CYTHON_AVOID_BORROWED_REFS
00186     #define CYTHON_AVOID_BORROWED_REFS 0
00187 #endif
00188 #ifndef CYTHON_ASSUME_SAFE_MACROS
00189     #define CYTHON_ASSUME_SAFE_MACROS 1
00190 #endif
00191 #ifndef CYTHON_UNPACK_METHODS
00192     #define CYTHON_UNPACK_METHODS 1
00193 #endif
00194 #if PY_VERSION_HEX >= 0x030B00A4
00195     #undef CYTHON_FAST_THREAD_STATE
00196     #define CYTHON_FAST_THREAD_STATE 0
00197 #elif !defined(CYTHON_FAST_THREAD_STATE)
00198     #define CYTHON_FAST_THREAD_STATE 1
00199 #endif
00200 #ifndef CYTHON_FAST_PYCALL
00201     #define CYTHON_FAST_PYCALL (PY_VERSION_HEX < 0x030B00A1)
00202 #endif
00203 #ifndef CYTHON_PEP489_MULTI_PHASE_INIT
00204     #define CYTHON_PEP489_MULTI_PHASE_INIT (PY_VERSION_HEX >= 0x03050000)
00205 #endif
00206 #ifndef CYTHON_USE_TP_FINALIZE
00207     #define CYTHON_USE_TP_FINALIZE (PY_VERSION_HEX >= 0x030400a1)
00208 #endif
00209 #ifndef CYTHON_USE_DICT_VERSIONS
00210     #define CYTHON_USE_DICT_VERSIONS (PY_VERSION_HEX >= 0x030600B1)
00211 #endif
00212 #if PY_VERSION_HEX >= 0x030B00A4
00213     #undef CYTHON_USE_EXC_INFO_STACK
00214     #define CYTHON_USE_EXC_INFO_STACK 0
00215 #elif !defined(CYTHON_USE_EXC_INFO_STACK)
00216     #define CYTHON_USE_EXC_INFO_STACK (PY_VERSION_HEX >= 0x030700A3)
00217 #endif
00218 #endif
00219 #if !defined(CYTHON_FAST_PYCCALL)
00220 #define CYTHON_FAST_PYCCALL (CYTHON_FAST_PYCALL && PY_VERSION_HEX >= 0x030600B1)
00221 #endif
00222 #if CYTHON_USE_PYLONG_INTERNALS
00223     #if PY_MAJOR_VERSION < 3
00224         #include "longintrepr.h"
00225     #endif
00226     #undef SHIFT
00227     #undef BASE
00228     #undef MASK
00229     #ifdef SIZEOF_VOID_P
00230         enum { __pyx_check_sizeof_voidp = 1 / (int)(sizeof(void*) == sizeof(void*)) };
00231     #endif
00232 #endif
00233 #ifndef __has_attribute
00234     #define __has_attribute(x) 0
00235 #endif
00236 #ifndef __has_cpp_attribute
00237     #define __has_cpp_attribute(x) 0
00238 #endif
00239 #ifndef CYTHON_RESTRICT
00240     #if defined(__GNUC__)
00241         #define CYTHON_RESTRICT __restrict__
00242     #elif defined(_MSC_VER) && _MSC_VER >= 1400
00243         #define CYTHON_RESTRICT __restrict
00244     #elif defined(__STDC_VERSION__) && __STDC_VERSION__ >= 199901L
00245         #define CYTHON_RESTRICT restrict
00246     #else
00247         #define CYTHON_RESTRICT
00248     #endif
00249 #endif
00250 #ifndef CYTHON_UNUSED
00251 # if defined(__GNUC__)
00252 #   if !(defined(__cplusplus)) || (__GNUC__ > 3 || (__GNUC__ == 3 && __GNUC_MINOR__ >= 4))
00253 #     define CYTHON_UNUSED __attribute__((__unused__))
00254 #   else
00255 #     define CYTHON_UNUSED
00256 #   endif
00257 # else
00258 #   define CYTHON_UNUSED
00259 # endif

```

```

00256 # endif
00257 # elif defined(__ICC) || (defined(__INTEL_COMPILER) && !defined(_MSC_VER))
00258 # define CYTHON_UNUSED __attribute__ ((__unused__))
00259 # else
00260 # define CYTHON_UNUSED
00261 # endif
00262 #endif
00263 #ifndef CYTHON_MAYBE_UNUSED_VAR
00264 # if defined(__cplusplus)
00265     template<class T> void CYTHON_MAYBE_UNUSED_VAR( const T& ) { }
00266 # else
00267 # define CYTHON_MAYBE_UNUSED_VAR(x) (void)(x)
00268 # endif
00269 #endif
00270 #ifndef CYTHON_NCP_UNUSED
00271 # if CYTHON_COMPILING_IN_CPYTHON
00272 # define CYTHON_NCP_UNUSED
00273 # else
00274 # define CYTHON_NCP_UNUSED CYTHON_UNUSED
00275 # endif
00276 #endif
00277 #define __Pyx_void_to_None(void_result) ((void)(void_result), Py_INCREF(Py_None), Py_None)
00278 #ifndef _MSC_VER
00279     #ifndef _MSC_STDINT_H_
00280         #if _MSC_VER < 1300
00281             typedef unsigned char    uint8_t;
00282             typedef unsigned int      uint32_t;
00283         #else
00284             typedef unsigned __int8   uint8_t;
00285             typedef unsigned __int32   uint32_t;
00286         #endif
00287     #endif
00288 #else
00289     #include <stdint.h>
00290 #endif
00291 #ifndef CYTHON_FALLTHROUGH
00292     #if defined(__cplusplus) && __cplusplus >= 201103L
00293         #if __has_cpp_attribute(fallthrough)
00294             #define CYTHON_FALLTHROUGH [[fallthrough]]
00295         #elif __has_cpp_attribute(clang::fallthrough)
00296             #define CYTHON_FALLTHROUGH [[clang::fallthrough]]
00297         #elif __has_cpp_attribute(gnu::fallthrough)
00298             #define CYTHON_FALLTHROUGH [[gnu::fallthrough]]
00299         #endif
00300     #endif
00301     #ifndef CYTHON_FALLTHROUGH
00302         #if __has_attribute(fallthrough)
00303             #define CYTHON_FALLTHROUGH __attribute__((fallthrough))
00304         #else
00305             #define CYTHON_FALLTHROUGH
00306         #endif
00307     #endif
00308     #if defined(__clang__) && defined(__apple_build_version__)
00309         #if __apple_build_version__ < 7000000
00310             #undef CYTHON_FALLTHROUGH
00311             #define CYTHON_FALLTHROUGH
00312         #endif
00313     #endif
00314 #endif
00315 #ifndef __cplusplus
00316 #error "Cython files generated with the C++ option must be compiled with a C++ compiler."
00317 #endif
00318 #ifndef CYTHON_INLINE
00319     #if defined(__clang__)
00320         #define CYTHON_INLINE __inline__ __attribute__ ((__unused__))
00321     #else
00322         #define CYTHON_INLINE inline
00323     #endif
00324 #endif
00325 #endif
00326 template<typename T>
00327 void __Pyx_call_destructor(T& x) {
00328     x.~T();
00329 }
00330 template<typename T>
00331 class __Pyx_FakeReference {
00332 public:
00333     __Pyx_FakeReference() : ptr(NULL) { }
00334     __Pyx_FakeReference(const T& ref) : ptr(const_cast<T*>(&ref)) { }
00335     T *operator->() { return ptr; }
00336     T *operator&() { return ptr; }
00337     operator T&() { return *ptr; }
00338     template<typename U> bool operator ==(U other) { return *ptr == other; }
00339     template<typename U> bool operator !=(U other) { return *ptr != other; }
00340 private:
00341     T *ptr;
00342 };

```



```

00343
00344 #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x02070600 && !defined(Py_OptimizeFlag)
00345     #define Py_OptimizeFlag 0
00346 #endif
00347 #define __PYX_BUILD_PY_SSIZE_T "n"
00348 #define CYTHON_FORMAT_SSIZE_T "z"
00349 #if PY_MAJOR_VERSION < 3
00350     #define __Pyx_BUILTIN_MODULE_NAME "__builtin__"
00351     #define __Pyx_PyCode_New(a, k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)\
00352         PyCode_New(a+k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)
00353     #define __Pyx_DefaultClassType PyClass_Type
00354 #else
00355     #define __Pyx_BUILTIN_MODULE_NAME "builtins"
00356     #define __Pyx_DefaultClassType PyType_Type
00357 #if PY_VERSION_HEX >= 0x030B00A1
00358     static CYTHON_INLINE PyCodeObject* __Pyx_PyCode_New(int a, int k, int l, int s, int f,
00359         PyObject *code, PyObject *c, PyObject *n, PyObject
00360         *v,
00361         PyObject *fv, PyObject *cell, PyObject *fn,
00362         PyObject *name, int fline, PyObject *lnos) {
00363         PyObject *kwds=NULL, *argcount=NULL, *posonlyargcount=NULL, *konlyargcount=NULL;
00364         PyObject *nlocals=NULL, *stacksize=NULL, *flags=NULL, *replace=NULL, *call_result=NULL,
00365         *empty=NULL;
00366         const char *fn_cstr=NULL;
00367         const char *name_cstr=NULL;
00368         PyCodeObject* co=NULL;
00369         PyObject *type, *value, *traceback;
00370         PyErr_Fetch(&type, &value, &traceback);
00371         if (!kwds=PyDict_New()) goto end;
00372         if (!(argcount=PyLong_FromLong(a))) goto end;
00373         if (PyDict_SetItemString(kwds, "co_argcount", argcount) != 0) goto end;
00374         if (!(posonlyargcount=PyLong_FromLong(0))) goto end;
00375         if (PyDict_SetItemString(kwds, "co_posonlyargcount", posonlyargcount) != 0) goto end;
00376         if (!(konlyargcount=PyLong_FromLong(k))) goto end;
00377         if (PyDict_SetItemString(kwds, "co_konlyargcount", konlyargcount) != 0) goto end;
00378         if (!(nlocals=PyLong_FromLong(l))) goto end;
00379         if (PyDict_SetItemString(kwds, "co_nlocals", nlocals) != 0) goto end;
00380         if (!(stacksize=PyLong_FromLong(s))) goto end;
00381         if (PyDict_SetItemString(kwds, "co_stacksize", stacksize) != 0) goto end;
00382         if (!(flags=PyLong_FromLong(f))) goto end;
00383         if (PyDict_SetItemString(kwds, "co_flags", flags) != 0) goto end;
00384         if (PyDict_SetItemString(kwds, "co_code", code) != 0) goto end;
00385         if (PyDict_SetItemString(kwds, "co_consts", c) != 0) goto end;
00386         if (PyDict_SetItemString(kwds, "co_names", n) != 0) goto end;
00387         if (PyDict_SetItemString(kwds, "co_varnames", v) != 0) goto end;
00388         if (PyDict_SetItemString(kwds, "co_freevars", fv) != 0) goto end;
00389         if (PyDict_SetItemString(kwds, "co_cellvars", cell) != 0) goto end;
00390         if (PyDict_SetItemString(kwds, "co_linetable", lnos) != 0) goto end;
00391         if (!(fn_cstr=PyUnicode_AsUTF8AndSize(fn, NULL))) goto end;
00392         if (!(name_cstr=PyUnicode_AsUTF8AndSize(name, NULL))) goto end;
00393         if (!(co = PyCode_NewEmpty(fn_cstr, name_cstr, fline))) goto end;
00394         if (!(replace = PyObject_GetAttrString((PyObject*)co, "replace"))) goto cleanup_code_too;
00395         if (!(empty = PyTuple_New(0)) goto cleanup_code_too; // unfortunately __pyx_empty_tuple isn't
00396         available here
00397         if (!(call_result = PyObject_Call(replace, empty, kwds))) goto cleanup_code_too;
00398         Py_XDECREF((PyObject*)co);
00399         co = (PyCodeObject*)call_result;
00400         call_result = NULL;
00401         if (0) {
00402             cleanup_code_too:
00403             Py_XDECREF((PyObject*)co);
00404             co = NULL;
00405         }
00406         end:
00407         Py_XDECREF(kwds);
00408         Py_XDECREF(argcount);
00409         Py_XDECREF(posonlyargcount);
00410         Py_XDECREF(konlyargcount);
00411         Py_XDECREF(nlocals);
00412         Py_XDECREF(stacksize);
00413         Py_XDECREF(replace);
00414         Py_XDECREF(call_result);
00415         Py_XDECREF(empty);
00416         if (type) {
00417             PyErr_Restore(type, value, traceback);
00418         }
00419         return co;
00420     }
00421 #else
00422     #define __Pyx_PyCode_New(a, k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)\
00423         PyCode_New(a, k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)
00424 #endif
00425 #define __Pyx_DefaultClassType PyType_Type
00426 #endif
00427 #ifndef Py_TPFLAGS_CHECKTYPES
00428 #define Py_TPFLAGS_CHECKTYPES 0
00429 #endif

```

```

00427 #ifndef Py_TPFLAGS_HAVE_INDEX
00428     #define Py_TPFLAGS_HAVE_INDEX 0
00429 #endif
00430 #ifndef Py_TPFLAGS_HAVE_NEWBUFFER
00431     #define Py_TPFLAGS_HAVE_NEWBUFFER 0
00432 #endif
00433 #ifndef Py_TPFLAGS_HAVE_FINALIZE
00434     #define Py_TPFLAGS_HAVE_FINALIZE 0
00435 #endif
00436 #ifndef METH_STACKLESS
00437     #define METH_STACKLESS 0
00438 #endif
00439 #if PY_VERSION_HEX <= 0x030700A3 || !defined(METH_FASTCALL)
00440     #ifndef METH_FASTCALL
00441         #define METH_FASTCALL 0x80
00442     #endif
00443     typedef PyObject *(*__Pyx_PyCFunctionFast) (PyObject *self, PyObject *const *args, Py_ssize_t
nargs);
00444     typedef PyObject *(*__Pyx_PyCFunctionFastWithKeywords) (PyObject *self, PyObject *const *args,
Py_ssize_t nargs, PyObject *kwnames);
00445 #else
00446     #define __Pyx_PyCFunctionFast _PyCFunctionFast
00447     #define __Pyx_PyCFunctionFastWithKeywords _PyCFunctionFastWithKeywords
00448 #endif
00449 #if CYTHON_FAST_PYCCALL
00450 #define __Pyx_PyFastCFunction_Check(func) \
00451     ((PyCFunction_Check(func) && (METH_FASTCALL == (PyCFunction_GET_FLAGS(func) & ~(METH_CLASS |
METH_STATIC | METH_COEXIST | METH_KEYWORDS | METH_STACKLESS))))))
00452 #else
00453 #define __Pyx_PyFastCFunction_Check(func) 0
00454 #endif
00455 #if CYTHON_COMPILING_IN_PYPY && !defined(PyObject_Malloc)
00456     #define PyObject_Malloc(s)    PyMem_Malloc(s)
00457     #define PyObject_Free(p)      PyMem_Free(p)
00458     #define PyObject_Realloc(p)   PyMem_Realloc(p)
00459 #endif
00460 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX < 0x030400A1
00461     #define PyMem_RawMalloc(n)     PyMem_Malloc(n)
00462     #define PyMem_RawRealloc(p, n) PyMem_Realloc(p, n)
00463     #define PyMem_RawFree(p)      PyMem_Free(p)
00464 #endif
00465 #if CYTHON_COMPILING_IN_PYSTON
00466     #define __Pyx_PyCode_HasFreeVars(co) PyCode_HasFreeVars(co)
00467     #define __Pyx_PyFrame_SetLineNumber(frame, lineno) PyFrame_SetLineNumber(frame, lineno)
00468 #else
00469     #define __Pyx_PyCode_HasFreeVars(co) (PyCode_GetNumFree(co) > 0)
00470     #define __Pyx_PyFrame_SetLineNumber(frame, lineno) (frame)->f_lineno = (lineno)
00471 #endif
00472 #if !CYTHON_FAST_THREAD_STATE || PY_VERSION_HEX < 0x02070000
00473     #define __Pyx_PyThreadState_Current PyThreadState_GET()
00474     #elif PY_VERSION_HEX >= 0x03060000
00475     #define __Pyx_PyThreadState_Current _PyThreadState_UncheckedGet()
00476     #elif PY_VERSION_HEX >= 0x03000000
00477     #define __Pyx_PyThreadState_Current PyThreadState_GET()
00478     #else
00479     #define __Pyx_PyThreadState_Current _PyThreadState_Current
00480 #endif
00481 #if PY_VERSION_HEX < 0x030700A2 && !defined(PyThread_tss_create) && !defined(Py_tss_NEEDS_INIT)
00482     #include "pythread.h"
00483     #define Py_tss_NEEDS_INIT 0
00484     typedef int Py_tss_t;
00485     static CYTHON_INLINE int PyThread_tss_create(Py_tss_t *key) {
00486         *key = PyThread_create_key();
00487         return 0;
00488     }
00489     static CYTHON_INLINE Py_tss_t * PyThread_tss_alloc(void) {
00490         Py_tss_t *key = (Py_tss_t *)PyObject_Malloc(sizeof(Py_tss_t));
00491         *key = Py_tss_NEEDS_INIT;
00492         return key;
00493     }
00494     static CYTHON_INLINE void PyThread_tss_free(Py_tss_t *key) {
00495         PyObject_Free(key);
00496     }
00497     static CYTHON_INLINE int PyThread_tss_is_created(Py_tss_t *key) {
00498         return *key != Py_tss_NEEDS_INIT;
00499     }
00500     static CYTHON_INLINE void PyThread_tss_delete(Py_tss_t *key) {
00501         PyThread_delete_key(*key);
00502         *key = Py_tss_NEEDS_INIT;
00503     }
00504     static CYTHON_INLINE int PyThread_tss_set(Py_tss_t *key, void *value) {
00505         return PyThread_set_key_value(*key, value);
00506     }
00507     static CYTHON_INLINE void * PyThread_tss_get(Py_tss_t *key) {
00508         return PyThread_get_key_value(*key);
00509     }
00510 #endif
00511 #endif

```

```

00512 #if CYTHON_COMPILING_IN_CPYTHON || defined(_PyDict_NewPresized)
00513 #define __Pyx_PyDict_NewPresized(n) ((n <= 8) ? PyDict_New() : _PyDict_NewPresized(n))
00514 #else
00515 #define __Pyx_PyDict_NewPresized(n) PyDict_New()
00516 #endif
00517 #if PY_MAJOR_VERSION >= 3 || CYTHON_FUTURE_DIVISION
00518 #define __Pyx_PyNumber_Divide(x, y) PyNumber_TrueDivide(x, y)
00519 #define __Pyx_PyNumber_InPlaceDivide(x, y) PyNumber_InPlaceTrueDivide(x, y)
00520 #else
00521 #define __Pyx_PyNumber_Divide(x, y) PyNumber_Divide(x, y)
00522 #define __Pyx_PyNumber_InPlaceDivide(x, y) PyNumber_InPlaceDivide(x, y)
00523 #endif
00524 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x030500A1 && CYTHON_USE_UNICODE_INTERNALS
00525 #define __Pyx_PyDict_GetItemStr(dict, name) _PyDict_GetItem_KnownHash(dict, name, ((PyASCIIObject *)
name)->hash)
00526 #else
00527 #define __Pyx_PyDict_GetItemStr(dict, name) PyDict_GetItem(dict, name)
00528 #endif
00529 #if PY_VERSION_HEX > 0x03030000 && defined(PyUnicode_KIND)
00530 #define CYTHON_PEP393_ENABLED 1
00531 #if defined(PyUnicode_IS_READY)
00532 #define __Pyx_PyUnicode_READY(op) (likely(PyUnicode_IS_READY(op)) ? \
0 : _PyUnicode_Ready((PyObject *) (op)))
00533 #else
00534 #define __Pyx_PyUnicode_READY(op) (0)
00535 #endif
00536 #define __Pyx_PyUnicode_GET_LENGTH(u) PyUnicode_GET_LENGTH(u)
00537 #define __Pyx_PyUnicode_READ_CHAR(u, i) PyUnicode_READ_CHAR(u, i)
00538 #define __Pyx_PyUnicode_MAX_CHAR_VALUE(u) PyUnicode_MAX_CHAR_VALUE(u)
00539 #define __Pyx_PyUnicode_KIND(u) PyUnicode_KIND(u)
00540 #define __Pyx_PyUnicode_DATA(u) PyUnicode_DATA(u)
00541 #define __Pyx_PyUnicode_READ(k, d, i) PyUnicode_READ(k, d, i)
00542 #define __Pyx_PyUnicode_WRITE(k, d, i, ch) PyUnicode_WRITE(k, d, i, ch)
00543 #if defined(PyUnicode_IS_READY) && defined(PyUnicode_GET_SIZE)
00544 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03090000
00545 #define __Pyx_PyUnicode_IS_TRUE(u) (0 != (likely(PyUnicode_IS_READY(u)) ?
PyUnicode_GET_LENGTH(u) : ((PyCompactUnicodeObject *) (u))->wstr_length))
00546 #else
00547 #define __Pyx_PyUnicode_IS_TRUE(u) (0 != (likely(PyUnicode_IS_READY(u)) ?
PyUnicode_GET_LENGTH(u) : PyUnicode_GET_SIZE(u)))
00548 #endif
00549 #else
00550 #define __Pyx_PyUnicode_IS_TRUE(u) (0 != PyUnicode_GET_LENGTH(u))
00551 #endif
00552 #else
00553 #define CYTHON_PEP393_ENABLED 0
00554 #define PyUnicode_1BYTE_KIND 1
00555 #define PyUnicode_2BYTE_KIND 2
00556 #define PyUnicode_4BYTE_KIND 4
00557 #define __Pyx_PyUnicode_READY(op) (0)
00558 #define __Pyx_PyUnicode_GET_LENGTH(u) PyUnicode_GET_SIZE(u)
00559 #define __Pyx_PyUnicode_READ_CHAR(u, i) ((Py_UCS4) (PyUnicode_AS_UNICODE(u)[i]))
00560 #define __Pyx_PyUnicode_MAX_CHAR_VALUE(u) ((sizeof(Py_UNICODE) == 2) ? 65535 : 1114111)
00561 #define __Pyx_PyUnicode_KIND(u) (sizeof(Py_UNICODE))
00562 #define __Pyx_PyUnicode_DATA(u) ((void*) PyUnicode_AS_UNICODE(u))
00563 #define __Pyx_PyUnicode_READ(k, d, i) ((void) (k), (Py_UCS4) ((Py_UNICODE*) d)[i])
00564 #define __Pyx_PyUnicode_WRITE(k, d, i, ch) ((void) (k)), ((Py_UNICODE*) d)[i] = ch
00565 #define __Pyx_PyUnicode_IS_TRUE(u) (0 != PyUnicode_GET_SIZE(u))
00566 #endif
00567 #endif
00568 #if CYTHON_COMPILING_IN_PYPY
00569 #define __Pyx_PyUnicode_Concat(a, b) PyNumber_Add(a, b)
00570 #define __Pyx_PyUnicode_ConcatSafe(a, b) PyNumber_Add(a, b)
00571 #else
00572 #define __Pyx_PyUnicode_Concat(a, b) PyUnicode_Concat(a, b)
00573 #define __Pyx_PyUnicode_ConcatSafe(a, b) ((unlikely((a) == Py_None) || unlikely((b) == Py_None)) ? \
PyNumber_Add(a, b) : __Pyx_PyUnicode_Concat(a, b))
00574 #endif
00575 #if CYTHON_COMPILING_IN_PYPY && !defined(PyUnicode_Contains)
00576 #define PyUnicode_Contains(u, s) PySequence_Contains(u, s)
00577 #endif
00578 #if CYTHON_COMPILING_IN_PYPY && !defined(PyByteArray_Check)
00579 #define PyByteArray_Check(obj) PyObject_TypeCheck(obj, &PyByteArray_Type)
00580 #endif
00581 #if CYTHON_COMPILING_IN_PYPY && !defined(PyObject_Format)
00582 #define PyObject_Format(obj, fmt) PyObject_CallMethod(obj, "__format__", "O", fmt)
00583 #endif
00584 #define __Pyx_PyString_FormatSafe(a, b) ((unlikely((a) == Py_None || (PyString_Check(b) &&
!PyString_CheckExact(b)))) ? PyNumber_Remainder(a, b) : __Pyx_PyString_Format(a, b))
00585 #define __Pyx_PyUnicode_FormatSafe(a, b) ((unlikely((a) == Py_None || (PyUnicode_Check(b) &&
!PyUnicode_CheckExact(b)))) ? PyNumber_Remainder(a, b) : PyUnicode_Format(a, b))
00586 #if PY_MAJOR_VERSION >= 3
00587 #define __Pyx_PyString_Format(a, b) PyUnicode_Format(a, b)
00588 #else
00589 #define __Pyx_PyString_Format(a, b) PyString_Format(a, b)
00590 #endif
00591 #if PY_MAJOR_VERSION < 3 && !defined(PyObject_ASCII)
00592 #define PyObject_ASCII(o) PyObject_Repr(o)
00593 #endif

```

```

00594 #endif
00595 #if PY_MAJOR_VERSION >= 3
00596     #define PyBaseString_Type          PyUnicode_Type
00597     #define PyStringObject              PyUnicodeObject
00598     #define PyString_Type                PyUnicode_Type
00599     #define PyString_Check              PyUnicode_Check
00600     #define PyString_CheckExact         PyUnicode_CheckExact
00601 #ifndef PyObject_Unicode
00602     #define PyObject_Unicode            PyObject_Str
00603 #endif
00604 #endif
00605 #if PY_MAJOR_VERSION >= 3
00606     #define __Pyx_PyBaseString_Check(obj) PyUnicode_Check(obj)
00607     #define __Pyx_PyBaseString_CheckExact(obj) PyUnicode_CheckExact(obj)
00608 #else
00609     #define __Pyx_PyBaseString_Check(obj) (PyString_Check(obj) || PyUnicode_Check(obj))
00610     #define __Pyx_PyBaseString_CheckExact(obj) (PyString_CheckExact(obj) || PyUnicode_CheckExact(obj))
00611 #endif
00612 #ifndef PySet_CheckExact
00613     #define PySet_CheckExact(obj)      (Py_TYPE(obj) == &PySet_Type)
00614 #endif
00615 #if PY_VERSION_HEX >= 0x030900A4
00616     #define __Pyx_SET_REFCNT(obj, refcnt) Py_SET_REFCNT(obj, refcnt)
00617     #define __Pyx_SET_SIZE(obj, size) Py_SET_SIZE(obj, size)
00618 #else
00619     #define __Pyx_SET_REFCNT(obj, refcnt) Py_REFCNT(obj) = (refcnt)
00620     #define __Pyx_SET_SIZE(obj, size) Py_SIZE(obj) = (size)
00621 #endif
00622 #if CYTHON_ASSUME_SAFE_MACROS
00623     #define __Pyx_PySequence_SIZE(seq) Py_SIZE(seq)
00624 #else
00625     #define __Pyx_PySequence_SIZE(seq) PySequence_Size(seq)
00626 #endif
00627 #if PY_MAJOR_VERSION >= 3
00628     #define PyIntObject                  PyLongObject
00629     #define PyInt_Type                    PyLong_Type
00630     #define PyInt_Check(op)              PyLong_Check(op)
00631     #define PyInt_CheckExact(op)          PyLong_CheckExact(op)
00632     #define PyInt_FromString              PyLong_FromString
00633     #define PyInt_FromUnicode             PyLong_FromUnicode
00634     #define PyInt_FromLong                PyLong_FromLong
00635     #define PyInt_FromSize_t              PyLong_FromSize_t
00636     #define PyInt_FromSsize_t             PyLong_FromSsize_t
00637     #define PyInt_AsLong                   PyLong_AsLong
00638     #define PyInt_AS_LONG                  PyLong_AS_LONG
00639     #define PyInt_AsSsize_t               PyLong_AsSsize_t
00640     #define PyInt_AsUnsignedLongMask       PyLong_AsUnsignedLongMask
00641     #define PyInt_AsUnsignedLongLongMask   PyLong_AsUnsignedLongLongMask
00642     #define PyNumber_Int                   PyNumber_Long
00643 #endif
00644 #if PY_MAJOR_VERSION >= 3
00645     #define PyBoolObject                  PyLongObject
00646 #endif
00647 #if PY_MAJOR_VERSION >= 3 && CYTHON_COMPILING_IN_PYPY
00648     #ifndef PyUnicode_InternFromString
00649         #define PyUnicode_InternFromString(s) PyUnicode_FromString(s)
00650     #endif
00651 #endif
00652 #if PY_VERSION_HEX < 0x030200A4
00653     typedef long Py_hash_t;
00654     #define __Pyx_PyInt_FromHash_t PyInt_FromLong
00655     #define __Pyx_PyInt_AsHash_t    __Pyx_PyIndex_AsHash_t
00656 #else
00657     #define __Pyx_PyInt_FromHash_t PyInt_FromSsize_t
00658     #define __Pyx_PyInt_AsHash_t    __Pyx_PyIndex_AsSsize_t
00659 #endif
00660 #if PY_MAJOR_VERSION >= 3
00661     #define __Pyx_PyMethod_New(func, self, klass) ((self) ? ((void)(klass), PyMethod_New(func, self)) :
    __Pyx_NewRef(func))
00662 #else
00663     #define __Pyx_PyMethod_New(func, self, klass) PyMethod_New(func, self, klass)
00664 #endif
00665 #if CYTHON_USE_ASYNC_SLOTS
00666     #if PY_VERSION_HEX >= 0x030500B1
00667         #define __Pyx_PyAsyncMethodsStruct PyAsyncMethods
00668         #define __Pyx_PyType_AsAsync(obj) (Py_TYPE(obj)->tp_as_async)
00669     #else
00670         #define __Pyx_PyType_AsAsync(obj) ((__Pyx_PyAsyncMethodsStruct*) (Py_TYPE(obj)->tp_reserved))
00671     #endif
00672 #else
00673     #define __Pyx_PyType_AsAsync(obj) NULL
00674 #endif
00675 #ifndef __Pyx_PyAsyncMethodsStruct
00676     typedef struct {
00677         unaryfunc am_await;
00678         unaryfunc am_aiter;
00679         unaryfunc am_anext;

```

```

00680     } __Pyx_PyAsyncMethodsStruct;
00681 #endif
00682
00683 #if defined(WIN32) || defined(MS_WINDOWS)
00684     #define _USE_MATH_DEFINES
00685 #endif
00686 #include <math.h>
00687 #ifndef NAN
00688 #define __PYX_NAN() ((float) NAN)
00689 #else
00690 static CYTHON_INLINE float __PYX_NAN() {
00691     float value;
00692     memset(&value, 0xFF, sizeof(value));
00693     return value;
00694 }
00695 #endif
00696 #if defined(__CYGWIN__) && defined(_LDBL_EQ_DBL)
00697 #define __Pyx_trunc1 trunc
00698 #else
00699 #define __Pyx_trunc1 trunc1
00700 #endif
00701
00702 #define __PYX_MARK_ERR_POS(f_index, lineno) \
00703 { __pyx_filename = __pyx_f[f_index]; (void)__pyx_filename; __pyx_lineno = lineno; \
(void)__pyx_lineno; __pyx_clineno = __LINE__; (void)__pyx_clineno; }
00704 #define __PYX_ERR(f_index, lineno, Ln_error) \
00705 { __PYX_MARK_ERR_POS(f_index, lineno) goto Ln_error; }
00706
00707 #ifndef __PYX_EXTERN_C
00708 #ifdef __cplusplus
00709     #define __PYX_EXTERN_C extern "C"
00710 #else
00711     #define __PYX_EXTERN_C extern
00712 #endif
00713 #endif
00714
00715 #define __PYX_HAVE__PyClical
00716 #define __PYX_HAVE_API__PyClical
00717 /* Early includes */
00718 #include "ios"
00719 #include "new"
00720 #include "stdexcept"
00721 #include "typeinfo"
00722 #include <vector>
00723 #include "PyClical.h"
00724 #include <string.h>
00725 #include <string>
00726 #ifdef _OPENMP
00727 #include <omp.h>
00728 #endif /* _OPENMP */
00729
00730 #if defined(PYREX_WITHOUT_ASSERTIONS) && !defined(CYTHON_WITHOUT_ASSERTIONS)
00731 #define CYTHON_WITHOUT_ASSERTIONS
00732 #endif
00733
00734 typedef struct {PyObject **p; const char *s; const Py_ssize_t n; const char* encoding;
00735                const char is_unicode; const char is_str; const char intern; } __Pyx_StringTabEntry;
00736
00737 #define __PYX_DEFAULT_STRING_ENCODING_IS_ASCII 0
00738 #define __PYX_DEFAULT_STRING_ENCODING_IS_UTF8 0
00739 #define __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT (PY_MAJOR_VERSION >= 3 &&
__PYX_DEFAULT_STRING_ENCODING_IS_UTF8)
00740 #define __PYX_DEFAULT_STRING_ENCODING ""
00741 #define __Pyx_PyObject_FromString __Pyx_PyBytes_FromString
00742 #define __Pyx_PyObject_FromStringAndSize __Pyx_PyBytes_FromStringAndSize
00743 #define __Pyx_uchar_cast(c) ((unsigned char)c)
00744 #define __Pyx_long_cast(x) ((long)x)
00745 #define __Pyx_fits_Py_ssize_t(v, type, is_signed) ( \
00746     (sizeof(type) < sizeof(Py_ssize_t)) || \
00747     (sizeof(type) > sizeof(Py_ssize_t) && \
00748         likely(v < (type)PY_SSIZE_T_MAX || \
00749             v == (type)PY_SSIZE_T_MAX) && \
00750     (!is_signed || likely(v > (type)PY_SSIZE_T_MIN || \
00751         v == (type)PY_SSIZE_T_MIN))) || \
00752     (sizeof(type) == sizeof(Py_ssize_t) && \
00753         (is_signed || likely(v < (type)PY_SSIZE_T_MAX || \
00754             v == (type)PY_SSIZE_T_MAX))) )
00755 static CYTHON_INLINE int __Pyx_is_valid_index(Py_ssize_t i, Py_ssize_t limit) {
00756     return (size_t) i < (size_t) limit;
00757 }
00758 #if defined (__cplusplus) && __cplusplus >= 201103L
00759     #include <cstdint>
00760     #define __Pyx_sst_abs(value) std::abs(value)
00761 #elif SIZEOF_INT >= SIZEOF_SIZE_T
00762     #define __Pyx_sst_abs(value) abs(value)
00763 #elif SIZEOF_LONG >= SIZEOF_SIZE_T
00764     #define __Pyx_sst_abs(value) labs(value)

```

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00765 #elif defined (_MSC_VER)
00766     #define __Pyx_sst_abs(value) ((Py_ssize_t)_abs64(value))
00767 #elif defined (__STDC_VERSION__) && __STDC_VERSION__ >= 199901L
00768     #define __Pyx_sst_abs(value) llabs(value)
00769 #elif defined (__GNUC__)
00770     #define __Pyx_sst_abs(value) __builtin_llabs(value)
00771 #else
00772     #define __Pyx_sst_abs(value) ((value<0) ? -value : value)
00773 #endif
00774 static CYTHON_INLINE const char* __Pyx_PyObject_AsString(PyObject*);
00775 static CYTHON_INLINE const char* __Pyx_PyObject_AsStringAndSize(PyObject*, Py_ssize_t* length);
00776 #define __Pyx_PyByteArray_FromString(s) PyByteArray_FromStringAndSize((const char*)s, strlen((const
char*)s))
00777 #define __Pyx_PyByteArray_FromStringAndSize(s, l) PyByteArray_FromStringAndSize((const char*)s, l)
00778 #define __Pyx_PyBytes_FromString PyBytes_FromString
00779 #define __Pyx_PyBytes_FromStringAndSize PyBytes_FromStringAndSize
00780 static CYTHON_INLINE PyObject* __Pyx_PyUnicode_FromString(const char*);
00781 #if PY_MAJOR_VERSION < 3
00782     #define __Pyx_PyStr_FromString __Pyx_PyBytes_FromString
00783     #define __Pyx_PyStr_FromStringAndSize __Pyx_PyBytes_FromStringAndSize
00784 #else
00785     #define __Pyx_PyStr_FromString __Pyx_PyUnicode_FromString
00786     #define __Pyx_PyStr_FromStringAndSize __Pyx_PyUnicode_FromStringAndSize
00787 #endif
00788 #define __Pyx_PyBytes_AsWritableString(s) ((char*) PyBytes_AS_STRING(s))
00789 #define __Pyx_PyBytes_AsWritableSString(s) ((signed char*) PyBytes_AS_STRING(s))
00790 #define __Pyx_PyBytes_AsWritableUString(s) ((unsigned char*) PyBytes_AS_STRING(s))
00791 #define __Pyx_PyBytes_AsString(s) ((const char*) PyBytes_AS_STRING(s))
00792 #define __Pyx_PyBytes_AsSSString(s) ((const signed char*) PyBytes_AS_STRING(s))
00793 #define __Pyx_PyBytes_AsUString(s) ((const unsigned char*) PyBytes_AS_STRING(s))
00794 #define __Pyx_PyObject_AsWritableString(s) ((char*) __Pyx_PyObject_AsString(s))
00795 #define __Pyx_PyObject_AsWritableSString(s) ((signed char*) __Pyx_PyObject_AsString(s))
00796 #define __Pyx_PyObject_AsWritableUString(s) ((unsigned char*) __Pyx_PyObject_AsString(s))
00797 #define __Pyx_PyObject_AsSSString(s) ((const signed char*) __Pyx_PyObject_AsString(s))
00798 #define __Pyx_PyObject_AsUString(s) ((const unsigned char*) __Pyx_PyObject_AsString(s))
00799 #define __Pyx_PyObject_FromCString(s) __Pyx_PyObject_FromString((const char*)s)
00800 #define __Pyx_PyBytes_FromCString(s) __Pyx_PyBytes_FromString((const char*)s)
00801 #define __Pyx_PyByteArray_FromCString(s) __Pyx_PyByteArray_FromString((const char*)s)
00802 #define __Pyx_PyStr_FromCString(s) __Pyx_PyStr_FromString((const char*)s)
00803 #define __Pyx_PyUnicode_FromCString(s) __Pyx_PyUnicode_FromString((const char*)s)
00804 static CYTHON_INLINE size_t __Pyx_Py_UNICODE_strlen(const Py_UNICODE *u) {
00805     const Py_UNICODE *u_end = u;
00806     while (*u_end++);
00807     return (size_t)(u_end - u - 1);
00808 }
00809 #define __Pyx_PyUnicode_FromUnicode(u) PyUnicode_FromUnicode(u, __Pyx_Py_UNICODE_strlen(u))
00810 #define __Pyx_PyUnicode_FromUnicodeAndLength PyUnicode_FromUnicode
00811 #define __Pyx_PyUnicode_AsUnicode PyUnicode_AsUnicode
00812 #define __Pyx_NewRef(obj) (Py_INCREF(obj), obj)
00813 #define __Pyx_Owned_Py_None(b) __Pyx_NewRef(Py_None)
00814 static CYTHON_INLINE PyObject * __Pyx_PyBool_FromLong(long b);
00815 static CYTHON_INLINE int __Pyx_PyObject_IsTrue(PyObject*);
00816 static CYTHON_INLINE int __Pyx_PyObject_IsTrueAndDecref(PyObject*);
00817 static CYTHON_INLINE PyObject* __Pyx_PyNumber_IntOrLong(PyObject* x);
00818 #define __Pyx_PySequence_Tuple(obj) \
00819     (likely(PyTuple_CheckExact(obj)) ? __Pyx_NewRef(obj) : PySequence_Tuple(obj))
00820 static CYTHON_INLINE Py_ssize_t __Pyx_PyIndex_AsSsize_t(PyObject*);
00821 static CYTHON_INLINE PyObject * __Pyx_PyInt_FromSize_t(size_t);
00822 static CYTHON_INLINE Py_hash_t __Pyx_PyIndex_AsHash_t(PyObject*);
00823 #if CYTHON_ASSUME_SAFE_MACROS
00824 #define __pyx_PyFloat_AsDouble(x) (PyFloat_CheckExact(x) ? PyFloat_AS_DOUBLE(x) : PyFloat_AsDouble(x))
00825 #else
00826 #define __pyx_PyFloat_AsDouble(x) PyFloat_AsDouble(x)
00827 #endif
00828 #define __pyx_PyFloat_AsFloat(x) ((float) __pyx_PyFloat_AsDouble(x))
00829 #if PY_MAJOR_VERSION >= 3
00830 #define __Pyx_PyNumber_Int(x) (PyLong_CheckExact(x) ? __Pyx_NewRef(x) : PyNumber_Long(x))
00831 #else
00832 #define __Pyx_PyNumber_Int(x) (PyInt_CheckExact(x) ? __Pyx_NewRef(x) : PyNumber_Int(x))
00833 #endif
00834 #define __Pyx_PyNumber_Float(x) (PyFloat_CheckExact(x) ? __Pyx_NewRef(x) : PyNumber_Float(x))
00835 #if PY_MAJOR_VERSION < 3 && __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
00836 static int __Pyx_sys_getdefaultencoding_not_ascii;
00837 static int __Pyx_init_sys_getdefaultencoding_params(void) {
00838     PyObject* sys;
00839     PyObject* default_encoding = NULL;
00840     PyObject* ascii_chars_u = NULL;
00841     PyObject* ascii_chars_b = NULL;
00842     const char* default_encoding_c;
00843     sys = PyImport_ImportModule("sys");
00844     if (!sys) goto bad;
00845     default_encoding = PyObject_CallMethod(sys, (char*) "getdefaultencoding", NULL);
00846     Py_DECREF(sys);
00847     if (!default_encoding) goto bad;
00848     default_encoding_c = PyBytes_AsString(default_encoding);
00849     if (!default_encoding_c) goto bad;
00850     if (strcmp(default_encoding_c, "ascii") == 0) {

```

```

00851     __Pyx_sys_getdefaultencoding_not_ascii = 0;
00852 } else {
00853     char ascii_chars[128];
00854     int c;
00855     for (c = 0; c < 128; c++) {
00856         ascii_chars[c] = c;
00857     }
00858     __Pyx_sys_getdefaultencoding_not_ascii = 1;
00859     ascii_chars_u = PyUnicode_DecodeASCII(ascii_chars, 128, NULL);
00860     if (!ascii_chars_u) goto bad;
00861     ascii_chars_b = PyUnicode_AsEncodedString(ascii_chars_u, default_encoding_c, NULL);
00862     if (!ascii_chars_b || !PyBytes_Check(ascii_chars_b) || memcmp(ascii_chars,
PyBytes_AS_STRING(ascii_chars_b), 128) != 0) {
00863         PyErr_Format(
00864             PyExc_ValueError,
00865             "This module compiled with c_string_encoding=ascii, but default encoding '%.200s' is
not a superset of ascii.",
00866             default_encoding_c);
00867         goto bad;
00868     }
00869     Py_DECREF(ascii_chars_u);
00870     Py_DECREF(ascii_chars_b);
00871 }
00872 Py_DECREF(default_encoding);
00873 return 0;
00874 bad:
00875     Py_XDECREF(default_encoding);
00876     Py_XDECREF(ascii_chars_u);
00877     Py_XDECREF(ascii_chars_b);
00878     return -1;
00879 }
00880 #endif
00881 #if __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT && PY_MAJOR_VERSION >= 3
00882 #define __Pyx_PyUnicode_FromStringAndSize(c_str, size) PyUnicode_DecodeUTF8(c_str, size, NULL)
00883 #else
00884 #define __Pyx_PyUnicode_FromStringAndSize(c_str, size) PyUnicode_Decode(c_str, size,
__PYX_DEFAULT_STRING_ENCODING, NULL)
00885 #if __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT
00886 static char* __PYX_DEFAULT_STRING_ENCODING;
00887 static int __Pyx_init_sys_getdefaultencoding_params(void) {
00888     PyObject* sys;
00889     PyObject* default_encoding = NULL;
00890     char* default_encoding_c;
00891     sys = PyImport_ImportModule("sys");
00892     if (!sys) goto bad;
00893     default_encoding = PyObject_CallMethod(sys, (char*) (const char*) "getdefaultencoding", NULL);
00894     Py_DECREF(sys);
00895     if (!default_encoding) goto bad;
00896     default_encoding_c = PyBytes_AsString(default_encoding);
00897     if (!default_encoding_c) goto bad;
00898     __PYX_DEFAULT_STRING_ENCODING = (char*) malloc(strlen(default_encoding_c) + 1);
00899     if (!__PYX_DEFAULT_STRING_ENCODING) goto bad;
00900     strcpy(__PYX_DEFAULT_STRING_ENCODING, default_encoding_c);
00901     Py_DECREF(default_encoding);
00902     return 0;
00903 bad:
00904     Py_XDECREF(default_encoding);
00905     return -1;
00906 }
00907 #endif
00908 #endif
00909
00910
00911 /* Test for GCC > 2.95 */
00912 #if defined(__GNUC__) && (__GNUC__ > 2 || (__GNUC__ == 2 && (__GNUC_MINOR__ > 95)))
00913 #define likely(x) __builtin_expect(!!(x), 1)
00914 #define unlikely(x) __builtin_expect(!!(x), 0)
00915 #else /* !__GNUC__ or GCC < 2.95 */
00916 #define likely(x) (x)
00917 #define unlikely(x) (x)
00918 #endif /* __GNUC__ */
00919 static CYTHON_INLINE void __Pyx_pretend_to_initialize(void* ptr) { (void)ptr; }
00920
00921 static PyObject* __pyx_m = NULL;
00922 static PyObject* __pyx_d;
00923 static PyObject* __pyx_b;
00924 static PyObject* __pyx_cython_runtime = NULL;
00925 static PyObject* __pyx_empty_tuple;
00926 static PyObject* __pyx_empty_bytes;
00927 static PyObject* __pyx_empty_unicode;
00928 static int __pyx_lineno;
00929 static int __pyx_clineno = 0;
00930 static const char* __pyx_cfilenm = __FILE__;
00931 static const char* __pyx_filename;
00932
00933
00934 static const char* __pyx_f[] = {

```



```

00935     "PyClicl.pyx",
00936     "stringsource",
00937 };
00938
00939 /*--- Type declarations ---*/
00940 struct __pyx_obj_8PyClicl_index_set;
00941 struct __pyx_obj_8PyClicl_clifford;
00942 struct __pyx_obj_8PyClicl__pyx_scope_struct____iter____;
00943 struct __pyx_opt_args_8PyClicl_approx_equal;
00944 struct __pyx_opt_args_8PyClicl_sqrt;
00945 struct __pyx_opt_args_8PyClicl_log;
00946 struct __pyx_opt_args_8PyClicl_cos;
00947 struct __pyx_opt_args_8PyClicl_acos;
00948 struct __pyx_opt_args_8PyClicl_acosh;
00949 struct __pyx_opt_args_8PyClicl_sin;
00950 struct __pyx_opt_args_8PyClicl_asin;
00951 struct __pyx_opt_args_8PyClicl_asinh;
00952 struct __pyx_opt_args_8PyClicl_tan;
00953 struct __pyx_opt_args_8PyClicl_atan;
00954 struct __pyx_opt_args_8PyClicl_atanh;
00955 struct __pyx_opt_args_8PyClicl_random_clifford;
00956
00957 /* "PyClicl.pyx":1359
00958 *     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
00959 *
00960 * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):          # ««««««««
00961 *     """
00962 *     Test for approximate equality of multivectors.
00963 */
00964 struct __pyx_opt_args_8PyClicl_approx_equal {
00965     int __pyx_n;
00966     PyObject *threshold;
00967     PyObject *tol;
00968 };
00969
00970 /* "PyClicl.pyx":1591
00971 *     return clifford().wrap( glucat.complexifier(toClifford(obj)) )
00972 *
00973 * cpdef inline sqrt(obj, i = None):          # ««««««««
00974 *     """
00975 *     Square root of multivector with optional complexifier.
00976 */
00977 struct __pyx_opt_args_8PyClicl_sqrt {
00978     int __pyx_n;
00979     PyObject *i;
00980 };
00981
00982 /* "PyClicl.pyx":1628
00983 *     return clifford().wrap( glucat.exp(toClifford(obj)) )
00984 *
00985 * cpdef inline log(obj,i = None):          # ««««««««
00986 *     """
00987 *     Natural logarithm of multivector with optional complexifier.
00988 */
00989 struct __pyx_opt_args_8PyClicl_log {
00990     int __pyx_n;
00991     PyObject *i;
00992 };
00993
00994 /* "PyClicl.pyx":1651
00995 *     return clifford().wrap( glucat.log(toClifford(obj)) )
00996 *
00997 * cpdef inline cos(obj,i = None):          # ««««««««
00998 *     """
00999 *     Cosine of multivector with optional complexifier.
01000 */
01001 struct __pyx_opt_args_8PyClicl_cos {
01002     int __pyx_n;
01003     PyObject *i;
01004 };
01005
01006 /* "PyClicl.pyx":1668
01007 *     return clifford().wrap( glucat.cos(toClifford(obj)) )
01008 *
01009 * cpdef inline acos(obj,i = None):          # ««««««««
01010 *     """
01011 *     Inverse cosine of multivector with optional complexifier.
01012 */
01013 struct __pyx_opt_args_8PyClicl_acos {
01014     int __pyx_n;
01015     PyObject *i;
01016 };
01017
01018 /* "PyClicl.pyx":1705
01019 *     return clifford().wrap( glucat.cosh(toClifford(obj)) )
01020 *
01021 * cpdef inline acosh(obj,i = None):          # ««««««««

```



```

01022 *      """
01023 *      Inverse hyperbolic cosine of multivector with optional complexifier.
01024 */
01025 struct __pyx_opt_args_8PyClicl_acosh {
01026     int __pyx_n;
01027     PyObject *i;
01028 };
01029
01030 /* "PyClicl.pyx":1728
01031 *      return clifford().wrap( glucat.acosh(toClifford(obj)) )
01032 *
01033 * cpdef inline sin(obj,i = None):          # ««««««««
01034 *      """
01035 *      Sine of multivector with optional complexifier.
01036 */
01037 struct __pyx_opt_args_8PyClicl_sin {
01038     int __pyx_n;
01039     PyObject *i;
01040 };
01041
01042 /* "PyClicl.pyx":1747
01043 *      return clifford().wrap( glucat.sin(toClifford(obj)) )
01044 *
01045 * cpdef inline asin(obj,i = None):          # ««««««««
01046 *      """
01047 *      Inverse sine of multivector with optional complexifier.
01048 */
01049 struct __pyx_opt_args_8PyClicl_asin {
01050     int __pyx_n;
01051     PyObject *i;
01052 };
01053
01054 /* "PyClicl.pyx":1782
01055 *      return clifford().wrap( glucat.sinh(toClifford(obj)) )
01056 *
01057 * cpdef inline asinh(obj,i = None):          # ««««««««
01058 *      """
01059 *      Inverse hyperbolic sine of multivector with optional complexifier.
01060 */
01061 struct __pyx_opt_args_8PyClicl_asinh {
01062     int __pyx_n;
01063     PyObject *i;
01064 };
01065
01066 /* "PyClicl.pyx":1801
01067 *      return clifford().wrap( glucat.asinh(toClifford(obj)) )
01068 *
01069 * cpdef inline tan(obj,i = None):          # ««««««««
01070 *      """
01071 *      Tangent of multivector with optional complexifier.
01072 */
01073 struct __pyx_opt_args_8PyClicl_tan {
01074     int __pyx_n;
01075     PyObject *i;
01076 };
01077
01078 /* "PyClicl.pyx":1818
01079 *      return clifford().wrap( glucat.tan(toClifford(obj)) )
01080 *
01081 * cpdef inline atan(obj,i = None):          # ««««««««
01082 *      """
01083 *      Inverse tangent of multivector with optional complexifier.
01084 */
01085 struct __pyx_opt_args_8PyClicl_atan {
01086     int __pyx_n;
01087     PyObject *i;
01088 };
01089
01090 /* "PyClicl.pyx":1847
01091 *      return clifford().wrap( glucat.tanh(toClifford(obj)) )
01092 *
01093 * cpdef inline atanh(obj,i = None):          # ««««««««
01094 *      """
01095 *      Inverse hyperbolic tangent of multivector with optional complexifier.
01096 */
01097 struct __pyx_opt_args_8PyClicl_atanh {
01098     int __pyx_n;
01099     PyObject *i;
01100 };
01101
01102 /* "PyClicl.pyx":1864
01103 *      return clifford().wrap( glucat.atanh(toClifford(obj)) )
01104 *
01105 * cpdef inline random_clifford(index_set ixt, fill = 1.0):          # ««««««««
01106 *      """
01107 *      Random multivector within a frame.
01108 */

```

```

01109 struct __pyx_opt_args_8PyClical_random_clifford {
01110     int __pyx_n;
01111     PyObject *fill;
01112 };
01113
01114 /* "PyClical.pyx":38
01115 *
01116 * # Forward reference
01117 * cdef class index_set          # ««««««««
01118 *
01119 * cdef inline IndexSet toIndexSet(obj):
01120 */
01121 struct __pyx_obj_8PyClical_index_set {
01122     PyObject_HEAD
01123     struct __pyx_vtabstruct_8PyClical_index_set *__pyx_vtab;
01124     IndexSet *instance;
01125 };
01126
01127
01128 /* "PyClical.pyx":532
01129 *
01130 * # Forward reference.
01131 * cdef class clifford          # ««««««««
01132 *
01133 * cdef inline Clifford toClifford(obj):
01134 */
01135 struct __pyx_obj_8PyClical_clifford {
01136     PyObject_HEAD
01137     struct __pyx_vtabstruct_8PyClical_clifford *__pyx_vtab;
01138     Clifford *instance;
01139 };
01140
01141
01142 /* "PyClical.pyx":229
01143 *         return self.instance.getitem(idx)
01144 *
01145 *     def __iter__(self):          # ««««««««
01146 *         """
01147 *         Iterate over the indices of an index_set.
01148 */
01149 struct __pyx_obj_8PyClical__pyx_scope_struct__iter__ {
01150     PyObject_HEAD
01151     PyObject *__pyx_v_idx;
01152     struct __pyx_obj_8PyClical_index_set *__pyx_v_self;
01153     PyObject *__pyx_t_0;
01154     Py_ssize_t __pyx_t_1;
01155     PyObject *(*__pyx_t_2)(PyObject *);
01156 };
01157
01158
01159
01160 /* "PyClical.pyx":46
01161 *     return index_set(obj).instance[0]
01162 *
01163 * cdef class index_set:          # ««««««««
01164 *     """
01165 *     Python class index_set wraps C++ class IndexSet.
01166 */
01167
01168 struct __pyx_vtabstruct_8PyClical_index_set {
01169     PyObject *(*wrap)(struct __pyx_obj_8PyClical_index_set *, IndexSet);
01170     IndexSet (*unwrap)(struct __pyx_obj_8PyClical_index_set *);
01171     PyObject *(*copy)(struct __pyx_obj_8PyClical_index_set *, int __pyx_skip_dispatch);
01172 };
01173
01174 static struct __pyx_vtabstruct_8PyClical_index_set *__pyx_vtabptr_8PyClical_index_set;
01175 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_9index_set_wrap(struct __pyx_obj_8PyClical_index_set
*, IndexSet);
01176 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_9index_set_unwrap(struct __pyx_obj_8PyClical_index_set
*);
01177
01178
01179 /* "PyClical.pyx":537
01180 *     return clifford(obj).instance[0]
01181 *
01182 * cdef class clifford:          # ««««««««
01183 *     """
01184 *     Python class clifford wraps C++ class Clifford.
01185 */
01186
01187 struct __pyx_vtabstruct_8PyClical_clifford {
01188     PyObject *(*wrap)(struct __pyx_obj_8PyClical_clifford *, Clifford);
01189     Clifford (*unwrap)(struct __pyx_obj_8PyClical_clifford *);
01190     PyObject *(*copy)(struct __pyx_obj_8PyClical_clifford *, int __pyx_skip_dispatch);
01191 };
01192
01193 static struct __pyx_vtabstruct_8PyClical_clifford *__pyx_vtabptr_8PyClical_clifford;
01194 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_8clifford_wrap(struct __pyx_obj_8PyClical_clifford *,
Clifford);

```

```

01193 static CYTHON_INLINE Clifford __pyx_f_8PyClical_8clifford_unwrap(struct __pyx_obj_8PyClical_clifford
01194 *);
01194
01195 /* --- Runtime support code (head) --- */
01196 /* Refnanny.proto */
01197 #ifndef CYTHON_REFNANNY
01198 #define CYTHON_REFNANNY 0
01199 #endif
01200 #if CYTHON_REFNANNY
01201 typedef struct {
01202     void (*INCRREF)(void*, PyObject*, int);
01203     void (*DECREF)(void*, PyObject*, int);
01204     void (*GOTREF)(void*, PyObject*, int);
01205     void (*GIVEREF)(void*, PyObject*, int);
01206     void* (*SetupContext)(const char*, int, const char*);
01207     void (*FinishContext)(void**);
01208 } __Pyx_RefNannyAPIStruct;
01209 static __Pyx_RefNannyAPIStruct *__Pyx_RefNanny = NULL;
01210 static __Pyx_RefNannyAPIStruct *__Pyx_RefNannyImportAPI(const char *modname);
01211 #define __Pyx_RefNannyDeclarations void *__pyx_refnanny = NULL;
01212 #ifndef WITH_THREAD
01213 #define __Pyx_RefNannySetupContext(name, acquire_gil)\
01214     if (acquire_gil) {\
01215         PyGILState_STATE __pyx_gilstate_save = PyGILState_Ensure();\
01216         __pyx_refnanny = __Pyx_RefNanny->SetupContext((name), __LINE__, __FILE__);\
01217         PyGILState_Release(__pyx_gilstate_save);\
01218     } else {\
01219         __pyx_refnanny = __Pyx_RefNanny->SetupContext((name), __LINE__, __FILE__);\
01220     }
01221 #else
01222 #define __Pyx_RefNannySetupContext(name, acquire_gil)\
01223     __pyx_refnanny = __Pyx_RefNanny->SetupContext((name), __LINE__, __FILE__)
01224 #endif
01225 #define __Pyx_RefNannyFinishContext()\
01226     __Pyx_RefNanny->FinishContext(&__pyx_refnanny)
01227 #define __Pyx_INCREF(r) __Pyx_RefNanny->INCRREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01228 #define __Pyx_DECREF(r) __Pyx_RefNanny->DECREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01229 #define __Pyx_GOTREF(r) __Pyx_RefNanny->GOTREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01230 #define __Pyx_GIVEREF(r) __Pyx_RefNanny->GIVEREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01231 #define __Pyx_XINCRREF(r) do { if ((r) != NULL) {__Pyx_INCREF(r); } } while(0)
01232 #define __Pyx_XDECREF(r) do { if ((r) != NULL) {__Pyx_DECREF(r); } } while(0)
01233 #define __Pyx_XGOTREF(r) do { if ((r) != NULL) {__Pyx_GOTREF(r); } } while(0)
01234 #define __Pyx_XGIVEREF(r) do { if ((r) != NULL) {__Pyx_GIVEREF(r); } } while(0)
01235 #else
01236 #define __Pyx_RefNannyDeclarations
01237 #define __Pyx_RefNannySetupContext(name, acquire_gil)
01238 #define __Pyx_RefNannyFinishContext()
01239 #define __Pyx_INCREF(r) Py_INCREF(r)
01240 #define __Pyx_DECREF(r) Py_DECREF(r)
01241 #define __Pyx_GOTREF(r)
01242 #define __Pyx_GIVEREF(r)
01243 #define __Pyx_XINCRREF(r) Py_XINCRREF(r)
01244 #define __Pyx_XDECREF(r) Py_XDECREF(r)
01245 #define __Pyx_XGOTREF(r)
01246 #define __Pyx_XGIVEREF(r)
01247 #endif
01248 #define __Pyx_XDECREF_SET(r, v) do {\
01249     PyObject *tmp = (PyObject *) r;\
01250     r = v; __Pyx_XDECREF(tmp);\
01251 } while (0)
01252 #define __Pyx_DECREF_SET(r, v) do {\
01253     PyObject *tmp = (PyObject *) r;\
01254     r = v; __Pyx_DECREF(tmp);\
01255 } while (0)
01256 #define __Pyx_CLEAR(r) do { PyObject* tmp = ((PyObject*)(r)); r = NULL; __Pyx_DECREF(tmp); } while(0)
01257 #define __Pyx_XCLEAR(r) do { if ((r) != NULL) {PyObject* tmp = ((PyObject*)(r)); r = NULL; __Pyx_DECREF(tmp); } } while(0)
01258
01259 /* PyObjectGetAttrStr.proto */
01260 #if CYTHON_USE_TYPE_SLOTS
01261 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStr(PyObject* obj, PyObject* attr_name);
01262 #else
01263 #define __Pyx_PyObject_GetAttrStr(o,n) PyObject_GetAttr(o,n)
01264 #endif
01265
01266 /* GetBuiltinName.proto */
01267 static PyObject *__Pyx_GetBuiltinName(PyObject *name);
01268
01269 /* PyCFunctionFastCall.proto */
01270 #if CYTHON_FAST_PYCCALL
01271 static CYTHON_INLINE PyObject* __Pyx_PyCFunction_FastCall(PyObject *func, PyObject **args, Py_ssize_t nargs);
01272 #else
01273 #define __Pyx_PyCFunction_FastCall(func, args, nargs) (assert(0), NULL)
01274 #endif
01275

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01276 /* PyFunctionFastCall.proto */
01277 #if CYTHON_FAST_PYCALL
01278 #define __Pyx_PyFunction_FastCall(func, args, nargs)\
01279     __Pyx_PyFunction_FastCallDict((func), (args), (nargs), NULL)
01280 #if 1 || PY_VERSION_HEX < 0x030600B1
01281 static PyObject * __Pyx_PyFunction_FastCallDict(PyObject *func, PyObject **args, Py_ssize_t nargs,
01282     PyObject *kwargs);
01283 #else
01283 #define __Pyx_PyFunction_FastCallDict(func, args, nargs, kwargs) __PyFunction_FastCallDict(func, args,
01284     nargs, kwargs)
01284 #endif
01285 #define __Pyx_BUILD_ASSERT_EXPR(cond)\
01286     (sizeof(char [1 - 2*(cond)]) - 1)
01287 #ifndef Py_MEMBER_SIZE
01288 #define Py_MEMBER_SIZE(type, member) sizeof(((type *)0)->member)
01289 #endif
01290 #if CYTHON_FAST_PYCALL
01291 static size_t __pyx_pyframe_localsplus_offset = 0;
01292 #include "frameobject.h"
01293 #define __Pxy_PyFrame_Initialize_Offsets()\
01294     ((void)__Pyx_BUILD_ASSERT_EXPR(sizeof(PyFrameObject) == offsetof(PyFrameObject, f_localsplus) +
01295     Py_MEMBER_SIZE(PyFrameObject, f_localsplus)),\
01296     (void)(__pyx_pyframe_localsplus_offset = ((size_t)PyFrame_Type.tp_basicsize) -
01297     Py_MEMBER_SIZE(PyFrameObject, f_localsplus)))
01298 #define __Pyx_PyFrame_GetLocalsplus(frame)\
01299     (assert(__pyx_pyframe_localsplus_offset), (PyObject *)((char *) (frame)) +
01300     __pyx_pyframe_localsplus_offset))
01301 #endif // CYTHON_FAST_PYCALL
01302 #endif
01303
01304 /* PyObjectCall.proto */
01305 #if CYTHON_COMPILING_IN_CPYTHON
01306 static CYTHON_INLINE PyObject* __Pyx_PyObject_Call(PyObject *func, PyObject *arg, PyObject *kw);
01307 #else
01307 #define __Pyx_PyObject_Call(func, arg, kw) PyObject_Call(func, arg, kw)
01308 #endif
01309
01310 /* PyObjectCallMethO.proto */
01311 #if CYTHON_COMPILING_IN_CPYTHON
01312 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallMethO(PyObject *func, PyObject *arg);
01313 #endif
01314
01315 /* PyObjectCallOneArg.proto */
01316 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg);
01317
01318 /* PyThreadStateGet.proto */
01319 #if CYTHON_FAST_THREAD_STATE
01320 #define __Pyx_PyThreadState_declare PyThreadState *__pyx_tstate;
01321 #define __Pyx_PyThreadState_assign __pyx_tstate = __Pyx_PyThreadState_Current;
01322 #define __Pyx_PyErr_Occurred() __pyx_tstate->curexc_type
01323 #else
01324 #define __Pyx_PyThreadState_declare
01325 #define __Pyx_PyThreadState_assign
01326 #define __Pyx_PyErr_Occurred() PyErr_Occurred()
01327 #endif
01328
01329 /* PyErrFetchRestore.proto */
01330 #if CYTHON_FAST_THREAD_STATE
01331 #define __Pyx_PyErr_Clear() __Pyx_ErrRestore(NULL, NULL, NULL)
01332 #define __Pyx_ErrRestoreWithState(type, value, tb) __Pyx_ErrRestoreInState(PyThreadState_GET(), type,
01333     value, tb)
01334 #define __Pyx_ErrFetchWithState(type, value, tb) __Pyx_ErrFetchInState(PyThreadState_GET(), type,
01335     value, tb)
01336 #define __Pyx_ErrRestore(type, value, tb) __Pyx_ErrRestoreInState(__pyx_tstate, type, value, tb)
01337 #define __Pyx_ErrFetch(type, value, tb) __Pyx_ErrFetchInState(__pyx_tstate, type, value, tb)
01338 static CYTHON_INLINE void __Pyx_ErrRestoreInState(PyThreadState *tstate, PyObject *type, PyObject
01339     *value, PyObject *tb);
01340 static CYTHON_INLINE void __Pyx_ErrFetchInState(PyThreadState *tstate, PyObject **type, PyObject
01341     **value, PyObject **tb);
01342 #if CYTHON_COMPILING_IN_CPYTHON
01343 #define __Pyx_PyErr_SetNone(exc) (Py_INCREF(exc), __Pyx_ErrRestore((exc), NULL, NULL))
01344 #else
01344 #define __Pyx_PyErr_SetNone(exc) PyErr_SetNone(exc)
01345 #endif
01346 #define __Pyx_PyErr_Clear() PyErr_Clear()
01347 #define __Pyx_PyErr_SetNone(exc) PyErr_SetNone(exc)
01348 #define __Pyx_ErrRestoreWithState(type, value, tb) PyErr_Restore(type, value, tb)
01349 #define __Pyx_ErrFetchWithState(type, value, tb) PyErr_Fetch(type, value, tb)
01350 #define __Pyx_ErrRestoreInState(tstate, type, value, tb) PyErr_Restore(type, value, tb)
01351 #define __Pyx_ErrFetchInState(tstate, type, value, tb) PyErr_Fetch(type, value, tb)
01352 #define __Pyx_ErrRestore(type, value, tb) PyErr_Restore(type, value, tb)
01353 #define __Pyx_ErrFetch(type, value, tb) PyErr_Fetch(type, value, tb)
01354 #endif
01355
01356 /* WriteUnraisableException.proto */
01357 static void __Pyx_WriteUnraisable(const char *name, int clineno,

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01354                                     int lineno, const char *filename,
01355                                     int full_traceback, int nogil);
01356
01357 /* PyDictVersioning.proto */
01358 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_TYPE_SLOTS
01359 #define __PYX_DICT_VERSION_INIT ((PY_UINT64_T) -1)
01360 #define __PYX_GET_DICT_VERSION(dict) (((PyDictObject*)(dict))>ma_version_tag)
01361 #define __PYX_UPDATE_DICT_CACHE(dict, value, cache_var, version_var)\
01362     (version_var) = __PYX_GET_DICT_VERSION(dict);\
01363     (cache_var) = (value);\
01364 #define __PYX_PY_DICT_LOOKUP_IF_MODIFIED(VAR, DICT, LOOKUP) {\
01365     static PY_UINT64_T __pyx_dict_version = 0;\
01366     static PyObject * __pyx_dict_cached_value = NULL;\
01367     if (likely(__PYX_GET_DICT_VERSION(DICT) == __pyx_dict_version)) {\
01368         (VAR) = __pyx_dict_cached_value;\
01369     } else {\
01370         (VAR) = __pyx_dict_cached_value = (LOOKUP);\
01371         __pyx_dict_version = __PYX_GET_DICT_VERSION(DICT);\
01372     }\
01373 }
01374 static CYTHON_INLINE PY_UINT64_T __Pyx_get_tp_dict_version(PyObject *obj);
01375 static CYTHON_INLINE PY_UINT64_T __Pyx_get_object_dict_version(PyObject *obj);
01376 static CYTHON_INLINE int __Pyx_object_dict_version_matches(PyObject* obj, PY_UINT64_T tp_dict_version,
PY_UINT64_T obj_dict_version);
01377 #else
01378 #define __PYX_GET_DICT_VERSION(dict) (0)
01379 #define __PYX_UPDATE_DICT_CACHE(dict, value, cache_var, version_var)
01380 #define __PYX_PY_DICT_LOOKUP_IF_MODIFIED(VAR, DICT, LOOKUP) (VAR) = (LOOKUP);
01381 #endif
01382
01383 /* PyObjectCallNoArg.proto */
01384 #if CYTHON_COMPILING_IN_CPYTHON
01385 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallNoArg(PyObject *func);
01386 #else
01387 #define __Pyx_PyObject_CallNoArg(func) __Pyx_PyObject_Call(func, __pyx_empty_tuple, NULL)
01388 #endif
01389
01390 /* RaiseDoubleKeywords.proto */
01391 static void __Pyx_RaiseDoubleKeywordsError(const char* func_name, PyObject* kw_name);
01392
01393 /* ParseKeywords.proto */
01394 static int __Pyx_ParseOptionalKeywords(PyObject *kwds, PyObject **argnames[],\
PyObject *kwds2, PyObject *values[], Py_ssize_t num_pos_args,\
const char* function_name);
01395
01396
01397
01398 /* RaiseArgTupleInvalid.proto */
01399 static void __Pyx_RaiseArgtupleInvalid(const char* func_name, int exact,
01400 Py_ssize_t num_min, Py_ssize_t num_max, Py_ssize_t num_found);
01401
01402 /* GetModuleGlobalName.proto */
01403 #if CYTHON_USE_DICT_VERSIONS
01404 #define __Pyx_GetModuleGlobalName(var, name) {\
01405     static PY_UINT64_T __pyx_dict_version = 0;\
01406     static PyObject * __pyx_dict_cached_value = NULL;\
01407     (var) = (likely(__pyx_dict_version == __PYX_GET_DICT_VERSION(__pyx_d))) ?\
01408         (likely(__pyx_dict_cached_value) ? __Pyx_NewRef(__pyx_dict_cached_value) :\
__Pyx_GetBuiltinName(name)) :\
__Pyx_GetModuleGlobalName(name, &__pyx_dict_version, &__pyx_dict_cached_value);\
01409 }
01410 #define __Pyx_GetModuleGlobalNameUncached(var, name) {\
01411     PY_UINT64_T __pyx_dict_version;\
01412     PyObject * __pyx_dict_cached_value;\
01413     (var) = __Pyx_GetModuleGlobalName(name, &__pyx_dict_version, &__pyx_dict_cached_value);\
01414 }
01415 static PyObject * __Pyx__GetModuleGlobalName(PyObject *name, PY_UINT64_T *dict_version, PyObject
**dict_cached_value);
01416 #else
01417 #define __Pyx_GetModuleGlobalName(var, name) (var) = __Pyx__GetModuleGlobalName(name)
01418 #define __Pyx_GetModuleGlobalNameUncached(var, name) (var) = __Pyx__GetModuleGlobalName(name)
01419 static CYTHON_INLINE PyObject * __Pyx__GetModuleGlobalName(PyObject *name);
01420 #endif
01421
01422
01423 /* GetTopmostException.proto */
01424 #if CYTHON_USE_EXC_INFO_STACK
01425 static _PyErr_StackItem * __Pyx_PyErr_GetTopmostException(PyThreadState *tstate);
01426 #endif
01427
01428 /* SaveResetException.proto */
01429 #if CYTHON_FAST_THREAD_STATE
01430 #define __Pyx_ExceptionSave(type, value, tb) __Pyx__ExceptionSave(__pyx_tstate, type, value, tb)
01431 static CYTHON_INLINE void __Pyx__ExceptionSave(PyThreadState *tstate, PyObject **type, PyObject
**value, PyObject **tb);
01432 #define __Pyx_ExceptionReset(type, value, tb) __Pyx__ExceptionReset(__pyx_tstate, type, value, tb)
01433 static CYTHON_INLINE void __Pyx__ExceptionReset(PyThreadState *tstate, PyObject *type, PyObject
*value, PyObject *tb);
01434 #else
01435 #define __Pyx_ExceptionSave(type, value, tb) PyErr_GetExcInfo(type, value, tb)

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01436 #define __Pyx_ExceptionReset(type, value, tb) PyErr_SetExcInfo(type, value, tb)
01437 #endif
01438
01439 /* PyErrExceptionMatches.proto */
01440 #if CYTHON_FAST_THREAD_STATE
01441 #define __Pyx_PyErr_ExceptionMatches(err) __Pyx_PyErr_ExceptionMatchesInState(__pyx_tstate, err)
01442 static CYTHON_INLINE int __Pyx_PyErr_ExceptionMatches(PyThreadState* tstate, PyObject* err);
01443 #else
01444 #define __Pyx_PyErr_ExceptionMatches(err) PyErr_ExceptionMatches(err)
01445 #endif
01446
01447 /* GetException.proto */
01448 #if CYTHON_FAST_THREAD_STATE
01449 #define __Pyx_GetException(type, value, tb) __Pyx_GetException(__pyx_tstate, type, value, tb)
01450 static int __Pyx_GetException(PyThreadState* tstate, PyObject** type, PyObject** value, PyObject** tb);
01451 #else
01452 static int __Pyx_GetException(PyObject** type, PyObject** value, PyObject** tb);
01453 #endif
01454
01455 /* RaiseException.proto */
01456 static void __Pyx_Raise(PyObject* type, PyObject* value, PyObject* tb, PyObject* cause);
01457
01458 /* PyObjectCall2Args.proto */
01459 static CYTHON_UNUSED PyObject* __Pyx_PyObject_Call2Args(PyObject* function, PyObject* arg1, PyObject* arg2);
01460
01461 /* PyIntBinop.proto */
01462 #if !CYTHON_COMPILING_IN_PYPY
01463 static PyObject* __Pyx_PyInt_AddObjC(PyObject* op1, PyObject* op2, long intval, int inplace, int zerodivision_check);
01464 #else
01465 #define __Pyx_PyInt_AddObjC(op1, op2, intval, inplace, zerodivision_check)\
01466     (inplace ? PyNumber_InPlaceAdd(op1, op2) : PyNumber_Add(op1, op2))
01467 #endif
01468
01469 /* PySequenceContains.proto */
01470 static CYTHON_INLINE int __Pyx_PySequence_ContainsTF(PyObject* item, PyObject* seq, int eq) {
01471     int result = PySequence_Contains(seq, item);
01472     return unlikely(result < 0) ? result : (result == (eq == Py_EQ));
01473 }
01474
01475 /* IncludeCppStringH.proto */
01476 #include <string>
01477
01478 /* decode_c_string_utf16.proto */
01479 static CYTHON_INLINE PyObject* __Pyx_PyUnicode_DecodeUTF16(const char* s, Py_ssize_t size, const char* errors) {
01480     int byteorder = 0;
01481     return PyUnicode_DecodeUTF16(s, size, errors, &byteorder);
01482 }
01483 static CYTHON_INLINE PyObject* __Pyx_PyUnicode_DecodeUTF16LE(const char* s, Py_ssize_t size, const char* errors) {
01484     int byteorder = -1;
01485     return PyUnicode_DecodeUTF16(s, size, errors, &byteorder);
01486 }
01487 static CYTHON_INLINE PyObject* __Pyx_PyUnicode_DecodeUTF16BE(const char* s, Py_ssize_t size, const char* errors) {
01488     int byteorder = 1;
01489     return PyUnicode_DecodeUTF16(s, size, errors, &byteorder);
01490 }
01491
01492 /* decode_c_bytes.proto */
01493 static CYTHON_INLINE PyObject* __Pyx_decode_c_bytes(
01494     const char* cstring, Py_ssize_t length, Py_ssize_t start, Py_ssize_t stop,
01495     const char* encoding, const char* errors,
01496     PyObject* (*decode_func)(const char* s, Py_ssize_t size, const char* errors));
01497
01498 /* decode_cpp_string.proto */
01499 static CYTHON_INLINE PyObject* __Pyx_decode_cpp_string(
01500     std::string cppstring, Py_ssize_t start, Py_ssize_t stop,
01501     const char* encoding, const char* errors,
01502     PyObject* (*decode_func)(const char* s, Py_ssize_t size, const char* errors)) {
01503     return __Pyx_decode_c_bytes(
01504         cppstring.data(), cppstring.size(), start, stop, encoding, errors, decode_func);
01505 }
01506
01507 /* SwapException.proto */
01508 #if CYTHON_FAST_THREAD_STATE
01509 #define __Pyx_ExceptionSwap(type, value, tb) __Pyx_ExceptionSwap(__pyx_tstate, type, value, tb)
01510 static CYTHON_INLINE void __Pyx_ExceptionSwap(PyThreadState* tstate, PyObject** type, PyObject** value, PyObject** tb);
01511 #else
01512 static CYTHON_INLINE void __Pyx_ExceptionSwap(PyObject** type, PyObject** value, PyObject** tb);
01513 #endif
01514
01515 /* SetItemInt.proto */

```

```

01516 #define __Pyx_SetItemInt(o, i, v, type, is_signed, to_py_func, is_list, wraparound, boundscheck)\
01517     (__Pyx_fits_Py_ssize_t(i, type, is_signed) ?\
01518     __Pyx_SetItemInt_Fast(o, (Py_ssize_t)i, v, is_list, wraparound, boundscheck) :\
01519     (is_list ? (PyErr_SetString(PyExc_IndexError, "list assignment index out of range"), -1) :\
01520     __Pyx_SetItemInt_Generic(o, to_py_func(i), v)))
01521 static int __Pyx_SetItemInt_Generic(PyObject *o, PyObject *j, PyObject *v);
01522 static CYTHON_INLINE int __Pyx_SetItemInt_Fast(PyObject *o, Py_ssize_t i, PyObject *v,
01523     int is_list, int wraparound, int boundscheck);
01524
01525 /* ArgTypeTest.proto */
01526 #define __Pyx_ArgTypeTest(obj, type, none_allowed, name, exact)\
01527     ((likely((Py_TYPE(obj) == type) | (none_allowed && (obj == Py_None)))) ? 1 :\
01528     __Pyx_ArgTypeTest(obj, type, name, exact))
01529 static int __Pyx_ArgTypeTest(PyObject *obj, PyTypeObject *type, const char *name, int exact);
01530
01531 /* Import.proto */
01532 static PyObject *__Pyx_Import(PyObject *name, PyObject *from_list, int level);
01533
01534 /* IncludeStringH.proto */
01535 #include <string.h>
01536
01537 /* PyObject_GenericGetAttrNoDict.proto */
01538 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
01539 static CYTHON_INLINE PyObject* __Pyx_PyObject_GenericGetAttrNoDict(PyObject* obj, PyObject*
    attr_name);
01540 #else
01541 #define __Pyx_PyObject_GenericGetAttrNoDict PyObject_GenericGetAttr
01542 #endif
01543
01544 /* PyObject_GenericGetAttr.proto */
01545 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
01546 static PyObject* __Pyx_PyObject_GenericGetAttr(PyObject* obj, PyObject* attr_name);
01547 #else
01548 #define __Pyx_PyObject_GenericGetAttr PyObject_GenericGetAttr
01549 #endif
01550
01551 /* SetVTable.proto */
01552 static int __Pyx_SetVtable(PyObject *dict, void *vtable);
01553
01554 /* PyObjectGetAttrStrNoError.proto */
01555 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStrNoError(PyObject* obj, PyObject* attr_name);
01556
01557 /* SetupReduce.proto */
01558 static int __Pyx_setup_reduce(PyObject* type_obj);
01559
01560 /* BytesEquals.proto */
01561 static CYTHON_INLINE int __Pyx_PyBytes_Equals(PyObject* s1, PyObject* s2, int equals);
01562
01563 /* UnicodeEquals.proto */
01564 static CYTHON_INLINE int __Pyx_PyUnicode_Equals(PyObject* s1, PyObject* s2, int equals);
01565
01566 /* CLineInTraceback.proto */
01567 #ifdef CYTHON_CLINE_IN_TRACEBACK
01568 #define __Pyx_CLineForTraceback(tstate, c_line) (((CYTHON_CLINE_IN_TRACEBACK)) ? c_line : 0)
01569 #else
01570 static int __Pyx_CLineForTraceback(PyThreadState *tstate, int c_line);
01571 #endif
01572
01573 /* CodeObjectCache.proto */
01574 typedef struct {
01575     PyCodeObject* code_object;
01576     int code_line;
01577 } __Pyx_CodeObjectCacheEntry;
01578 struct __Pyx_CodeObjectCache {
01579     int count;
01580     int max_count;
01581     __Pyx_CodeObjectCacheEntry* entries;
01582 };
01583 static struct __Pyx_CodeObjectCache __pyx_code_cache = {0,0,NULL};
01584 static int __pyx_bisect_code_objects(__Pyx_CodeObjectCacheEntry* entries, int count, int code_line);
01585 static PyCodeObject* __pyx_find_code_object(int code_line);
01586 static void __pyx_insert_code_object(int code_line, PyCodeObject* code_object);
01587
01588 /* AddTraceback.proto */
01589 static void __Pyx_AddTraceback(const char *funcname, int c_line,
    int py_line, const char *filename);
01590
01591
01592 /* GCCDiagnostics.proto */
01593 #if defined(__GNUC__) && (__GNUC__ > 4 || (__GNUC__ == 4 && __GNUC_MINOR__ >= 6))
01594 #define __Pyx_HAS_GCC_DIAGNOSTIC
01595 #endif
01596
01597 /* CppExceptionConversion.proto */
01598 #ifndef __Pyx_CppExn2PyErr
01599 #include <new>
01600 #include <typeinfo>
01601 #include <stdexcept>

```



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01602 #include <ios>
01603 static void __Pyx_CppExn2PyErr() {
01604     try {
01605         if (PyErr_Occurred())
01606             ; // let the latest Python exn pass through and ignore the current one
01607         else
01608             throw;
01609     } catch (const std::bad_alloc& exn) {
01610         PyErr_SetString(PyExc_MemoryError, exn.what());
01611     } catch (const std::bad_cast& exn) {
01612         PyErr_SetString(PyExc_TypeError, exn.what());
01613     } catch (const std::bad_typeid& exn) {
01614         PyErr_SetString(PyExc_TypeError, exn.what());
01615     } catch (const std::domain_error& exn) {
01616         PyErr_SetString(PyExc_ValueError, exn.what());
01617     } catch (const std::invalid_argument& exn) {
01618         PyErr_SetString(PyExc_ValueError, exn.what());
01619     } catch (const std::ios_base::failure& exn) {
01620         PyErr_SetString(PyExc_IOError, exn.what());
01621     } catch (const std::out_of_range& exn) {
01622         PyErr_SetString(PyExc_IndexError, exn.what());
01623     } catch (const std::overflow_error& exn) {
01624         PyErr_SetString(PyExc_OverflowError, exn.what());
01625     } catch (const std::range_error& exn) {
01626         PyErr_SetString(PyExc_ArithmeticError, exn.what());
01627     } catch (const std::underflow_error& exn) {
01628         PyErr_SetString(PyExc_ArithmeticError, exn.what());
01629     } catch (const std::exception& exn) {
01630         PyErr_SetString(PyExc_RuntimeError, exn.what());
01631     }
01632     catch (...)
01633     {
01634         PyErr_SetString(PyExc_RuntimeError, "Unknown exception");
01635     }
01636 }
01637 #endif
01638
01639 /* CIntFromPy.proto */
01640 static CYTHON_INLINE int __Pyx_PyInt_As_int(PyObject *);
01641
01642 /* CIntToPy.proto */
01643 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_int(int value);
01644
01645 /* CIntToPy.proto */
01646 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_long(long value);
01647
01648 /* CIntFromPy.proto */
01649 static CYTHON_INLINE long __Pyx_PyInt_As_long(PyObject *);
01650
01651 /* FastTypeChecks.proto */
01652 #if CYTHON_COMPILING_IN_CPYTHON
01653 #define __Pyx_TypeCheck(obj, type) __Pyx_IsSubtype(Py_TYPE(obj), (PyTypeObject *)type)
01654 static CYTHON_INLINE int __Pyx_IsSubtype(PyTypeObject *a, PyTypeObject *b);
01655 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches(PyObject *err, PyObject *type);
01656 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches2(PyObject *err, PyObject *type1, PyObject
    *type2);
01657 #else
01658 #define __Pyx_TypeCheck(obj, type) PyObject_TypeCheck(obj, (PyTypeObject *)type)
01659 #define __Pyx_PyErr_GivenExceptionMatches(err, type) PyErr_GivenExceptionMatches(err, type)
01660 #define __Pyx_PyErr_GivenExceptionMatches2(err, type1, type2) (PyErr_GivenExceptionMatches(err, type1)
    || PyErr_GivenExceptionMatches(err, type2))
01661 #endif
01662 #define __Pyx_PyException_Check(obj) __Pyx_TypeCheck(obj, PyExc_Exception)
01663
01664 /* FetchCommonType.proto */
01665 static PyTypeObject* __Pyx_FetchCommonType(PyTypeObject* type);
01666
01667 /* PyObjectGetMethod.proto */
01668 static int __Pyx_PyObject_GetMethod(PyObject *obj, PyObject *name, PyObject **method);
01669
01670 /* PyObjectCallMethod1.proto */
01671 static PyObject* __Pyx_PyObject_CallMethod1(PyObject* obj, PyObject* method_name, PyObject* arg);
01672
01673 /* CoroutineBase.proto */
01674 typedef PyObject *(*__pyx_coroutine_body_t)(PyObject *, PyThreadState *, PyObject *);
01675 #if CYTHON_USE_EXC_INFO_STACK
01676 #define __Pyx_ExcInfoStruct __PyErr_StackItem
01677 #else
01678 typedef struct {
01679     PyObject *exc_type;
01680     PyObject *exc_value;
01681     PyObject *exc_traceback;
01682 } __Pyx_ExcInfoStruct;
01683 #endif
01684 typedef struct {
01685     PyObject_HEAD
01686     __pyx_coroutine_body_t body;

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01687     PyObject *closure;
01688     __Pyx_ExcInfoStruct gi_exc_state;
01689     PyObject *gi_weakreflist;
01690     PyObject *classobj;
01691     PyObject *yieldfrom;
01692     PyObject *gi_name;
01693     PyObject *gi_qualname;
01694     PyObject *gi_modulename;
01695     PyObject *gi_code;
01696     PyObject *gi_frame;
01697     int resume_label;
01698     char is_running;
01699 } __pyx_CoroutineObject;
01700 static __pyx_CoroutineObject *__Pyx__Coroutine_New(
01701     PyTypeObject *type, __pyx_coroutine_body_t body, PyObject *code, PyObject *closure,
01702     PyObject *name, PyObject *qualname, PyObject *module_name);
01703 static __pyx_CoroutineObject *__Pyx__Coroutine_NewInit(
01704     __pyx_CoroutineObject *gen, __pyx_coroutine_body_t body, PyObject *code, PyObject
01705     *closure,
01706     PyObject *name, PyObject *qualname, PyObject *module_name);
01707 static CYTHON_INLINE void __Pyx_Coroutine_ExceptionClear(__Pyx_ExcInfoStruct *self);
01708 static int __Pyx_Coroutine_clear(PyObject *self);
01709 static PyObject *__Pyx_Coroutine_Send(PyObject *self, PyObject *value);
01710 static PyObject *__Pyx_Coroutine_Close(PyObject *self);
01711 static PyObject *__Pyx_Coroutine_Throw(PyObject *gen, PyObject *args);
01712 #if CYTHON_USE_EXC_INFO_STACK
01713 #define __Pyx_Coroutine_SwapException(self)
01714 #define __Pyx_Coroutine_ResetAndClearException(self)
01715     __Pyx_Coroutine_ExceptionClear(&(self)->gi_exc_state)
01716 #else
01717 #define __Pyx_Coroutine_SwapException(self) {\
01718     __Pyx_ExceptionSwap(&(self)->gi_exc_state.exc_type, &(self)->gi_exc_state.exc_value,\
01719     &(self)->gi_exc_state.exc_traceback);\
01720     __Pyx_Coroutine_ResetFrameBackpointer(&(self)->gi_exc_state);\
01721 }
01722 #define __Pyx_Coroutine_ResetAndClearException(self) {\
01723     __Pyx_ExceptionReset((self)->gi_exc_state.exc_type, (self)->gi_exc_state.exc_value,\
01724     (self)->gi_exc_state.exc_traceback);\
01725     (self)->gi_exc_state.exc_type = (self)->gi_exc_state.exc_value =
01726     (self)->gi_exc_state.exc_traceback = NULL;\
01727 }
01728 #endif
01729 #if CYTHON_FAST_THREAD_STATE
01730 #define __Pyx_PyGen_FetchStopIterationValue(pvalue)\
01731     __Pyx_PyGen_FetchStopIterationValue(__pyx_tstate, pvalue)
01732 #else
01733 #define __Pyx_PyGen_FetchStopIterationValue(pvalue)\
01734     __Pyx_PyGen_FetchStopIterationValue(__Pyx_PyThreadState_Current, pvalue)
01735 #endif
01736 static int __Pyx_PyGen_FetchStopIterationValue(PyThreadState *tstate, PyObject **pvalue);
01737 static CYTHON_INLINE void __Pyx_Coroutine_ResetFrameBackpointer(__Pyx_ExcInfoStruct *exc_state);
01738
01739 /* PatchModuleWithCoroutine.proto */
01740 static PyObject* __Pyx_Coroutine_patch_module(PyObject* module, const char* py_code);
01741
01742 /* PatchGeneratorABC.proto */
01743 static int __Pyx_patch_abc(void);
01744
01745 /* Generator.proto */
01746 #define __Pyx_Generator_USED
01747 static PyTypeObject *__pyx_GeneratorType = 0;
01748 #define __Pyx_Generator_CheckExact(obj) (Py_TYPE(obj) == __pyx_GeneratorType)
01749 #define __Pyx_Generator_New(body, code, closure, name, qualname, module_name)\
01750     __Pyx__Coroutine_New(__pyx_GeneratorType, body, code, closure, name, qualname, module_name)
01751 static PyObject *__Pyx_Generator_Next(PyObject *self);
01752 static int __pyx_Generator_init(void);
01753
01754 /* CheckBinaryVersion.proto */
01755 static int __Pyx_check_binary_version(void);
01756
01757 /* InitStrings.proto */
01758 static int __Pyx_InitStrings(__Pyx_StringTabEntry *t);
01759
01760 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_9index_set_wrap(struct __pyx_obj_8PyClical_index_set
01761 * __pyx_v_self, IndexSet __pyx_v_other); /* proto*/
01762 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_9index_set_unwrap(struct __pyx_obj_8PyClical_index_set
01763 * __pyx_v_self); /* proto*/
01764 static PyObject *__pyx_f_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set * __pyx_v_self,
01765 int __pyx_skip_dispatch); /* proto*/
01766 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_8clifford_wrap(struct __pyx_obj_8PyClical_clifford
01767 * __pyx_v_self, Clifford __pyx_v_other); /* proto*/
01768 static CYTHON_INLINE Clifford __pyx_f_8PyClical_8clifford_unwrap(struct __pyx_obj_8PyClical_clifford
01769 * __pyx_v_self); /* proto*/
01770 static PyObject *__pyx_f_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford * __pyx_v_self,
01771 int __pyx_skip_dispatch); /* proto*/
01772
01773 /* Module declarations from 'libc++.vector' */

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01763
01764 /* Module declarations from 'glucat' */
01765
01766 /* Module declarations from 'libc.string' */
01767
01768 /* Module declarations from 'libcpp.string' */
01769
01770 /* Module declarations from 'PyClicl' */
01771 static PyTypeObject *__pyx_ptype_8PyClicl_index_set = 0;
01772 static PyTypeObject *__pyx_ptype_8PyClicl_clifford = 0;
01773 static PyTypeObject *__pyx_ptype_8PyClicl__pyx_scope_struct__iter__ = 0;
01774 static CYTHON_INLINE IndexSet __pyx_f_8PyClicl_toIndexSet(PyObject *); /*proto*/
01775 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_compare(PyObject *, PyObject *, int
__pyx_skip_dispatch); /*proto*/
01776 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_min_neg(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01777 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_max_pos(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01778 static CYTHON_INLINE std::vector<scalar_t> __pyx_f_8PyClicl_list_to_vector(PyObject *); /*proto*/
01779 static CYTHON_INLINE Clifford __pyx_f_8PyClicl_toClifford(PyObject *); /*proto*/
01780 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_error_squared_tol(PyObject *, int
__pyx_skip_dispatch); /*proto*/
01781 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_error_squared(PyObject *, PyObject *, PyObject *, int
__pyx_skip_dispatch); /*proto*/
01782 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_approx_equal(PyObject *, PyObject *, int
__pyx_skip_dispatch, struct __pyx_opt_args_8PyClicl_approx_equal *__pyx_optional_args); /*proto*/
01783 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_inv(PyObject *, int __pyx_skip_dispatch); /*proto*/
01784 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_scalar(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01785 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_real(PyObject *, int __pyx_skip_dispatch); /*proto*/
01786 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_imag(PyObject *, int __pyx_skip_dispatch); /*proto*/
01787 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_pure(PyObject *, int __pyx_skip_dispatch); /*proto*/
01788 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_even(PyObject *, int __pyx_skip_dispatch); /*proto*/
01789 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_odd(PyObject *, int __pyx_skip_dispatch); /*proto*/
01790 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_involute(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01791 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_reverse(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01792 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_conj(PyObject *, int __pyx_skip_dispatch); /*proto*/
01793 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_quad(PyObject *, int __pyx_skip_dispatch); /*proto*/
01794 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_norm(PyObject *, int __pyx_skip_dispatch); /*proto*/
01795 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_abs(PyObject *, int __pyx_skip_dispatch); /*proto*/
01796 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_max_abs(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01797 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_pow(PyObject *, PyObject *, int __pyx_skip_dispatch);
/*proto*/
01798 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_outer_pow(PyObject *, PyObject *, int
__pyx_skip_dispatch); /*proto*/
01799 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_complexifier(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01800 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_sqrt(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_sqrt *__pyx_optional_args); /*proto*/
01801 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_exp(PyObject *, int __pyx_skip_dispatch); /*proto*/
01802 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_log(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_log *__pyx_optional_args); /*proto*/
01803 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_cos(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_cos *__pyx_optional_args); /*proto*/
01804 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_acos(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_acos *__pyx_optional_args); /*proto*/
01805 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_cosh(PyObject *, int __pyx_skip_dispatch); /*proto*/
01806 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_acosh(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_acosh *__pyx_optional_args); /*proto*/
01807 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_sin(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_sin *__pyx_optional_args); /*proto*/
01808 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_asin(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_asin *__pyx_optional_args); /*proto*/
01809 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_sinh(PyObject *, int __pyx_skip_dispatch); /*proto*/
01810 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_asinh(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_asinh *__pyx_optional_args); /*proto*/
01811 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_tan(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_tan *__pyx_optional_args); /*proto*/
01812 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_atan(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_atan *__pyx_optional_args); /*proto*/
01813 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_tanh(PyObject *, int __pyx_skip_dispatch); /*proto*/
01814 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_atanh(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClicl_atanh *__pyx_optional_args); /*proto*/
01815 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_random_clifford(struct __pyx_obj_8PyClicl_index_set
*, int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClicl_random_clifford *__pyx_optional_args);
/*proto*/
01816 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_cga3(PyObject *, int __pyx_skip_dispatch); /*proto*/
01817 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_cga3std(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01818 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_agc3(PyObject *, int __pyx_skip_dispatch); /*proto*/
01819 static CYTHON_INLINE PyObject *__pyx_convert_PyObject_string_to_py_std_in_string(std::string const
&); /*proto*/
01820 static CYTHON_INLINE PyObject *__pyx_convert_PyUnicode_string_to_py_std_in_string(std::string const
&); /*proto*/

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01821 static CYTHON_INLINE PyObject *__pyx_convert_PyStr_string_to_py_std__in_string(std::string const &);
01822 /*proto*/
01823 static CYTHON_INLINE PyObject *__pyx_convert_PyBytes_string_to_py_std__in_string(std::string const &);
01824 /*proto*/
01825 static CYTHON_INLINE PyObject *__pyx_convert_PyByteArray_string_to_py_std__in_string(std::string const
&); /*proto*/
01824 #define __Pyx_MODULE_NAME "PyClical"
01825 extern int __pyx_module_is_main_PyClical;
01826 int __pyx_module_is_main_PyClical = 0;
01827
01828 /* Implementation of 'PyClical' */
01829 static PyObject *__pyx_builtin_IndexError;
01830 static PyObject *__pyx_builtin_RuntimeError;
01831 static PyObject *__pyx_builtin_TypeError;
01832 static PyObject *__pyx_builtin_ValueError;
01833 static PyObject *__pyx_builtin_NotImplemented;
01834 static PyObject *__pyx_builtin_range;
01835 static PyObject *__pyx_builtin_xrange;
01836 static const char __pyx_k_[] = ".";
01837 static const char __pyx_k_e[] = "e";
01838 static const char __pyx_k_i[] = "i";
01839 static const char __pyx_k_m[] = "m";
01840 static const char __pyx_k_p[] = "p";
01841 static const char __pyx_k_q[] = "q";
01842 static const char __pyx_k_2[] = " ";
01843 static const char __pyx_k_5[] = ":";
01844 static const char __pyx_k_6[] = "\n\t";
01845 static const char __pyx_k_7[] = "(";
01846 static const char __pyx_k_8[] = ", ";
01847 static const char __pyx_k_9[] = ").";
01848 static const char __pyx_k_cl[] = "cl";
01849 static const char __pyx_k_pi[] = "pi";
01850 static const char __pyx_k_abc[] = "abc";
01851 static const char __pyx_k_cos[] = "cos";
01852 static const char __pyx_k_exp[] = "exp";
01853 static const char __pyx_k_frm[] = "frm";
01854 static const char __pyx_k_inv[] = "inv";
01855 static const char __pyx_k_ist[] = "ist";
01856 static const char __pyx_k_ixt[] = "ixt";
01857 static const char __pyx_k_lhs[] = "lhs";
01858 static const char __pyx_k_log[] = "log";
01859 static const char __pyx_k_max[] = "max";
01860 static const char __pyx_k_min[] = "min";
01861 static const char __pyx_k_obj[] = "obj";
01862 static const char __pyx_k_odd[] = "odd";
01863 static const char __pyx_k_pow[] = "pow";
01864 static const char __pyx_k_rhs[] = "rhs";
01865 static const char __pyx_k_sin[] = "sin";
01866 static const char __pyx_k_tan[] = "tan";
01867 static const char __pyx_k_tau[] = "tau";
01868 static const char __pyx_k_tol[] = "tol";
01869 static const char __pyx_k_Real[] = "Real";
01870 static const char __pyx_k_acos[] = "acos";
01871 static const char __pyx_k_args[] = "args";
01872 static const char __pyx_k_asin[] = "asin";
01873 static const char __pyx_k_atan[] = "atan";
01874 static const char __pyx_k_conj[] = "conj";
01875 static const char __pyx_k_copy[] = "copy";
01876 static const char __pyx_k_cosh[] = "cosh";
01877 static const char __pyx_k_even[] = "even";
01878 static const char __pyx_k_fill[] = "fill";
01879 static const char __pyx_k_from[] = " from ";
01880 static const char __pyx_k_iter[] = "__iter__";
01881 static const char __pyx_k_main[] = "__main__";
01882 static const char __pyx_k_math[] = "math";
01883 static const char __pyx_k_name[] = "__name__";
01884 static const char __pyx_k_norm[] = "norm";
01885 static const char __pyx_k_pure[] = "pure";
01886 static const char __pyx_k_quad[] = "quad";
01887 static const char __pyx_k_send[] = "send";
01888 static const char __pyx_k_sinh[] = "sinh";
01889 static const char __pyx_k_sqrt[] = "sqrt";
01890 static const char __pyx_k_tanh[] = "tanh";
01891 static const char __pyx_k_test[] = "_test";
01892 static const char __pyx_k_UTF_8[] = "UTF-8";
01893 static const char __pyx_k_acosh[] = "acosh";
01894 static const char __pyx_k_asinh[] = "asinh";
01895 static const char __pyx_k_atanh[] = "atanh";
01896 static const char __pyx_k_close[] = "close";
01897 static const char __pyx_k_grade[] = "grade";
01898 static const char __pyx_k_istpq[] = "istpq";
01899 static const char __pyx_k_nbar3[] = "nbar3";
01900 static const char __pyx_k_ninf3[] = "ninf3";
01901 static const char __pyx_k_other[] = "other";
01902 static const char __pyx_k_range[] = "range";
01903 static const char __pyx_k_throw[] = "throw";
01904 static const char __pyx_k_using[] = " using (";

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01905 static const char __pyx_k_utf_8[] = "utf-8";
01906 static const char __pyx_k_value[] = " value ";
01907 static const char __pyx_k_encode[] = "encode";
01908 static const char __pyx_k_import[] = "__import__";
01909 static const char __pyx_k_reduce[] = "__reduce__";
01910 static const char __pyx_k_scalar[] = "scalar";
01911 static const char __pyx_k_test_2[] = "__test__";
01912 static const char __pyx_k_xrange[] = "xrange";
01913 static const char __pyx_k_doctest[] = "doctest";
01914 static const char __pyx_k_invalid[] = " invalid ";
01915 static const char __pyx_k_numbers[] = "numbers";
01916 static const char __pyx_k_reverse[] = "reverse";
01917 static const char __pyx_k_testmod[] = "testmod";
01918 static const char __pyx_k_version[] = "__version__";
01919 static const char __pyx_k_Integral[] = "Integral";
01920 static const char __pyx_k_PyClical[] = "PyClical";
01921 static const char __pyx_k_Sequence[] = "Sequence";
01922 static const char __pyx_k_as_frame[] = " as frame:\n\t";
01923 static const char __pyx_k_clifford[] = "clifford";
01924 static const char __pyx_k_getstate[] = "__getstate__";
01925 static const char __pyx_k_involute[] = "involute";
01926 static const char __pyx_k_setstate[] = "__setstate__";
01927 static const char __pyx_k_to_frame[] = " to frame ";
01928 static const char __pyx_k_TypeError[] = "TypeError";
01929 static const char __pyx_k_index_set[] = "index_set";
01930 static const char __pyx_k_outer_pow[] = "outer_pow";
01931 static const char __pyx_k_reduce_ex[] = "__reduce_ex__";
01932 static const char __pyx_k_threshold[] = "threshold";
01933 static const char __pyx_k_IndexError[] = "IndexError";
01934 static const char __pyx_k_ValueError[] = "ValueError";
01935 static const char __pyx_k_pyx_vtable[] = "__pyx_vtable__";
01936 static const char __pyx_k_collections[] = "collections";
01937 static const char __pyx_k_e_line_1936[] = "e (line 1936)";
01938 static const char __pyx_k_PyClical_pyx[] = "PyClical.pyx";
01939 static const char __pyx_k_RuntimeError[] = "RuntimeError";
01940 static const char __pyx_k_abs_line_1522[] = "abs (line 1522)";
01941 static const char __pyx_k_cos_line_1651[] = "cos (line 1651)";
01942 static const char __pyx_k_exp_line_1614[] = "exp (line 1614)";
01943 static const char __pyx_k_inv_line_1378[] = "inv (line 1378)";
01944 static const char __pyx_k_log_line_1628[] = "log (line 1628)";
01945 static const char __pyx_k_odd_line_1446[] = "odd (line 1446)";
01946 static const char __pyx_k_pow_line_1543[] = "pow (line 1543)";
01947 static const char __pyx_k_reduce_cython[] = "__reduce_cython__";
01948 static const char __pyx_k_sin_line_1728[] = "sin (line 1728)";
01949 static const char __pyx_k_tan_line_1801[] = "tan (line 1801)";
01950 static const char __pyx_k_using_invalid[] = " using invalid ";
01951 static const char __pyx_k_Cannot_reframe[] = "Cannot reframe";
01952 static const char __pyx_k_NotImplemented[] = "NotImplemented";
01953 static const char __pyx_k_Not_applicable[] = "Not applicable.";
01954 static const char __pyx_k_acos_line_1668[] = "acos (line 1668)";
01955 static const char __pyx_k_agc3_line_1893[] = "agc3 (line 1893)";
01956 static const char __pyx_k_asin_line_1747[] = "asin (line 1747)";
01957 static const char __pyx_k_atan_line_1818[] = "atan (line 1818)";
01958 static const char __pyx_k_cga3_line_1873[] = "cga3 (line 1873)";
01959 static const char __pyx_k_conj_line_1485[] = "conj (line 1485)";
01960 static const char __pyx_k_cosh_line_1689[] = "cosh (line 1689)";
01961 static const char __pyx_k_even_line_1437[] = "even (line 1437)";
01962 static const char __pyx_k_imag_line_1415[] = "imag (line 1415)";
01963 static const char __pyx_k_invalid_string[] = " invalid string ";
01964 static const char __pyx_k_norm_line_1511[] = "norm (line 1511)";
01965 static const char __pyx_k_pure_line_1426[] = "pure (line 1426)";
01966 static const char __pyx_k_quad_line_1500[] = "quad (line 1500)";
01967 static const char __pyx_k_real_line_1404[] = "real (line 1404)";
01968 static const char __pyx_k_scalar_epsilon[] = "scalar_epsilon";
01969 static const char __pyx_k_sinh_line_1768[] = "sinh (line 1768)";
01970 static const char __pyx_k_sqrt_line_1591[] = "sqrt (line 1591)";
01971 static const char __pyx_k_tanh_line_1835[] = "tanh (line 1835)";
01972 static const char __pyx_k_acosh_line_1705[] = "acosh (line 1705)";
01973 static const char __pyx_k_asinh_line_1782[] = "asinh (line 1782)";
01974 static const char __pyx_k_atanh_line_1847[] = "atanh (line 1847)";
01975 static const char __pyx_k_istpq_line_1949[] = "istpq (line 1949)";
01976 static const char __pyx_k_setstate_cython[] = "__setstate_cython__";
01977 static const char __pyx_k_compare_line_492[] = "compare (line 492)";
01978 static const char __pyx_k_index_set_iter[] = "index_set.__iter__";
01979 static const char __pyx_k_max_pos_line_513[] = "max_pos (line 513)";
01980 static const char __pyx_k_min_neg_line_504[] = "min_neg (line 504)";
01981 static const char __pyx_k_scalar_line_1393[] = "scalar (line 1393)";
01982 static const char __pyx_k_cga3std_line_1882[] = "cga3std (line 1882)";
01983 static const char __pyx_k_max_abs_line_1531[] = "max_abs (line 1531)";
01984 static const char __pyx_k_reverse_line_1470[] = "reverse (line 1470)";
01985 static const char __pyx_k_cline_in_traceback[] = "cline_in_traceback";
01986 static const char __pyx_k_involute_line_1455[] = "involute (line 1455)";
01987 static const char __pyx_k_outer_pow_line_1567[] = "outer_pow (line 1567)";
01988 static const char __pyx_k_clifford_inv_line_926[] = "clifford.inv (line 926)";
01989 static const char __pyx_k_clifford_pow_line_980[] = "clifford.pow (line 980)";
01990 static const char __pyx_k_approx_equal_line_1359[] = "approx_equal (line 1359)";
01991 static const char __pyx_k_clifford_abs_line_1175[] = "clifford.abs (line 1175)";

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01992 static const char __pyx_k_clifford_copy_line_556[] = "clifford.copy (line 556)";
01993 static const char __pyx_k_clifford_odd_line_1070[] = "clifford.odd (line 1070)";
01994 static const char __pyx_k_complexifier_line_1576[] = "complexifier (line 1576)";
01995 static const char __pyx_k_index_set_copy_line_65[] = "index_set.copy (line 65)";
01996 static const char __pyx_k_index_set_max_line_351[] = "index_set.max (line 351)";
01997 static const char __pyx_k_index_set_min_line_342[] = "index_set.min (line 342)";
01998 static const char __pyx_k_clifford_conj_line_1138[] = "clifford.conj (line 1138)";
01999 static const char __pyx_k_clifford_even_line_1061[] = "clifford.even (line 1061)";
02000 static const char __pyx_k_clifford_norm_line_1164[] = "clifford.norm (line 1164)";
02001 static const char __pyx_k_clifford_pure_line_1050[] = "clifford.pure (line 1050)";
02002 static const char __pyx_k_clifford_quad_line_1153[] = "clifford.quad (line 1153)";
02003 static const char __pyx_k_error_squared_line_1346[] = "error_squared (line 1346)";
02004 static const char __pyx_k_Unary_print_clifford_l_1[] = "\n          Unary -. \n\n          >>
print(-clifford(\"{1}\")\n          -{1}\n          ";
02005 static const char __pyx_k_clifford_or_line_939[] = "clifford.__or__ (line 939)";
02006 static const char __pyx_k_clifford_frame_line_1224[] = "clifford.frame (line 1224)";
02007 static const char __pyx_k_clifford_hidden_doctests[] = "clifford.hidden_doctests";
02008 static const char __pyx_k_clifford_isinf_line_1206[] = "clifford.isinf (line 1206)";
02009 static const char __pyx_k_clifford_isnan_line_1215[] = "clifford.isnan (line 1215)";
02010 static const char __pyx_k_index_set_count_line_315[] = "index_set.count (line 315)";
02011 static const char __pyx_k_clifford_add_line_740[] = "clifford.__add__ (line 740)";
02012 static const char __pyx_k_clifford_and_line_836[] = "clifford.__and__ (line 836)";
02013 static const char __pyx_k_clifford_ior_line_950[] = "clifford.__ior__ (line 950)";
02014 static const char __pyx_k_clifford_mod_line_806[] = "clifford.__mod__ (line 806)";
02015 static const char __pyx_k_clifford_mul_line_780[] = "clifford.__mul__ (line 780)";
02016 static const char __pyx_k_clifford_neg_line_722[] = "clifford.__neg__ (line 722)";
02017 static const char __pyx_k_clifford_pos_line_731[] = "clifford.__pos__ (line 731)";
02018 static const char __pyx_k_clifford_pow_line_961[] = "clifford.__pow__ (line 961)";
02019 static const char __pyx_k_clifford_sub_line_760[] = "clifford.__sub__ (line 760)";
02020 static const char __pyx_k_clifford_xor_line_866[] = "clifford.__xor__ (line 866)";
02021 static const char __pyx_k_clifford_reframe_line_649[] = "clifford.reframe (line 649)";
02022 static const char __pyx_k_clifford_scalar_line_1039[] = "clifford.scalar (line 1039)";
02023 static const char __pyx_k_index_set_or_line_293[] = "index_set.__or__ (line 293)";
02024 static const char __pyx_k_index_set_hidden_doctests[] = "index_set.hidden_doctests";
02025 static const char __pyx_k_random_clifford_line_1864[] = "random_clifford (line 1864)";
02026 static const char __pyx_k_Cannot_take_vector_part_of[] = "Cannot take vector part of ";
02027 static const char __pyx_k_Unary_print_clifford_l_1_2[] = "\n          Unary +. \n\n          >>
print(+clifford(\"{1}\")\n          {1}\n          ";
02028 static const char __pyx_k_clifford_iadd_line_751[] = "clifford.__iadd__ (line 751)";
02029 static const char __pyx_k_clifford_iand_line_851[] = "clifford.__iand__ (line 851)";
02030 static const char __pyx_k_clifford_idiv_line_911[] = "clifford.__idiv__ (line 911)";
02031 static const char __pyx_k_clifford_imod_line_821[] = "clifford.__imod__ (line 821)";
02032 static const char __pyx_k_clifford_imul_line_793[] = "clifford.__imul__ (line 793)";
02033 static const char __pyx_k_clifford_isub_line_771[] = "clifford.__isub__ (line 771)";
02034 static const char __pyx_k_clifford_iter_line_638[] = "clifford.__iter__ (line 638)";
02035 static const char __pyx_k_clifford_ixor_line_881[] = "clifford.__ixor__ (line 881)";
02036 static const char __pyx_k_clifford_str_line_1244[] = "clifford.__str__ (line 1244)";
02037 static const char __pyx_k_clifford_max_abs_line_1184[] = "clifford.max_abs (line 1184)";
02038 static const char __pyx_k_clifford_reverse_line_1123[] = "clifford.reverse (line 1123)";
02039 static const char __pyx_k_index_set_and_line_271[] = "index_set.__and__ (line 271)";
02040 static const char __pyx_k_index_set_ior_line_304[] = "index_set.__ior__ (line 304)";
02041 static const char __pyx_k_index_set_str_line_395[] = "index_set.__str__ (line 395)";
02042 static const char __pyx_k_index_set_xor_line_249[] = "index_set.__xor__ (line 249)";
02043 static const char __pyx_k_clifford_call_line_1020[] = "clifford.__call__ (line 1020)";
02044 static const char __pyx_k_clifford_repr_line_1235[] = "clifford.__repr__ (line 1235)";
02045 static const char __pyx_k_clifford_involute_line_1107[] = "clifford.involute (line 1107)";
02046 static const char __pyx_k_error_squared_tol_line_1337[] = "error_squared_tol (line 1337)";
02047 static const char __pyx_k_index_set_iand_line_282[] = "index_set.__iand__ (line 282)";
02048 static const char __pyx_k_index_set_iter_line_229[] = "index_set.__iter__ (line 229)";
02049 static const char __pyx_k_index_set_ixor_line_260[] = "index_set.__ixor__ (line 260)";
02050 static const char __pyx_k_index_set_repr_line_384[] = "index_set.__repr__ (line 384)";
02051 static const char __pyx_k_clifford_outer_pow_line_1004[] = "clifford.outer_pow (line 1004)";
02052 static const char __pyx_k_clifford_truncated_line_1195[] = "clifford.truncated (line 1195)";
02053 static const char __pyx_k_index_set_count_neg_line_324[] = "index_set.count_neg (line 324)";
02054 static const char __pyx_k_index_set_count_pos_line_333[] = "index_set.count_pos (line 333)";
02055 static const char __pyx_k_clifford_getitem_line_707[] = "clifford.__getitem__ (line 707)";
02056 static const char __pyx_k_clifford_truediv_line_896[] = "clifford.__truediv__ (line 896)";
02057 static const char __pyx_k_index_set_invert_line_240[] = "index_set.__invert__ (line 240)";
02058 static const char __pyx_k_Abbreviation_for_index_set_qp[] = "\n          Abbreviation for
index_set({-q,...p}).\n\n          >> print(istpq(2,3))\n          {-3,-2,-1,1,2}\n          ";
02059 static const char __pyx_k_Conjugation_reverse_o_involute[] = "\n          Conjugation, reverse o
involute == involute o reverse.\n\n          >> print((clifford(\"{1}\")).conj())\n          -{1}\n\n
>> print((clifford(\"{2}\") * clifford(\"{1}\")).conj())\n          {1,2}\n\n          >>
print((clifford(\"{1}\") * clifford(\"{2}\")).conj())\n          -{1,2}\n\n          >>
print(clifford(\"1+{1}+{1,2}\").conj())\n          1-{1}-{1,2}\n\n          ";
02060 static const char __pyx_k_Geometric_product_x_clifford_2[] = "\n          Geometric product.\n\n
>> x = clifford(2); x *= clifford(\"{2}\"); print(x)\n          2{2}\n\n          >> x = clifford(\"{1}\");
x *= clifford(\"{2}\"); print(x)\n          {1,2}\n\n          >> x = clifford(\"{1}\"); x *=
clifford(\"{1,2}\"); print(x)\n          {2}\n\n          ";
02061 static const char __pyx_k_Geometric_sum_print_clifford_1[] = "\n          Geometric sum.\n\n
print(clifford(1) + clifford(\"{2}\"))\n          1+{2}\n\n          >> print(clifford(\"{1}\") +
clifford(\"{2}\"))\n          {1}+{2}\n\n          ";
02062 static const char __pyx_k_Hyperbolic_sine_of_multivector[] = "\n          Hyperbolic sine of
multivector.\n\n
>> x=clifford(\"{1,2}\") * pi/2; print(sinh(x))\n          {1,2}\n\n          >>
x=clifford(\"{1,2}\") * pi/6; print(sinh(x))\n          0.5{1,2}\n\n          ";
02063 static const char __pyx_k_Inner_product_print_clifford_1[] = "\n          Inner product.\n\n
print(clifford(\"{1}\") & clifford(\"{2}\"))\n          0\n\n          >> print(clifford(2) &

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    clifford("\{2}\")\n          0\n          >> print(clifford("\{1}\") & clifford("\{1}\"))\n          1\n
    >> print(clifford("\{1}\") & clifford("\{1,2}\"))\n          {2}\n          ";
02064 static const char __pyx_k_Inverse_tangent_of_multivector[] = "\n          Inverse tangent of multivector
with optional complexifier.\n\n          >> s=index_set({1,2,3}); x=clifford("\{1}\");
print(tan(atan(x,s),s))\n          {1}\n          >> x=clifford("\{1}\"); print(tan(atan(x)))\n          {1}\n          ";
02065 static const char __pyx_k_Iterate_over_the_indices_of_an[] = "\n          Iterate over the indices of an
index_set.\n\n          >> for i in index_set({-3,4,7}):print(i, end=",")\n          -3,4,7,\n          ";
02066 static const char __pyx_k_Maximum_member_index_set_1_1_2[] = "\n          Maximum member.\n\n          >>
index_set({-1,1,2}).max()\n          2\n          ";
02067 static const char __pyx_k_Maximum_positive_index_or_0_if[] = "\n          Maximum positive index, or 0 if
none.\n\n          >> max_pos(index_set({1,2}))\n          2\n          ";
02068 static const char __pyx_k_Minimum_member_index_set_1_1_2[] = "\n          Minimum member.\n\n          >>
index_set({-1,1,2}).min()\n          -1\n          ";
02069 static const char __pyx_k_Minimum_negative_index_or_0_if[] = "\n          Minimum negative index, or 0 if
none.\n\n          >> min_neg(index_set({1,2}))\n          0\n          ";
02070 static const char __pyx_k_Odd_part_of_multivector_sum_of[] = "\n          Odd part of multivector, sum
of odd grade terms.\n\n          >> print(clifford("\{1+1+{1,2}\").odd())\n          {1}\n          ";
02071 static const char __pyx_k_Outer_product_power_x_clifford[] = "\n          Outer product power.\n\n          >>
x=clifford("\{2+{1}\"); print(x.outer_pow(0))\n          1\n          >> x=clifford("\{2+{1}\");
print(x.outer_pow(1))\n          2+{1}\n          >> x=clifford("\{2+{1}\"); print(x.outer_pow(2))\n          1+3{1}+3{1,2}\n          ";
02072 static const char __pyx_k_Outer_product_print_clifford_1[] = "\n          Outer product.\n\n          >>
print(clifford("\{1}\") ^ clifford("\{2}\"))\n          {1,2}\n          >> print(clifford(2) ^
clifford("\{2}\"))\n          2{2}\n          >> print(clifford("\{1}\") ^ clifford("\{1}\"))\n          0\n
>> print(clifford("\{1}\") ^ clifford("\{1,2}\"))\n          0\n          ";
02073 static const char __pyx_k_Power_self_to_the_m_x_clifford[] = "\n          Power: self to the m.\n\n          >>
x=clifford("\{1}\"); print(x ** 2)\n          1\n          >> x=clifford("\{2}\"); print(x ** 2)\n          4\n
>> x=clifford("\{2+{1}\"); print(x ** 0)\n          1\n          >> x=clifford("\{2+{1}\");
print(x ** 1)\n          2+{1}\n          >> x=clifford("\{2+{1}\"); print(x ** 2)\n          5+4{1}\n
>> i=clifford("\{1,2}\"); print(exp(pi/2) * (i ** i))\n          1\n          ";
02074 static const char __pyx_k_Pure_part_print_clifford_1_1_1[] = "\n          Pure part.\n\n          >>
print(clifford("\{1+{1}+{1,2}\").pure())\n          {1}+{1,2}\n          >>
print(clifford("\{1,2}\").pure())\n          {1,2}\n          ";
02075 static const char __pyx_k_Quadratic_form_rev_x_x_0_print[] = "\n          Quadratic form ==
(rev(x)*x)(0).\n\n          >> print(clifford("\{1+{1}+{1,2}\").quad())\n          3.0\n          >>
print(clifford("\{1+{-1}+{1,2}+{1,2,3}\").quad())\n          2.0\n          ";
02076 static const char __pyx_k_Quadratic_norm_error_tolerance[] = "\n          Quadratic norm error tolerance
relative to a specific multivector.\n\n          >> print(error_squared_tol(clifford("\{1}\")) * 3.0 -
error_squared_tol(clifford("\{1} - 2{2} + 3{3}\")))\n          0.0\n          ";
02077 static const char __pyx_k_Set_complement_not_print_index[] = "\n          Set complement: not.\n\n          >>
print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))\n
{-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32}\n
";
02078 static const char __pyx_k_Set_union_or_print_index_set_1[] = "\n          Set union: or.\n\n          >>
print(index_set({1}) | index_set({2}))\n          {1,2}\n          >> print(index_set({1,2}) |
index_set({2}))\n          {1,2}\n          ";
02079 static const char __pyx_k_Transform_left_hand_side_using[] = "\n          Transform left hand side,
using right hand side as a transformation.\n\n          >> x=clifford("\{1,2}\") * pi/2;
y=clifford("\{1}\"); print(y|x)\n          -{1}\n          >> x=clifford("\{1,2}\") * pi/2;
y=clifford("\{1}\"); print(y|exp(x))\n          -{1}\n          ";
02080 static const char __pyx_k_clifford_vector_part_line_1079[] = "clifford.vector_part (line 1079)";
02081 static const char __pyx_k_index_set__getitem__line_191[] = "index_set.__getitem__ (line 191)";
02082 static const char __pyx_k_index_set__setitem__line_179[] = "index_set.__setitem__ (line 179)";
02083 static const char __pyx_k_lexicographic_compare_eg_3_4_5[] = "\n          \"lexicographic compare\" eg.
{3,4,5} is less than {3,7,8};\n          -1 if a<b, +1 if a>b, 0 if a==b.\n\n          >>
compare(index_set({1,2}), index_set({-1,3}))\n          -1\n          >>
compare(index_set({-1,4}), index_set({-1,3}))\n          1\n          ";
02084 static const char __pyx_k_Abbreviation_for_clifford_index[] = "\n          Abbreviation for
clifford(index_set(obj)).\n\n          >> print(e(1))\n          {1}\n          >> print(e(-1))\n          {-1}\n          >>
print(e(0))\n          1\n          ";
02085 static const char __pyx_k_Absolute_value_of_multivector_m[] = "\n          Absolute value of multivector:
multivector 2-norm.\n\n          >> abs(clifford("\{1+{-1}+{1,2}+{1,2,3}\"))\n          2.0\n          ";
02086 static const char __pyx_k_Absolute_value_square_root_of_n[] = "\n          Absolute value: square root
of norm.\n\n          >> clifford("\{1+{-1}+{1,2}+{1,2,3}\").abs()\n          2.0\n          ";
02087 static const char __pyx_k_Cardinality_number_of_indices_i[] = "\n          Cardinality: Number of
indices included in set.\n\n          >> index_set({-1,1,2}).count()\n          3\n          ";
02088 static const char __pyx_k_Check_if_a_multivector_contains[] = "\n          Check if a multivector
contains any infinite values.\n\n          >> clifford().isinf()\n          False\n          ";
02089 static const char __pyx_k_Contraction_print_clifford_1_cl[] = "\n          Contraction.\n\n          >>
print(clifford("\{1}\") % clifford("\{2}\"))\n          0\n          >> print(clifford(2) %
clifford("\{2}\"))\n          2{2}\n          >> print(clifford("\{1}\") % clifford("\{1}\"))\n          1\n
>> print(clifford("\{1}\") % clifford("\{1,2}\"))\n          {2}\n          ";
02090 static const char __pyx_k_Contraction_x_clifford_1_x_clif[] = "\n          Contraction.\n\n          >> x
= clifford("\{1}\"); x %= clifford("\{2}\"); print(x)\n          0\n          >> x = clifford(2); x %=
clifford("\{2}\"); print(x)\n          2{2}\n          >> x = clifford("\{1}\"); x %= clifford("\{1}\");
print(x)\n          1\n          >> x = clifford("\{1}\"); x %= clifford("\{1,2}\"); print(x)\n          {2}\n
";
02091 static const char __pyx_k_Convert_CGA3_null_vector_to_Euc[] = "\n          Convert CGA3 null vector to
Euclidean 3D vector using Doran and Lasenby definition.\n\n          >> x=clifford("\{2{1}+9{2}+{3}\");
print(agc3(cga3(x)))\n          2{1}+9{2}+{3}\n          >> x=clifford("\{2{1}+9{2}+{3}\");
print(agc3(cga3(x))-x)\n          0\n          ";
02092 static const char __pyx_k_Convert_CGA3_null_vector_to_sta[] = "\n          Convert CGA3 null vector to
standard conformal null vector using Doran and Lasenby definition.\n\n          >>
x=clifford("\{2{1}+9{2}+{3}\"); print(cga3std(cga3(x)))\n          87{-1}+4{1}+18{2}+2{3}+85{4}\n          >>
x=clifford("\{2{1}+9{2}+{3}\"); print(cga3std(cga3(x))-cga3(x))\n          0\n          ";
02093 static const char __pyx_k_Convert_Euclidean_3D_multivector[] = "\n          Convert Euclidean 3D multivector

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to Conformal Geometric Algebra using Doran and Lasenby definition.\n\n    >>
x=clifford("\2{1}+9{2}+{3}\"); print(cga3(x))\n    87{-1}+4{1}+18{2}+2{3}+85{4}\n    ";
02094 static const char __pyx_k_Copy_this_clifford_object_x_cli[] = "\n    Copy this clifford
object.\n\n    >> x=clifford("\1{2}\"); y=x.copy(); print(y)\n    {2}\n    ";
02095 static const char __pyx_k_Copy_this_index_set_object_s_in[] = "\n    Copy this index_set
object.\n\n    >> s=index_set(1); t=s.copy(); print(t)\n    {1}\n    ";
02096 static const char __pyx_k_Cosine_of_multivector_with_opti[] = "\n    Cosine of multivector with
optional complexifier.\n\n    >> x=clifford("\1{2}\"); print(cos(acos(x), "\1{2,3}\"))\n    {1,2}\n
>> x=clifford("\1{2}\"); print(cos(acos(x))\n    {1,2}\n    ";
02097 static const char __pyx_k_Even_part_of_multivector_sum_of[] = "\n    Even part of multivector, sum
of even grade terms.\n\n    >> print(clifford("\1+{1}+{1,2}\").even())\n    1+{1,2}\n
";
02098 static const char __pyx_k_Exponential_of_multivector_x_cl[] = "\n    Exponential of multivector.\n\n
>> x=clifford("\1{2}\") * pi/4; print(exp(x))\n    0.7071+0.7071{1,2}\n    >> x=clifford("\1{2}\") *
pi/2; print(exp(x))\n    {1,2}\n    ";
02099 static const char __pyx_k_Geometric_difference_print_clif[] = "\n    Geometric difference.\n\n
>> print(clifford(1) - clifford("\2{2}\"))\n    1-{2}\n    >> print(clifford("\1{1}\") -
clifford("\2{2}\"))\n    {1}-{2}\n    ";
02100 static const char __pyx_k_Geometric_difference_x_clifford[] = "\n    Geometric difference.\n\n
>> x = clifford(1); x -= clifford("\2{2}\"); print(x)\n    1-{2}\n    ";
02101 static const char __pyx_k_Geometric_multiplicative_invers[] = "\n    Geometric multiplicative
inverse.\n\n    >> x = clifford("\1{1}\"); print(x.inv())\n    {1}\n    >> x = clifford(2);
print(x.inv())\n    0.5\n    >> x = clifford("\1{2}\"); print(x.inv())\n    -{1,2}\n
";
02102 static const char __pyx_k_Geometric_product_print_cliffor[] = "\n    Geometric product.\n\n
>> print(clifford("\1{1}\") * clifford("\2{2}\"))\n    {1,2}\n    >> print(clifford(2) *
clifford("\2{2}\"))\n    2{2}\n    >> print(clifford("\1{1}\") * clifford("\1{2}\"))\n
{2}\n    ";
02103 static const char __pyx_k_Geometric_quotient_print_cliffo[] = "\n    Geometric quotient.\n\n
>> print(clifford("\1{1}\") / clifford("\2{2}\"))\n    {1,2}\n    >> print(clifford(2) /
clifford("\2{2}\"))\n    2{2}\n    >> print(clifford("\1{1}\") / clifford("\1{1}\"))\n    1\n
>> print(clifford("\1{1}\") / clifford("\1{2}\"))\n    -{2}\n    ";
02104 static const char __pyx_k_Geometric_quotient_x_clifford_l[] = "\n    Geometric quotient.\n\n
>> x = clifford("\1{1}\"); x /= clifford("\2{2}\"); print(x)\n    {1,2}\n    >> x = clifford(2);
x /= clifford("\2{2}\"); print(x)\n    2{2}\n    >> x = clifford("\1{1}\"); x /=
clifford("\1{1}\"); print(x)\n    1\n    >> x = clifford("\1{1}\"); x /= clifford("\1{2}\");
print(x)\n    -{2}\n    ";
02105 static const char __pyx_k_Geometric_sum_x_clifford_l_x_cl[] = "\n    Geometric sum.\n\n    >>
x = clifford(1); x += clifford("\2{2}\"); print(x)\n    1+{2}\n    ";
02106 static const char __pyx_k_Get_the_value_of_an_index_set_o[] = "\n    Get the value of an index_set
object at an index.\n\n    >> index_set({1})[1]\n    True\n    >> index_set({1})[2]\n
False\n    >> index_set({2})[-1]\n    False\n    >> index_set({2})[1]\n    False\n
>> index_set({2})[2]\n    True\n    >> index_set({2})[33]\n    False\n    ";
02107 static const char __pyx_k_Hyperbolic_cosine_of_multivecto[] = "\n    Hyperbolic cosine of
multivector.\n\n    >> x=clifford("\1{2}\") * pi; print(cosh(x))\n    -1\n    >>
x=clifford("\1{2,3}\"); print(cosh(acos(x))\n    {1,2,3}\n    >> x=clifford("\1{2}\");
print(cosh(acos(x))\n    {1,2}\n    ";
02108 static const char __pyx_k_Hyperbolic_tangent_of_multivect[] = "\n    Hyperbolic tangent of
multivector.\n\n    >> x=clifford("\1{2}\") * pi/4; print(tanh(x))\n    {1,2}\n    ";
02109 static const char __pyx_k_Imaginary_part_deprecated_alway[] = "\n    Imaginary part: deprecated
(always 0).\n\n    >> imag(clifford("\1+{1}+{1,2}\"))\n    0.0\n    >> imag(clifford("\1{2}\"))\n
0.0\n    ";
02110 static const char __pyx_k_Inner_product_x_clifford_l_x_cl[] = "\n    Inner product.\n\n    >>
x = clifford("\1{1}\"); x &= clifford("\2{2}\"); print(x)\n    0\n    >> x = clifford(2); x &=
clifford("\2{2}\"); print(x)\n    0\n    >> x = clifford("\1{1}\"); x &= clifford("\1{1}\");
print(x)\n    1\n    >> x = clifford("\1{1}\"); x &= clifford("\1{2}\"); print(x)\n
{2}\n    ";
02111 static const char __pyx_k_Integer_power_of_multivector_ob[] = "\n    Integer power of multivector: obj
to the m.\n\n    >> x=clifford("\1{1}\"); print(pow(x,2))\n    1\n    >> x=clifford("\2{2}\");
print(pow(x,2))\n    4\n    >> x=clifford("\2+{1}\"); print(pow(x,0))\n    1\n    >>
x=clifford("\2+{1}\"); print(pow(x,1))\n    2+{1}\n    >> x=clifford("\2+{1}\"); print(pow(x,2))\n
5+4{1}\n    >> print(pow(clifford("\1+{1}+{1,2}\"),3))\n    1+3{1}+3{1,2}\n    >>
i=clifford("\1{2}\"); print(exp(pi/2) * pow(i, i))\n    1\n    ";
02112 static const char __pyx_k_Inverse_cosine_of_multivector_w[] = "\n    Inverse cosine of multivector
with optional complexifier.\n\n    >> x=clifford("\1{2}\"); print(cos(acos(x), "\1{2,3}\"))\n
{1,2}\n    >> x=clifford("\1{2}\"); print(cos(acos(x), "\{-1,1,2,3,4}\"))\n    {1,2}\n    >>
print(acos(0) / pi)\n    0.5\n    >> x=clifford("\1{2}\"); print(cos(acos(x))\n    {1,2}\n    ";
02113 static const char __pyx_k_Inverse_hyperbolic_cosine_of_mu[] = "\n    Inverse hyperbolic cosine of
multivector with optional complexifier.\n\n    >> print(acos(0, "\{-2,-1,1}\"))\n    1.571{-2,-1,1}\n
>> x=clifford("\1{2,3}\"); print(cosh(acos(x), "\{-1,1,2,3,4}\"))\n    {1,2,3}\n    >>
print(acos(0))\n    1.571{-1}\n    >> x=clifford("\1{2,3}\"); print(cosh(acos(x))\n    {1,2,3}\n
>> x=clifford("\1{2}\"); print(cosh(acos(x))\n    {1,2}\n    ";
02114 static const char __pyx_k_Inverse_hyperbolic_sine_of_mult[] = "\n    Inverse hyperbolic sine of
multivector with optional complexifier.\n\n    >> x=clifford("\1{2}\"); print(asinh(x, "\1{2,3}\") *
2/pi)\n    {1,2}\n    >> x=clifford("\1{2}\"); print(asinh(x) * 2/pi)\n    {1,2}\n    >>
x=clifford("\1{2}\") / 2; print(asinh(x) * 6/pi)\n    {1,2}\n    ";
02115 static const char __pyx_k_Inverse_hyperbolic_tangent_of_m[] = "\n    Inverse hyperbolic tangent of
multivector with optional complexifier.\n\n    >> s=index_set({1,2,3}); x=clifford("\1{2}\");
print(tanh(atanh(x,s))\n    {1,2}\n    >> x=clifford("\1{2}\"); print(tanh(atanh(x))\n    {1,2}\n
";
02116 static const char __pyx_k_Inverse_sine_of_multivector_wit[] = "\n    Inverse sine of multivector with
optional complexifier.\n\n    >> s="\{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n    {-1}\n    >>
s="\{-1}\"; x=clifford(s); print(asin(sin(x,s), "\{-2,-1,1}\"))\n    {-1}\n    >> print(asin(1) / pi)\n
0.5\n    >> x=clifford("\1{2,3}\"); print(asin(sin(x))\n    {1,2,3}\n    ";
02117 static const char __pyx_k_Main_involution_each_i_is_repla[] = "\n    Main involution, each {i} is
replaced by -{i} in each term,\n    eg. clifford("\1{1}\") -> -clifford("\1{1}\").\n\n    >>
print(clifford("\1{1}\").involute())\n    -{1}\n    >> print((clifford("\2{2}\") *

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    clifford("\{1}\").involute()\n          -{1,2}\n          >> print((clifford("\{1}\") *
    clifford("\{2}\").involute()\n          {1,2}\n          >>
    print(clifford("\{1+{1}+{1,2}\").involute()\n          1-{1}+{1,2}\n          ";
02118 static const char __pyx_k_Maximum_absolute_value_of_coord[] = "\n          Maximum absolute value of
coordinates multivector: multivector infinity-norm.\n          >>
max_abs(clifford("\{1+{-1}+{1,2}+{1,2,3}\")\n          1.0\n          >> max_abs(clifford("\{3+2{1}+{1,2}\")\n
3.0\n          ";
02119 static const char __pyx_k_Maximum_of_absolute_values_of_c[] = "\n          Maximum of absolute values of
components of multivector: multivector infinity norm.\n          >>
clifford("\{1+{-1}+{1,2}+{1,2,3}\").max_abs()\n          1.0\n          >>
clifford("\{3+2{1}+{1,2}\").max_abs()\n          3.0\n          ";
02120 static const char __pyx_k_Natural_logarithm_of_multivector[] = "\n          Natural logarithm of multivector
with optional complexifier.\n          >> x=clifford("\{-1}\"); print((log(x,\{-1}\") * 2/pi))\n
{-1}\n          >> x=clifford("\{1,2}\"); print((log(x,\{1,2,3}\") * 2/pi))\n          {1,2}\n          >>
x=clifford("\{1,2}\"); print((log(x) * 2/pi))\n          {1,2}\n          >> x=clifford("\{1,2}\");
print((log(x,\{1,2}\") * 2/pi))\n          Traceback (most recent call last):\n          ... \n          RuntimeError:
check_complex(val, i): i is not a valid complexifier for val\n          ";
02121 static const char __pyx_k_Norm_sum_of_squares_of_coordinates[] = "\n          Norm == sum of squares of
coordinates.\n          >> clifford("\{1+{1}+{1,2}\").norm()\n          3.0\n          >>
clifford("\{1+{-1}+{1,2}+{1,2,3}\").norm()\n          4.0\n          ";
02122 static const char __pyx_k_Not_applicable_for_a_in_cliffor[] = "\n          Not applicable.\n          >>
for a in clifford(index_set((-3,4,7))):print(a, end=",\n")\n          Traceback (most recent call
last):\n          ... \n          TypeError: Not applicable.\n          ";
02123 static const char __pyx_k_Number_of_negative_indices_incl[] = "\n          Number of negative indices
included in set.\n          >> index_set((-1,1,2)).count_neg()\n          1\n          ";
02124 static const char __pyx_k_Number_of_positive_indices_incl[] = "\n          Number of positive indices
included in set.\n          >> index_set((-1,1,2)).count_pos()\n          2\n          ";
02125 static const char __pyx_k_Outer_product_power_of_multivec[] = "\n          Outer product power of
multivector.\n          >> print(outer_pow(clifford("\{1+{1}+{1,2}\"),3))\n          1+3{1}+3{1,2}\n          ";
02126 static const char __pyx_k_Outer_product_x_clifford_l_x_cl[] = "\n          Outer product.\n          >>
x = clifford("\{1}\"); x ^= clifford("\{2}\"); print(x)\n          {1,2}\n          >> x = clifford(2); x
^= clifford("\{2}\"); print(x)\n          2{2}\n          >> x = clifford("\{1}\"); x ^=
clifford("\{1}\"); print(x)\n          0\n          >> x = clifford("\{1}\"); x ^= clifford("\{1,2}\");
print(x)\n          0\n          ";
02127 static const char __pyx_k_Pure_grade_vector_part_print_cl[] = "\n          Pure grade-vector part.\n          >>
print(clifford("\{1}\") (1))\n          {1}\n          >> print(clifford("\{1}\") (0))\n          0\n
>> print(clifford("\{1+{1}+{1,2}\") (0))\n          1\n          >> print(clifford("\{1+{1}+{1,2}\") (1))\n
{1}\n          >> print(clifford("\{1+{1}+{1,2}\") (2))\n          {1,2}\n          >>
print(clifford("\{1+{1}+{1,2}\") (3))\n          0\n          ";
02128 static const char __pyx_k_Pure_part_print_pure_clifford_l[] = "\n          Pure part.\n          >>
print(pure(clifford("\{1+{1}+{1,2}\")\n          {1}+{1,2}\n          >> print(pure(clifford("\{1,2}\")\n
{1,2}\n          ";
02129 static const char __pyx_k_Put_self_into_a_larger_frame_co[] = "\n          Put self into a larger frame,
containing the union of self.frame() and index set ixt.\n          This can be used to make
multiplication faster, by multiplying within a common frame.\n          >>
clifford("\{2+3{1}\").reframe(index_set({1,2,3}))\n          clifford("\{2+3{1}\")\n          >>
s=index_set({1,2,3});t=index_set((-3,-2,-1));x=random_clifford(s); x.reframe(t).frame() == (s|t);\n
True\n          ";
02130 static const char __pyx_k_Random_multivector_within_a_fra[] = "\n          Random multivector within a
frame.\n          >> print(random_clifford(index_set((-3,-1,2))).frame())\n          {-3,-1,2}\n          ";
02131 static const char __pyx_k_Real_part_synonym_for_scalar_pa[] = "\n          Real part: synonym for scalar
part.\n          >> real(clifford("\{1+{1}+{1,2}\")\n          1.0\n          >> real(clifford("\{1,2}\")\n
0.0\n          ";
02132 static const char __pyx_k_Relative_or_absolute_error_usin[] = "\n          Relative or absolute error using
the quadratic norm.\n          >> err2=sqrt(scalar_epsilon*scalar_epsilon)\n          >>
print(error_squared(clifford("\{1}\"), clifford("\{1}\"), err2))\n          0.0\n          >>
print(error_squared(clifford("\{1\}-3{2}+4{3}\"), clifford("\{1}\"), err2))\n          25.0\n          ";
02133 static const char __pyx_k_Remove_all_terms_of_self_with_r[] = "\n          Remove all terms of self with
relative size smaller than limit.\n          >> clifford("\{1e8+{1}+1e-8{1,2}\").truncated(1.0e-6)\n
clifford("\{1000000000}\")\n          >> clifford("\{1e4+{1}+1e-4{1,2}\").truncated(1.0e-6)\n
clifford("\{10000+{1}\")\n          ";
02134 static const char __pyx_k_Reversion_eg_1_2_2_1_print_reve[] = "\n          Reversion, eg. {1}*{2} ->
{2}*{1}\n          >> print(reverse(clifford("\{1}\")\n          {1}\n          >> print(reverse(clifford("\{2}\") *
clifford("\{1}\")\n          {1,2}\n          >> print(reverse(clifford("\{1}\") * clifford("\{2}\")\n
-{1,2}\n          >> print(reverse(clifford("\{1+{1}+{1,2}\")\n          1+{1}-{1,2}\n          ";
02135 static const char __pyx_k_Reversion_eg_clifford_l_cliffor[] = "\n          Reversion, eg.
clifford("\{1}\")*clifford("\{2}\") -> clifford("\{2}\")*clifford("\{1}\").\n          >>
print(clifford("\{1}\").reverse()\n          {1}\n          >> print((clifford("\{2}\") *
clifford("\{1}\").reverse())\n          {1,2}\n          >> print((clifford("\{1}\") *
clifford("\{2}\").reverse())\n          -{1,2}\n          >> print(clifford("\{1+{1}+{1,2}\").reverse())\n
1+{1}-{1,2}\n          ";
02136 static const char __pyx_k_Scalar_part_clifford_l_1_2_sc[] = "\n          Scalar part.\n          >>
clifford("\{1+{1}+{1,2}\").scalar()\n          1.0\n          >> clifford("\{1,2}\").scalar()\n
0.0\n          ";
02137 static const char __pyx_k_Scalar_part_scalar_clifford_l_1[] = "\n          Scalar part.\n          >>
scalar(clifford("\{1+{1}+{1,2}\")\n          1.0\n          >> scalar(clifford("\{1,2}\")\n          0.0\n          ";
02138 static const char __pyx_k_Set_intersection_and_print_inde[] = "\n          Set intersection: and.\n          >>
print(index_set({1}) & index_set({2}))\n          {2}\n          >> print(index_set({1,2}) &
index_set({2}))\n          {2}\n          ";
02139 static const char __pyx_k_Set_intersection_and_x_index_se[] = "\n          Set intersection: and.\n          >>
x = index_set({1}); x &= index_set({2}); print(x)\n          {2}\n          >> x = index_set({1,2}); x
&= index_set({2}); print(x)\n          {2}\n          ";
02140 static const char __pyx_k_Set_the_value_of_an_index_set_o[] = "\n          Set the value of an index_set
object at index idx to value val.\n          >> s=index_set({1}); s[2] = True; print(s)\n
{1,2}\n          >> s=index_set({1,2}); s[1] = False; print(s)\n          {2}\n          ";
02141 static const char __pyx_k_Set_union_or_x_index_set_l_x_in[] = "\n          Set union: or.\n          >>
x = index_set({1}); x |= index_set({2}); print(x)\n          {1,2}\n          >> x = index_set({1,2}); x

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| = index_set({2}); print(x)\n          {1,2}\n          ";
02142 static const char __pyx_k_Sign_of_geometric_product_of_tw[] = "\n          Sign of geometric product of
two Clifford basis elements.\n\n          >> s = index_set({1,2}); t=index_set({-1});
s.sign_of_mult(t)\n          1\n          ";
02143 static const char __pyx_k_Sign_of_geometric_square_of_a_C[] = "\n          Sign of geometric square of a
Clifford basis element.\n\n          >> s = index_set({1,2}); s.sign_of_square()\n          -1\n          ";
02144 static const char __pyx_k_Sine_of_multivector_with_option[] = "\n          Sine of multivector with optional
complexifier.\n\n          >> s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n          {-1}\n          >>
s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),\"{-2,-1,1}\"))\n          {-1}\n          >>
x=clifford(\"{1,2,3}\"); print(asin(sin(x))\n          {1,2,3}\n          ";
02145 static const char __pyx_k_Square_root_of_1_which_commutates[] = "\n          Square root of -1 which commutes
with all members of the frame of the given multivector.\n\n          >>
print(complexifier(clifford(index_set({1})))\n          {1,2,3}\n          >>
print(complexifier(clifford(index_set({-1})))\n          {-1}\n          >> print(complexifier(index_set({1})))\n
{1,2,3}\n          >> print(complexifier(index_set({-1})))\n          {-1}\n          ";
02146 static const char __pyx_k_Square_root_of_multivector_with[] = "\n          Square root of multivector with
optional complexifier.\n\n          >> print(sqrt(-1))\n          {-1}\n          >> print(sqrt(clifford(\"2{-1}\")))\n
1+{-1}\n          >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)\n          {1,2,3}\n          -1\n
>> j=sqrt(-1,\"{1,2,3}\"); print(j); print(j*j)\n          {1,2,3}\n          -1\n          ";
02147 static const char __pyx_k_Subalgebra_generated_by_all_gen[] = "\n          Subalgebra generated by all
generators of terms of given multivector.\n\n          >>
print(clifford(\"1+3{-1}+2{1,2}+4{-2,7}\".frame())\n          {-2,-1,1,2,7}\n          >>
s=clifford(\"1+3{-1}+2{1,2}+4{-2,7}\".frame()); type(s)\n          <class 'PyClical.index_set'>\n
";
02148 static const char __pyx_k_Subscripting_map_from_index_set[] = "\n          Subscripting: map from index
set to scalar coordinate.\n\n          >> clifford(\"{1}\")[index_set(1)]\n          1.0\n          >>
clifford(\"{1}\")[index_set({1})]\n          1.0\n          >> clifford(\"{1}\")[index_set({1,2})]\n
0.0\n          >> clifford(\"2{1,2}\")[index_set({1,2})]\n          2.0\n          ";
02149 static const char __pyx_k_Symmetric_set_difference_exclus[] = "\n          Symmetric set difference:
exclusive or.\n\n          >> print(index_set({1}) ^ index_set({2}))\n          {1,2}\n          >>
print(index_set({1,2}) ^ index_set({2}))\n          {1}\n          ";
02150 static const char __pyx_k_Tangent_of_multivector_with_opt[] = "\n          Tangent of multivector with
optional complexifier.\n\n          >> x=clifford(\"{1,2}\"); print(tan(x,\"{1,2,3}\"))\n          0.7616{1,2}\n
>> x=clifford(\"{1,2}\"); print(tan(x))\n          0.7616{1,2}\n          ";
02151 static const char __pyx_k_Test_for_approximate_equality_o[] = "\n          Test for approximate equality of
multivectors.\n\n          >> err2=sqrt(epsilon)*epsilon\n          >>
print(approx_equal(clifford(\"{1}\"), clifford(\"1{1}\"))\n          True\n          >>
print(approx_equal(clifford(\"1{1}-3{2}+4{3}\"), clifford(\"{1}\"))\n          False\n          >>
print(approx_equal(clifford(\"1{1}-3{2}+4{3}+0.001\"), clifford(\"1{1}-3{2}+4{3}\"), err2, err2))\n
False\n          >> print(approx_equal(clifford(\"1{1}-3{2}+4{3}+1.0e-30\"), clifford(\"1{1}-3{2}+4{3}\"),
err2, err2))\n          True\n          ";
02152 static const char __pyx_k_Tests_for_functions_that_Doctest[] = "\n          Tests for functions that Doctest
cannot see.\n\n          For index_set.__cinit__: Construct index_set.\n\n          >> print(index_set(1))\n
{1}\n          >> print(index_set({1,2}))\n          {1,2}\n          >> print(index_set(index_set({1,2})))\n
{1,2}\n          >> print(index_set({1,2}))\n          {1,2}\n          >> print(index_set({1,2,1}))\n
{1,2}\n          >> print(index_set(\"{\"}))\n          {}\n          >>
print(index_set(\"{\"}))\n          Traceback (most recent call last):\n          ... ValueError: Cannot
initialize index_set object from invalid string '{'.\n          >> print(index_set(\"{1}\"))\n          Traceback
(most recent call last):\n          ... ValueError: Cannot initialize index_set object from invalid
string '{1'.\n          >> print(index_set(\"{1,2,100}\"))\n          Traceback (most recent call last):\n
... ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.\n          >>
print(index_set({1,2,100}))\n          Traceback (most recent call last):\n          ... IndexError: Cannot
initialize index_set object from invalid {1, 2, 100}.\n          >> print(index_set([1,2]))\n          Traceback
(most recent call last):\n          ... TypeError: Cannot initialize index_set object from <class
'list'>.\n          For index_set.__richcmp__: Compare two objects of class index_set.\n          >>
index_set(1) == index_set({1})\n          True\n          >> index_set({1}) != index_set({1})\n          False\n
>> index_set({1}) != index_set({2})\n          True\n          >> index_set({1}) == index_set({2})\n          False\n
>> index_set({1}) < index_set({2})\n          True\n          >> index_set({1}) <= index_set({2})\n          True\n
>> index_set({1}) > index_set({2})\n          False\n          >> index_set({1}) >= index_set({2})\n          False\n
>> None == index_set({1,2})\n          False\n          >> None != index_set({1,2})\n          True\n          >> None <
index_set({1,2})\n          False\n          >> None <= index_set({1,2})\n          False\n          >> None >
index_set({1,2})\n          False\n          >> None >= index_set({1,2})\n          False\n          >> "index_set({1,2}) ==
None\n          False\n          >> index_set({1,2}) != None\n          True\n          >> index_set({1,2}) < None\n
False\n          >> index_set({1,2}) <= None\n          False\n          >> index_set({1,2}) > None\n          False\n
>> index_set({1,2}) >= None\n          False\n          ";
02153 static const char __pyx_k_The_informal_string_representat[] = "\n          The
\\342\\200\\234informal\\342\\200\\235 string representation of self.\n\n          >>
index_set({1,2}).__str__()\n          '{1,2}'\n          >> str(index_set({1,2}))\n          '{1,2}'\n
";
02154 static const char __pyx_k_The_official_string_representat[] = "\n          The
\\342\\200\\234official\\342\\200\\235 string representation of self.\n\n          >>
index_set({1,2}).__repr__()\n          'index_set({1,2})'\n          >> repr(index_set({1,2}))\n
'index_set({1,2})'\n          ";
02155 static const char __pyx_k_This_comparison_operator_is_not[] = "This comparison operator is not
implemented for ";
02156 static const char __pyx_k_Vector_part_of_multivector_as_a[] = "\n          Vector part of
multivector, as a Python list, with respect to frm.\n\n          >>
print(clifford(\"1+2{1}+3{2}+4{1,2}\".vector_part())\n          [2.0, 3.0]\n          >>
print(clifford(\"1+2{1}+3{2}+4{1,2}\".vector_part(index_set({-1,1,2})))\n          [0.0, 2.0, 3.0]\n
";
02157 static const char __pyx_k_index_set_sign_of_mult_line_366[] = "index_set.sign_of_mult (line
366)";
02158 static const char __pyx_k_norm_sum_of_squares_of_coordina[] = "\n          norm == sum of squares of
coordinates.\n\n          >> norm(clifford(\"1+{1}+{1,2}\"))\n          3.0\n          >>
norm(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n          4.0\n          ";
02159 static const char __pyx_k_Cannot_initialize_clifford_objec[] = "Cannot initialize clifford
object from";

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02160         static const char __pyx_k_Cannot_initialize_index_set_obje[] = "Cannot initialize index_set
object from";
02161         static const char __pyx_k_Conjugation_reverse_o_involute_2[] = "\n    Conjugation, reverse o
involute == involute o reverse.\n\n    >> print(conj(clifford(\"{1}\")))\n    -{1}\n    >>
print(conj(clifford(\"{2}\") * clifford(\"{1}\")))\n    {1,2}\n    >> print(conj(clifford(\"{1}\") *
clifford(\"{2}\")))\n    -{1,2}\n    >> print(conj(clifford(\"1+{1}+{1,2}\")))\n    1-{1}-{1,2}\n
";
02162         static const char __pyx_k_Odd_part_of_multivector_sum_of_2[] = "\n    Odd part of multivector,
sum of odd grade terms.\n\n    >> print(odd(clifford(\"1+{1}+{1,2}\")))\n    {1}\n    ";
02163         static const char __pyx_k_Power_self_to_the_m_x_clifford_2[] = "\n    Power: self to the
m.\n\n    >> x=clifford(\"{1}\"); print(x.pow(2))\n    1\n    >> x=clifford(\"2\");
print(x.pow(2))\n    4\n    >> x=clifford(\"2+{1}\"); print(x.pow(0))\n    1\n    >>
x=clifford(\"2+{1}\"); print(x.pow(1))\n    2+{1}\n    >> x=clifford(\"2+{1}\");
print(x.pow(2))\n    5+4{1}\n    >> print(clifford(\"1+{1}+{1,2}\").pow(3))\n    1+3{1}+3{1,2}\n
    >> i=clifford(\"{1,2}\"); print(exp(pi/2) * i.pow(i))\n    1\n    ";
02164         static const char __pyx_k_Quadratic_form_rev_x_x_0_print_2[] = "\n    Quadratic form ==
(rev(x)*x)(0).\n\n    >> print(quad(clifford(\"1+{1}+{1,2}\")))\n    3.0\n    >>
print(quad(clifford(\"1+{-1}+{1,2}+{1,2,3}\")))\n    2.0\n    ";
02165         static const char __pyx_k_Transform_left_hand_side_using_2[] = "\n    Transform left hand
side, using right hand side as a transformation.\n\n    >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); y|=x; print(y)\n    -{1}\n    >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); y|=exp(x); print(y)\n    -{1}\n    ";
02166         static const char __pyx_k_clifford_hidden_doctests_line_12[] = "clifford_hidden_doctests (line
1253)";
02167         static const char __pyx_k_index_set_hidden_doctests_line_4[] = "index_set_hidden_doctests
(line 406)";
02168         static const char __pyx_k_index_set_sign_of_square_line_37[] = "index_set.sign_of_square (line
375)";
02169         static const char __pyx_k_no_default_reduce_due_to_non[] = "no default __reduce__ due to
non-trivial __cinit__";
02170         static const char __pyx_k_Check_if_a_multivector_contains_2[] = "\n    Check if a
multivector contains any IEEE NaN values.\n\n    >> clifford().isnan()\n    False\n    ";
02171         static const char __pyx_k_Even_part_of_multivector_sum_of_2[] = "\n    Even part of
multivector, sum of even grade terms.\n\n    >> print(even(clifford(\"1+{1}+{1,2}\")))\n    1+{1,2}\n
";
02172         static const char __pyx_k_Geometric_multiplicative_invers_2[] = "\n    Geometric
multiplicative inverse.\n\n    >> print(inv(clifford(\"{1}\")))\n    {1}\n    >>
print(inv(clifford(\"{-1}\")))\n    -{-1}\n    >> print(inv(clifford(\"{-2,-1}\")))\n    -{-2,-1}\n
    >> print(inv(clifford(\"{-1}+{1}\")))\n    nan\n    ";
02173         static const char __pyx_k_Main_involution_each_i_is_repla_2[] = "\n    Main involution, each
{i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})\n\n    >>
print(involute(clifford(\"{1}\")))\n    -{1}\n    >> print(involute(clifford(\"{2}\") *
clifford(\"{1}\")))\n    -{1,2}\n    >> print(involute(clifford(\"{1}\") * clifford(\"{2}\")))\n
    {1,2}\n    >> print(involute(clifford(\"1+{1}+{1,2}\")))\n    1-{1}+{1,2}\n    ";
02174         static const char __pyx_k_Symmetric_set_difference_exclus_2[] = "\n    Symmetric set
difference: exclusive or.\n\n    >> x = index_set({1}); x ^= index_set({2}); print(x)\n
    {1,2}\n    >> x = index_set({1,2}); x ^= index_set({2}); print(x)\n    {1}\n    ";
02175         static const char __pyx_k_Tests_for_functions_that_Doctes_2[] = "\n    Tests for functions
that Doctest cannot see.\n\n    For clifford.__cinit__: Construct an object of type clifford.\n\n
    >> print(clifford(2))\n    2\n    >> print(clifford(2.0))\n    2\n    >> print(clifford(1.0e-1))\n
    0.1\n    >> print(clifford(\"2\"))\n    2\n    >> print(clifford(\"2{1,2,3}\"))\n    2{1,2,3}\n
    >> print(clifford(clifford(\"2{1,2,3}\")))\n    2{1,2,3}\n    >> print(clifford(\"{-1}\"))\n
    -{1}\n    >> print(clifford(2, index_set({1,2})))\n    2{1,2}\n    >> print(clifford([2,3], index_set({1,2})))\n
    2{1}+3{2}\n    >> print(clifford([1,2]))\n    Traceback (most recent call last):\n    ...
TypeError: Cannot initialize clifford object from <class 'list'>.\n    >> print(clifford(None))\n
    Traceback (most recent call last):\n    ...
TypeError: Cannot initialize clifford object from
<class 'NoneType'>.\n    >> print(clifford(None, [1,2]))\n    Traceback (most recent call last):\n
...
TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).\n
    >> print(clifford([1,2], [1,2]))\n    Traceback (most recent call last):\n    ...
TypeError:
Cannot initialize clifford object from (<class 'list'>, <class 'list'>).\n    >>
print(clifford(\"\"))\n    Traceback (most recent call last):\n    ...
ValueError: Cannot
initialize clifford object from invalid string \".\n    >> print(clifford(\"{\")\n    Traceback (most
recent call last):\n    ...
ValueError: Cannot initialize clifford object from invalid string
'{'.\n    >> print(clifford(\"{1}\")\n    Traceback (most recent call last):\n    ...
ValueError: Cannot initialize clifford object from invalid string '{1'.\n    >>
print(clifford(\"+\"))\n    Traceback (most recent call last):\n    ...
ValueError: Cannot
initialize clifford object from invalid string '+'.
    >> print(clifford(\"-\"))\n    Traceback
(most recent call last):\n    ...
ValueError: Cannot initialize clifford object from
invalid string '-'.
    >> print(clifford(\"{1}+\"))\n    Traceback (most recent call last):\n    ...
ValueError: Cannot initialize clifford object from invalid string '{1}+'.
    >> print(clifford(\"{1}+{1}\"))\n    Traceback (most recent call last):\n    ...
ValueError: Cannot initialize clifford object from invalid string '{1}+{1}\n    For
clifford.__richcmp__: Compare objects of type clifford.\n\n    >> clifford(\"{1}\") ==
clifford(\"{1}\")\n    True\n    >> clifford(\"{1}\") != clifford(\"1.0{1}\")\n    False\n    >>
clifford(\"{1}\") != clifford(\"1.0\")\n    True\n    >> clifford(\"{1,2}\") == None\n    False\n
    >> clifford(\"{1,2}\") != None\n    True\n    >> None == clifford(\"{1,2}\")\n    False\n    >> None
!= clifford(\"{1,2}\")\n    True\n    ";
02176         static const char __pyx_k_The_informal_string_representat_2[] = "\n    The
\342\200\234informal\342\200\235 string representation of self.\n\n    >>
clifford(\"1+3{-1}+2{1,2}+4{-2,7}\").__str__()\n    '1+3{-1}+2{1,2}+4{-2,7}'\n    ";
02177         static const char __pyx_k_The_official_string_representat_2[] = "\n    The
\342\200\234official\342\200\235 string representation of self.\n\n    >>
clifford(\"1+3{-1}+2{1,2}+4{-2,7}\").__repr__()\n    'clifford(\"1+3{-1}+2{1,2}+4{-2,7}\")'\n
";
02178         static PyObject *__pyx_kp_u_;
02179         static PyObject *__pyx_kp_u_Abbreviation_for_clifford_index;
02180         static PyObject *__pyx_kp_u_Abbreviation_for_index_set_q_p;
02181         static PyObject *__pyx_kp_u_Absolute_value_of_multivector_m;
02182         static PyObject *__pyx_kp_u_Absolute_value_square_root_of_n;

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02183     static PyObject *__pyx_kp_u_Cannot_initialize_clifford_objec;
02184     static PyObject *__pyx_kp_u_Cannot_initialize_index_set_obje;
02185     static PyObject *__pyx_kp_u_Cannot_reframe;
02186     static PyObject *__pyx_kp_u_Cannot_take_vector_part_of;
02187     static PyObject *__pyx_kp_u_Cardinality_Number_of_indices_i;
02188     static PyObject *__pyx_kp_u_Check_if_a_multivector_contains;
02189     static PyObject *__pyx_kp_u_Check_if_a_multivector_contains_2;
02190     static PyObject *__pyx_kp_u_Conjugation_reverse_o_involute;
02191     static PyObject *__pyx_kp_u_Conjugation_reverse_o_involute_2;
02192     static PyObject *__pyx_kp_u_Contraction_print_clifford_1_cl;
02193     static PyObject *__pyx_kp_u_Contraction_x_clifford_1_x_clif;
02194     static PyObject *__pyx_kp_u_Convert_CGA3_null_vector_to_Euc;
02195     static PyObject *__pyx_kp_u_Convert_CGA3_null_vector_to_sta;
02196     static PyObject *__pyx_kp_u_Convert_Euclidean_3D_multivecto;
02197     static PyObject *__pyx_kp_u_Copy_this_clifford_object_x_cli;
02198     static PyObject *__pyx_kp_u_Copy_this_index_set_object_s_in;
02199     static PyObject *__pyx_kp_u_Cosine_of_multivector_with_opti;
02200     static PyObject *__pyx_kp_u_Even_part_of_multivector_sum_of;
02201     static PyObject *__pyx_kp_u_Even_part_of_multivector_sum_of_2;
02202     static PyObject *__pyx_kp_u_Exponential_of_multivector_x_cl;
02203     static PyObject *__pyx_kp_u_Geometric_difference_print_clif;
02204     static PyObject *__pyx_kp_u_Geometric_difference_x_clifford;
02205     static PyObject *__pyx_kp_u_Geometric_multiplicative_invers;
02206     static PyObject *__pyx_kp_u_Geometric_multiplicative_invers_2;
02207     static PyObject *__pyx_kp_u_Geometric_product_print_cliffor;
02208     static PyObject *__pyx_kp_u_Geometric_product_x_clifford_2;
02209     static PyObject *__pyx_kp_u_Geometric_quotient_print_cliffo;
02210     static PyObject *__pyx_kp_u_Geometric_quotient_x_clifford_1;
02211     static PyObject *__pyx_kp_u_Geometric_sum_print_clifford_1;
02212     static PyObject *__pyx_kp_u_Geometric_sum_x_clifford_1_x_cl;
02213     static PyObject *__pyx_kp_u_Get_the_value_of_an_index_set_o;
02214     static PyObject *__pyx_kp_u_Hyperbolic_cosine_of_multivecto;
02215     static PyObject *__pyx_kp_u_Hyperbolic_sine_of_multivector;
02216     static PyObject *__pyx_kp_u_Hyperbolic_tangent_of_multivect;
02217     static PyObject *__pyx_kp_u_Imaginary_part_deprecated_alway;
02218     static PyObject *__pyx_n_s_IndexError;
02219     static PyObject *__pyx_kp_u_Inner_product_print_clifford_1;
02220     static PyObject *__pyx_kp_u_Inner_product_x_clifford_1_x_cl;
02221     static PyObject *__pyx_kp_u_Integer_power_of_multivector_ob;
02222     static PyObject *__pyx_n_s_Integral;
02223     static PyObject *__pyx_kp_u_Inverse_cosine_of_multivector_w;
02224     static PyObject *__pyx_kp_u_Inverse_hyperbolic_cosine_of_mu;
02225     static PyObject *__pyx_kp_u_Inverse_hyperbolic_sine_of_mult;
02226     static PyObject *__pyx_kp_u_Inverse_hyperbolic_tangent_of_m;
02227     static PyObject *__pyx_kp_u_Inverse_sine_of_multivector_wit;
02228     static PyObject *__pyx_kp_u_Inverse_tangent_of_multivector;
02229     static PyObject *__pyx_kp_u_Iterate_over_the_indices_of_an;
02230     static PyObject *__pyx_kp_u_Main_involution_each_i_is_repla;
02231     static PyObject *__pyx_kp_u_Main_involution_each_i_is_repla_2;
02232     static PyObject *__pyx_kp_u_Maximum_absolute_value_of_coord;
02233     static PyObject *__pyx_kp_u_Maximum_member_index_set_1_1_2;
02234     static PyObject *__pyx_kp_u_Maximum_of_absolute_values_of_c;
02235     static PyObject *__pyx_kp_u_Maximum_positive_index_or_0_if;
02236     static PyObject *__pyx_kp_u_Minimum_member_index_set_1_1_2;
02237     static PyObject *__pyx_kp_u_Minimum_negative_index_or_0_if;
02238     static PyObject *__pyx_kp_u_Natural_logarithm_of_multivecto;
02239     static PyObject *__pyx_kp_u_Norm_sum_of_squares_of_coordina;
02240     static PyObject *__pyx_n_s_NotImplemented;
02241     static PyObject *__pyx_kp_u_Not_applicable;
02242     static PyObject *__pyx_kp_u_Not_applicable_for_a_in_cliffor;
02243     static PyObject *__pyx_kp_u_Number_of_negative_indices_incl;
02244     static PyObject *__pyx_kp_u_Number_of_positive_indices_incl;
02245     static PyObject *__pyx_kp_u_Odd_part_of_multivector_sum_of;
02246     static PyObject *__pyx_kp_u_Odd_part_of_multivector_sum_of_2;
02247     static PyObject *__pyx_kp_u_Outer_product_power_of_multivect;
02248     static PyObject *__pyx_kp_u_Outer_product_power_x_clifford;
02249     static PyObject *__pyx_kp_u_Outer_product_print_clifford_1;
02250     static PyObject *__pyx_kp_u_Outer_product_x_clifford_1_x_cl;
02251     static PyObject *__pyx_kp_u_Power_self_to_the_m_x_clifford;
02252     static PyObject *__pyx_kp_u_Power_self_to_the_m_x_clifford_2;
02253     static PyObject *__pyx_kp_u_Pure_grade_vector_part_print_cl;
02254     static PyObject *__pyx_kp_u_Pure_part_print_clifford_1_1_1;
02255     static PyObject *__pyx_kp_u_Pure_part_print_pure_clifford_1;
02256     static PyObject *__pyx_kp_u_Put_self_into_a_larger_frame_co;
02257     static PyObject *__pyx_n_s_PyClical;
02258     static PyObject *__pyx_kp_s_PyClical_pyx;
02259     static PyObject *__pyx_kp_u_Quadratic_form_rev_x_x_0_print;
02260     static PyObject *__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2;
02261     static PyObject *__pyx_kp_u_Quadratic_norm_error_tolerance;
02262     static PyObject *__pyx_kp_u_Random_multivector_within_a_fra;
02263     static PyObject *__pyx_n_s_Real;
02264     static PyObject *__pyx_kp_u_Real_part_synonym_for_scalar_pa;
02265     static PyObject *__pyx_kp_u_Relative_or_absolute_error_usin;
02266     static PyObject *__pyx_kp_u_Remove_all_terms_of_self_with_r;
02267     static PyObject *__pyx_kp_u_Reversion_eg_1_2_2_1_print_reve;
02268     static PyObject *__pyx_kp_u_Reversion_eg_clifford_1_cliffor;
02269     static PyObject *__pyx_n_s_RuntimeError;

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02270 static PyObject *__pyx_kp_u_Scalar_part_clifford_1_1_1_2_sc;
02271 static PyObject *__pyx_kp_u_Scalar_part_scalar_clifford_1_1;
02272 static PyObject *__pyx_n_s_Sequence;
02273 static PyObject *__pyx_kp_u_Set_complement_not_print_index;
02274 static PyObject *__pyx_kp_u_Set_intersection_and_print_index;
02275 static PyObject *__pyx_kp_u_Set_intersection_and_x_index_set;
02276 static PyObject *__pyx_kp_u_Set_the_value_of_an_index_set_o;
02277 static PyObject *__pyx_kp_u_Set_union_or_print_index_set_1;
02278 static PyObject *__pyx_kp_u_Set_union_or_x_index_set_1_x_in;
02279 static PyObject *__pyx_kp_u_Sign_of_geometric_product_of_tw;
02280 static PyObject *__pyx_kp_u_Sign_of_geometric_square_of_a_C;
02281 static PyObject *__pyx_kp_u_Sine_of_multivector_with_option;
02282 static PyObject *__pyx_kp_u_Square_root_of_1_which_commutates;
02283 static PyObject *__pyx_kp_u_Square_root_of_multivector_with;
02284 static PyObject *__pyx_kp_u_Subalgebra_generated_by_all_gen;
02285 static PyObject *__pyx_kp_u_Subscripting_map_from_index_set;
02286 static PyObject *__pyx_kp_u_Symmetric_set_difference_exclus;
02287 static PyObject *__pyx_kp_u_Symmetric_set_difference_exclus_2;
02288 static PyObject *__pyx_kp_u_Tangent_of_multivector_with_opt;
02289 static PyObject *__pyx_kp_u_Test_for_approximate_equality_o;
02290 static PyObject *__pyx_kp_u_Tests_for_functions_that_Doctes;
02291 static PyObject *__pyx_kp_u_Tests_for_functions_that_Doctes_2;
02292 static PyObject *__pyx_kp_u_The_informal_string_representat;
02293 static PyObject *__pyx_kp_u_The_informal_string_representat_2;
02294 static PyObject *__pyx_kp_u_The_official_string_representat;
02295 static PyObject *__pyx_kp_u_The_official_string_representat_2;
02296 static PyObject *__pyx_kp_u_This_comparison_operator_is_not;
02297 static PyObject *__pyx_kp_u_Transform_left_hand_side_using;
02298 static PyObject *__pyx_kp_u_Transform_left_hand_side_using_2;
02299 static PyObject *__pyx_n_s_TypeError;
02300 static PyObject *__pyx_kp_u_UTF_8;
02301 static PyObject *__pyx_kp_u_Unary_print_clifford_1_1;
02302 static PyObject *__pyx_kp_u_Unary_print_clifford_1_1_2;
02303 static PyObject *__pyx_n_s_ValueError;
02304 static PyObject *__pyx_kp_u_Vector_part_of_multivector_as_a;
02305 static PyObject *__pyx_kp_u_2;
02306 static PyObject *__pyx_kp_u_5;
02307 static PyObject *__pyx_kp_u_6;
02308 static PyObject *__pyx_kp_u_7;
02309 static PyObject *__pyx_kp_u_8;
02310 static PyObject *__pyx_kp_u_9;
02311 static PyObject *__pyx_n_s_abc;
02312 static PyObject *__pyx_kp_u_abs_line_1522;
02313 static PyObject *__pyx_n_s_acos;
02314 static PyObject *__pyx_kp_u_acos_line_1668;
02315 static PyObject *__pyx_n_s_acosh;
02316 static PyObject *__pyx_kp_u_acosh_line_1705;
02317 static PyObject *__pyx_kp_u_agc3_line_1893;
02318 static PyObject *__pyx_kp_u_approx_equal_line_1359;
02319 static PyObject *__pyx_n_s_args;
02320 static PyObject *__pyx_kp_u_as_frame;
02321 static PyObject *__pyx_n_s_asin;
02322 static PyObject *__pyx_kp_u_asin_line_1747;
02323 static PyObject *__pyx_n_s_asinh;
02324 static PyObject *__pyx_kp_u_asinh_line_1782;
02325 static PyObject *__pyx_n_s_atan;
02326 static PyObject *__pyx_kp_u_atan_line_1818;
02327 static PyObject *__pyx_n_s_atanh;
02328 static PyObject *__pyx_kp_u_atanh_line_1847;
02329 static PyObject *__pyx_kp_u_cga3_line_1873;
02330 static PyObject *__pyx_kp_u_cga3std_line_1882;
02331 static PyObject *__pyx_n_s_cl;
02332 static PyObject *__pyx_n_s_clifford;
02333 static PyObject *__pyx_kp_u_clifford__add__line_740;
02334 static PyObject *__pyx_kp_u_clifford__and__line_836;
02335 static PyObject *__pyx_kp_u_clifford__call__line_1020;
02336 static PyObject *__pyx_kp_u_clifford__getitem__line_707;
02337 static PyObject *__pyx_kp_u_clifford__iadd__line_751;
02338 static PyObject *__pyx_kp_u_clifford__iand__line_851;
02339 static PyObject *__pyx_kp_u_clifford__idiv__line_911;
02340 static PyObject *__pyx_kp_u_clifford__imod__line_821;
02341 static PyObject *__pyx_kp_u_clifford__imul__line_793;
02342 static PyObject *__pyx_kp_u_clifford__ior__line_950;
02343 static PyObject *__pyx_kp_u_clifford__isub__line_771;
02344 static PyObject *__pyx_kp_u_clifford__iter__line_638;
02345 static PyObject *__pyx_kp_u_clifford__ixor__line_881;
02346 static PyObject *__pyx_kp_u_clifford__mod__line_806;
02347 static PyObject *__pyx_kp_u_clifford__mul__line_780;
02348 static PyObject *__pyx_kp_u_clifford__neg__line_722;
02349 static PyObject *__pyx_kp_u_clifford__or__line_939;
02350 static PyObject *__pyx_kp_u_clifford__pos__line_731;
02351 static PyObject *__pyx_kp_u_clifford__pow__line_961;
02352 static PyObject *__pyx_kp_u_clifford__repr__line_1235;
02353 static PyObject *__pyx_kp_u_clifford__str__line_1244;
02354 static PyObject *__pyx_kp_u_clifford__sub__line_760;
02355 static PyObject *__pyx_kp_u_clifford__truediv__line_896;
02356 static PyObject *__pyx_kp_u_clifford__xor__line_866;

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02357     static PyObject *__pyx_kp_u_clifford_abs_line_1175;
02358     static PyObject *__pyx_kp_u_clifford_conj_line_1138;
02359     static PyObject *__pyx_kp_u_clifford_copy_line_556;
02360     static PyObject *__pyx_kp_u_clifford_even_line_1061;
02361     static PyObject *__pyx_kp_u_clifford_frame_line_1224;
02362     static PyObject *__pyx_n_s_clifford_hidden_doctests;
02363     static PyObject *__pyx_kp_u_clifford_hidden_doctests_line_12;
02364     static PyObject *__pyx_kp_u_clifford_inv_line_926;
02365     static PyObject *__pyx_kp_u_clifford_involute_line_1107;
02366     static PyObject *__pyx_kp_u_clifford_isinf_line_1206;
02367     static PyObject *__pyx_kp_u_clifford_isnan_line_1215;
02368     static PyObject *__pyx_kp_u_clifford_max_abs_line_1184;
02369     static PyObject *__pyx_kp_u_clifford_norm_line_1164;
02370     static PyObject *__pyx_kp_u_clifford_odd_line_1070;
02371     static PyObject *__pyx_kp_u_clifford_outer_pow_line_1004;
02372     static PyObject *__pyx_kp_u_clifford_pow_line_980;
02373     static PyObject *__pyx_kp_u_clifford_pure_line_1050;
02374     static PyObject *__pyx_kp_u_clifford_quad_line_1153;
02375     static PyObject *__pyx_kp_u_clifford_reframe_line_649;
02376     static PyObject *__pyx_kp_u_clifford_reverse_line_1123;
02377     static PyObject *__pyx_kp_u_clifford_scalar_line_1039;
02378     static PyObject *__pyx_kp_u_clifford_truncated_line_1195;
02379     static PyObject *__pyx_kp_u_clifford_vector_part_line_1079;
02380     static PyObject *__pyx_n_s_cline_in_traceback;
02381     static PyObject *__pyx_n_s_close;
02382     static PyObject *__pyx_n_s_collections;
02383     static PyObject *__pyx_kp_u_compare_line_492;
02384     static PyObject *__pyx_kp_u_complexifier_line_1576;
02385     static PyObject *__pyx_n_s_conj;
02386     static PyObject *__pyx_kp_u_conj_line_1485;
02387     static PyObject *__pyx_n_s_copy;
02388     static PyObject *__pyx_n_s_cos;
02389     static PyObject *__pyx_kp_u_cos_line_1651;
02390     static PyObject *__pyx_n_s_cosh;
02391     static PyObject *__pyx_kp_u_cosh_line_1689;
02392     static PyObject *__pyx_n_s_doctest;
02393     static PyObject *__pyx_n_s_e;
02394     static PyObject *__pyx_kp_u_e_line_1936;
02395     static PyObject *__pyx_n_s_encode;
02396     static PyObject *__pyx_kp_u_error_squared_line_1346;
02397     static PyObject *__pyx_kp_u_error_squared_tol_line_1337;
02398     static PyObject *__pyx_n_s_even;
02399     static PyObject *__pyx_kp_u_even_line_1437;
02400     static PyObject *__pyx_n_s_exp;
02401     static PyObject *__pyx_kp_u_exp_line_1614;
02402     static PyObject *__pyx_n_s_fill;
02403     static PyObject *__pyx_n_s_frm;
02404     static PyObject *__pyx_kp_u_from;
02405     static PyObject *__pyx_n_s_getstate;
02406     static PyObject *__pyx_n_s_grade;
02407     static PyObject *__pyx_n_s_i;
02408     static PyObject *__pyx_kp_u_imag_line_1415;
02409     static PyObject *__pyx_n_s_import;
02410     static PyObject *__pyx_n_s_index_set;
02411     static PyObject *__pyx_kp_u_index_set_and_line_271;
02412     static PyObject *__pyx_kp_u_index_set_getitem_line_191;
02413     static PyObject *__pyx_kp_u_index_set_iand_line_282;
02414     static PyObject *__pyx_kp_u_index_set_invert_line_240;
02415     static PyObject *__pyx_kp_u_index_set_ior_line_304;
02416     static PyObject *__pyx_n_s_index_set_iter;
02417     static PyObject *__pyx_kp_u_index_set_iter_line_229;
02418     static PyObject *__pyx_kp_u_index_set_ixor_line_260;
02419     static PyObject *__pyx_kp_u_index_set_or_line_293;
02420     static PyObject *__pyx_kp_u_index_set_repr_line_384;
02421     static PyObject *__pyx_kp_u_index_set_setitem_line_179;
02422     static PyObject *__pyx_kp_u_index_set_str_line_395;
02423     static PyObject *__pyx_kp_u_index_set_xor_line_249;
02424     static PyObject *__pyx_kp_u_index_set_copy_line_65;
02425     static PyObject *__pyx_kp_u_index_set_count_line_315;
02426     static PyObject *__pyx_kp_u_index_set_count_neg_line_324;
02427     static PyObject *__pyx_kp_u_index_set_count_pos_line_333;
02428     static PyObject *__pyx_n_s_index_set_hidden_doctests;
02429     static PyObject *__pyx_kp_u_index_set_hidden_doctests_line_4;
02430     static PyObject *__pyx_kp_u_index_set_max_line_351;
02431     static PyObject *__pyx_kp_u_index_set_min_line_342;
02432     static PyObject *__pyx_kp_u_index_set_sign_of_mult_line_366;
02433     static PyObject *__pyx_kp_u_index_set_sign_of_square_line_37;
02434     static PyObject *__pyx_n_s_inv;
02435     static PyObject *__pyx_kp_u_inv_line_1378;
02436     static PyObject *__pyx_kp_u_invalid;
02437     static PyObject *__pyx_kp_u_invalid_string;
02438     static PyObject *__pyx_n_s_involute;
02439     static PyObject *__pyx_kp_u_involute_line_1455;
02440     static PyObject *__pyx_n_s_ist;
02441     static PyObject *__pyx_n_s_istpq;
02442     static PyObject *__pyx_kp_u_istpq_line_1949;
02443     static PyObject *__pyx_n_s_iter;

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02444     static PyObject *__pyx_n_s_ixt;
02445     static PyObject *__pyx_kp_u_lexicographic_compare_eg_3_4_5;
02446     static PyObject *__pyx_n_s_lhs;
02447     static PyObject *__pyx_n_s_log;
02448     static PyObject *__pyx_kp_u_log_line_1628;
02449     static PyObject *__pyx_n_s_m;
02450     static PyObject *__pyx_n_s_main;
02451     static PyObject *__pyx_n_u_main;
02452     static PyObject *__pyx_n_s_math;
02453     static PyObject *__pyx_n_s_max;
02454     static PyObject *__pyx_kp_u_max_abs_line_1531;
02455     static PyObject *__pyx_kp_u_max_pos_line_513;
02456     static PyObject *__pyx_n_s_min;
02457     static PyObject *__pyx_kp_u_min_neg_line_504;
02458     static PyObject *__pyx_n_s_name;
02459     static PyObject *__pyx_n_s_nbar3;
02460     static PyObject *__pyx_n_s_ninf3;
02461     static PyObject *__pyx_kp_s_no_default__reduce__due_to_non;
02462     static PyObject *__pyx_n_s_norm;
02463     static PyObject *__pyx_kp_u_norm_line_1511;
02464     static PyObject *__pyx_kp_u_norm_sum_of_squares_of_coordina;
02465     static PyObject *__pyx_n_s_numbers;
02466     static PyObject *__pyx_n_s_obj;
02467     static PyObject *__pyx_n_s_odd;
02468     static PyObject *__pyx_kp_u_odd_line_1446;
02469     static PyObject *__pyx_n_s_other;
02470     static PyObject *__pyx_n_s_outer_pow;
02471     static PyObject *__pyx_kp_u_outer_pow_line_1567;
02472     static PyObject *__pyx_n_s_p;
02473     static PyObject *__pyx_n_s_pi;
02474     static PyObject *__pyx_n_s_pow;
02475     static PyObject *__pyx_kp_u_pow_line_1543;
02476     static PyObject *__pyx_n_s_pure;
02477     static PyObject *__pyx_kp_u_pure_line_1426;
02478     static PyObject *__pyx_n_s_pyx_vtable;
02479     static PyObject *__pyx_n_s_q;
02480     static PyObject *__pyx_n_s_quad;
02481     static PyObject *__pyx_kp_u_quad_line_1500;
02482     static PyObject *__pyx_kp_u_random_clifford_line_1864;
02483     static PyObject *__pyx_n_s_range;
02484     static PyObject *__pyx_kp_u_real_line_1404;
02485     static PyObject *__pyx_n_s_reduce;
02486     static PyObject *__pyx_n_s_reduce_cython;
02487     static PyObject *__pyx_n_s_reduce_ex;
02488     static PyObject *__pyx_n_s_reverse;
02489     static PyObject *__pyx_kp_u_reverse_line_1470;
02490     static PyObject *__pyx_n_s_rhs;
02491     static PyObject *__pyx_n_s_scalar;
02492     static PyObject *__pyx_n_s_scalar_epsilon;
02493     static PyObject *__pyx_kp_u_scalar_line_1393;
02494     static PyObject *__pyx_n_s_send;
02495     static PyObject *__pyx_n_s_setstate;
02496     static PyObject *__pyx_n_s_setstate_cython;
02497     static PyObject *__pyx_n_s_sin;
02498     static PyObject *__pyx_kp_u_sin_line_1728;
02499     static PyObject *__pyx_n_s_sinh;
02500     static PyObject *__pyx_kp_u_sinh_line_1768;
02501     static PyObject *__pyx_n_s_sqrt;
02502     static PyObject *__pyx_kp_u_sqrt_line_1591;
02503     static PyObject *__pyx_n_s_tan;
02504     static PyObject *__pyx_kp_u_tan_line_1801;
02505     static PyObject *__pyx_n_s_tanh;
02506     static PyObject *__pyx_kp_u_tanh_line_1835;
02507     static PyObject *__pyx_n_s_tau;
02508     static PyObject *__pyx_n_s_test;
02509     static PyObject *__pyx_n_s_test_2;
02510     static PyObject *__pyx_n_s_testmod;
02511     static PyObject *__pyx_n_s_threshold;
02512     static PyObject *__pyx_n_s_throw;
02513     static PyObject *__pyx_kp_u_to_frame;
02514     static PyObject *__pyx_n_s_tol;
02515     static PyObject *__pyx_kp_u_using;
02516     static PyObject *__pyx_kp_u_using_invalid;
02517     static PyObject *__pyx_kp_u_utf_8;
02518     static PyObject *__pyx_kp_u_value;
02519     static PyObject *__pyx_n_s_version;
02520     static PyObject *__pyx_n_s_xrange;
02521     static PyObject *__pyx_pf_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self); /* proto */
02522     static int __pyx_pf_8PyClical_9index_set_2_cinit__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, PyObject *__pyx_v_other); /* proto */
02523     static void __pyx_pf_8PyClical_9index_set_4_dealloc__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self); /* proto */
02524     static PyObject *__pyx_pf_8PyClical_9index_set_6_richcmp__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op); /* proto */
02525     static int __pyx_pf_8PyClical_9index_set_8_setitem__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, PyObject *__pyx_v_idx, PyObject *__pyx_v_val); /* proto */

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02526 static PyObject * __pyx_pf_8PyClical_9index_set_10_getitem__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self, PyObject * __pyx_v_idx); /* proto */
02527 static int __pyx_pf_8PyClical_9index_set_12_contains__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self, PyObject * __pyx_v_idx); /* proto */
02528 static PyObject * __pyx_pf_8PyClical_9index_set_14_iter__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02529 static PyObject * __pyx_pf_8PyClical_9index_set_17_invert__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02530 static PyObject * __pyx_pf_8PyClical_9index_set_19_xor__(PyObject * __pyx_v_lhs, PyObject
    * __pyx_v_rhs); /* proto */
02531 static PyObject * __pyx_pf_8PyClical_9index_set_21_ixor__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self, PyObject * __pyx_v_rhs); /* proto */
02532 static PyObject * __pyx_pf_8PyClical_9index_set_23_and__(PyObject * __pyx_v_lhs, PyObject
    * __pyx_v_rhs); /* proto */
02533 static PyObject * __pyx_pf_8PyClical_9index_set_25_iand__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self, PyObject * __pyx_v_rhs); /* proto */
02534 static PyObject * __pyx_pf_8PyClical_9index_set_27_or__(PyObject * __pyx_v_lhs, PyObject * __pyx_v_rhs);
    /* proto */
02535 static PyObject * __pyx_pf_8PyClical_9index_set_29_ior__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self, PyObject * __pyx_v_rhs); /* proto */
02536 static PyObject * __pyx_pf_8PyClical_9index_set_31_count__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02537 static PyObject * __pyx_pf_8PyClical_9index_set_33_count_neg__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02538 static PyObject * __pyx_pf_8PyClical_9index_set_35_count_pos__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02539 static PyObject * __pyx_pf_8PyClical_9index_set_37_min__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02540 static PyObject * __pyx_pf_8PyClical_9index_set_39_max__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02541 static PyObject * __pyx_pf_8PyClical_9index_set_41_hash_fn__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02542 static PyObject * __pyx_pf_8PyClical_9index_set_43_sign_of_mult__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self, PyObject * __pyx_v_rhs); /* proto */
02543 static PyObject * __pyx_pf_8PyClical_9index_set_45_sign_of_square__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02544 static PyObject * __pyx_pf_8PyClical_9index_set_47_repr__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02545 static PyObject * __pyx_pf_8PyClical_9index_set_49_str__(struct __pyx_obj_8PyClical_index_set
    * __pyx_v_self); /* proto */
02546 static PyObject * __pyx_pf_8PyClical_9index_set_51_reduce_cython__(CYTHON_UNUSED struct
    __pyx_obj_8PyClical_index_set * __pyx_v_self); /* proto */
02547 static PyObject * __pyx_pf_8PyClical_9index_set_53_setstate_cython__(CYTHON_UNUSED struct
    __pyx_obj_8PyClical_index_set * __pyx_v_self, CYTHON_UNUSED PyObject * __pyx_v__pyx_state); /* proto */
02548 static PyObject * __pyx_pf_8PyClical_index_set_hidden_doctests(CYTHON_UNUSED PyObject * __pyx_self); /*
    proto */
02549 static PyObject * __pyx_pf_8PyClical_2compare(CYTHON_UNUSED PyObject * __pyx_self, PyObject
    * __pyx_v_lhs, PyObject * __pyx_v_rhs); /* proto */
02550 static PyObject * __pyx_pf_8PyClical_4min_neg(CYTHON_UNUSED PyObject * __pyx_self, PyObject
    * __pyx_v_obj); /* proto */
02551 static PyObject * __pyx_pf_8PyClical_6max_pos(CYTHON_UNUSED PyObject * __pyx_self, PyObject
    * __pyx_v_obj); /* proto */
02552 static PyObject * __pyx_pf_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford * __pyx_v_self);
    /* proto */
02553 static int __pyx_pf_8PyClical_8clifford_2_cinit__(struct __pyx_obj_8PyClical_clifford * __pyx_v_self,
    PyObject * __pyx_v_other, PyObject * __pyx_v_ixt); /* proto */
02554 static void __pyx_pf_8PyClical_8clifford_4_dealloc__(struct __pyx_obj_8PyClical_clifford
    * __pyx_v_self); /* proto */
02555 static int __pyx_pf_8PyClical_8clifford_6_contains__(CYTHON_UNUSED struct
    __pyx_obj_8PyClical_clifford * __pyx_v_self, CYTHON_UNUSED PyObject * __pyx_v_x); /* proto */
02556 static PyObject * __pyx_pf_8PyClical_8clifford_8_iter__(CYTHON_UNUSED struct
    __pyx_obj_8PyClical_clifford * __pyx_v_self); /* proto */
02557 static PyObject * __pyx_pf_8PyClical_8clifford_10_reframe(struct __pyx_obj_8PyClical_clifford
    * __pyx_v_self, PyObject * __pyx_v_ixt); /* proto */
02558 static PyObject * __pyx_pf_8PyClical_8clifford_12_richcmp__(struct __pyx_obj_8PyClical_clifford
    * __pyx_v_lhs, PyObject * __pyx_v_rhs, int __pyx_v_op); /* proto */
02559 static PyObject * __pyx_pf_8PyClical_8clifford_14_getitem__(struct __pyx_obj_8PyClical_clifford
    * __pyx_v_self, PyObject * __pyx_v_ixt); /* proto */
02560 static PyObject * __pyx_pf_8PyClical_8clifford_16_neg__(struct __pyx_obj_8PyClical_clifford
    * __pyx_v_self); /* proto */
02561 static PyObject * __pyx_pf_8PyClical_8clifford_18_pos__(struct __pyx_obj_8PyClical_clifford
    * __pyx_v_self); /* proto */
02562 static PyObject * __pyx_pf_8PyClical_8clifford_20_add__(PyObject * __pyx_v_lhs, PyObject * __pyx_v_rhs);
    /* proto */
02563 static PyObject * __pyx_pf_8PyClical_8clifford_22_iadd__(struct __pyx_obj_8PyClical_clifford
    * __pyx_v_self, PyObject * __pyx_v_rhs); /* proto */
02564 static PyObject * __pyx_pf_8PyClical_8clifford_24_sub__(PyObject * __pyx_v_lhs, PyObject * __pyx_v_rhs);
    /* proto */
02565 static PyObject * __pyx_pf_8PyClical_8clifford_26_isub__(struct __pyx_obj_8PyClical_clifford
    * __pyx_v_self, PyObject * __pyx_v_rhs); /* proto */
02566 static PyObject * __pyx_pf_8PyClical_8clifford_28_mul__(PyObject * __pyx_v_lhs, PyObject * __pyx_v_rhs);
    /* proto */
02567 static PyObject * __pyx_pf_8PyClical_8clifford_30_imul__(struct __pyx_obj_8PyClical_clifford
    * __pyx_v_self, PyObject * __pyx_v_rhs); /* proto */
02568 static PyObject * __pyx_pf_8PyClical_8clifford_32_mod__(PyObject * __pyx_v_lhs, PyObject * __pyx_v_rhs);
    /* proto */
02569 static PyObject * __pyx_pf_8PyClical_8clifford_34_imod__(struct __pyx_obj_8PyClical_clifford

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    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02570 static PyObject *__pyx_pf_8PyClical_8clifford_36_and__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02571 static PyObject *__pyx_pf_8PyClical_8clifford_38_iand__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02572 static PyObject *__pyx_pf_8PyClical_8clifford_40_xor__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02573 static PyObject *__pyx_pf_8PyClical_8clifford_42_ixor__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02574 static PyObject *__pyx_pf_8PyClical_8clifford_44_truediv__(PyObject *__pyx_v_lhs, PyObject
    *__pyx_v_rhs); /* proto */
02575 #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
02576 static PyObject *__pyx_pf_8PyClical_8clifford_46_idiv__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02577 #endif
02578 static PyObject *__pyx_pf_8PyClical_8clifford_48inv(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02579 static PyObject *__pyx_pf_8PyClical_8clifford_50_or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02580 static PyObject *__pyx_pf_8PyClical_8clifford_52_ior__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02581 static PyObject *__pyx_pf_8PyClical_8clifford_54_pow__(PyObject *__pyx_v_self, PyObject *__pyx_v_m,
    CYTHON_UNUSED PyObject *__pyx_v_dummy); /* proto */
02582 static PyObject *__pyx_pf_8PyClical_8clifford_56pow(struct __pyx_obj_8PyClical_clifford *__pyx_v_self,
    PyObject *__pyx_v_m); /* proto */
02583 static PyObject *__pyx_pf_8PyClical_8clifford_58outer_pow(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_m); /* proto */
02584 static PyObject *__pyx_pf_8PyClical_8clifford_60_call__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_grade); /* proto */
02585 static PyObject *__pyx_pf_8PyClical_8clifford_62scalar(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02586 static PyObject *__pyx_pf_8PyClical_8clifford_64pure(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02587 static PyObject *__pyx_pf_8PyClical_8clifford_66even(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02588 static PyObject *__pyx_pf_8PyClical_8clifford_68odd(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02589 static PyObject *__pyx_pf_8PyClical_8clifford_70vector_part(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_frm); /* proto */
02590 static PyObject *__pyx_pf_8PyClical_8clifford_72involute(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02591 static PyObject *__pyx_pf_8PyClical_8clifford_74reverse(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02592 static PyObject *__pyx_pf_8PyClical_8clifford_76conj(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02593 static PyObject *__pyx_pf_8PyClical_8clifford_78quad(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02594 static PyObject *__pyx_pf_8PyClical_8clifford_80norm(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02595 static PyObject *__pyx_pf_8PyClical_8clifford_82abs(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02596 static PyObject *__pyx_pf_8PyClical_8clifford_84max_abs(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02597 static PyObject *__pyx_pf_8PyClical_8clifford_86truncated(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_limit); /* proto */
02598 static PyObject *__pyx_pf_8PyClical_8clifford_88isinf(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02599 static PyObject *__pyx_pf_8PyClical_8clifford_90isnan(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02600 static PyObject *__pyx_pf_8PyClical_8clifford_92frame(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02601 static PyObject *__pyx_pf_8PyClical_8clifford_94_repr__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02602 static PyObject *__pyx_pf_8PyClical_8clifford_96_str__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02603 static PyObject *__pyx_pf_8PyClical_8clifford_98_reduce_cython__(CYTHON_UNUSED struct
    __pyx_obj_8PyClical_clifford *__pyx_v_self); /* proto */
02604 static PyObject *__pyx_pf_8PyClical_8clifford_100_setstate_cython__(CYTHON_UNUSED struct
    __pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state); /* proto */
02605 static PyObject *__pyx_pf_8PyClical_8clifford_hidden_doctests(CYTHON_UNUSED PyObject *__pyx_self); /*
    proto */
02606 static PyObject *__pyx_pf_8PyClical_10error_squared_tol(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02607 static PyObject *__pyx_pf_8PyClical_12error_squared(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold); /* proto */
02608 static PyObject *__pyx_pf_8PyClical_14approx_equal(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold, PyObject *__pyx_v_tol); /* proto */
02609 static PyObject *__pyx_pf_8PyClical_16lnv(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02610 static PyObject *__pyx_pf_8PyClical_18scalar(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02611 static PyObject *__pyx_pf_8PyClical_20real(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02612 static PyObject *__pyx_pf_8PyClical_22imag(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02613 static PyObject *__pyx_pf_8PyClical_24pure(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */

```



```

02614 static PyObject *__pyx_pf_8PyClical_26even(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02615 static PyObject *__pyx_pf_8PyClical_28odd(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02616 static PyObject *__pyx_pf_8PyClical_30involute(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj); /* proto */
02617 static PyObject *__pyx_pf_8PyClical_32reverse(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj); /* proto */
02618 static PyObject *__pyx_pf_8PyClical_34conj(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02619 static PyObject *__pyx_pf_8PyClical_36quad(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02620 static PyObject *__pyx_pf_8PyClical_38norm(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02621 static PyObject *__pyx_pf_8PyClical_40abs(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02622 static PyObject *__pyx_pf_8PyClical_42max_abs(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj); /* proto */
02623 static PyObject *__pyx_pf_8PyClical_44pow(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_m); /* proto */
02624 static PyObject *__pyx_pf_8PyClical_46outer_pow(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj, PyObject *__pyx_v_m); /* proto */
02625 static PyObject *__pyx_pf_8PyClical_48complexifier(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj); /* proto */
02626 static PyObject *__pyx_pf_8PyClical_50sqrt(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02627 static PyObject *__pyx_pf_8PyClical_52exp(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02628 static PyObject *__pyx_pf_8PyClical_54log(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02629 static PyObject *__pyx_pf_8PyClical_56cos(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02630 static PyObject *__pyx_pf_8PyClical_58acos(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02631 static PyObject *__pyx_pf_8PyClical_60cosh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02632 static PyObject *__pyx_pf_8PyClical_62acosh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02633 static PyObject *__pyx_pf_8PyClical_64sin(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02634 static PyObject *__pyx_pf_8PyClical_66asin(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02635 static PyObject *__pyx_pf_8PyClical_68sinh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02636 static PyObject *__pyx_pf_8PyClical_70asinh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02637 static PyObject *__pyx_pf_8PyClical_72tan(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02638 static PyObject *__pyx_pf_8PyClical_74atan(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02639 static PyObject *__pyx_pf_8PyClical_76tanh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02640 static PyObject *__pyx_pf_8PyClical_78atanh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
PyObject *__pyx_v_i); /* proto */
02641 static PyObject *__pyx_pf_8PyClical_80random_clifford(CYTHON_UNUSED PyObject *__pyx_self, struct
__pyx_obj_8PyClical_index_set *__pyx_v_ixt, PyObject *__pyx_v_fill); /* proto */
02642 static PyObject *__pyx_pf_8PyClical_82cga3(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02643 static PyObject *__pyx_pf_8PyClical_84cga3std(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj); /* proto */
02644 static PyObject *__pyx_pf_8PyClical_86agc3(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
/* proto */
02645 static PyObject *__pyx_pf_8PyClical_88e(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj); /*
proto */
02646 static PyObject *__pyx_pf_8PyClical_90istpq(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_p,
PyObject *__pyx_v_q); /* proto */
02647 static PyObject *__pyx_pf_8PyClical_92_test(CYTHON_UNUSED PyObject *__pyx_self); /* proto */
02648 static PyObject *__pyx_tp_new_8PyClical_index_set(PyTypeObject *t, PyObject *a, PyObject *k);
/*proto*/
02649 static PyObject *__pyx_tp_new_8PyClical_clifford(PyTypeObject *t, PyObject *a, PyObject *k); /*proto*/
02650 static PyObject *__pyx_tp_new_8PyClical__pyx_scope_struct__iter__(PyTypeObject *t, PyObject *a,
PyObject *k); /*proto*/
02651 static PyObject *__pyx_float_0_0;
02652 static PyObject *__pyx_float_1_0;
02653 static PyObject *__pyx_float_4_0;
02654 static PyObject *__pyx_float_8_0;
02655 static PyObject *__pyx_int_0;
02656 static PyObject *__pyx_int_1;
02657 static PyObject *__pyx_int_4;
02658 static PyObject *__pyx_int_neg_1;
02659 static PyObject *__pyx_tuple__3;
02660 static PyObject *__pyx_tuple__4;
02661 static PyObject *__pyx_tuple__10;
02662 static PyObject *__pyx_tuple__11;
02663 static PyObject *__pyx_tuple__12;
02664 static PyObject *__pyx_tuple__15;
02665 static PyObject *__pyx_tuple__16;

```

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02666 static PyObject *__pyx_tuple__18;
02667 static PyObject *__pyx_tuple__20;
02668 static PyObject *__pyx_tuple__21;
02669 static PyObject *__pyx_tuple__22;
02670 static PyObject *__pyx_codeobj__13;
02671 static PyObject *__pyx_codeobj__14;
02672 static PyObject *__pyx_codeobj__17;
02673 static PyObject *__pyx_codeobj__19;
02674 static PyObject *__pyx_codeobj__23;
02675 /* Late includes */
02676
02677 /* "PyClical.pyx":40
02678 * cdef class index_set
02679 *
02680 * cdef inline IndexSet toIndexSet(obj):          # ««««««««
02681 *     """
02682 *     Return the C++ IndexSet instance wrapped by index_set(obj).
02683 */
02684
02685 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_toIndexSet(PyObject *__pyx_v_obj) {
02686     IndexSet __pyx_r;
02687     __Pyx_RefNannyDeclarations
02688     PyObject *__pyx_t_1 = NULL;
02689     int __pyx_lineno = 0;
02690     const char *__pyx_filename = NULL;
02691     int __pyx_clineno = 0;
02692     __Pyx_RefNannySetupContext("toIndexSet", 0);
02693
02694     /* "PyClical.pyx":44
02695     *     Return the C++ IndexSet instance wrapped by index_set(obj).
02696     *     """
02697     *     return index_set(obj).instance[0]          # ««««««««
02698     *
02699     * cdef class index_set:
02700     */
02701     __pyx_t_1 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_index_set, __pyx_v_obj);
02702     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 44, __pyx_L1_error)
02703     __Pyx_GOTREF(__pyx_t_1);
02704     __pyx_r = (((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1->instance[0]));
02705     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
02706     goto __pyx_L0;
02707
02708     /* "PyClical.pyx":40
02709     * cdef class index_set
02710     *
02711     * cdef inline IndexSet toIndexSet(obj):          # ««««««««
02712     *     """
02713     *     Return the C++ IndexSet instance wrapped by index_set(obj).
02714     */
02715
02716     /* function exit code */
02717     __pyx_L1_error:;
02718     __Pyx_XDECREF(__pyx_t_1);
02719     __Pyx_WriteUnraisable("PyClical.toIndexSet", __pyx_clineno, __pyx_lineno, __pyx_filename, 1, 0);
02720     __pyx_L0:;
02721     __Pyx_RefNannyFinishContext();
02722     return __pyx_r;
02723 }
02724
02725 /* "PyClical.pyx":52
02726 *     cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
02727 *
02728 *     cdef inline wrap(index_set self, IndexSet other):          # ««««««««
02729 *         """
02730 *         Wrap an instance of the C++ class IndexSet.
02731 */
02732
02733 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_9index_set_wrap(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, IndexSet __pyx_v_other) {
02734     PyObject *__pyx_r = NULL;
02735     __Pyx_RefNannyDeclarations
02736     __Pyx_RefNannySetupContext("wrap", 0);
02737
02738     /* "PyClical.pyx":56
02739     *     Wrap an instance of the C++ class IndexSet.
02740     *     """
02741     *     self.instance[0] = other          # ««««««««
02742     *     return self
02743     *
02744     */
02745     (__pyx_v_self->instance[0]) = __pyx_v_other;
02746
02747     /* "PyClical.pyx":57
02748     *     """
02749     *     self.instance[0] = other
02750     *     return self          # ««««««««

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02751 *
02752 *      cdef inline IndexSet unwrap(index_set self):
02753 */
02754 __Pyx_XDECREF(__pyx_r);
02755 __Pyx_INCREF((PyObject *)__pyx_v_self);
02756 __pyx_r = (PyObject *)__pyx_v_self;
02757 goto __pyx_L0;
02758
02759 /* "PyClical.pyx":52
02760 *      cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
02761 *
02762 *      cdef inline wrap(index_set self, IndexSet other):          # ««««««««
02763 *          """
02764 *          Wrap an instance of the C++ class IndexSet.
02765 */
02766
02767 /* function exit code */
02768 __pyx_L0:;
02769 __Pyx_XGIVEREF(__pyx_r);
02770 __Pyx_RefNannyFinishContext();
02771 return __pyx_r;
02772 }
02773
02774 /* "PyClical.pyx":59
02775 *      return self
02776 *
02777 *      cdef inline IndexSet unwrap(index_set self):          # ««««««««
02778 *          """
02779 *          Return the wrapped C++ IndexSet instance.
02780 */
02781
02782 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_9index_set_unwrap(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
02783     IndexSet __pyx_r;
02784     __Pyx_RefNannyDeclarations
02785     __Pyx_RefNannySetupContext("unwrap", 0);
02786
02787 /* "PyClical.pyx":63
02788 *          Return the wrapped C++ IndexSet instance.
02789 *          """
02790 *          return self.instance[0]          # ««««««««
02791 *
02792 *      cpdef copy(index_set self):
02793 */
02794 __pyx_r = (__pyx_v_self->instance[0]);
02795 goto __pyx_L0;
02796
02797 /* "PyClical.pyx":59
02798 *      return self
02799 *
02800 *      cdef inline IndexSet unwrap(index_set self):          # ««««««««
02801 *          """
02802 *          Return the wrapped C++ IndexSet instance.
02803 */
02804
02805 /* function exit code */
02806 __pyx_L0:;
02807 __Pyx_RefNannyFinishContext();
02808 return __pyx_r;
02809 }
02810
02811 /* "PyClical.pyx":65
02812 *          return self.instance[0]
02813 *
02814 *      cpdef copy(index_set self):          # ««««««««
02815 *          """
02816 *          Copy this index_set object.
02817 */
02818
02819 static PyObject *__pyx_pw_8PyClical_9index_set_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused); /*proto*/
02820 static PyObject *__pyx_f_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set *__pyx_v_self,
int __pyx_skip_dispatch) {
02821     PyObject *__pyx_r = NULL;
02822     __Pyx_RefNannyDeclarations
02823     PyObject *__pyx_t_1 = NULL;
02824     PyObject *__pyx_t_2 = NULL;
02825     PyObject *__pyx_t_3 = NULL;
02826     PyObject *__pyx_t_4 = NULL;
02827     int __pyx_lineno = 0;
02828     const char *__pyx_filename = NULL;
02829     int __pyx_clineno = 0;
02830     __Pyx_RefNannySetupContext("copy", 0);
02831     /* Check if called by wrapper */
02832     if (unlikely(__pyx_skip_dispatch));
02833     /* Check if overridden in Python */
02834     else if (unlikely((Py_TYPE((PyObject *)__pyx_v_self)->tp_dictoffset != 0) || (Py_TYPE((PyObject

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*)__pyx_v_self))->tp_flags & (Py_TPFLAGS_IS_ABSTRACT | Py_TPFLAGS_HEAPTYPE)))) {
02835     #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
02836     static PY_UINT64_T __pyx_tp_dict_version = __PYX_DICT_VERSION_INIT, __pyx_obj_dict_version =
__PYX_DICT_VERSION_INIT;
02837     if (unlikely(!__Pyx_object_dict_version_matches(((PyObject *)__pyx_v_self), __pyx_tp_dict_version,
__pyx_obj_dict_version))) {
02838         PY_UINT64_T __pyx_type_dict_guard = __Pyx_get_tp_dict_version(((PyObject *)__pyx_v_self));
02839         #endif
02840         __pyx_t_1 = __Pyx_PyObject_GetAttrStr(((PyObject *)__pyx_v_self), __pyx_n_s_copy); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 65, __pyx_L1_error)
02841         __Pyx_GOTREF(__pyx_t_1);
02842         if (!PyCFunction_Check(__pyx_t_1) || (PyCFunction_GET_FUNCTION(__pyx_t_1) !=
(PyCFunction)(void*)__pyx_pw_8PyClical_9index_set_lcopy)) {
02843             __Pyx_XDECREF(__pyx_r);
02844             __Pyx_INCREF(__pyx_t_1);
02845             __pyx_t_3 = __pyx_t_1; __pyx_t_4 = NULL;
02846             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_3))) {
02847                 __pyx_t_4 = PyMethod_GET_SELF(__pyx_t_3);
02848                 if (likely(__pyx_t_4)) {
02849                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
02850                     __Pyx_INCREF(__pyx_t_4);
02851                     __Pyx_INCREF(function);
02852                     __Pyx_DECREF_SET(__pyx_t_3, function);
02853                 }
02854             }
02855             __pyx_t_2 = (__pyx_t_4) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_4) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
02856             __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
02857             if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 65, __pyx_L1_error)
02858             __Pyx_GOTREF(__pyx_t_2);
02859             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
02860             __pyx_r = __pyx_t_2;
02861             __pyx_t_2 = 0;
02862             __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
02863             goto __pyx_L0;
02864         }
02865         #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
02866         __pyx_tp_dict_version = __Pyx_get_tp_dict_version(((PyObject *)__pyx_v_self));
02867         __pyx_obj_dict_version = __Pyx_get_object_dict_version(((PyObject *)__pyx_v_self));
02868         if (unlikely(__pyx_type_dict_guard != __pyx_tp_dict_version)) {
02869             __pyx_tp_dict_version = __pyx_obj_dict_version = __PYX_DICT_VERSION_INIT;
02870         }
02871         #endif
02872         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
02873         #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
02874     }
02875     #endif
02876 }
02877
02878 /* "PyClical.pyx":72
02879 *     {1}
02880 *     """
02881 *     return index_set(self) # ««««««««
02882 *
02883 *     def __cinit__(self, other = 0):
02884 */
02885     __Pyx_XDECREF(__pyx_r);
02886     __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_index_set), ((PyObject
*)__pyx_v_self)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 72, __pyx_L1_error)
02887     __Pyx_GOTREF(__pyx_t_1);
02888     __pyx_r = __pyx_t_1;
02889     __pyx_t_1 = 0;
02890     goto __pyx_L0;
02891
02892 /* "PyClical.pyx":65
02893 *     return self.instance[0]
02894 *
02895 *     cpdef copy(index_set self): # ««««««««
02896 *     """
02897 *     Copy this index_set object.
02898 */
02899
02900 /* function exit code */
02901 __pyx_L1_error:;
02902 __Pyx_XDECREF(__pyx_t_1);
02903 __Pyx_XDECREF(__pyx_t_2);
02904 __Pyx_XDECREF(__pyx_t_3);
02905 __Pyx_XDECREF(__pyx_t_4);
02906 __Pyx_AddTraceback("PyClical.index_set.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
02907 __pyx_r = 0;
02908 __pyx_L0:;
02909 __Pyx_XGIVEREF(__pyx_r);
02910 __Pyx_RefNannyFinishContext();
02911 return __pyx_r;
02912 }
02913
02914 /* Python wrapper */

```

```

02915 static PyObject *__pyx_pw_8PyClical_9index_set_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
    *unused); /*proto*/
02916 static char __pyx_doc_8PyClical_9index_set_copy[] = "\n          Copy this index_set object.\n\n
    >> s=index_set(1); t=s.copy(); print(t)\n          {1}\n          ";
02917 static PyObject *__pyx_pw_8PyClical_9index_set_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
    *unused) {
02918     PyObject *__pyx_r = 0;
02919     __Pyx_RefNannyDeclarations
02920     __Pyx_RefNannySetupContext("copy (wrapper)", 0);
02921     __pyx_r = __pyx_pf_8PyClical_9index_set_copy(((struct __pyx_obj_8PyClical_index_set
    *)__pyx_v_self));
02922
02923     /* function exit code */
02924     __Pyx_RefNannyFinishContext();
02925     return __pyx_r;
02926 }
02927
02928 static PyObject *__pyx_pf_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self) {
02929     PyObject *__pyx_r = NULL;
02930     __Pyx_RefNannyDeclarations
02931     PyObject *__pyx_t_1 = NULL;
02932     int __pyx_lineno = 0;
02933     const char *__pyx_filename = NULL;
02934     int __pyx_clineno = 0;
02935     __Pyx_RefNannySetupContext("copy", 0);
02936     __Pyx_XDECREF(__pyx_r);
02937     __pyx_t_1 = __pyx_f_8PyClical_9index_set_copy(__pyx_v_self, 1); if (unlikely(!__pyx_t_1))
    __PYX_ERR(0, 65, __pyx_L1_error)
02938     __Pyx_GOTREF(__pyx_t_1);
02939     __pyx_r = __pyx_t_1;
02940     __pyx_t_1 = 0;
02941     goto __pyx_L0;
02942
02943     /* function exit code */
02944     __pyx_L1_error:;
02945     __Pyx_XDECREF(__pyx_t_1);
02946     __Pyx_AddTraceback("PyClical.index_set.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
02947     __pyx_r = NULL;
02948     __pyx_L0:;
02949     __Pyx_XGIVEREF(__pyx_r);
02950     __Pyx_RefNannyFinishContext();
02951     return __pyx_r;
02952 }
02953
02954 /* "PyClical.pyx":74
02955 *         return index_set(self)
02956 *
02957 *     def __cinit__(self, other = 0):          # ««««««««
02958 *     """
02959 *         Construct an object of type index_set.
02960 */
02961
02962 /* Python wrapper */
02963 static int __pyx_pw_8PyClical_9index_set_3__cinit__(PyObject *__pyx_v_self, PyObject *__pyx_args,
    PyObject *__pyx_kwds); /*proto*/
02964 static int __pyx_pw_8PyClical_9index_set_3__cinit__(PyObject *__pyx_v_self, PyObject *__pyx_args,
    PyObject *__pyx_kwds) {
02965     PyObject *__pyx_v_other = 0;
02966     int __pyx_lineno = 0;
02967     const char *__pyx_filename = NULL;
02968     int __pyx_clineno = 0;
02969     int __pyx_r;
02970     __Pyx_RefNannyDeclarations
02971     __Pyx_RefNannySetupContext("__cinit__ (wrapper)", 0);
02972     {
02973         static PyObject** __pyx_pyargnames[] = {&__pyx_n_s_other,0};
02974         PyObject* values[1] = {0};
02975         values[0] = ((PyObject *)__pyx_int_0);
02976         if (unlikely(__pyx_kwds)) {
02977             Py_ssize_t kw_args;
02978             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
02979             switch (pos_args) {
02980                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
02981                     CYTHON_FALLTHROUGH;
02982                 case 0: break;
02983                 default: goto __pyx_L5_argtuple_error;
02984             }
02985             kw_args = PyDict_Size(__pyx_kwds);
02986             switch (pos_args) {
02987                 case 0:
02988                     if (kw_args > 0) {
02989                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_other);
02990                         if (value) { values[0] = value; kw_args--; }
02991                     }
02992             }
02993             if (unlikely(kw_args > 0)) {

```

```

02994         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values, pos_args,
02995             "__cinit__") < 0)) __PYX_ERR(0, 74, __pyx_L3_error)
02996     } else {
02997         switch (PyTuple_GET_SIZE(__pyx_args)) {
02998             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
02999             CYTHON_FALLTHROUGH;
03000             case 0: break;
03001             default: goto __pyx_L5_argtuple_error;
03002         }
03003     }
03004     __pyx_v_other = values[0];
03005 }
03006 goto __pyx_L4_argument_unpacking_done;
03007 __pyx_L5_argtuple_error:;
03008 __Pyx_RaiseArgtupleInvalid("__cinit__", 0, 0, 1, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0, 74,
03009 __pyx_L3_error)
03009 __pyx_L3_error:;
03010 __Pyx_AddTraceback("PyCliclal.index_set.__cinit__", __pyx_clineno, __pyx_lineno, __pyx_filename);
03011 __Pyx_RefNannyFinishContext();
03012 return -1;
03013 __pyx_L4_argument_unpacking_done:;
03014 __pyx_r = __pyx_pf_8PyCliclal_9index_set_2__cinit__((struct __pyx_obj_8PyCliclal_index_set
*)__pyx_v_self), __pyx_v_other);
03015
03016 /* function exit code */
03017 __Pyx_RefNannyFinishContext();
03018 return __pyx_r;
03019 }
03020
03021 static int __pyx_pf_8PyCliclal_9index_set_2__cinit__(struct __pyx_obj_8PyCliclal_index_set
*)__pyx_v_self, PyObject *__pyx_v_other) {
03022     PyObject *__pyx_v_error_msg_prefix = NULL;
03023     PyObject *__pyx_v_idx = NULL;
03024     PyObject *__pyx_v_bother = NULL;
03025     int __pyx_r;
03026     __Pyx_RefNannyDeclarations
03027     int __pyx_t_1;
03028     int __pyx_t_2;
03029     IndexSet *__pyx_t_3;
03030     PyObject *__pyx_t_4 = NULL;
03031     PyObject *__pyx_t_5 = NULL;
03032     int __pyx_t_6;
03033     int __pyx_t_7;
03034     PyObject *__pyx_t_8 = NULL;
03035     PyObject *__pyx_t_9 = NULL;
03036     PyObject *__pyx_t_10 = NULL;
03037     Py_ssize_t __pyx_t_11;
03038     PyObject *(*__pyx_t_12)(PyObject *);
03039     PyObject *__pyx_t_13 = NULL;
03040     PyObject *__pyx_t_14 = NULL;
03041     PyObject *__pyx_t_15 = NULL;
03042     PyObject *__pyx_t_16 = NULL;
03043     char *__pyx_t_17;
03044     int __pyx_lineno = 0;
03045     const char *__pyx_filename = NULL;
03046     int __pyx_clineno = 0;
03047     __Pyx_RefNannySetupContext("__cinit__", 0);
03048
03049     /* "PyCliclal.pyx":93
03050     *     {}
03051     *     """
03052     *     error_msg_prefix = "Cannot initialize index_set object from" # ««««««««
03053     *     if isinstance(other, index_set):
03054     *         self.instance = new IndexSet((<index_set>other).unwrap())
03055     */
03056     __Pyx_INCREF(__pyx_kp_u_Cannot_initialize_index_set_obje);
03057     __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_initialize_index_set_obje;
03058
03059     /* "PyCliclal.pyx":94
03060     *     """
03061     *     error_msg_prefix = "Cannot initialize index_set object from"
03062     *     if isinstance(other, index_set): # ««««««««
03063     *         self.instance = new IndexSet((<index_set>other).unwrap())
03064     *     elif isinstance(other, numbers.Integral):
03065     */
03066     __pyx_t_1 = __Pyx_TypeCheck(__pyx_v_other, __pyx_ptype_8PyCliclal_index_set);
03067     __pyx_t_2 = (__pyx_t_1 != 0);
03068     if (__pyx_t_2) {
03069
03070         /* "PyCliclal.pyx":95
03071         *     error_msg_prefix = "Cannot initialize index_set object from"
03072         *     if isinstance(other, index_set):
03073         *         self.instance = new IndexSet((<index_set>other).unwrap()) # ««««««««
03074         *     elif isinstance(other, numbers.Integral):
03075         *         self.instance = new IndexSet(<int>other)
03076         */

```

```

03077     try {
03078         __pyx_t_3 = new IndexSet(__pyx_f_8PyClical_9index_set_unwrap(((struct
__pyx_obj_8PyClical_index_set *)__pyx_v_other)));
03079     } catch(...) {
03080         __Pyx_CppExn2PyErr();
03081         __PYX_ERR(0, 95, __pyx_L1_error)
03082     }
03083     __pyx_v_self->instance = __pyx_t_3;
03084
03085     /* "PyClical.pyx":94
03086     *
03087     *     error_msg_prefix = "Cannot initialize index_set object from"
03088     *     if isinstance(other, index_set): # ««««««««
03089     *         self.instance = new IndexSet((<index_set>other).unwrap())
03090     *     elif isinstance(other, numbers.Integral):
03091     */
03092     goto __pyx_L3;
03093 }
03094
03095 /* "PyClical.pyx":96
03096 *     if isinstance(other, index_set):
03097 *         self.instance = new IndexSet((<index_set>other).unwrap())
03098 *     elif isinstance(other, numbers.Integral): # ««««««««
03099 *         self.instance = new IndexSet(<int>other)
03100 *     elif isinstance(other, (set, frozenset)):
03101     */
03102     __Pyx_GetModuleGlobalName(__pyx_t_4, __pyx_n_s_numbers); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 96,
__pyx_L1_error)
03103     __Pyx_GOTREF(__pyx_t_4);
03104     __pyx_t_5 = __Pyx_PyObject_GetAttrStr(__pyx_t_4, __pyx_n_s_Integral); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 96, __pyx_L1_error)
03105     __Pyx_GOTREF(__pyx_t_5);
03106     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03107     __pyx_t_2 = PyObject_IsInstance(__pyx_v_other, __pyx_t_5); if (unlikely(__pyx_t_2 == ((int)-1)))
__PYX_ERR(0, 96, __pyx_L1_error)
03108     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03109     __pyx_t_1 = (__pyx_t_2 != 0);
03110     if (__pyx_t_1) {
03111
03112         /* "PyClical.pyx":97
03113         *         self.instance = new IndexSet((<index_set>other).unwrap())
03114         *     elif isinstance(other, numbers.Integral):
03115         *         self.instance = new IndexSet(<int>other) # ««««««««
03116         *     elif isinstance(other, (set, frozenset)):
03117         *         try:
03118         */
03119         __pyx_t_6 = __Pyx_PyInt_As_int(__pyx_v_other); if (unlikely((__pyx_t_6 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 97, __pyx_L1_error)
03120         try {
03121             __pyx_t_3 = new IndexSet(((int) __pyx_t_6));
03122         } catch(...) {
03123             __Pyx_CppExn2PyErr();
03124             __PYX_ERR(0, 97, __pyx_L1_error)
03125         }
03126         __pyx_v_self->instance = __pyx_t_3;
03127
03128         /* "PyClical.pyx":96
03129         *     if isinstance(other, index_set):
03130         *         self.instance = new IndexSet((<index_set>other).unwrap())
03131         *     elif isinstance(other, numbers.Integral): # ««««««««
03132         *         self.instance = new IndexSet(<int>other)
03133         *     elif isinstance(other, (set, frozenset)):
03134         */
03135         goto __pyx_L3;
03136     }
03137
03138     /* "PyClical.pyx":98
03139     *     elif isinstance(other, numbers.Integral):
03140     *         self.instance = new IndexSet(<int>other)
03141     *     elif isinstance(other, (set, frozenset)): # ««««««««
03142     *         try:
03143     *             self.instance = new IndexSet()
03144     */
03145     __pyx_t_2 = PySet_Check(__pyx_v_other);
03146     __pyx_t_7 = (__pyx_t_2 != 0);
03147     if (!__pyx_t_7) {
03148     } else {
03149         __pyx_t_1 = __pyx_t_7;
03150         goto __pyx_L4_bool_binop_done;
03151     }
03152     __pyx_t_7 = PyFrozenSet_Check(__pyx_v_other);
03153     __pyx_t_2 = (__pyx_t_7 != 0);
03154     __pyx_t_1 = __pyx_t_2;
03155     __pyx_L4_bool_binop_done;
03156     __pyx_t_2 = (__pyx_t_1 != 0);
03157     if (__pyx_t_2) {
03158

```

```

03159      /* "PyCliclcal.pyx":99
03160      *          self.instance = new IndexSet(<int>other)
03161      *          elif isinstance(other, (set, frozenset)):
03162      *              try:
03163      *                  self.instance = new IndexSet()
03164      *                  for idx in other:
03165      */
03166      {
03167          __Pyx_PyThreadState_declare
03168          __Pyx_PyThreadState_assign
03169          __Pyx_ExceptionSave(&__pyx_t_8, &__pyx_t_9, &__pyx_t_10);
03170          __Pyx_XGOTREF(__pyx_t_8);
03171          __Pyx_XGOTREF(__pyx_t_9);
03172          __Pyx_XGOTREF(__pyx_t_10);
03173      /*try:*/ {
03174
03175          /* "PyCliclcal.pyx":100
03176          *          elif isinstance(other, (set, frozenset)):
03177          *              try:
03178          *                  self.instance = new IndexSet()
03179          *                  for idx in other:
03180          *                      self[idx] = True
03181          */
03182          __pyx_t_3 = new IndexSet();
03183          __pyx_v_self->instance = __pyx_t_3;
03184
03185          /* "PyCliclcal.pyx":101
03186          *              try:
03187          *                  self.instance = new IndexSet()
03188          *                  for idx in other:
03189          *                      self[idx] = True
03190          *              except IndexError:
03191          */
03192          if (likely(PyList_CheckExact(__pyx_v_other)) || PyTuple_CheckExact(__pyx_v_other)) {
03193              __pyx_t_5 = __pyx_v_other; __Pyx_INCREF(__pyx_t_5); __pyx_t_11 = 0;
03194              __pyx_t_12 = NULL;
03195          } else {
03196              __pyx_t_11 = -1; __pyx_t_5 = PyObject_GetIter(__pyx_v_other); if (unlikely(!__pyx_t_5))
03197              __PYX_ERR(0, 101, __pyx_L6_error)
03198              __Pyx_GOTREF(__pyx_t_5);
03199              __pyx_t_12 = Py_TYPE(__pyx_t_5)->tp_iternext; if (unlikely(!__pyx_t_12)) __PYX_ERR(0, 101,
03200              __pyx_L6_error)
03201          }
03202          for (;;) {
03203              if (likely(!__pyx_t_12)) {
03204                  if (likely(PyList_CheckExact(__pyx_t_5))) {
03205                      if (__pyx_t_11 >= PyList_GET_SIZE(__pyx_t_5)) break;
03206                      #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
03207                      __pyx_t_4 = PyList_GET_ITEM(__pyx_t_5, __pyx_t_11); __Pyx_INCREF(__pyx_t_4);
03208                      __pyx_t_11++; if (unlikely(0 < 0)) __PYX_ERR(0, 101, __pyx_L6_error)
03209                      #else
03210                      __pyx_t_4 = PySequence_ITEM(__pyx_t_5, __pyx_t_11); __pyx_t_11++; if
03211                      (unlikely(!__pyx_t_4)) __PYX_ERR(0, 101, __pyx_L6_error)
03212                      __Pyx_GOTREF(__pyx_t_4);
03213                      #endif
03214                  } else {
03215                      if (__pyx_t_11 >= PyTuple_GET_SIZE(__pyx_t_5)) break;
03216                      #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
03217                      __pyx_t_4 = PyTuple_GET_ITEM(__pyx_t_5, __pyx_t_11); __Pyx_INCREF(__pyx_t_4);
03218                      __pyx_t_11++; if (unlikely(0 < 0)) __PYX_ERR(0, 101, __pyx_L6_error)
03219                      #else
03220                      __pyx_t_4 = PySequence_ITEM(__pyx_t_5, __pyx_t_11); __pyx_t_11++; if
03221                      (unlikely(!__pyx_t_4)) __PYX_ERR(0, 101, __pyx_L6_error)
03222                      __Pyx_GOTREF(__pyx_t_4);
03223                      #endif
03224                  } else {
03225                      __pyx_t_4 = __pyx_t_12(__pyx_t_5);
03226                      if (unlikely(!__pyx_t_4)) {
03227                          PyObject* exc_type = PyErr_Occurred();
03228                          if (exc_type) {
03229                              if (likely(__Pyx_PyErr_GivenExceptionMatches(exc_type, PyExc_StopIteration)))
03230                                  PyErr_Clear();
03231                              else __PYX_ERR(0, 101, __pyx_L6_error)
03232                          }
03233                          break;
03234                      }
03235                      __Pyx_GOTREF(__pyx_t_4);
03236                  }
03237                  __Pyx_XDECREF_SET(__pyx_v_idx, __pyx_t_4);
03238                  __pyx_t_4 = 0;
03239
03240          /* "PyCliclcal.pyx":102
03241          *          self.instance = new IndexSet()
03242          *          for idx in other:
03243          *              self[idx] = True
03244          *          except IndexError:

```



```

03239 *             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03240 */
03241         if (unlikely(PyObject_SetItem((PyObject *)__pyx_v_self, __pyx_v_idx, Py_True) < 0))
__PYX_ERR(0, 102, __pyx_L6_error)
03242
03243         /* "PyClical.pyx":101
03244 *             try:
03245 *                 self.instance = new IndexSet()
03246 *                 for idx in other:
03247 *                     self[idx] = True
03248 *             except IndexError:
03249 */
03250     }
03251     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03252
03253     /* "PyClical.pyx":99
03254 *         self.instance = new IndexSet(<int>other)
03255 *         elif isinstance(other, (set, frozenset)):
03256 *             try:
03257 *                 self.instance = new IndexSet()
03258 *                 for idx in other:
03259 */
03260     }
03261     __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
03262     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
03263     __Pyx_XDECREF(__pyx_t_10); __pyx_t_10 = 0;
03264     goto __pyx_L11_try_end;
03265     __pyx_L6_error:;
03266     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
03267     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
03268
03269     /* "PyClical.pyx":103
03270 *         for idx in other:
03271 *             self[idx] = True
03272 *         except IndexError:
03273 *             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03274 *         except (RuntimeError, TypeError):
03275 */
03276     __pyx_t_6 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_IndexError);
03277     if (__pyx_t_6) {
03278         __Pyx_AddTraceback("PyClical.index_set.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
03279         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_4, &__pyx_t_13) < 0) __PYX_ERR(0, 103,
__pyx_L8_except_error)
03280         __Pyx_GOTREF(__pyx_t_5);
03281         __Pyx_GOTREF(__pyx_t_4);
03282         __Pyx_GOTREF(__pyx_t_13);
03283
03284         /* "PyClical.pyx":104
03285 *             self[idx] = True
03286 *         except IndexError:
03287 *             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03288 *         except (RuntimeError, TypeError):
03289 *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03290 */
03291         __pyx_t_14 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_invalid); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 104, __pyx_L8_except_error)
03292         __Pyx_GOTREF(__pyx_t_14);
03293         __pyx_t_15 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 104,
__pyx_L8_except_error)
03294         __Pyx_GOTREF(__pyx_t_15);
03295         __pyx_t_16 = PyNumber_Add(__pyx_t_14, __pyx_t_15); if (unlikely(!__pyx_t_16)) __PYX_ERR(0,
104, __pyx_L8_except_error)
03296         __Pyx_GOTREF(__pyx_t_16);
03297         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03298         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03299         __pyx_t_15 = PyNumber_Add(__pyx_t_16, __pyx_kp_u_); if (unlikely(!__pyx_t_15)) __PYX_ERR(0,
104, __pyx_L8_except_error)
03300         __Pyx_GOTREF(__pyx_t_15);
03301         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03302         __pyx_t_16 = __Pyx_PyObject_CallOneArg(__pyx_builtin_IndexError, __pyx_t_15); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 104, __pyx_L8_except_error)
03303         __Pyx_GOTREF(__pyx_t_16);
03304         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03305         __Pyx_Raise(__pyx_t_16, 0, 0, 0);
03306         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03307         __PYX_ERR(0, 104, __pyx_L8_except_error)
03308     }
03309
03310     /* "PyClical.pyx":105
03311 *         except IndexError:
03312 *             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03313 *         except (RuntimeError, TypeError):
03314 *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03315 *         elif isinstance(other, str):
03316 */

```

```

03317     __pyx_t_6 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError) ||
__Pyx_PyErr_ExceptionMatches(__pyx_builtin_TypeError);
03318     if (__pyx_t_6) {
03319         __Pyx_AddTraceback("PyCliclcal.index_set.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
03320         if (__Pyx_GetException(&__pyx_t_13, &__pyx_t_4, &__pyx_t_5) < 0) __PYX_ERR(0, 105,
__pyx_L8_except_error)
03321         __Pyx_GOTREF(__pyx_t_13);
03322         __Pyx_GOTREF(__pyx_t_4);
03323         __Pyx_GOTREF(__pyx_t_5);
03324
03325         /* "PyCliclcal.pyx":106
03326         *         raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03327         *         except (RuntimeError, TypeError):
03328         *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")           #
03329         *             elif isinstance(other, str):
03330         *                 try:
03331         */
03332         __pyx_t_16 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_invalid); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 106, __pyx_L8_except_error)
03333         __Pyx_GOTREF(__pyx_t_16);
03334         __pyx_t_15 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 106,
__pyx_L8_except_error)
03335         __Pyx_GOTREF(__pyx_t_15);
03336         __pyx_t_14 = PyNumber_Add(__pyx_t_16, __pyx_t_15); if (unlikely(!__pyx_t_14)) __PYX_ERR(0,
106, __pyx_L8_except_error)
03337         __Pyx_GOTREF(__pyx_t_14);
03338         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03339         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03340         __pyx_t_15 = PyNumber_Add(__pyx_t_14, __pyx_kp_u_); if (unlikely(!__pyx_t_15)) __PYX_ERR(0,
106, __pyx_L8_except_error)
03341         __Pyx_GOTREF(__pyx_t_15);
03342         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03343         __pyx_t_14 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_15); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 106, __pyx_L8_except_error)
03344         __Pyx_GOTREF(__pyx_t_14);
03345         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03346         __Pyx_Raise(__pyx_t_14, 0, 0, 0);
03347         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03348         __PYX_ERR(0, 106, __pyx_L8_except_error)
03349     }
03350     goto __pyx_L8_except_error;
03351     __pyx_L8_except_error:;
03352
03353     /* "PyCliclcal.pyx":99
03354     *         self.instance = new IndexSet(<int>other)
03355     *         elif isinstance(other, (set, frozenset)):
03356     *             try:           # ««««««««
03357     *                 self.instance = new IndexSet()
03358     *                 for idx in other:
03359     */
03360         __Pyx_XGIVEREF(__pyx_t_8);
03361         __Pyx_XGIVEREF(__pyx_t_9);
03362         __Pyx_XGIVEREF(__pyx_t_10);
03363         __Pyx_ExceptionReset(__pyx_t_8, __pyx_t_9, __pyx_t_10);
03364         goto __pyx_L1_error;
03365         __pyx_L11_try_end:;
03366     }
03367
03368     /* "PyCliclcal.pyx":98
03369     *         elif isinstance(other, numbers.Integral):
03370     *             self.instance = new IndexSet(<int>other)
03371     *         elif isinstance(other, (set, frozenset)):           # ««««««««
03372     *             try:
03373     *                 self.instance = new IndexSet()
03374     */
03375     goto __pyx_L3;
03376 }
03377
03378 /* "PyCliclcal.pyx":107
03379 *         except (RuntimeError, TypeError):
03380 *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03381 *         elif isinstance(other, str):           # ««««««««
03382 *             try:
03383 *                 bother = other.encode("UTF-8")
03384 */
03385 __pyx_t_2 = PyUnicode_Check(__pyx_v_other);
03386 __pyx_t_1 = (__pyx_t_2 != 0);
03387 if (likely(__pyx_t_1)) {
03388
03389     /* "PyCliclcal.pyx":108
03390     *         raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03391     *         elif isinstance(other, str):
03392     *             try:           # ««««««««
03393     *                 bother = other.encode("UTF-8")
03394     *                 self.instance = new IndexSet(<char *>bother)

```

```

03395 */
03396 {
03397     __Pyx_PyThreadState_declare
03398     __Pyx_PyThreadState_assign
03399     __Pyx_ExceptionSave(&__pyx_t_10, &__pyx_t_9, &__pyx_t_8);
03400     __Pyx_XGOTREF(__pyx_t_10);
03401     __Pyx_XGOTREF(__pyx_t_9);
03402     __Pyx_XGOTREF(__pyx_t_8);
03403     /*try:*/ {
03404
03405         /* "PyClical.pyx":109
03406         *         elif isinstance(other, str):
03407         *             try:
03408         *                 bother = other.encode("UTF-8")           # ««««««««
03409         *                 self.instance = new IndexSet(<char *>bother)
03410         *             except RuntimeError:
03411         */
03412         __pyx_t_4 = __Pyx_PyObject_GetAttrStr(__pyx_v_other, __pyx_n_s_encode); if
(unlikely(!__pyx_t_4)) __PYX_ERR(0, 109, __pyx_L18_error)
03413         __Pyx_GOTREF(__pyx_t_4);
03414         __pyx_t_13 = NULL;
03415         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_4))) {
03416             __pyx_t_13 = PyMethod_GET_SELF(__pyx_t_4);
03417             if (likely(__pyx_t_13)) {
03418                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_4);
03419                 __Pyx_INCREF(__pyx_t_13);
03420                 __Pyx_INCREF(function);
03421                 __Pyx_DECREF_SET(__pyx_t_4, function);
03422             }
03423         }
03424         __pyx_t_5 = (__pyx_t_13) ? __Pyx_PyObject_Call2Args(__pyx_t_4, __pyx_t_13, __pyx_kp_u_UTF_8) :
__Pyx_PyObject_CallOneArg(__pyx_t_4, __pyx_kp_u_UTF_8);
03425         __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
03426         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 109, __pyx_L18_error)
03427         __Pyx_GOTREF(__pyx_t_5);
03428         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03429         __pyx_v_bother = __pyx_t_5;
03430         __pyx_t_5 = 0;
03431
03432         /* "PyClical.pyx":110
03433         *             try:
03434         *                 bother = other.encode("UTF-8")
03435         *                 self.instance = new IndexSet(<char *>bother)           # ««««««««
03436         *             except RuntimeError:
03437         *                 raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03438         */
03439         __pyx_t_17 = __Pyx_PyObject_AsWritableString(__pyx_v_bother); if (unlikely(!__pyx_t_17) &&
PyErr_Occurred()) __PYX_ERR(0, 110, __pyx_L18_error)
03440         try {
03441             __pyx_t_3 = new IndexSet(((char *)__pyx_t_17));
03442         } catch (...) {
03443             __Pyx_CppExn2PyErr();
03444             __PYX_ERR(0, 110, __pyx_L18_error)
03445         }
03446         __pyx_v_self->instance = __pyx_t_3;
03447
03448         /* "PyClical.pyx":108
03449         *                 raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03450         *             elif isinstance(other, str):
03451         *                 try:           # ««««««««
03452         *                     bother = other.encode("UTF-8")
03453         *                     self.instance = new IndexSet(<char *>bother)
03454         */
03455     }
03456     __Pyx_XDECREF(__pyx_t_10); __pyx_t_10 = 0;
03457     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
03458     __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
03459     goto __pyx_L23_try_end;
03460     __pyx_L18_error:;
03461     __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
03462     __Pyx_XDECREF(__pyx_t_14); __pyx_t_14 = 0;
03463     __Pyx_XDECREF(__pyx_t_15); __pyx_t_15 = 0;
03464     __Pyx_XDECREF(__pyx_t_16); __pyx_t_16 = 0;
03465     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
03466     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
03467
03468     /* "PyClical.pyx":111
03469     *         bother = other.encode("UTF-8")
03470     *         self.instance = new IndexSet(<char *>bother)
03471     *     except RuntimeError:           # ««««««««
03472     *         raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03473     *     else:
03474     */
03475     __pyx_t_6 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
03476     if (__pyx_t_6) {
03477         __Pyx_AddTraceback("PyClical.index_set.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);

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03478         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_4, &__pyx_t_13) < 0) __PYX_ERR(0, 111,
__pyx_L20_except_error)
03479         __Pyx_GOTREF(__pyx_t_5);
03480         __Pyx_GOTREF(__pyx_t_4);
03481         __Pyx_GOTREF(__pyx_t_13);
03482
03483         /* "PyClical.pyx":112
03484         *             self.instance = new IndexSet(<char *>bother)
03485         *             except RuntimeError:
03486         *                 raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03487         *             # ««««««««
03488         *             else:
03489         *                 raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03490         */
03490         __pyx_t_14 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_invalid_string); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 112, __pyx_L20_except_error)
03491         __Pyx_GOTREF(__pyx_t_14);
03492         __pyx_t_15 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 112,
__pyx_L20_except_error)
03493         __Pyx_GOTREF(__pyx_t_15);
03494         __pyx_t_16 = PyNumber_Add(__pyx_t_14, __pyx_t_15); if (unlikely(!__pyx_t_16)) __PYX_ERR(0,
112, __pyx_L20_except_error)
03495         __Pyx_GOTREF(__pyx_t_16);
03496         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03497         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03498         __pyx_t_15 = PyNumber_Add(__pyx_t_16, __pyx_kp_u); if (unlikely(!__pyx_t_15)) __PYX_ERR(0,
112, __pyx_L20_except_error)
03499         __Pyx_GOTREF(__pyx_t_15);
03500         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03501         __pyx_t_16 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_15); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 112, __pyx_L20_except_error)
03502         __Pyx_GOTREF(__pyx_t_16);
03503         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03504         __Pyx_Raise(__pyx_t_16, 0, 0, 0);
03505         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03506         __PYX_ERR(0, 112, __pyx_L20_except_error)
03507     }
03508     goto __pyx_L20_except_error;
03509     __pyx_L20_except_error:;
03510
03511     /* "PyClical.pyx":108
03512     *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03513     *             elif isinstance(other, str):
03514     *                 try:
03515     *                     # ««««««««
03516     *                     bother = other.encode("UTF-8")
03517     *                     self.instance = new IndexSet(<char *>bother)
03518     */
03518     __Pyx_XGIVEREF(__pyx_t_10);
03519     __Pyx_XGIVEREF(__pyx_t_9);
03520     __Pyx_XGIVEREF(__pyx_t_8);
03521     __Pyx_ExceptionReset(__pyx_t_10, __pyx_t_9, __pyx_t_8);
03522     goto __pyx_L1_error;
03523     __pyx_L23_try_end:;
03524 }
03525
03526 /* "PyClical.pyx":107
03527 *             except (RuntimeError, TypeError):
03528 *                 raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03529 *             elif isinstance(other, str):
03530 *                 try:
03531 *                     # ««««««««
03532 *                     bother = other.encode("UTF-8")
03533     */
03533     goto __pyx_L3;
03534 }
03535
03536 /* "PyClical.pyx":114
03537 *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03538 *             else:
03539 *                 raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03540 *             # ««««««««
03541 *             def __dealloc__(self):
03542     */
03543     /*else*/ {
03544         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_2); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 114, __pyx_L1_error)
03545         __Pyx_GOTREF(__pyx_t_13);
03546         __pyx_t_4 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other))); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 114, __pyx_L1_error)
03547         __Pyx_GOTREF(__pyx_t_4);
03548         __pyx_t_5 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0,
114, __pyx_L1_error)
03549         __Pyx_GOTREF(__pyx_t_5);
03550         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
03551         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03552         __pyx_t_4 = __Pyx_PyUnicode_Concat(__pyx_t_5, __pyx_kp_u); if (unlikely(!__pyx_t_4)) __PYX_ERR(0,
114, __pyx_L1_error)
03553         __Pyx_GOTREF(__pyx_t_4);

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```

03554     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03555     __pyx_t_5 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_4); if
(unlikely(!__pyx_t_5)) __PYX_ERR(0, 114, __pyx_L1_error)
03556     __Pyx_GOTREF(__pyx_t_5);
03557     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03558     __Pyx_Raise(__pyx_t_5, 0, 0, 0);
03559     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03560     __PYX_ERR(0, 114, __pyx_L1_error)
03561 }
03562 __pyx_L3:;
03563
03564 /* "PyClical.pyx":74
03565 *         return index_set(self)
03566 *
03567 *     def __cinit__(self, other = 0):                # ««««««««
03568 *         """
03569 *         Construct an object of type index_set.
03570 *
03571 */
03572 /* function exit code */
03573 __pyx_r = 0;
03574 goto __pyx_L0;
03575 __pyx_L1_error:;
03576 __Pyx_XDECREF(__pyx_t_4);
03577 __Pyx_XDECREF(__pyx_t_5);
03578 __Pyx_XDECREF(__pyx_t_13);
03579 __Pyx_XDECREF(__pyx_t_14);
03580 __Pyx_XDECREF(__pyx_t_15);
03581 __Pyx_XDECREF(__pyx_t_16);
03582 __Pyx_AddTraceback("PyClical.index_set.__cinit__", __pyx_clineno, __pyx_lineno, __pyx_filename);
03583 __pyx_r = -1;
03584 __pyx_L0:;
03585 __Pyx_XDECREF(__pyx_v_error_msg_prefix);
03586 __Pyx_XDECREF(__pyx_v_idx);
03587 __Pyx_XDECREF(__pyx_v_bother);
03588 __Pyx_RefNannyFinishContext();
03589 return __pyx_r;
03590 }
03591
03592 /* "PyClical.pyx":116
03593 *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03594 *
03595 *     def __dealloc__(self):                # ««««««««
03596 *         """
03597 *         Clean up by deallocating the instance of C++ class IndexSet.
03598 *
03599 */
03600 /* Python wrapper */
03601 static void __pyx_pw_8PyClical_9index_set_5__dealloc__(PyObject *__pyx_v_self); /*proto*/
03602 static void __pyx_pw_8PyClical_9index_set_5__dealloc__(PyObject *__pyx_v_self) {
03603     __Pyx_RefNannyDeclarations
03604     __Pyx_RefNannySetupContext("__dealloc__ (wrapper)", 0);
03605     __pyx_pf_8PyClical_9index_set_4__dealloc__(((struct __pyx_obj_8PyClical_index_set *)__pyx_v_self));
03606 }
03607 /* function exit code */
03608 __Pyx_RefNannyFinishContext();
03609 }
03610
03611 static void __pyx_pf_8PyClical_9index_set_4__dealloc__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
03612     __Pyx_RefNannyDeclarations
03613     __Pyx_RefNannySetupContext("__dealloc__", 0);
03614
03615     /* "PyClical.pyx":120
03616     *         Clean up by deallocating the instance of C++ class IndexSet.
03617     *         """
03618     *         del self.instance                # ««««««««
03619     *
03620     *     def __richcmp__(lhs, rhs, int op):
03621     */
03622     delete __pyx_v_self->instance;
03623
03624     /* "PyClical.pyx":116
03625     *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03626     *
03627     *     def __dealloc__(self):                # ««««««««
03628     *         """
03629     *         Clean up by deallocating the instance of C++ class IndexSet.
03630     *
03631 */
03632 /* function exit code */
03633 __Pyx_RefNannyFinishContext();
03634 }
03635
03636 /* "PyClical.pyx":122
03637 *         del self.instance
03638 *

```

```

03639 *      def __richcmp__(lhs, rhs, int op):          # ««««««««
03640 *          """
03641 *          Compare two objects of class index_set.
03642 */
03643
03644 /* Python wrapper */
03645 static PyObject *__pyx_pw_8PyClical_9index_set_7__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op); /*proto*/
03646 static PyObject *__pyx_pw_8PyClical_9index_set_7__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op) {
03647     PyObject *__pyx_r = 0;
03648     __Pyx_RefNannyDeclarations
03649     __Pyx_RefNannySetupContext("__richcmp__ (wrapper)", 0);
03650     __pyx_r = __pyx_pf_8PyClical_9index_set_6__richcmp__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_lhs), ((PyObject *)__pyx_v_rhs), ((int)__pyx_v_op));
03651
03652     /* function exit code */
03653     __Pyx_RefNannyFinishContext();
03654     return __pyx_r;
03655 }
03656
03657 static PyObject *__pyx_pf_8PyClical_9index_set_6__richcmp__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op) {
03658     PyObject *__pyx_v_eq = NULL;
03659     PyObject *__pyx_v_lt = NULL;
03660     PyObject *__pyx_r = NULL;
03661     __Pyx_RefNannyDeclarations
03662     int __pyx_t_1;
03663     int __pyx_t_2;
03664     int __pyx_t_3;
03665     PyObject *__pyx_t_4 = NULL;
03666     int __pyx_lineno = 0;
03667     const char *__pyx_filename = NULL;
03668     int __pyx_clineno = 0;
03669     __Pyx_RefNannySetupContext("__richcmp__", 0);
03670
03671     /* "PyClical.pyx":143
03672 *         False
03673 *         """
03674 *         if (lhs is None) or (rhs is None):          # ««««««««
03675 *             eq = bool(lhs is rhs)
03676 *             if op == 2: # ==
03677 */
03678     __pyx_t_2 = (((PyObject *)__pyx_v_lhs) == Py_None);
03679     __pyx_t_3 = (__pyx_t_2 != 0);
03680     if (!__pyx_t_3) {
03681     } else {
03682         __pyx_t_1 = __pyx_t_3;
03683         goto __pyx_L4_bool_binop_done;
03684     }
03685     __pyx_t_3 = (__pyx_v_rhs == Py_None);
03686     __pyx_t_2 = (__pyx_t_3 != 0);
03687     __pyx_t_1 = __pyx_t_2;
03688     __pyx_L4_bool_binop_done;
03689     if (__pyx_t_1) {
03690
03691         /* "PyClical.pyx":144
03692 *         """
03693 *         if (lhs is None) or (rhs is None):
03694 *             eq = bool(lhs is rhs)          # ««««««««
03695 *             if op == 2: # ==
03696 *                 return eq
03697 */
03698     __pyx_t_1 = (((PyObject *)__pyx_v_lhs) == __pyx_v_rhs);
03699     __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 144,
__pyx_L1_error)
03700     __Pyx_GOTREF(__pyx_t_4);
03701     __pyx_v_eq = __pyx_t_4;
03702     __pyx_t_4 = 0;
03703
03704     /* "PyClical.pyx":145
03705 *         if (lhs is None) or (rhs is None):
03706 *             eq = bool(lhs is rhs)
03707 *             if op == 2: # ==          # ««««««««
03708 *                 return eq
03709 *             elif op == 3: # !=
03710 */
03711     switch (__pyx_v_op) {
03712     case 2:
03713
03714         /* "PyClical.pyx":146
03715 *             eq = bool(lhs is rhs)
03716 *             if op == 2: # ==
03717 *                 return eq          # ««««««««
03718 *             elif op == 3: # !=
03719 *                 return not eq
03720 */

```

```

03721     __Pyx_XDECREF(__pyx_r);
03722     __Pyx_INCREF(__pyx_v_eq);
03723     __pyx_r = __pyx_v_eq;
03724     goto __pyx_L0;
03725
03726     /* "PyClical.pyx":145
03727  *         if (lhs is None) or (rhs is None):
03728  *             eq = bool(lhs is rhs)
03729  *             if op == 2: # ==                # ««««««««
03730  *                 return eq
03731  *             elif op == 3: # !=
03732  */
03733     break;
03734     case 3:
03735
03736     /* "PyClical.pyx":148
03737  *         return eq
03738  *         elif op == 3: # !=
03739  *             return not eq                # ««««««««
03740  *         else:
03741  *             if op == 0: # <
03742  */
03743     __Pyx_XDECREF(__pyx_r);
03744     __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_eq); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 148,
__pyx_L1_error)
03745     __pyx_t_4 = __Pyx_PyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 148,
__pyx_L1_error)
03746     __Pyx_GOTREF(__pyx_t_4);
03747     __pyx_r = __pyx_t_4;
03748     __pyx_t_4 = 0;
03749     goto __pyx_L0;
03750
03751     /* "PyClical.pyx":147
03752  *         if op == 2: # ==
03753  *             return eq
03754  *         elif op == 3: # !=                # ««««««««
03755  *             return not eq
03756  *         else:
03757  */
03758     break;
03759     default:
03760
03761     /* "PyClical.pyx":150
03762  *         return not eq
03763  *         else:
03764  *             if op == 0: # <                # ««««««««
03765  *                 return False
03766  *             elif op == 1: # <=
03767  */
03768     switch (__pyx_v_op) {
03769     case 0:
03770
03771     /* "PyClical.pyx":151
03772  *         else:
03773  *             if op == 0: # <
03774  *                 return False                # ««««««««
03775  *             elif op == 1: # <=
03776  *                 return eq
03777  */
03778     __Pyx_XDECREF(__pyx_r);
03779     __Pyx_INCREF(Py_False);
03780     __pyx_r = Py_False;
03781     goto __pyx_L0;
03782
03783     /* "PyClical.pyx":150
03784  *         return not eq
03785  *         else:
03786  *             if op == 0: # <                # ««««««««
03787  *                 return False
03788  *             elif op == 1: # <=
03789  */
03790     break;
03791     case 1:
03792
03793     /* "PyClical.pyx":153
03794  *         return False
03795  *         elif op == 1: # <=
03796  *             return eq                # ««««««««
03797  *         elif op == 4: # >
03798  *             return False
03799  */
03800     __Pyx_XDECREF(__pyx_r);
03801     __Pyx_INCREF(__pyx_v_eq);
03802     __pyx_r = __pyx_v_eq;
03803     goto __pyx_L0;
03804
03805     /* "PyClical.pyx":152

```

```

03806 *             if op == 0: # <
03807 *                 return False
03808 *             elif op == 1: # <=
03809 *                 return eq
03810 *             elif op == 4: # >
03811 */
03812     break;
03813     case 4:
03814
03815         /* "PyClicical.pyx":155
03816 *             return eq
03817 *             elif op == 4: # >
03818 *                 return False
03819 *             elif op == 5: # >=
03820 *                 return eq
03821 */
03822     __Pyx_XDECREF(__pyx_r);
03823     __Pyx_INCREF(Py_False);
03824     __pyx_r = Py_False;
03825     goto __pyx_L0;
03826
03827     /* "PyClicical.pyx":154
03828 *             elif op == 1: # <=
03829 *                 return eq
03830 *             elif op == 4: # >
03831 *                 return False
03832 *             elif op == 5: # >=
03833 */
03834     break;
03835     case 5:
03836
03837         /* "PyClicical.pyx":157
03838 *             return False
03839 *             elif op == 5: # >=
03840 *                 return eq
03841 *             else:
03842 *                 return NotImplemented
03843 */
03844     __Pyx_XDECREF(__pyx_r);
03845     __Pyx_INCREF(__pyx_v_eq);
03846     __pyx_r = __pyx_v_eq;
03847     goto __pyx_L0;
03848
03849     /* "PyClicical.pyx":156
03850 *             elif op == 4: # >
03851 *                 return False
03852 *             elif op == 5: # >=
03853 *                 return eq
03854 *             else:
03855 */
03856     break;
03857     default:
03858
03859         /* "PyClicical.pyx":159
03860 *             return eq
03861 *             else:
03862 *                 return NotImplemented
03863 *             else:
03864 *                 eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03865 */
03866     __Pyx_XDECREF(__pyx_r);
03867     __Pyx_INCREF(__pyx_builtin_NotImplemented);
03868     __pyx_r = __pyx_builtin_NotImplemented;
03869     goto __pyx_L0;
03870     break;
03871 }
03872 break;
03873 }
03874
03875 /* "PyClicical.pyx":143
03876 *     False
03877 *     """
03878 *     if (lhs is None) or (rhs is None):
03879 *         eq = bool(lhs is rhs)
03880 *         if op == 2: # ==
03881 */
03882 }
03883
03884 /* "PyClicical.pyx":161
03885 *             return NotImplemented
03886 *             else:
03887 *                 eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03888 *                 if op == 2: # ==
03889 *                     return eq
03890 */
03891 /*else*/ {
03892     __pyx_t_1 = (__pyx_f_8PyClicical_toIndexSet(((PyObject *)__pyx_v_lhs)) ==

```



```

__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs));
03893 __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 161,
__pyx_L1_error)
03894 __Pyx_GOTREF(__pyx_t_4);
03895 __pyx_v_eq = __pyx_t_4;
03896 __pyx_t_4 = 0;
03897
03898 /* "PyClical.pyx":162
03899 *     else:
03900 *         eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03901 *         if op == 2: # ==                # ««««««««
03902 *             return eq
03903 *         elif op == 3: # !=
03904 */
03905 switch (__pyx_v_op) {
03906     case 2:
03907
03908         /* "PyClical.pyx":163
03909 *         eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03910 *         if op == 2: # ==
03911 *             return eq                # ««««««««
03912 *         elif op == 3: # !=
03913 *             return not eq
03914 */
03915         __Pyx_XDECREF(__pyx_r);
03916         __Pyx_INCREF(__pyx_v_eq);
03917         __pyx_r = __pyx_v_eq;
03918         goto __pyx_L0;
03919
03920         /* "PyClical.pyx":162
03921 *         else:
03922 *         eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03923 *         if op == 2: # ==                # ««««««««
03924 *             return eq
03925 *         elif op == 3: # !=
03926 */
03927         break;
03928     case 3:
03929
03930         /* "PyClical.pyx":165
03931 *         return eq
03932 *         elif op == 3: # !=
03933 *             return not eq                # ««««««««
03934 *         else:
03935 *             lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03936 */
03937         __Pyx_XDECREF(__pyx_r);
03938         __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_eq); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 165,
__pyx_L1_error)
03939         __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 165,
__pyx_L1_error)
03940         __Pyx_GOTREF(__pyx_t_4);
03941         __pyx_r = __pyx_t_4;
03942         __pyx_t_4 = 0;
03943         goto __pyx_L0;
03944
03945         /* "PyClical.pyx":164
03946 *         if op == 2: # ==
03947 *             return eq
03948 *         elif op == 3: # !=                # ««««««««
03949 *             return not eq
03950 *         else:
03951 */
03952         break;
03953     default:
03954
03955         /* "PyClical.pyx":167
03956 *         return not eq
03957 *         else:
03958 *             lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )                # ««««««««
03959 *             if op == 0: # <
03960 *                 return lt
03961 */
03962         __pyx_t_1 = (__pyx_f_8PyClical_toIndexSet(((PyObject *)__pyx_v_lhs)) <
__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs));
03963         __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 167,
__pyx_L1_error)
03964         __Pyx_GOTREF(__pyx_t_4);
03965         __pyx_v_lt = __pyx_t_4;
03966         __pyx_t_4 = 0;
03967
03968         /* "PyClical.pyx":168
03969 *         else:
03970 *             lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03971 *             if op == 0: # <                # ««««««««
03972 *                 return lt
03973 *             elif op == 1: # <=

```

```

03974 */
03975     switch (__pyx_v_op) {
03976     case 0:
03977
03978         /* "PyClical.pyx":169
03979  *         lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03980  *         if op == 0: # <
03981  *             return lt # ««««««««
03982  *         elif op == 1: # <=
03983  *             return lt or eq
03984  */
03985         __Pyx_XDECREF(__pyx_r);
03986         __Pyx_INCREF(__pyx_v_lt);
03987         __pyx_r = __pyx_v_lt;
03988         goto __pyx_L0;
03989
03990         /* "PyClical.pyx":168
03991  *         else:
03992  *             lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03993  *             if op == 0: # <
03994  *                 return lt # ««««««««
03995  *             elif op == 1: # <=
03996  */
03997         break;
03998     case 1:
03999
04000         /* "PyClical.pyx":171
04001  *             return lt
04002  *             elif op == 1: # <=
04003  *                 return lt or eq # ««««««««
04004  *             elif op == 4: # >
04005  *                 return not (lt or eq)
04006  */
04007         __Pyx_XDECREF(__pyx_r);
04008         __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_lt); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 171,
__pyx_L1_error)
04009         if (!__pyx_t_1) {
04010         } else {
04011             __Pyx_INCREF(__pyx_v_lt);
04012             __pyx_t_4 = __pyx_v_lt;
04013             goto __pyx_L6_bool_binop_done;
04014         }
04015         __Pyx_INCREF(__pyx_v_eq);
04016         __pyx_t_4 = __pyx_v_eq;
04017         __pyx_L6_bool_binop_done;
04018         __pyx_r = __pyx_t_4;
04019         __pyx_t_4 = 0;
04020         goto __pyx_L0;
04021
04022         /* "PyClical.pyx":170
04023  *         if op == 0: # <
04024  *             return lt
04025  *         elif op == 1: # <=
04026  *             return lt or eq # ««««««««
04027  *         elif op == 4: # >
04028  */
04029         break;
04030     case 4:
04031
04032         /* "PyClical.pyx":173
04033  *             return lt or eq
04034  *             elif op == 4: # >
04035  *                 return not (lt or eq) # ««««««««
04036  *             elif op == 5: # >=
04037  *                 return not lt
04038  */
04039         __Pyx_XDECREF(__pyx_r);
04040         __pyx_t_2 = __Pyx_PyObject_IsTrue(__pyx_v_lt); if (unlikely(__pyx_t_2 < 0)) __PYX_ERR(0, 173,
__pyx_L1_error)
04041         if (!__pyx_t_2) {
04042         } else {
04043             __pyx_t_1 = __pyx_t_2;
04044             goto __pyx_L8_bool_binop_done;
04045         }
04046         __pyx_t_2 = __Pyx_PyObject_IsTrue(__pyx_v_eq); if (unlikely(__pyx_t_2 < 0)) __PYX_ERR(0, 173,
__pyx_L1_error)
04047         __pyx_t_1 = __pyx_t_2;
04048         __pyx_L8_bool_binop_done;
04049         __pyx_t_4 = __Pyx_PyBool_FromLong((__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 173,
__pyx_L1_error)
04050         __Pyx_GOTREF(__pyx_t_4);
04051         __pyx_r = __pyx_t_4;
04052         __pyx_t_4 = 0;
04053         goto __pyx_L0;
04054
04055         /* "PyClical.pyx":172
04056  *             elif op == 1: # <=

```

```

04057 *             return lt or eq
04058 *             elif op == 4: # >             # ««««««««
04059 *             return not (lt or eq)
04060 *             elif op == 5: # >=
04061 */
04062     break;
04063     case 5:
04064
04065         /* "PyClical.pyx":175
04066 *             return not (lt or eq)
04067 *             elif op == 5: # >=
04068 *             return not lt             # ««««««««
04069 *             else:
04070 *             return NotImplemented
04071 */
04072     __Pyx_XDECREF(__pyx_r);
04073     __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_lt); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 175,
__pyx_L1_error)
04074     __pyx_t_4 = __Pyx_PyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 175,
__pyx_L1_error)
04075     __Pyx_GOTREF(__pyx_t_4);
04076     __pyx_r = __pyx_t_4;
04077     __pyx_t_4 = 0;
04078     goto __pyx_L0;
04079
04080     /* "PyClical.pyx":174
04081 *             elif op == 4: # >
04082 *             return not (lt or eq)
04083 *             elif op == 5: # >=             # ««««««««
04084 *             return not lt
04085 *             else:
04086 */
04087     break;
04088     default:
04089
04090     /* "PyClical.pyx":177
04091 *             return not lt
04092 *             else:
04093 *             return NotImplemented             # ««««««««
04094 *
04095 *     def __setitem__(self, idx, val):
04096 */
04097     __Pyx_XDECREF(__pyx_r);
04098     __Pyx_INCREF(__pyx_builtin_NotImplemented);
04099     __pyx_r = __pyx_builtin_NotImplemented;
04100     goto __pyx_L0;
04101     break;
04102 }
04103 break;
04104 }
04105 }
04106
04107 /* "PyClical.pyx":122
04108 *     del self.instance
04109 *
04110 *     def __richcmp__(lhs, rhs, int op):             # ««««««««
04111 *     """
04112 *     Compare two objects of class index_set.
04113 */
04114
04115 /* function exit code */
04116 __pyx_L1_error:;
04117 __Pyx_XDECREF(__pyx_t_4);
04118 __Pyx_AddTraceback("PyClical.index_set.__richcmp__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04119 __pyx_r = NULL;
04120 __pyx_L0:;
04121 __Pyx_XDECREF(__pyx_v_eq);
04122 __Pyx_XDECREF(__pyx_v_lt);
04123 __Pyx_XGIVEREF(__pyx_r);
04124 __Pyx_RefNannyFinishContext();
04125 return __pyx_r;
04126 }
04127
04128 /* "PyClical.pyx":179
04129 *     return NotImplemented
04130 *
04131 *     def __setitem__(self, idx, val):             # ««««««««
04132 *     """
04133 *     Set the value of an index_set object at index idx to value val.
04134 */
04135
04136 /* Python wrapper */
04137 static int __pyx_pw_8PyClical_9index_set_9__setitem__(PyObject *__pyx_v_self, PyObject *__pyx_v_idx,
PyObject *__pyx_v_val); /*proto*/
04138 static char __pyx_doc_8PyClical_9index_set_8__setitem__[] = "\n        Set the value of an index_set
object at index idx to value val.\n\n        >> s=index_set({1}); s[2] = True; print(s)\n
{1,2}\n        >> s=index_set({1,2}); s[1] = False; print(s)\n        {2}\n        ";

```

```

04139 #if CYTHON_COMPILING_IN_CPYTHON
04140 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_8__setitem__;
04141 #endif
04142 static int __pyx_pw_8PyClical_9index_set_9__setitem__(PyObject *__pyx_v_self, PyObject *__pyx_v_idx,
PyObject *__pyx_v_val) {
04143     int __pyx_r;
04144     __Pyx_RefNannyDeclarations
04145     __Pyx_RefNannySetupContext("__setitem__ (wrapper)", 0);
04146     __pyx_r = __pyx_pf_8PyClical_9index_set_8__setitem__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_idx), ((PyObject *)__pyx_v_val));
04147
04148     /* function exit code */
04149     __Pyx_RefNannyFinishContext();
04150     return __pyx_r;
04151 }
04152
04153 static int __pyx_pf_8PyClical_9index_set_8__setitem__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_idx, PyObject *__pyx_v_val) {
04154     int __pyx_r;
04155     __Pyx_RefNannyDeclarations
04156     int __pyx_t_1;
04157     int __pyx_t_2;
04158     int __pyx_lineno = 0;
04159     const char *__pyx_filename = NULL;
04160     int __pyx_clineno = 0;
04161     __Pyx_RefNannySetupContext("__setitem__", 0);
04162
04163     /* "PyClical.pyx":188
04164     *         {2}
04165     *         """
04166     *         self.instance.set(idx, val)           # ««««««««
04167     *         return
04168     */
04170     __pyx_t_1 = __Pyx_PyInt_As_int(__pyx_v_idx); if (unlikely((__pyx_t_1 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 188, __pyx_L1_error)
04171     __pyx_t_2 = __Pyx_PyInt_As_int(__pyx_v_val); if (unlikely((__pyx_t_2 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 188, __pyx_L1_error)
04172     try {
04173         __pyx_v_self->instance->set(__pyx_t_1, __pyx_t_2);
04174     } catch (...) {
04175         __Pyx_CppExn2PyErr();
04176         __PYX_ERR(0, 188, __pyx_L1_error)
04177     }
04178
04179     /* "PyClical.pyx":189
04180     *         """
04181     *         self.instance.set(idx, val)
04182     *         return           # ««««««««
04183     *
04184     *         def __getitem__(self, idx):
04185     */
04186     __pyx_r = 0;
04187     goto __pyx_L0;
04188
04189     /* "PyClical.pyx":179
04190     *         return NotImplemented
04191     *
04192     *         def __setitem__(self, idx, val):           # ««««««««
04193     *         """
04194     *         Set the value of an index_set object at index idx to value val.
04195     */
04196
04197     /* function exit code */
04198     __pyx_L1_error:;
04199     __Pyx_AddTraceback("PyClical.index_set.__setitem__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04200     __pyx_r = -1;
04201     __pyx_L0:;
04202     __Pyx_RefNannyFinishContext();
04203     return __pyx_r;
04204 }
04205
04206 /* "PyClical.pyx":191
04207 *         return
04208 *
04209 *         def __getitem__(self, idx):           # ««««««««
04210 *         """
04211 *         Get the value of an index_set object at an index.
04212     */
04213
04214     /* Python wrapper */
04215     static PyObject *__pyx_pw_8PyClical_9index_set_11__getitem__(PyObject *__pyx_v_self, PyObject
*__pyx_v_idx); /*proto*/
04216     static char __pyx_doc_8PyClical_9index_set_10__getitem__[] = "\n        Get the value of an index_set
object at an index.\n\n        >> index_set({1})[1]\n            True\n        >> index_set({1})[2]\n
False\n        >> index_set({2})[-1]\n            False\n        >> index_set({2})[1]\n            False\n
>> index_set({2})[2]\n            True\n        >> index_set({2})[33]\n            False\n        ";

```

```

04217 #if CYTHON_COMPILING_IN_CPYTHON
04218 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_10__getitem__;
04219 #endif
04220 static PyObject *__pyx_pw_8PyClical_9index_set_11__getitem__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_idx) {
04221     PyObject *__pyx_r = 0;
04222     __Pyx_RefNannyDeclarations
04223     __Pyx_RefNannySetupContext("__getitem__ (wrapper)", 0);
04224     __pyx_r = __pyx_pf_8PyClical_9index_set_10__getitem__(((struct __pyx_obj_8PyClical_index_set
    *)__pyx_v_self), ((PyObject *)__pyx_v_idx));
04225
04226     /* function exit code */
04227     __Pyx_RefNannyFinishContext();
04228     return __pyx_r;
04229 }
04230
04231 static PyObject *__pyx_pf_8PyClical_9index_set_10__getitem__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, PyObject *__pyx_v_idx) {
04232     PyObject *__pyx_r = NULL;
04233     __Pyx_RefNannyDeclarations
04234     int __pyx_t_1;
04235     PyObject *__pyx_t_2 = NULL;
04236     int __pyx_lineno = 0;
04237     const char *__pyx_filename = NULL;
04238     int __pyx_clineno = 0;
04239     __Pyx_RefNannySetupContext("__getitem__", 0);
04240
04241     /* "PyClical.pyx":208
04242     *         False
04243     *         """
04244     *         return self.instance.getitem(idx) # ««««««««
04245     *
04246     *     def __contains__(self, idx):
04247     */
04248     __Pyx_XDECREF(__pyx_r);
04249     __pyx_t_1 = __Pyx_PyInt_As_int(__pyx_v_idx); if (unlikely((__pyx_t_1 == (int)-1) &&
    PyErr_Occurred())) __PYX_ERR(0, 208, __pyx_L1_error)
04250     __pyx_t_2 = __Pyx_PyBool_FromLong(__pyx_v_self->instance->operator[](__pyx_t_1)); if
    (unlikely(!__pyx_t_2)) __PYX_ERR(0, 208, __pyx_L1_error)
04251     __Pyx_GOTREF(__pyx_t_2);
04252     __pyx_r = __pyx_t_2;
04253     __pyx_t_2 = 0;
04254     goto __pyx_L0;
04255
04256     /* "PyClical.pyx":191
04257     *         return
04258     *
04259     *     def __getitem__(self, idx): # ««««««««
04260     *         """
04261     *         Get the value of an index_set object at an index.
04262     */
04263
04264     /* function exit code */
04265     __pyx_L1_error:;
04266     __Pyx_XDECREF(__pyx_t_2);
04267     __Pyx_AddTraceback("PyClical.index_set.__getitem__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04268     __pyx_r = NULL;
04269     __pyx_L0:;
04270     __Pyx_XGIVEREF(__pyx_r);
04271     __Pyx_RefNannyFinishContext();
04272     return __pyx_r;
04273 }
04274
04275 /* "PyClical.pyx":210
04276 *         return self.instance.getitem(idx)
04277 *
04278 *     def __contains__(self, idx): # ««««««««
04279 *         """
04280 *         Check that an index_set object contains the index idx: idx in self.
04281     */
04282
04283 /* Python wrapper */
04284 static int __pyx_pw_8PyClical_9index_set_13__contains__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_idx); /*proto*/
04285 static char __pyx_doc_8PyClical_9index_set_12__contains__[] = "\n        Check that an index_set
    object contains the index idx: idx in self.\n\n        >> 1 in index_set({1})\n            True\n
    >> 2 in index_set({1})\n            False\n        >> -1 in index_set({2})\n            False\n
    >> 1 in index_set({2})\n            False\n        >> 2 in index_set({2})\n            True\n
    >> 33 in index_set({2})\n            False\n        ";
04286 #if CYTHON_COMPILING_IN_CPYTHON
04287 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_12__contains__;
04288 #endif
04289 static int __pyx_pw_8PyClical_9index_set_13__contains__(PyObject *__pyx_v_self, PyObject *__pyx_v_idx)
    {
04290     int __pyx_r;
04291     __Pyx_RefNannyDeclarations
04292     __Pyx_RefNannySetupContext("__contains__ (wrapper)", 0);

```

```

04293 __pyx_r = __pyx_pf_8PyClical_9index_set_12__contains__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_idx));
04294
04295 /* function exit code */
04296 __Pyx_RefNannyFinishContext();
04297 return __pyx_r;
04298 }
04299
04300 static int __pyx_pf_8PyClical_9index_set_12__contains__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_idx) {
04301     int __pyx_r;
04302     __Pyx_RefNannyDeclarations
04303     int __pyx_t_1;
04304     int __pyx_lineno = 0;
04305     const char *__pyx_filename = NULL;
04306     int __pyx_clineno = 0;
04307     __Pyx_RefNannySetupContext("__contains__", 0);
04308
04309     /* "PyClical.pyx":227
04310     *         False
04311     *         """
04312     *         return self.instance.getitem(idx) # ««««««««
04313     *
04314     *     def __iter__(self):
04315     */
04316     __pyx_t_1 = __Pyx_PyInt_As_int(__pyx_v_idx); if (unlikely((__pyx_t_1 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 227, __pyx_L1_error)
04317     __pyx_r = __pyx_v_self->instance->operator[](__pyx_t_1);
04318     goto __pyx_L0;
04319
04320     /* "PyClical.pyx":210
04321     *         return self.instance.getitem(idx)
04322     *
04323     *     def __contains__(self, idx): # ««««««««
04324     *         """
04325     *         Check that an index_set object contains the index idx: idx in self.
04326     */
04327
04328     /* function exit code */
04329     __pyx_L1_error:;
04330     __Pyx_AddTraceback("PyClical.index_set.__contains__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04331     __pyx_r = -1;
04332     __pyx_L0:;
04333     __Pyx_RefNannyFinishContext();
04334     return __pyx_r;
04335 }
04336
04337 static PyObject *__pyx_gb_8PyClical_9index_set_16generator(__pyx_CoroutineObject *__pyx_generator,
CYTHON_UNUSED PyThreadState *__pyx_tstate, PyObject *__pyx_sent_value); /* proto */
04338
04339 /* "PyClical.pyx":229
04340 *         return self.instance.getitem(idx)
04341 *
04342 *     def __iter__(self): # ««««««««
04343 *         """
04344 *         Iterate over the indices of an index_set.
04345     */
04346
04347 /* Python wrapper */
04348 static PyObject *__pyx_pw_8PyClical_9index_set_15__iter__(PyObject *__pyx_v_self); /*proto*/
04349 static char __pyx_doc_8PyClical_9index_set_14__iter__[] = "\n        Iterate over the indices of an\nindex_set.\n\n        >> for i in index_set({-3,4,7}):print(i, end=\",\")\n        -3,4,7,\n    ";
04350 #if CYTHON_COMPILING_IN_CPYTHON
04351 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_14__iter__;
04352 #endif
04353 static PyObject *__pyx_pw_8PyClical_9index_set_15__iter__(PyObject *__pyx_v_self) {
04354     PyObject *__pyx_r = 0;
04355     __Pyx_RefNannyDeclarations
04356     __Pyx_RefNannySetupContext("__iter__ (wrapper)", 0);
04357     __pyx_r = __pyx_pf_8PyClical_9index_set_14__iter__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
04358
04359     /* function exit code */
04360     __Pyx_RefNannyFinishContext();
04361     return __pyx_r;
04362 }
04363
04364 static PyObject *__pyx_pf_8PyClical_9index_set_14__iter__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
04365     struct __pyx_obj_8PyClical__pyx_scope_struct__iter__ *__pyx_cur_scope;
04366     PyObject *__pyx_r = NULL;
04367     __Pyx_RefNannyDeclarations
04368     int __pyx_lineno = 0;
04369     const char *__pyx_filename = NULL;
04370     int __pyx_clineno = 0;
04371     __Pyx_RefNannySetupContext("__iter__", 0);
04372     __pyx_cur_scope = (struct __pyx_obj_8PyClical__pyx_scope_struct__iter__
*)__pyx_tp_new_8PyClical__pyx_scope_struct__iter__(__pyx_ptype_8PyClical__pyx_scope_struct__iter__,

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__pyx_empty_tuple, NULL);
04372 if (unlikely(!__pyx_cur_scope)) {
04373     __pyx_cur_scope = ((struct __pyx_obj_8PyClical__pyx_scope_struct__iter__ *)Py_None);
04374     __Pyx_INCREF(Py_None);
04375     __PYX_ERR(0, 229, __pyx_L1_error)
04376 } else {
04377     __Pyx_GOTREF(__pyx_cur_scope);
04378 }
04379 __pyx_cur_scope->__pyx_v_self = __pyx_v_self;
04380 __Pyx_INCREF((PyObject *)__pyx_cur_scope->__pyx_v_self);
04381 __Pyx_GIVEREF((PyObject *)__pyx_cur_scope->__pyx_v_self);
04382 {
04383     __pyx_CoroutineObject *gen = __Pyx_Generator_New((__pyx_coroutine_body_t)
__pyx_gb_8PyClical_9index_set_16generator, NULL, (PyObject *) __pyx_cur_scope, __pyx_n_s_iter,
__pyx_n_s_index_set__iter, __pyx_n_s_PyClical); if (unlikely(!gen)) __PYX_ERR(0, 229, __pyx_L1_error)
04384     __Pyx_DECREF(__pyx_cur_scope);
04385     __Pyx_RefNannyFinishContext();
04386     return (PyObject *) gen;
04387 }
04388
04389 /* function exit code */
04390 __pyx_L1_error:;
04391 __Pyx_AddTraceback("PyClical.index_set.__iter__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04392 __pyx_r = NULL;
04393 __Pyx_DECREF((PyObject *)__pyx_cur_scope);
04394 __Pyx_XGIVEREF(__pyx_r);
04395 __Pyx_RefNannyFinishContext();
04396 return __pyx_r;
04397 }
04398
04399 static PyObject *__pyx_gb_8PyClical_9index_set_16generator(__pyx_CoroutineObject *__pyx_generator,
CYTHON_UNUSED PyThreadState *__pyx_tstate, PyObject *__pyx_sent_value) /* generator body */
04400 {
04401     struct __pyx_obj_8PyClical__pyx_scope_struct__iter__ *__pyx_cur_scope = ((struct
__pyx_obj_8PyClical__pyx_scope_struct__iter__ *)__pyx_generator->closure);
04402     PyObject *__pyx_r = NULL;
04403     PyObject *__pyx_t_1 = NULL;
04404     PyObject *__pyx_t_2 = NULL;
04405     PyObject *__pyx_t_3 = NULL;
04406     PyObject *__pyx_t_4 = NULL;
04407     Py_ssize_t __pyx_t_5;
04408     PyObject *(*__pyx_t_6)(PyObject *);
04409     int __pyx_t_7;
04410     int __pyx_t_8;
04411     int __pyx_lineno = 0;
04412     const char *__pyx_filename = NULL;
04413     int __pyx_clineno = 0;
04414     __Pyx_RefNannyDeclarations
04415     __Pyx_RefNannySetupContext("__iter__", 0);
04416     switch (__pyx_generator->resume_label) {
04417         case 0: goto __pyx_L3_first_run;
04418         case 1: goto __pyx_L7_resume_from_yield;
04419         default: /* CPython raises the right error here */
04420             __Pyx_RefNannyFinishContext();
04421             return NULL;
04422     }
04423     __pyx_L3_first_run:;
04424     if (unlikely(!__pyx_sent_value)) __PYX_ERR(0, 229, __pyx_L1_error)
04425
04426     /* "PyClical.pyx":236
04427     *         -3,4,7,
04428     *         """
04429     *         for idx in range(self.min(), self.max()+1):           # ««««««««
04430     *             if idx in self:
04431     *                 yield idx
04432     */
04433     __pyx_t_2 = __Pyx_PyObject_GetAttrStr((PyObject *)__pyx_cur_scope->__pyx_v_self, __pyx_n_s_min);
04434     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 236, __pyx_L1_error)
04435     __Pyx_GOTREF(__pyx_t_2);
04436     __pyx_t_3 = NULL;
04437     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_2))) {
04438         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_2);
04439         if (likely(__pyx_t_3)) {
04440             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_2);
04441             __Pyx_INCREF(__pyx_t_3);
04442             __Pyx_INCREF(function);
04443             __Pyx_DECREF_SET(__pyx_t_2, function);
04444         }
04445         __pyx_t_1 = (__pyx_t_3) ? __Pyx_PyObject_CallOneArg(__pyx_t_2, __pyx_t_3) :
__Pyx_PyObject_CallNoArg(__pyx_t_2);
04446         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
04447         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 236, __pyx_L1_error)
04448         __Pyx_GOTREF(__pyx_t_1);
04449         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04450         __pyx_t_3 = __Pyx_PyObject_GetAttrStr((PyObject *)__pyx_cur_scope->__pyx_v_self, __pyx_n_s_max);
04451         if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 236, __pyx_L1_error)

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04451     __Pyx_GOTREF(__pyx_t_3);
04452     __pyx_t_4 = NULL;
04453     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
04454         __pyx_t_4 = PyMethod_GET_SELF(__pyx_t_3);
04455         if (likely(__pyx_t_4)) {
04456             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
04457             __Pyx_INCREF(__pyx_t_4);
04458             __Pyx_INCREF(function);
04459             __Pyx_DECREF_SET(__pyx_t_3, function);
04460         }
04461     }
04462     __pyx_t_2 = (__pyx_t_4) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_4) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
04463     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
04464     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 236, __pyx_l1_error)
04465     __Pyx_GOTREF(__pyx_t_2);
04466     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
04467     __pyx_t_3 = __Pyx_PyInt_AddObjC(__pyx_t_2, __pyx_int_1, 1, 0, 0); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_l1_error)
04468     __Pyx_GOTREF(__pyx_t_3);
04469     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04470     __pyx_t_2 = PyTuple_New(2); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 236, __pyx_l1_error)
04471     __Pyx_GOTREF(__pyx_t_2);
04472     __Pyx_GIVEREF(__pyx_t_1);
04473     PyTuple_SET_ITEM(__pyx_t_2, 0, __pyx_t_1);
04474     __Pyx_GIVEREF(__pyx_t_3);
04475     PyTuple_SET_ITEM(__pyx_t_2, 1, __pyx_t_3);
04476     __pyx_t_1 = 0;
04477     __pyx_t_3 = 0;
04478     __pyx_t_3 = __Pyx_PyObject_Call(__pyx_builtin_range, __pyx_t_2, NULL); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_l1_error)
04479     __Pyx_GOTREF(__pyx_t_3);
04480     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04481     if (likely(PyList_CheckExact(__pyx_t_3)) || PyTuple_CheckExact(__pyx_t_3)) {
04482         __pyx_t_2 = __pyx_t_3; __Pyx_INCREF(__pyx_t_2); __pyx_t_5 = 0;
04483         __pyx_t_6 = NULL;
04484     } else {
04485         __pyx_t_5 = -1; __pyx_t_2 = PyObject_GetIter(__pyx_t_3); if (unlikely(!__pyx_t_2)) __PYX_ERR(0,
236, __pyx_l1_error)
04486         __Pyx_GOTREF(__pyx_t_2);
04487         __pyx_t_6 = Py_TYPE(__pyx_t_2)->tp_iternext; if (unlikely(!__pyx_t_6)) __PYX_ERR(0, 236,
__pyx_l1_error)
04488     }
04489     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
04490     for (;;) {
04491         if (likely(!__pyx_t_6)) {
04492             if (likely(PyList_CheckExact(__pyx_t_2))) {
04493                 if (__pyx_t_5 >= PyList_GET_SIZE(__pyx_t_2)) break;
04494                 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
04495                 __pyx_t_3 = PyList_GET_ITEM(__pyx_t_2, __pyx_t_5); __Pyx_INCREF(__pyx_t_3); __pyx_t_5++; if
(unlikely(0 < 0)) __PYX_ERR(0, 236, __pyx_l1_error)
04496                 #else
04497                 __pyx_t_3 = PySequence_ITEM(__pyx_t_2, __pyx_t_5); __pyx_t_5++; if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_l1_error)
04498                 __Pyx_GOTREF(__pyx_t_3);
04499                 #endif
04500             } else {
04501                 if (__pyx_t_5 >= PyTuple_GET_SIZE(__pyx_t_2)) break;
04502                 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
04503                 __pyx_t_3 = PyTuple_GET_ITEM(__pyx_t_2, __pyx_t_5); __Pyx_INCREF(__pyx_t_3); __pyx_t_5++; if
(unlikely(0 < 0)) __PYX_ERR(0, 236, __pyx_l1_error)
04504                 #else
04505                 __pyx_t_3 = PySequence_ITEM(__pyx_t_2, __pyx_t_5); __pyx_t_5++; if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_l1_error)
04506                 __Pyx_GOTREF(__pyx_t_3);
04507                 #endif
04508             }
04509         } else {
04510             __pyx_t_3 = __pyx_t_6(__pyx_t_2);
04511             if (unlikely(!__pyx_t_3)) {
04512                 PyObject* exc_type = PyErr_Occurred();
04513                 if (exc_type) {
04514                     if (likely(__Pyx_PyErr_GivenExceptionMatches(exc_type, PyExc_StopIteration))) PyErr_Clear();
04515                     else __PYX_ERR(0, 236, __pyx_l1_error)
04516                 }
04517                 break;
04518             }
04519             __Pyx_GOTREF(__pyx_t_3);
04520         }
04521         __Pyx_XGOTREF(__pyx_cur_scope->__pyx_v_idx);
04522         __Pyx_XDECREF_SET(__pyx_cur_scope->__pyx_v_idx, __pyx_t_3);
04523         __Pyx_GIVEREF(__pyx_t_3);
04524         __pyx_t_3 = 0;
04525     }
04526     /* "PyClical.pyx":237
04527     *
04528     *     for idx in range(self.min(), self.max()+1):

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04529 *             if idx in self:             # ««««««««
04530 *                 yield idx
04531 *
04532 */
04533 __pyx_t_7 = (__Pyx_PySequence_ContainsTF(__pyx_cur_scope->__pyx_v_idx, ((PyObject
*)__pyx_cur_scope->__pyx_v_self), Py_EQ)); if (unlikely(__pyx_t_7 < 0)) __PYX_ERR(0, 237,
__pyx_L1_error)
04534 __pyx_t_8 = (__pyx_t_7 != 0);
04535 if (__pyx_t_8) {
04536
04537     /* "PyClical.pyx":238
04538     *         for idx in range(self.min(), self.max()+1):
04539     *             if idx in self:
04540     *                 yield idx             # ««««««««
04541     *
04542     *     def __invert__(self):
04543     */
04544     __Pyx_INCREF(__pyx_cur_scope->__pyx_v_idx);
04545     __pyx_r = __pyx_cur_scope->__pyx_v_idx;
04546     __Pyx_XGIVEREF(__pyx_t_2);
04547     __pyx_cur_scope->__pyx_t_0 = __pyx_t_2;
04548     __pyx_cur_scope->__pyx_t_1 = __pyx_t_5;
04549     __pyx_cur_scope->__pyx_t_2 = __pyx_t_6;
04550     __Pyx_XGIVEREF(__pyx_r);
04551     __Pyx_RefNannyFinishContext();
04552     __Pyx_Coroutine_ResetAndClearException(__pyx_generator);
04553     /* return from generator, yielding value */
04554     __pyx_generator->resume_label = 1;
04555     return __pyx_r;
04556     __pyx_L7_resume_from_yield:;
04557     __pyx_t_2 = __pyx_cur_scope->__pyx_t_0;
04558     __pyx_cur_scope->__pyx_t_0 = 0;
04559     __Pyx_XGOTREF(__pyx_t_2);
04560     __pyx_t_5 = __pyx_cur_scope->__pyx_t_1;
04561     __pyx_t_6 = __pyx_cur_scope->__pyx_t_2;
04562     if (unlikely(!__pyx_sent_value)) __PYX_ERR(0, 238, __pyx_L1_error)
04563
04564     /* "PyClical.pyx":237
04565     *         """
04566     *         for idx in range(self.min(), self.max()+1):
04567     *             if idx in self:             # ««««««««
04568     *                 yield idx
04569     *
04570     */
04571     }
04572
04573     /* "PyClical.pyx":236
04574     *         -3,4,7,
04575     *         """
04576     *         for idx in range(self.min(), self.max()+1):             # ««««««««
04577     *             if idx in self:
04578     *                 yield idx
04579     */
04580     }
04581     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04582     CYTHON_MAYBE_UNUSED_VAR(__pyx_cur_scope);
04583
04584     /* "PyClical.pyx":229
04585     *         return self.instance.getitem(idx)
04586     *
04587     *     def __iter__(self):             # ««««««««
04588     *         """
04589     *         Iterate over the indices of an index_set.
04590     */
04591
04592     /* function exit code */
04593     PyErr_SetNone(PyExc_StopIteration);
04594     goto __pyx_L0;
04595     __pyx_L1_error:;
04596     __Pyx_XDECREF(__pyx_t_1);
04597     __Pyx_XDECREF(__pyx_t_2);
04598     __Pyx_XDECREF(__pyx_t_3);
04599     __Pyx_XDECREF(__pyx_t_4);
04600     __Pyx_AddTraceback("__iter__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04601     __pyx_L0:;
04602     __Pyx_XDECREF(__pyx_r); __pyx_r = 0;
04603     #if !CYTHON_USE_EXC_INFO_STACK
04604     __Pyx_Coroutine_ResetAndClearException(__pyx_generator);
04605     #endif
04606     __pyx_generator->resume_label = -1;
04607     __Pyx_Coroutine_clear((PyObject*)__pyx_generator);
04608     __Pyx_RefNannyFinishContext();
04609     return __pyx_r;
04610 }
04611
04612 /* "PyClical.pyx":240
04613 *         yield idx

```

```

04614 *
04615 *     def __invert__(self):          # ««««««««
04616 *         """
04617 *             Set complement: not.
04618 */
04619
04620 /* Python wrapper */
04621 static PyObject *__pyx_pw_8PyClical_9index_set_18__invert__(PyObject *__pyx_v_self); /*proto*/
04622 static char __pyx_doc_8PyClical_9index_set_17__invert__[] = "\n
    >>>
    print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))\n
    {-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32}\n
    ";
04623 #if CYTHON_COMPILING_IN_CPYTHON
04624 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_17__invert__;
04625 #endif
04626 static PyObject *__pyx_pw_8PyClical_9index_set_18__invert__(PyObject *__pyx_v_self) {
04627     PyObject *__pyx_r = 0;
04628     __Pyx_RefNannyDeclarations
04629     __Pyx_RefNannySetupContext("__invert__ (wrapper)", 0);
04630     __pyx_r = __pyx_pf_8PyClical_9index_set_17__invert__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
04631
04632 /* function exit code */
04633 __Pyx_RefNannyFinishContext();
04634 return __pyx_r;
04635 }
04636
04637 static PyObject *__pyx_pf_8PyClical_9index_set_17__invert__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
04638     PyObject *__pyx_r = NULL;
04639     __Pyx_RefNannyDeclarations
04640     PyObject *__pyx_t_1 = NULL;
04641     PyObject *__pyx_t_2 = NULL;
04642     int __pyx_lineno = 0;
04643     const char *__pyx_filename = NULL;
04644     int __pyx_clineno = 0;
04645     __Pyx_RefNannySetupContext("__invert__", 0);
04646
04647 /* "PyClical.pyx":247
04648 *
04649 *     """
04650 *         return index_set().wrap( self.instance.invert() )          # ««««««««
04651 *
04652 *     def __xor__(lhs, rhs):
04653 */
04654     __Pyx_XDECREF(__pyx_r);
04655     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 247, __pyx_L1_error)
04656     __Pyx_GOTREF(__pyx_t_1);
04657     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
__pyx_v_self->instance->operator~()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 247, __pyx_L1_error)
04658     __Pyx_GOTREF(__pyx_t_2);
04659     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
04660     __pyx_r = __pyx_t_2;
04661     __pyx_t_2 = 0;
04662     goto __pyx_L0;
04663
04664 /* "PyClical.pyx":240
04665 *         yield idx
04666 *
04667 *     def __invert__(self):          # ««««««««
04668 *         """
04669 *             Set complement: not.
04670 */
04671
04672 /* function exit code */
04673 __pyx_L1_error:;
04674     __Pyx_XDECREF(__pyx_t_1);
04675     __Pyx_XDECREF(__pyx_t_2);
04676     __Pyx_AddTraceback("PyClical.index_set.__invert__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04677     __pyx_r = NULL;
04678     __pyx_L0:;
04679     __Pyx_XGIVEREF(__pyx_r);
04680     __Pyx_RefNannyFinishContext();
04681     return __pyx_r;
04682 }
04683
04684 /* "PyClical.pyx":249
04685 *         return index_set().wrap( self.instance.invert() )
04686 *
04687 *     def __xor__(lhs, rhs):          # ««««««««
04688 *         """
04689 *             Symmetric set difference: exclusive or.
04690 */
04691

```

```

04692 /* Python wrapper */
04693 static PyObject *__pyx_pw_8PyClical_9index_set_20__xor__(PyObject *__pyx_v_lhs, PyObject
    *__pyx_v_rhs); /*proto*/
04694 static char __pyx_doc_8PyClical_9index_set_19__xor__[] = "\n          Symmetric set difference:
    exclusive or.\n\n          >> print(index_set({1}) ^ index_set({2}))\n          {1,2}\n          >>
    print(index_set({1,2}) ^ index_set({2}))\n          {1}\n          ";
04695 #if CYTHON_COMPILING_IN_CPYTHON
04696 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_19__xor__;
04697 #endif
04698 static PyObject *__pyx_pw_8PyClical_9index_set_20__xor__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
    {
04699     PyObject *__pyx_r = 0;
04700     __Pyx_RefNannyDeclarations
04701     __Pyx_RefNannySetupContext("__xor__ (wrapper)", 0);
04702     __pyx_r = __pyx_pf_8PyClical_9index_set_19__xor__((PyObject *)__pyx_v_lhs), ((PyObject
    *)__pyx_v_rhs));
04703
04704     /* function exit code */
04705     __Pyx_RefNannyFinishContext();
04706     return __pyx_r;
04707 }
04708
04709 static PyObject *__pyx_pf_8PyClical_9index_set_19__xor__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
    {
04710     PyObject *__pyx_r = NULL;
04711     __Pyx_RefNannyDeclarations
04712     PyObject *__pyx_t_1 = NULL;
04713     PyObject *__pyx_t_2 = NULL;
04714     int __pyx_lineno = 0;
04715     const char *__pyx_filename = NULL;
04716     int __pyx_clineno = 0;
04717     __Pyx_RefNannySetupContext("__xor__", 0);
04718
04719     /* "PyClical.pyx":258
04720     *         {1}
04721     *         """
04722     *         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) ) # ««««««««
04723     *
04724     *         def __ixor__(self, rhs):
04725     */
04726     __Pyx_XDECREF(__pyx_r);
04727     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 258, __pyx_L1_error)
04728     __Pyx_GOTREF(__pyx_t_1);
04729     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
    (__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs) ^ __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
    (unlikely(!__pyx_t_2)) __PYX_ERR(0, 258, __pyx_L1_error)
04730     __Pyx_GOTREF(__pyx_t_2);
04731     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
04732     __pyx_r = __pyx_t_2;
04733     __pyx_t_2 = 0;
04734     goto __pyx_L0;
04735
04736     /* "PyClical.pyx":249
04737     *         return index_set().wrap( self.instance.invert() )
04738     *
04739     *         def __xor__(lhs, rhs): # ««««««««
04740     *         """
04741     *         Symmetric set difference: exclusive or.
04742     */
04743
04744     /* function exit code */
04745     __pyx_L1_error:;
04746     __Pyx_XDECREF(__pyx_t_1);
04747     __Pyx_XDECREF(__pyx_t_2);
04748     __Pyx_AddTraceback("PyClical.index_set.__xor__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04749     __pyx_r = NULL;
04750     __pyx_L0:;
04751     __Pyx_XGIVEREF(__pyx_r);
04752     __Pyx_RefNannyFinishContext();
04753     return __pyx_r;
04754 }
04755
04756 /* "PyClical.pyx":260
04757 *         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
04758 *
04759 *         def __ixor__(self, rhs): # ««««««««
04760 *         """
04761 *         Symmetric set difference: exclusive or.
04762     */
04763
04764 /* Python wrapper */
04765 static PyObject *__pyx_pw_8PyClical_9index_set_22__ixor__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_rhs); /*proto*/
04766 static char __pyx_doc_8PyClical_9index_set_21__ixor__[] = "\n          Symmetric set difference:
    exclusive or.\n\n          >> x = index_set({1}); x ^= index_set({2}); print(x)\n          {1,2}\n          >>
    x = index_set({1,2}); x ^= index_set({2}); print(x)\n          {1}\n          ";

```

```

04767 #if CYTHON_COMPILING_IN_CPYTHON
04768 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_21__ixor__
04769 #endif
04770 static PyObject *__pyx_pw_8PyClical_9index_set_22__ixor__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_rhs) {
04771     PyObject *__pyx_r = 0;
04772     __Pyx_RefNannyDeclarations
04773     __Pyx_RefNannySetupContext("__ixor__ (wrapper)", 0);
04774     __pyx_r = __pyx_pf_8PyClical_9index_set_21__ixor__((struct __pyx_obj_8PyClical_index_set
    *)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
04775
04776     /* function exit code */
04777     __Pyx_RefNannyFinishContext();
04778     return __pyx_r;
04779 }
04780
04781 static PyObject *__pyx_pf_8PyClical_9index_set_21__ixor__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, PyObject *__pyx_v_rhs) {
04782     PyObject *__pyx_r = NULL;
04783     __Pyx_RefNannyDeclarations
04784     PyObject *__pyx_t_1 = NULL;
04785     int __pyx_lineno = 0;
04786     const char *__pyx_filename = NULL;
04787     int __pyx_clineno = 0;
04788     __Pyx_RefNannySetupContext("__ixor__", 0);
04789
04790     /* "PyClical.pyx":269
04791     *     {1}
04792     *     """
04793     *     return self.wrap( self.unwrap() ^ toIndexSet(rhs) )           # ««««««««
04794     *
04795     *     def __and__(lhs, rhs):
04796     */
04797     __Pyx_XDECREF(__pyx_r);
04798     __pyx_t_1 = __pyx_f_8PyClical_9index_set_wrap(__pyx_v_self,
    (__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self) ^ __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 269, __pyx_L1_error)
04799     __Pyx_GOTREF(__pyx_t_1);
04800     __pyx_r = __pyx_t_1;
04801     __pyx_t_1 = 0;
04802     goto __pyx_L0;
04803
04804     /* "PyClical.pyx":260
04805     *     return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
04806     *
04807     *     def __ixor__(self, rhs):           # ««««««««
04808     *     """
04809     *     Symmetric set difference: exclusive or.
04810     */
04811
04812     /* function exit code */
04813     __pyx_L1_error:;
04814     __Pyx_XDECREF(__pyx_t_1);
04815     __Pyx_AddTraceback("PyClical.index_set.__ixor__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04816     __pyx_r = NULL;
04817     __pyx_L0:;
04818     __Pyx_XGIVEREF(__pyx_r);
04819     __Pyx_RefNannyFinishContext();
04820     return __pyx_r;
04821 }
04822
04823 /* "PyClical.pyx":271
04824 *     return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
04825 *
04826 *     def __and__(lhs, rhs):           # ««««««««
04827 *     """
04828 *     Set intersection: and.
04829     */
04830
04831     /* Python wrapper */
04832     static PyObject *__pyx_pw_8PyClical_9index_set_24__and__(PyObject *__pyx_v_lhs, PyObject
    *__pyx_v_rhs); /*proto*/
04833     static char __pyx_doc_8PyClical_9index_set_23__and__[] = "\n        Set intersection: and.\n\n
    >> print(index_set({1}) & index_set({2}))\n        {}\n        >> print(index_set({1,2}) &
    index_set({2}))\n        {2}\n        ";
04834 #if CYTHON_COMPILING_IN_CPYTHON
04835 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_23__and__;
04836 #endif
04837 static PyObject *__pyx_pw_8PyClical_9index_set_24__and__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
    {
04838     PyObject *__pyx_r = 0;
04839     __Pyx_RefNannyDeclarations
04840     __Pyx_RefNannySetupContext("__and__ (wrapper)", 0);
04841     __pyx_r = __pyx_pf_8PyClical_9index_set_23__and__((PyObject *)__pyx_v_lhs), ((PyObject
    *)__pyx_v_rhs));
04842
04843     /* function exit code */

```

```

04844  __Pyx_RefNannyFinishContext();
04845  return __pyx_r;
04846 }
04847
04848 static PyObject *__pyx_pf_8PyClical_9index_set_23__and__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
04849 {
04850     PyObject *__pyx_r = NULL;
04851     __Pyx_RefNannyDeclarations
04852     PyObject *__pyx_t_1 = NULL;
04853     PyObject *__pyx_t_2 = NULL;
04854     int __pyx_lineno = 0;
04855     const char *__pyx_filename = NULL;
04856     int __pyx_clineno = 0;
04857     __Pyx_RefNannySetupContext("__and__", 0);
04858
04859     /* "PyClical.pyx":280
04860     {2}
04861     """
04862     return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) ) # ««««««««
04863
04864     def __iand__(self, rhs):
04865     */
04866     __Pyx_XDECREF(__pyx_r);
04867     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 280, __pyx_L1_error)
04868     __Pyx_GOTREF(__pyx_t_1);
04869     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
(__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs) & __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 280, __pyx_L1_error)
04870     __Pyx_GOTREF(__pyx_t_2);
04871     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
04872     __pyx_r = __pyx_t_2;
04873     __pyx_t_2 = 0;
04874     goto __pyx_L0;
04875
04876     /* "PyClical.pyx":271
04877     return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
04878
04879     def __and__(lhs, rhs): # ««««««««
04880     """
04881     Set intersection: and.
04882     */
04883
04884     /* function exit code */
04885     __pyx_L1_error:;
04886     __Pyx_XDECREF(__pyx_t_1);
04887     __Pyx_XDECREF(__pyx_t_2);
04888     __Pyx_AddTraceback("PyClical.index_set.__and__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04889     __pyx_r = NULL;
04890     __pyx_L0:;
04891     __Pyx_XGIVEREF(__pyx_r);
04892     __Pyx_RefNannyFinishContext();
04893     return __pyx_r;
04894 }
04895
04896 /* "PyClical.pyx":282
04897     return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
04898
04899     def __iand__(self, rhs): # ««««««««
04900     """
04901     Set intersection: and.
04902     */
04903
04904     /* Python wrapper */
04905     static PyObject *__pyx_pw_8PyClical_9index_set_26__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
04906     static char __pyx_doc_8PyClical_9index_set_25__iand__[] = "\n          Set intersection: and.\n\n
>> x = index_set({1}); x &= index_set({2}); print(x)\n          {}\n          >> x = index_set({1,2}); x
&= index_set({2}); print(x)\n          {2}\n          ";
04907 #if CYTHON_COMPILING_IN_CPYTHON
04908 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_25__iand__;
04909 #endif
04910 static PyObject *__pyx_pw_8PyClical_9index_set_26__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
04911     PyObject *__pyx_r = 0;
04912     __Pyx_RefNannyDeclarations
04913     __Pyx_RefNannySetupContext("__iand__ (wrapper)", 0);
04914     __pyx_r = __pyx_pf_8PyClical_9index_set_25__iand__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
04915
04916     /* function exit code */
04917     __Pyx_RefNannyFinishContext();
04918     return __pyx_r;
04919 }
04920
04921 static PyObject *__pyx_pf_8PyClical_9index_set_25__iand__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_rhs) {

```

```

04921 PyObject *__pyx_r = NULL;
04922 __Pyx_RefNannyDeclarations
04923 PyObject *__pyx_t_1 = NULL;
04924 int __pyx_lineno = 0;
04925 const char *__pyx_filename = NULL;
04926 int __pyx_clineno = 0;
04927 __Pyx_RefNannySetupContext("__iand__", 0);
04928
04929 /* "PyClicl.pyx":291
04930 *      {2}
04931 *      """
04932 *      return self.wrap( self.unwrap() & toIndexSet(rhs) )          # ««««««««
04933 *
04934 *      def __or__(lhs, rhs):
04935 */
04936 __Pyx_XDECREF(__pyx_r);
04937 __pyx_t_1 = __pyx_f_8PyClicl_9index_set_wrap(__pyx_v_self,
  (__pyx_f_8PyClicl_9index_set_unwrap(__pyx_v_self) & __pyx_f_8PyClicl_toIndexSet(__pyx_v_rhs))); if
  (unlikely(!__pyx_t_1)) __PYX_ERR(0, 291, __pyx_L1_error)
04938 __Pyx_GOTREF(__pyx_t_1);
04939 __pyx_r = __pyx_t_1;
04940 __pyx_t_1 = 0;
04941 goto __pyx_L0;
04942
04943 /* "PyClicl.pyx":282
04944 *      return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
04945 *
04946 *      def __iand__(self, rhs):          # ««««««««
04947 *      """
04948 *      Set intersection: and.
04949 */
04950
04951 /* function exit code */
04952 __pyx_L1_error:;
04953 __Pyx_XDECREF(__pyx_t_1);
04954 __Pyx_AddTraceback("PyClicl.index_set.__iand__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04955 __pyx_r = NULL;
04956 __pyx_L0:;
04957 __Pyx_XGIVEREF(__pyx_r);
04958 __Pyx_RefNannyFinishContext();
04959 return __pyx_r;
04960 }
04961
04962 /* "PyClicl.pyx":293
04963 *      return self.wrap( self.unwrap() & toIndexSet(rhs) )
04964 *
04965 *      def __or__(lhs, rhs):          # ««««««««
04966 *      """
04967 *      Set union: or.
04968 */
04969
04970 /* Python wrapper */
04971 static PyObject *__pyx_pw_8PyClicl_9index_set_28__or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
04972 /*proto*/
04973 static char __pyx_doc_8PyClicl_9index_set_27__or__[] = "\n          Set union: or.\n\n          >>
  print(index_set({1}) | index_set({2}))\n          {1,2}\n          >> print(index_set({1,2}) |
  index_set({2}))\n          {1,2}\n          ";
04974 #if CYTHON_COMPILING_IN_CPYTHON
04975 struct wrapperbase __pyx_wrapperbase_8PyClicl_9index_set_27__or__;
04976 #endif
04977 static PyObject *__pyx_pw_8PyClicl_9index_set_28__or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
  {
04978     PyObject *__pyx_r = 0;
04979     __Pyx_RefNannyDeclarations
04980     __Pyx_RefNannySetupContext("__or__ (wrapper)", 0);
04981     __pyx_r = __pyx_pf_8PyClicl_9index_set_27__or__(((PyObject *)__pyx_v_lhs), ((PyObject
  *)__pyx_v_rhs));
04982
04983     /* function exit code */
04984     __Pyx_RefNannyFinishContext();
04985     return __pyx_r;
04986 }
04987 static PyObject *__pyx_pf_8PyClicl_9index_set_27__or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
  {
04988     PyObject *__pyx_r = NULL;
04989     __Pyx_RefNannyDeclarations
04990     PyObject *__pyx_t_1 = NULL;
04991     PyObject *__pyx_t_2 = NULL;
04992     int __pyx_lineno = 0;
04993     const char *__pyx_filename = NULL;
04994     int __pyx_clineno = 0;
04995     __Pyx_RefNannySetupContext("__or__", 0);
04996
04997     /* "PyClicl.pyx":302
04998 *      {1,2}
04999 *      """

```

```

05000 *         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )           # ««««««««
05001 *
05002 *     def __ior__(self, rhs):
05003 */
05004     __Pyx_XDECREF(__pyx_r);
05005     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 302, __pyx_L1_error)
05006     __Pyx_GOTREF(__pyx_t_1);
05007     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
(__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs) | __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 302, __pyx_L1_error)
05008     __Pyx_GOTREF(__pyx_t_2);
05009     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
05010     __pyx_r = __pyx_t_2;
05011     __pyx_t_2 = 0;
05012     goto __pyx_L0;
05013
05014 /* "PyClical.pyx":293
05015 *         return self.wrap( self.unwrap() & toIndexSet(rhs) )
05016 *
05017 *     def __or__(lhs, rhs):           # ««««««««
05018 *         """
05019 *         Set union: or.
05020 */
05021
05022 /* function exit code */
05023 __pyx_L1_error;
05024 __Pyx_XDECREF(__pyx_t_1);
05025 __Pyx_XDECREF(__pyx_t_2);
05026 __Pyx_AddTraceback("PyClical.index_set.__or__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05027 __pyx_r = NULL;
05028 __pyx_L0;
05029 __Pyx_XGIVEREF(__pyx_r);
05030 __Pyx_RefNannyFinishContext();
05031 return __pyx_r;
05032 }
05033
05034 /* "PyClical.pyx":304
05035 *         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
05036 *
05037 *     def __ior__(self, rhs):           # ««««««««
05038 *         """
05039 *         Set union: or.
05040 */
05041
05042 /* Python wrapper */
05043 static PyObject *__pyx_pw_8PyClical_9index_set_30__ior__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
05044 static char __pyx_doc_8PyClical_9index_set_29__ior__[] = "\n        Set union: or.\n\n        >> x =
index_set({1}); x |= index_set({2}); print(x)\n        {1,2}\n        >> x = index_set({1,2}); x |=
index_set({2}); print(x)\n        {1,2}\n        ";
05045 #if CYTHON_COMPILING_IN_CPYTHON
05046 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_29__ior__;
05047 #endif
05048 static PyObject *__pyx_pw_8PyClical_9index_set_30__ior__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
05049     PyObject *__pyx_r = 0;
05050     __Pyx_RefNannyDeclarations
05051     __Pyx_RefNannySetupContext("__ior__ (wrapper)", 0);
05052     __pyx_r = __pyx_pf_8PyClical_9index_set_29__ior__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
05053
05054 /* function exit code */
05055 __Pyx_RefNannyFinishContext();
05056 return __pyx_r;
05057 }
05058
05059 static PyObject *__pyx_pf_8PyClical_9index_set_29__ior__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_rhs) {
05060     PyObject *__pyx_r = NULL;
05061     __Pyx_RefNannyDeclarations
05062     PyObject *__pyx_t_1 = NULL;
05063     int __pyx_lineno = 0;
05064     const char *__pyx_filename = NULL;
05065     int __pyx_clineno = 0;
05066     __Pyx_RefNannySetupContext("__ior__", 0);
05067
05068 /* "PyClical.pyx":313
05069 *         {1,2}
05070 *         """
05071 *         return self.wrap( self.unwrap() | toIndexSet(rhs) )           # ««««««««
05072 *
05073 *     def count(self):
05074 */
05075     __Pyx_XDECREF(__pyx_r);
05076     __pyx_t_1 = __pyx_f_8PyClical_9index_set_wrap(__pyx_v_self,
(__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self) | __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if

```

```

(unlikely(!__pyx_t_1)) __PYX_ERR(0, 313, __pyx_L1_error)
05077 __Pyx_GOTREF(__pyx_t_1);
05078 __pyx_r = __pyx_t_1;
05079 __pyx_t_1 = 0;
05080 goto __pyx_L0;
05081
05082 /* "PyClical.pyx":304
05083 *      return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
05084 *
05085 *      def __ior__(self, rhs):          # ««««««««
05086 *          """
05087 *          Set union: or.
05088 */
05089
05090 /* function exit code */
05091 __pyx_L1_error:;
05092 __Pyx_XDECREF(__pyx_t_1);
05093 __Pyx_AddTraceback("PyClical.index_set.__ior__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05094 __pyx_r = NULL;
05095 __pyx_L0:;
05096 __Pyx_XGIVEREF(__pyx_r);
05097 __Pyx_RefNannyFinishContext();
05098 return __pyx_r;
05099 }
05100
05101 /* "PyClical.pyx":315
05102 *      return self.wrap( self.unwrap() | toIndexSet(rhs) )
05103 *
05104 *      def count(self):                # ««««««««
05105 *          """
05106 *          Cardinality: Number of indices included in set.
05107 */
05108
05109 /* Python wrapper */
05110 static PyObject *__pyx_pw_8PyClical_9index_set_32count(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused); /*proto*/
05111 static char __pyx_doc_8PyClical_9index_set_31count[] = "\n      Cardinality: Number of indices
included in set.\n\n      >> index_set({-1,1,2}).count()\n      3\n      ";
05112 static PyObject *__pyx_pf_8PyClical_9index_set_32count(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused) {
05113     PyObject *__pyx_r = 0;
05114     __Pyx_RefNannyDeclarations
05115     __Pyx_RefNannySetupContext("count (wrapper)", 0);
05116     __pyx_r = __pyx_pf_8PyClical_9index_set_31count(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05117
05118 /* function exit code */
05119 __Pyx_RefNannyFinishContext();
05120 return __pyx_r;
05121 }
05122
05123 static PyObject *__pyx_pf_8PyClical_9index_set_31count(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05124     PyObject *__pyx_r = NULL;
05125     __Pyx_RefNannyDeclarations
05126     PyObject *__pyx_t_1 = NULL;
05127     int __pyx_lineno = 0;
05128     const char *__pyx_filename = NULL;
05129     int __pyx_clineno = 0;
05130     __Pyx_RefNannySetupContext("count", 0);
05131
05132 /* "PyClical.pyx":322
05133 *          3
05134 *          """
05135 *          return self.instance.count()          # ««««««««
05136 *
05137 *          def count_neg(self):
05138 */
05139     __Pyx_XDECREF(__pyx_r);
05140     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->count()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 322, __pyx_L1_error)
05141     __Pyx_GOTREF(__pyx_t_1);
05142     __pyx_r = __pyx_t_1;
05143     __pyx_t_1 = 0;
05144     goto __pyx_L0;
05145
05146 /* "PyClical.pyx":315
05147 *      return self.wrap( self.unwrap() | toIndexSet(rhs) )
05148 *
05149 *      def count(self):                # ««««««««
05150 *          """
05151 *          Cardinality: Number of indices included in set.
05152 */
05153
05154 /* function exit code */
05155 __pyx_L1_error:;
05156 __Pyx_XDECREF(__pyx_t_1);

```



```

05157 __Pyx_AddTraceback("PyClical.index_set.count", __pyx_clineno, __pyx_lineno, __pyx_filename);
05158 __pyx_r = NULL;
05159 __pyx_L0:;
05160 __Pyx_XGIVEREF(__pyx_r);
05161 __Pyx_RefNannyFinishContext();
05162 return __pyx_r;
05163 }
05164
05165 /* "PyClical.pyx":324
05166 *         return self.instance.count()
05167 *
05168 *         def count_neg(self):          # ««««««««
05169 *             """
05170 *             Number of negative indices included in set.
05171 */
05172
05173 /* Python wrapper */
05174 static PyObject *__pyx_pw_8PyClical_9index_set_34count_neg(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05175 static char __pyx_doc_8PyClical_9index_set_33count_neg[] = "\n        Number of negative indices
included in set.\n\n        >> index_set({-1,1,2}).count_neg()\n        1\n        ";
05176 static PyObject *__pyx_pw_8PyClical_9index_set_34count_neg(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
05177     PyObject *__pyx_r = 0;
05178     __Pyx_RefNannyDeclarations
05179     __Pyx_RefNannySetupContext("count_neg (wrapper)", 0);
05180     __pyx_r = __pyx_pf_8PyClical_9index_set_33count_neg(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05181
05182     /* function exit code */
05183     __Pyx_RefNannyFinishContext();
05184     return __pyx_r;
05185 }
05186
05187 static PyObject *__pyx_pf_8PyClical_9index_set_33count_neg(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05188     PyObject *__pyx_r = NULL;
05189     __Pyx_RefNannyDeclarations
05190     PyObject *__pyx_t_1 = NULL;
05191     int __pyx_lineno = 0;
05192     const char *__pyx_filename = NULL;
05193     int __pyx_clineno = 0;
05194     __Pyx_RefNannySetupContext("count_neg", 0);
05195
05196     /* "PyClical.pyx":331
05197     *         1
05198     *         """
05199     *         return self.instance.count_neg()          # ««««««««
05200     *
05201     *         def count_pos(self):
05202     */
05203     __Pyx_XDECREF(__pyx_r);
05204     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->count_neg()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 331, __pyx_L1_error)
05205     __Pyx_GOTREF(__pyx_t_1);
05206     __pyx_r = __pyx_t_1;
05207     __pyx_t_1 = 0;
05208     goto __pyx_L0;
05209
05210     /* "PyClical.pyx":324
05211     *         return self.instance.count()
05212     *
05213     *         def count_neg(self):          # ««««««««
05214     *             """
05215     *             Number of negative indices included in set.
05216     */
05217
05218     /* function exit code */
05219     __pyx_L1_error:;
05220     __Pyx_XDECREF(__pyx_t_1);
05221     __Pyx_AddTraceback("PyClical.index_set.count_neg", __pyx_clineno, __pyx_lineno, __pyx_filename);
05222     __pyx_r = NULL;
05223     __pyx_L0:;
05224     __Pyx_XGIVEREF(__pyx_r);
05225     __Pyx_RefNannyFinishContext();
05226     return __pyx_r;
05227 }
05228
05229 /* "PyClical.pyx":333
05230 *         return self.instance.count_neg()
05231 *
05232 *         def count_pos(self):          # ««««««««
05233 *             """
05234 *             Number of positive indices included in set.
05235     */
05236
05237 /* Python wrapper */

```

```

05238 static PyObject *__pyx_pw_8PyClical_9index_set_36count_pos(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05239 static char __pyx_doc_8PyClical_9index_set_35count_pos[] = "\n          Number of positive indices
included in set.\n\n          >> index_set({-1,1,2}).count_pos()\n          2\n";
05240 static PyObject *__pyx_pw_8PyClical_9index_set_36count_pos(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
05241     PyObject *__pyx_r = 0;
05242     __Pyx_RefNannyDeclarations
05243     __Pyx_RefNannySetupContext("count_pos (wrapper)", 0);
05244     __pyx_r = __pyx_pf_8PyClical_9index_set_35count_pos(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05245
05246     /* function exit code */
05247     __Pyx_RefNannyFinishContext();
05248     return __pyx_r;
05249 }
05250
05251 static PyObject *__pyx_pf_8PyClical_9index_set_35count_pos(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05252     PyObject *__pyx_r = NULL;
05253     __Pyx_RefNannyDeclarations
05254     PyObject *__pyx_t_1 = NULL;
05255     int __pyx_lineno = 0;
05256     const char *__pyx_filename = NULL;
05257     int __pyx_clineno = 0;
05258     __Pyx_RefNannySetupContext("count_pos", 0);
05259
05260     /* "PyClical.pyx":340
05261     *         2
05262     *         """
05263     *         return self.instance.count_pos()          # ««««««««
05264     *
05265     *     def min(self):
05266     */
05267     __Pyx_XDECREF(__pyx_r);
05268     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->count_pos()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 340, __pyx_L1_error)
05269     __Pyx_GOTREF(__pyx_t_1);
05270     __pyx_r = __pyx_t_1;
05271     __pyx_t_1 = 0;
05272     goto __pyx_L0;
05273
05274     /* "PyClical.pyx":333
05275     *         return self.instance.count_neg()
05276     *
05277     *     def count_pos(self):          # ««««««««
05278     *         """
05279     *         Number of positive indices included in set.
05280     */
05281
05282     /* function exit code */
05283     __pyx_L1_error:;
05284     __Pyx_XDECREF(__pyx_t_1);
05285     __Pyx_AddTraceback("PyClical.index_set.count_pos", __pyx_clineno, __pyx_lineno, __pyx_filename);
05286     __pyx_r = NULL;
05287     __pyx_L0:;
05288     __Pyx_XGIVEREF(__pyx_r);
05289     __Pyx_RefNannyFinishContext();
05290     return __pyx_r;
05291 }
05292
05293 /* "PyClical.pyx":342
05294 *         return self.instance.count_pos()
05295 *
05296 *     def min(self):          # ««««««««
05297 *         """
05298 *         Minimum member.
05299     */
05300
05301 /* Python wrapper */
05302 static PyObject *__pyx_pw_8PyClical_9index_set_38min(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused); /*proto*/
05303 static char __pyx_doc_8PyClical_9index_set_37min[] = "\n          Minimum member.\n\n          >>
index_set({-1,1,2}).min()\n          -1\n";
05304 static PyObject *__pyx_pw_8PyClical_9index_set_38min(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused) {
05305     PyObject *__pyx_r = 0;
05306     __Pyx_RefNannyDeclarations
05307     __Pyx_RefNannySetupContext("min (wrapper)", 0);
05308     __pyx_r = __pyx_pf_8PyClical_9index_set_37min(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05309
05310     /* function exit code */
05311     __Pyx_RefNannyFinishContext();
05312     return __pyx_r;
05313 }
05314

```

```

05315 static PyObject *__pyx_pf_8PyClical_9index_set_37min(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self) {
05316     PyObject *__pyx_r = NULL;
05317     __Pyx_RefNannyDeclarations
05318     PyObject *__pyx_t_1 = NULL;
05319     int __pyx_lineno = 0;
05320     const char *__pyx_filename = NULL;
05321     int __pyx_clineno = 0;
05322     __Pyx_RefNannySetupContext("min", 0);
05323
05324     /* "PyClical.pyx":349
05325     *         -1
05326     *         """
05327     *         return self.instance.min() # «««««««
05328     *
05329     *     def max(self):
05330     */
05331     __Pyx_XDECREF(__pyx_r);
05332     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->min()); if (unlikely(!__pyx_t_1))
    PYX_ERR(0, 349, __pyx_L1_error)
05333     __Pyx_GOTREF(__pyx_t_1);
05334     __pyx_r = __pyx_t_1;
05335     __pyx_t_1 = 0;
05336     goto __pyx_L0;
05337
05338     /* "PyClical.pyx":342
05339     *         return self.instance.count_pos()
05340     *
05341     *     def min(self): # «««««««
05342     *         """
05343     *         Minimum member.
05344     */
05345
05346     /* function exit code */
05347     __pyx_L1_error:;
05348     __Pyx_XDECREF(__pyx_t_1);
05349     __Pyx_AddTraceback("PyClical.index_set.min", __pyx_clineno, __pyx_lineno, __pyx_filename);
05350     __pyx_r = NULL;
05351     __pyx_L0:;
05352     __Pyx_XGIVEREF(__pyx_r);
05353     __Pyx_RefNannyFinishContext();
05354     return __pyx_r;
05355 }
05356
05357 /* "PyClical.pyx":351
05358 *         return self.instance.min()
05359 *
05360 *     def max(self): # «««««««
05361 *         """
05362 *         Maximum member.
05363     */
05364
05365     /* Python wrapper */
05366 static PyObject *__pyx_pw_8PyClical_9index_set_40max(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
    *unused); /*proto*/
05367 static char __pyx_doc_8PyClical_9index_set_39max[] = "\n        Maximum member.\n\n        >>
    index_set({-1,1,2}).max()\n        2\n        ";
05368 static PyObject *__pyx_pw_8PyClical_9index_set_40max(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
    *unused) {
05369     PyObject *__pyx_r = 0;
05370     __Pyx_RefNannyDeclarations
05371     __Pyx_RefNannySetupContext("max (wrapper)", 0);
05372     __pyx_r = __pyx_pf_8PyClical_9index_set_39max(((struct __pyx_obj_8PyClical_index_set
    *)__pyx_v_self));
05373
05374     /* function exit code */
05375     __Pyx_RefNannyFinishContext();
05376     return __pyx_r;
05377 }
05378
05379 static PyObject *__pyx_pf_8PyClical_9index_set_39max(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self) {
05380     PyObject *__pyx_r = NULL;
05381     __Pyx_RefNannyDeclarations
05382     PyObject *__pyx_t_1 = NULL;
05383     int __pyx_lineno = 0;
05384     const char *__pyx_filename = NULL;
05385     int __pyx_clineno = 0;
05386     __Pyx_RefNannySetupContext("max", 0);
05387
05388     /* "PyClical.pyx":358
05389     *         2
05390     *         """
05391     *         return self.instance.max() # «««««««
05392     *
05393     *     def hash_fn(self):
05394     */

```

```

05395     __Pyx_XDECREF(__pyx_r);
05396     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->max()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 358, __pyx_L1_error)
05397     __Pyx_GOTREF(__pyx_t_1);
05398     __pyx_r = __pyx_t_1;
05399     __pyx_t_1 = 0;
05400     goto __pyx_L0;
05401
05402     /* "PyClical.pyx":351
05403     *         return self.instance.min()
05404     *
05405     *     def max(self):                # ««««««««
05406     *         """
05407     *         Maximum member.
05408     */
05409
05410     /* function exit code */
05411     __pyx_L1_error:;
05412     __Pyx_XDECREF(__pyx_t_1);
05413     __Pyx_AddTraceback("PyClical.index_set.max", __pyx_clineno, __pyx_lineno, __pyx_filename);
05414     __pyx_r = NULL;
05415     __pyx_L0:;
05416     __Pyx_XGIVEREF(__pyx_r);
05417     __Pyx_RefNannyFinishContext();
05418     return __pyx_r;
05419 }
05420
05421 /* "PyClical.pyx":360
05422 *         return self.instance.max()
05423 *
05424 *     def hash_fn(self):                # ««««««««
05425 *         """
05426 *         Hash function.
05427 */
05428
05429 /* Python wrapper */
05430 static PyObject * __pyx_pw_8PyClical_9index_set_42hash_fn(PyObject * __pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05431 static char __pyx_doc_8PyClical_9index_set_41hash_fn[] = "\n        Hash function.\n        ";
05432 static PyObject * __pyx_pw_8PyClical_9index_set_42hash_fn(PyObject * __pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
05433     PyObject * __pyx_r = 0;
05434     __Pyx_RefNannyDeclarations
05435     __Pyx_RefNannySetupContext("hash_fn (wrapper)", 0);
05436     __pyx_r = __pyx_pf_8PyClical_9index_set_41hash_fn(((struct __pyx_obj_8PyClical_index_set
*) __pyx_v_self));
05437
05438     /* function exit code */
05439     __Pyx_RefNannyFinishContext();
05440     return __pyx_r;
05441 }
05442
05443 static PyObject * __pyx_pf_8PyClical_9index_set_41hash_fn(struct __pyx_obj_8PyClical_index_set
* __pyx_v_self) {
05444     PyObject * __pyx_r = NULL;
05445     __Pyx_RefNannyDeclarations
05446     PyObject * __pyx_t_1 = NULL;
05447     int __pyx_lineno = 0;
05448     const char * __pyx_filename = NULL;
05449     int __pyx_clineno = 0;
05450     __Pyx_RefNannySetupContext("hash_fn", 0);
05451
05452     /* "PyClical.pyx":364
05453     *         Hash function.
05454     *
05455     *         return self.instance.hash_fn()                # ««««««««
05456     *
05457     *     def sign_of_mult(self, rhs):
05458     */
05459     __Pyx_XDECREF(__pyx_r);
05460     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->hash_fn()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 364, __pyx_L1_error)
05461     __Pyx_GOTREF(__pyx_t_1);
05462     __pyx_r = __pyx_t_1;
05463     __pyx_t_1 = 0;
05464     goto __pyx_L0;
05465
05466     /* "PyClical.pyx":360
05467     *         return self.instance.max()
05468     *
05469     *     def hash_fn(self):                # ««««««««
05470     *         """
05471     *         Hash function.
05472     */
05473
05474     /* function exit code */
05475     __pyx_L1_error:;

```

```

05476     __Pyx_XDECREF(__pyx_t_1);
05477     __Pyx_AddTraceback("PyClicl.index_set.hash_fn", __pyx_clineno, __pyx_lineno, __pyx_filename);
05478     __pyx_r = NULL;
05479     __pyx_L0:;
05480     __Pyx_XGIVEREF(__pyx_r);
05481     __Pyx_RefNannyFinishContext();
05482     return __pyx_r;
05483 }
05484
05485 /* "PyClicl.pyx":366
05486 *         return self.instance.hash_fn()
05487 *
05488 *         def sign_of_mult(self, rhs):
05489 *             """
05490 *             Sign of geometric product of two Clifford basis elements.
05491 */
05492
05493 /* Python wrapper */
05494 static PyObject *__pyx_pw_8PyClicl_9index_set_44sign_of_mult(PyObject *__pyx_v_self, PyObject
05495 *__pyx_v_rhs); /*proto*/
05496 static char __pyx_doc_8PyClicl_9index_set_43sign_of_mult[] = "\n        Sign of geometric product of
05497 two Clifford basis elements.\n\n        >> s = index_set({1,2}); t=index_set({-1});
05498 s.sign_of_mult(t)\n        1\n        ";
05499 static PyObject *__pyx_pw_8PyClicl_9index_set_44sign_of_mult(PyObject *__pyx_v_self, PyObject
05500 *__pyx_v_rhs) {
05501     PyObject *__pyx_r = 0;
05502     __Pyx_RefNannyDeclarations
05503     __Pyx_RefNannySetupContext("sign_of_mult (wrapper)", 0);
05504     __pyx_r = __pyx_pf_8PyClicl_9index_set_43sign_of_mult(((struct __pyx_obj_8PyClicl_index_set
05505 *)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
05506
05507 /* function exit code */
05508 __Pyx_RefNannyFinishContext();
05509 return __pyx_r;
05510 }
05511
05512 static PyObject *__pyx_pf_8PyClicl_9index_set_43sign_of_mult(struct __pyx_obj_8PyClicl_index_set
05513 *__pyx_v_self, PyObject *__pyx_v_rhs) {
05514     PyObject *__pyx_r = NULL;
05515     __Pyx_RefNannyDeclarations
05516     PyObject *__pyx_t_1 = NULL;
05517     int __pyx_lineno = 0;
05518     const char *__pyx_filename = NULL;
05519     int __pyx_clineno = 0;
05520     __Pyx_RefNannySetupContext("sign_of_mult", 0);
05521
05522 /* "PyClicl.pyx":373
05523 *         1
05524 *         """
05525 *         return self.instance.sign_of_mult(toIndexSet(rhs))
05526 *             # <<<<<<<<<
05527 *
05528 *         def sign_of_square(self):
05529 */
05530     __Pyx_XDECREF(__pyx_r);
05531     __pyx_t_1 =
05532     __Pyx_PyInt_From_int(__pyx_v_self->instance->sign_of_mult(__pyx_f_8PyClicl_toIndexSet(__pyx_v_rhs)));
05533     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 373, __pyx_L1_error)
05534     __Pyx_GOTREF(__pyx_t_1);
05535     __pyx_r = __pyx_t_1;
05536     __pyx_t_1 = 0;
05537     goto __pyx_L0;
05538
05539 /* "PyClicl.pyx":366
05540 *         return self.instance.hash_fn()
05541 *
05542 *         def sign_of_mult(self, rhs):
05543 *             """
05544 *             Sign of geometric product of two Clifford basis elements.
05545 */
05546
05547 /* function exit code */
05548 __pyx_L1_error:;
05549 __Pyx_XDECREF(__pyx_t_1);
05550 __Pyx_AddTraceback("PyClicl.index_set.sign_of_mult", __pyx_clineno, __pyx_lineno, __pyx_filename);
05551 __pyx_r = NULL;
05552 __pyx_L0:;
05553 __Pyx_XGIVEREF(__pyx_r);
05554 __Pyx_RefNannyFinishContext();
05555 return __pyx_r;
05556 }
05557
05558 /* "PyClicl.pyx":375
05559 *         return self.instance.sign_of_mult(toIndexSet(rhs))
05560 *
05561 *         def sign_of_square(self):
05562 *             # <<<<<<<<<
05563 *             """
05564 *             Sign of geometric square of a Clifford basis element.

```

```

05555 */
05556
05557 /* Python wrapper */
05558 static PyObject *__pyx_pw_8PyClicl_9index_set_46sign_of_square(PyObject *__pyx_v_self, CYTHON_UNUSED
05559 PyObject *unused); /*proto*/
05559 static char __pyx_doc_8PyClicl_9index_set_45sign_of_square[] = "\n          Sign of geometric square of
a Clifford basis element.\n\n          >> s = index_set({1,2}); s.sign_of_square()\n          -1\n";
05560 static PyObject *__pyx_pw_8PyClicl_9index_set_46sign_of_square(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
05561     PyObject *__pyx_r = 0;
05562     __Pyx_RefNannyDeclarations
05563     __Pyx_RefNannySetupContext("sign_of_square (wrapper)", 0);
05564     __pyx_r = __pyx_pf_8PyClicl_9index_set_45sign_of_square(((struct __pyx_obj_8PyClicl_index_set
*)__pyx_v_self));
05565
05566     /* function exit code */
05567     __Pyx_RefNannyFinishContext();
05568     return __pyx_r;
05569 }
05570
05571 static PyObject *__pyx_pf_8PyClicl_9index_set_45sign_of_square(struct __pyx_obj_8PyClicl_index_set
*__pyx_v_self) {
05572     PyObject *__pyx_r = NULL;
05573     __Pyx_RefNannyDeclarations
05574     PyObject *__pyx_t_1 = NULL;
05575     int __pyx_lineno = 0;
05576     const char *__pyx_filename = NULL;
05577     int __pyx_clineno = 0;
05578     __Pyx_RefNannySetupContext("sign_of_square", 0);
05579
05580     /* "PyClicl.pyx":382
05581     *         -1
05582     *         """
05583     *         return self.instance.sign_of_square() # <<<<<<<<
05584     *
05585     *     def __repr__(self):
05586     */
05587     __Pyx_XDECREF(__pyx_r);
05588     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->sign_of_square()); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 382, __pyx_L1_error)
05589     __Pyx_GOTREF(__pyx_t_1);
05590     __pyx_r = __pyx_t_1;
05591     __pyx_t_1 = 0;
05592     goto __pyx_L0;
05593
05594     /* "PyClicl.pyx":375
05595     *         return self.instance.sign_of_mult(toIndexSet(rhs))
05596     *
05597     *     def sign_of_square(self): # <<<<<<<<
05598     *         """
05599     *         Sign of geometric square of a Clifford basis element.
05600     */
05601
05602     /* function exit code */
05603     __pyx_L1_error:;
05604     __Pyx_XDECREF(__pyx_t_1);
05605     __Pyx_AddTraceback("PyClicl.index_set.sign_of_square", __pyx_clineno, __pyx_lineno,
__pyx_filename);
05606     __pyx_r = NULL;
05607     __pyx_L0:;
05608     __Pyx_XGIVEREF(__pyx_r);
05609     __Pyx_RefNannyFinishContext();
05610     return __pyx_r;
05611 }
05612
05613 /* "PyClicl.pyx":384
05614 *         return self.instance.sign_of_square()
05615 *
05616 *     def __repr__(self): # <<<<<<<<
05617 *         """
05618 *         The official string representation of self.
05619     */
05620
05621 /* Python wrapper */
05622 static PyObject *__pyx_pw_8PyClicl_9index_set_48__repr__(PyObject *__pyx_v_self); /*proto*/
05623 static char __pyx_doc_8PyClicl_9index_set_47__repr__[] = "\n          The
\342\200\234official\342\200\235 string representation of self.\n\n          >>
index_set({1,2}).__repr__()\n          'index_set({1,2})'\n          >> repr(index_set({1,2}))\n
'index_set({1,2})'\n          ";
05624 #if CYTHON_COMPILING_IN_CPYTHON
05625 struct wrapperbase __pyx_wrapperbase_8PyClicl_9index_set_47__repr__;
05626 #endif
05627 static PyObject *__pyx_pw_8PyClicl_9index_set_48__repr__(PyObject *__pyx_v_self) {
05628     PyObject *__pyx_r = 0;
05629     __Pyx_RefNannyDeclarations
05630     __Pyx_RefNannySetupContext("__repr__ (wrapper)", 0);

```

```

05631  __pyx_r = __pyx_pf_8PyClical_9index_set_47__repr__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05632
05633  /* function exit code */
05634  __Pyx_RefNannyFinishContext();
05635  return __pyx_r;
05636 }
05637
05638 static PyObject *__pyx_pf_8PyClical_9index_set_47__repr__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05639  PyObject *__pyx_r = NULL;
05640  __Pyx_RefNannyDeclarations
05641  PyObject *__pyx_t_1 = NULL;
05642  int __pyx_lineno = 0;
05643  const char *__pyx_filename = NULL;
05644  int __pyx_clineno = 0;
05645  __Pyx_RefNannySetupContext("__repr__", 0);
05646
05647  /* "PyClical.pyx":393
05648  *      'index_set({1,2})'
05649  *      """
05650  *      return index_set_to_repr( self.unwrap() ).decode()           # ««««««««
05651  *
05652  *      def __str__(self):
05653  */
05654  __Pyx_XDECREF(__pyx_r);
05655  __pyx_t_1 =
__Pyx_decode_cpp_string(index_set_to_repr(__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self)), 0,
PY_SSIZE_T_MAX, NULL, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 393, __pyx_L1_error)
05656  __Pyx_GOTREF(__pyx_t_1);
05657  __pyx_r = __pyx_t_1;
05658  __pyx_t_1 = 0;
05659  goto __pyx_L0;
05660
05661  /* "PyClical.pyx":384
05662  *      return self.instance.sign_of_square()
05663  *
05664  *      def __repr__(self):           # ««««««««
05665  *      """
05666  *      The official string representation of self.
05667  */
05668
05669  /* function exit code */
05670  __pyx_L1_error:;
05671  __Pyx_XDECREF(__pyx_t_1);
05672  __Pyx_AddTraceback("PyClical.index_set.__repr__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05673  __pyx_r = NULL;
05674  __pyx_L0:;
05675  __Pyx_XGIVEREF(__pyx_r);
05676  __Pyx_RefNannyFinishContext();
05677  return __pyx_r;
05678 }
05679
05680 /* "PyClical.pyx":395
05681 *      return index_set_to_repr( self.unwrap() ).decode()
05682 *
05683 *      def __str__(self):           # ««««««««
05684 *      """
05685 *      The informal string representation of self.
05686 */
05687
05688 /* Python wrapper */
05689 static PyObject *__pyx_pw_8PyClical_9index_set_50__str__(PyObject *__pyx_v_self); /*proto*/
05690 static char __pyx_doc_8PyClical_9index_set_49__str__[] = "\n      The
>> str(index_set({1,2}))\n      '1,2'\n";
05691 #if CYTHON_COMPILING_IN_CPYTHON
05692 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_49__str__;
05693 #endif
05694 static PyObject *__pyx_pw_8PyClical_9index_set_50__str__(PyObject *__pyx_v_self) {
05695  PyObject *__pyx_r = 0;
05696  __Pyx_RefNannyDeclarations
05697  __Pyx_RefNannySetupContext("__str__ (wrapper)", 0);
05698  __pyx_r = __pyx_pf_8PyClical_9index_set_49__str__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05699
05700  /* function exit code */
05701  __Pyx_RefNannyFinishContext();
05702  return __pyx_r;
05703 }
05704
05705 static PyObject *__pyx_pf_8PyClical_9index_set_49__str__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05706  PyObject *__pyx_r = NULL;
05707  __Pyx_RefNannyDeclarations
05708  PyObject *__pyx_t_1 = NULL;

```

```

05709     int __pyx_lineno = 0;
05710     const char *__pyx_filename = NULL;
05711     int __pyx_clineno = 0;
05712     __Pyx_RefNannySetupContext("__str__", 0);
05713
05714     /* "PyClical.pyx":404
05715     *         '{1,2}'
05716     *         """
05717     *         return index_set_to_str( self.unwrap() ).decode()           # ««««««««
05718     *
05719     * def index_set_hidden_doctests():
05720     */
05721     __Pyx_XDECREF(__pyx_r);
05722     __pyx_t_1 =
05723     __Pyx_decode_cpp_string(index_set_to_str(__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self)), 0,
05724     PY_SSIZE_T_MAX, NULL, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 404, __pyx_L1_error)
05725     __Pyx_GOTREF(__pyx_t_1);
05726     __pyx_r = __pyx_t_1;
05727     __pyx_t_1 = 0;
05728     goto __pyx_L0;
05729
05730     /* "PyClical.pyx":395
05731     *         return index_set_to_repr( self.unwrap() ).decode()
05732     *
05733     * def __str__(self):           # ««««««««
05734     *         """
05735     *         The informal string representation of self.
05736     */
05737     /* function exit code */
05738     __Pyx_XDECREF(__pyx_t_1);
05739     __Pyx_AddTraceback("PyClical.index_set.__str__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05740     __pyx_r = NULL;
05741     __pyx_L0:;
05742     __Pyx_XGIVEREF(__pyx_r);
05743     __Pyx_RefNannyFinishContext();
05744     return __pyx_r;
05745 }
05746
05747 /* "(tree fragment)":1
05748 * def __reduce_cython__(self):           # ««««««««
05749 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05750 * def __setstate_cython__(self, __pyx_state):
05751 */
05752
05753 /* Python wrapper */
05754 static PyObject *__pyx_pw_8PyClical_9index_set_52__reduce_cython__(PyObject *__pyx_v_self,
05755 CYTHON_UNUSED PyObject *unused); /*proto*/
05756 static PyObject *__pyx_pw_8PyClical_9index_set_52__reduce_cython__(PyObject *__pyx_v_self,
05757 CYTHON_UNUSED PyObject *unused) {
05758     PyObject *__pyx_r = 0;
05759     __Pyx_RefNannyDeclarations
05760     __Pyx_RefNannySetupContext("__reduce_cython__ (wrapper)", 0);
05761     __pyx_r = __pyx_pf_8PyClical_9index_set_51__reduce_cython__(((struct __pyx_obj_8PyClical_index_set
05762 *)__pyx_v_self));
05763
05764     /* function exit code */
05765     __Pyx_RefNannyFinishContext();
05766     return __pyx_r;
05767 }
05768
05769 static PyObject *__pyx_pf_8PyClical_9index_set_51__reduce_cython__(CYTHON_UNUSED struct
05770 __pyx_obj_8PyClical_index_set *__pyx_v_self) {
05771     PyObject *__pyx_r = NULL;
05772     __Pyx_RefNannyDeclarations
05773     PyObject *__pyx_t_1 = NULL;
05774     int __pyx_lineno = 0;
05775     const char *__pyx_filename = NULL;
05776     int __pyx_clineno = 0;
05777     __Pyx_RefNannySetupContext("__reduce_cython__", 0);
05778
05779     /* "(tree fragment)":2
05780     * def __reduce_cython__(self):
05781     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")           # ««««««««
05782     * def __setstate_cython__(self, __pyx_state):
05783     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05784     */
05785     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__3, NULL); if
05786     (unlikely(!__pyx_t_1)) __PYX_ERR(1, 2, __pyx_L1_error)
05787     __Pyx_GOTREF(__pyx_t_1);
05788     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
05789     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
05790     __PYX_ERR(1, 2, __pyx_L1_error)
05791
05792     /* "(tree fragment)":1
05793     * def __reduce_cython__(self):           # ««««««««

```



```

05789 *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05790 * def __setstate_cython__(self, __pyx_state):
05791 */
05792
05793 /* function exit code */
05794 __pyx_L1_error:;
05795 __Pyx_XDECREF(__pyx_t_1);
05796 __Pyx_AddTraceback("PyClical.index_set.__reduce_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
05797 __pyx_r = NULL;
05798 __Pyx_XGIVEREF(__pyx_r);
05799 __Pyx_RefNannyFinishContext();
05800 return __pyx_r;
05801 }
05802
05803 /* "(tree fragment)":3
05804 * def __reduce_cython__(self):
05805 *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05806 * def __setstate_cython__(self, __pyx_state):          # ««««««««
05807 *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05808 */
05809
05810 /* Python wrapper */
05811 static PyObject *__pyx_pw_8PyClical_9index_set_54__setstate_cython__(PyObject *__pyx_v_self, PyObject
*__pyx_v__pyx_state); /*proto*/
05812 static PyObject *__pyx_pw_8PyClical_9index_set_54__setstate_cython__(PyObject *__pyx_v_self, PyObject
*__pyx_v__pyx_state) {
05813     PyObject *__pyx_r = 0;
05814     __Pyx_RefNannyDeclarations
05815     __Pyx_RefNannySetupContext("__setstate_cython__ (wrapper)", 0);
05816     __pyx_r = __pyx_pf_8PyClical_9index_set_53__setstate_cython__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v__pyx_state));
05817
05818 /* function exit code */
05819 __Pyx_RefNannyFinishContext();
05820 return __pyx_r;
05821 }
05822
05823 static PyObject *__pyx_pf_8PyClical_9index_set_53__setstate_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_index_set *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state) {
05824     PyObject *__pyx_r = NULL;
05825     __Pyx_RefNannyDeclarations
05826     PyObject *__pyx_t_1 = NULL;
05827     int __pyx_lineno = 0;
05828     const char *__pyx_filename = NULL;
05829     int __pyx_clineno = 0;
05830     __Pyx_RefNannySetupContext("__setstate_cython__", 0);
05831
05832 /* "(tree fragment)":4
05833 *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05834 * def __setstate_cython__(self, __pyx_state):
05835 *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")          # ««««««««
05836 */
05837     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple_4, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 4, __pyx_L1_error)
05838     __Pyx_GOTREF(__pyx_t_1);
05839     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
05840     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
05841     __PYX_ERR(1, 4, __pyx_L1_error)
05842
05843 /* "(tree fragment)":3
05844 * def __reduce_cython__(self):
05845 *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05846 * def __setstate_cython__(self, __pyx_state):          # ««««««««
05847 *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05848 */
05849
05850 /* function exit code */
05851 __pyx_L1_error:;
05852 __Pyx_XDECREF(__pyx_t_1);
05853 __Pyx_AddTraceback("PyClical.index_set.__setstate_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
05854 __pyx_r = NULL;
05855 __Pyx_XGIVEREF(__pyx_r);
05856 __Pyx_RefNannyFinishContext();
05857 return __pyx_r;
05858 }
05859
05860 /* "PyClical.pyx":406
05861 *      return index_set_to_str( self.unwrap() ).decode()
05862 *
05863 * def index_set_hidden_doctests():          # ««««««««
05864 *     """
05865 *     Tests for functions that Doctest cannot see.
05866 */
05867
05868 /* Python wrapper */

```

```

05869 static PyObject *__pyx_pw_8PyClical_lindex_set_hidden_doctests(PyObject *__pyx_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05870 static char __pyx_doc_8PyClical_index_set_hidden_doctests[] = "\n    Tests for functions that Doctest
cannot see.\n\n    For index_set.__cinit__: Construct index_set.\n\n    >> print(index_set(1))\n
{1}\n    >> print(index_set({1,2}))\n    {1,2}\n    >> print(index_set(index_set({1,2})))\n    {1,2}\n
>> print(index_set({1,2}))\n    {1,2}\n    >> print(index_set({1,2,1}))\n    {1,2}\n    >>
print(index_set({1,2,1}))\n    {1,2}\n    >> print(index_set(\"\"))\n    {}\n    >>
print(index_set(\"{\"}))\n    Traceback (most recent call last):\n    ... \n    ValueError: Cannot
initialize index_set object from invalid string '{'.\n    >> print(index_set(\"{1}\"))\n    Traceback
(most recent call last):\n    ... \n    ValueError: Cannot initialize index_set object from invalid
string '{1'.\n    >> print(index_set(\"{1,2,100}\"))\n    Traceback (most recent call last):\n
... \n    ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.\n    >>
print(index_set({1,2,100}))\n    Traceback (most recent call last):\n    ... \n    IndexError: Cannot
initialize index_set object from invalid {1, 2, 100}.\n    >> print(index_set([1,2]))\n    Traceback
(most recent call last):\n    ... \n    TypeError: Cannot initialize index_set object from <class
'list'>.\n\n    For index_set.__richcmp__: Compare two objects of class index_set.\n\n    >>
index_set(1) == index_set({1})\n    True\n    >> index_set({1}) != index_set({1})\n    False\n    >>
index_set({1}) != index_set({2})\n    True\n    >> index_set({1}) == index_set({2})\n    False\n    >>
index_set({1}) < index_set({2})\n    True\n    >> index_set({1}) <= index_set({2})\n    True\n    >>
index_set({1}) > index_set({2})\n    False\n    >> index_set({1}) >= index_set({2})\n    False\n    >>
None == index_set({1,2})\n    False\n    >> None != index_set({1,2})\n    True\n    >> None <
index_set({1,2})\n    False\n    >> None <= index_set({1,2})\n    False\n    >> None >
index_set({1,2})\n    False\n    >> None >= index_set({1,2})\n    False\n    >> "index_set({1,2}) ==
None\n    False\n    >> index_set({1,2}) != None\n    True\n    >> index_set({1,2}) < None\n
False\n    >> index_set({1,2}) <= None\n    False\n    >> index_set({1,2}) > None\n    False\n    >>
index_set({1,2}) >= None\n    False\n    ";
05871 static PyMethodDef __pyx_mdef_8PyClical_lindex_set_hidden_doctests =
{"index_set_hidden_doctests", (PyCFunction)__pyx_pw_8PyClical_lindex_set_hidden_doctests, METH_NOARGS,
__pyx_doc_8PyClical_index_set_hidden_doctests};
05872 static PyObject *__pyx_pw_8PyClical_lindex_set_hidden_doctests(PyObject *__pyx_self,
CYTHON_UNUSED PyObject *unused) {
05873     PyObject *__pyx_r = 0;
05874     __Pyx_RefNannyDeclarations
05875     __Pyx_RefNannySetupContext("index_set_hidden_doctests (wrapper)", 0);
05876     __pyx_r = __pyx_pf_8PyClical_index_set_hidden_doctests(__pyx_self);
05877
05878     /* function exit code */
05879     __Pyx_RefNannyFinishContext();
05880     return __pyx_r;
05881 }
05882
05883 static PyObject *__pyx_pf_8PyClical_index_set_hidden_doctests(CYTHON_UNUSED PyObject
*__pyx_self) {
05884     PyObject *__pyx_r = NULL;
05885     __Pyx_RefNannyDeclarations
05886     __Pyx_RefNannySetupContext("index_set_hidden_doctests", 0);
05887
05888     /* "PyClical.pyx":490
05889     *     False
05890     *     """
05891     *     return          # ««««««««
05892     *
05893     * cpdef inline compare(lhs,rhs):
05894     */
05895         __Pyx_XDECREF(__pyx_r);
05896         __pyx_r = Py_None; __Pyx_INCREF(Py_None);
05897         goto __pyx_L0;
05898
05899     /* "PyClical.pyx":406
05900     *     return index_set_to_str( self.unwrap() ).decode()
05901     *
05902     * def index_set_hidden_doctests():          # ««««««««
05903     *     """
05904     *     Tests for functions that Doctest cannot see.
05905     */
05906
05907     /* function exit code */
05908     __pyx_L0:;
05909     __Pyx_XGIVEREF(__pyx_r);
05910     __Pyx_RefNannyFinishContext();
05911     return __pyx_r;
05912 }
05913
05914     /* "PyClical.pyx":492
05915     *     return
05916     *
05917     * cpdef inline compare(lhs,rhs):          # ««««««««
05918     *     """
05919     *     "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
05920     */
05921
05922     static PyObject *__pyx_pw_8PyClical_3compare(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kws); /*proto*/
05923     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_compare(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, CYTHON_UNUSED int __pyx_skip_dispatch) {
05924         PyObject *__pyx_r = NULL;
05925         __Pyx_RefNannyDeclarations

```

```

05926     PyObject *__pyx_t_1 = NULL;
05927     int __pyx_lineno = 0;
05928     const char *__pyx_filename = NULL;
05929     int __pyx_clineno = 0;
05930     __Pyx_RefNannySetupContext("compare", 0);
05931
05932     /* "PyClical.pyx":502
05933     *      1
05934     *      """
05935     *      return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )           # ««««««««
05936     *
05937     * cpdef inline min_neg(obj):
05938     */
05939     __Pyx_XDECREF(__pyx_r);
05940     __pyx_t_1 = __Pyx_PyInt_From_int(compare(__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs),
__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 502,
__pyx_L1_error)
05941     __Pyx_GOTREF(__pyx_t_1);
05942     __pyx_r = __pyx_t_1;
05943     __pyx_t_1 = 0;
05944     goto __pyx_L0;
05945
05946     /* "PyClical.pyx":492
05947     *      return
05948     *
05949     * cpdef inline compare(lhs,rhs):           # ««««««««
05950     *      """
05951     *      "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
05952     */
05953
05954     /* function exit code */
05955     __pyx_L1_error:;
05956     __Pyx_XDECREF(__pyx_t_1);
05957     __Pyx_AddTraceback("PyClical.compare", __pyx_clineno, __pyx_lineno, __pyx_filename);
05958     __pyx_r = 0;
05959     __pyx_L0:;
05960     __Pyx_XGIVEREF(__pyx_r);
05961     __Pyx_RefNannyFinishContext();
05962     return __pyx_r;
05963 }
05964
05965     /* Python wrapper */
05966     static PyObject *__pyx_pw_8PyClical_3compare(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
05967     static char __pyx_doc_8PyClical_2compare[] = "\n    \"lexicographic compare\" eg. {3,4,5} is
less than {3,7,8};\n    -1 if a<b, +1 if a>b, 0 if a==b.\n\n    >>
compare(index_set({1,2}),index_set({-1,3}))\n    -1\n    >>
compare(index_set({-1,4}),index_set({-1,3}))\n    1\n    ";
05968     static PyObject *__pyx_pw_8PyClical_3compare(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
05969         PyObject *__pyx_v_lhs = 0;
05970         PyObject *__pyx_v_rhs = 0;
05971         int __pyx_lineno = 0;
05972         const char *__pyx_filename = NULL;
05973         int __pyx_clineno = 0;
05974         PyObject *__pyx_r = 0;
05975         __Pyx_RefNannyDeclarations
05976         __Pyx_RefNannySetupContext("compare (wrapper)", 0);
05977         {
05978             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_lhs,&__pyx_n_s_rhs,0};
05979             PyObject* values[2] = {0,0};
05980             if (unlikely(__pyx_kwds)) {
05981                 Py_ssize_t kw_args;
05982                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
05983                 switch (pos_args) {
05984                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
05985                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
05986                     case 0: break;
05987                     default: goto __pyx_L5_argtuple_error;
05988                 }
05989                 kw_args = PyDict_Size(__pyx_kwds);
05990                 switch (pos_args) {
05991                     case 0:
05992                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_lhs)) != 0))
05993                             kw_args--;
05994                         else goto __pyx_L5_argtuple_error;
05995                         CYTHON_FALLTHROUGH;
05996                     case 1:
05997                         if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_rhs)) != 0))
05998                             kw_args--;
05999                         else {
06000                             __Pyx_RaiseArgtupleInvalid("compare", 1, 2, 2, 1); __PYX_ERR(0, 492, __pyx_L3_error)
06001                         }
06002                     }
06003                 if (unlikely(kw_args > 0)) {

```

```

06004         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
pos_args, "compare") < 0)) __PYX_ERR(0, 492, __pyx_L3_error)
06005     }
06006     } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
06007         goto __pyx_L5_argtuple_error;
06008     } else {
06009         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
06010         values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
06011     }
06012     __pyx_v_lhs = values[0];
06013     __pyx_v_rhs = values[1];
06014 }
06015 goto __pyx_L4_argument_unpacking_done;
06016 __pyx_L5_argtuple_error:;
06017 __Pyx_RaiseArgtupleInvalid("compare", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0,
492, __pyx_L3_error)
06018 __pyx_L3_error:;
06019 __Pyx_AddTraceback("PyClical.compare", __pyx_clineno, __pyx_lineno, __pyx_filename);
06020 __Pyx_RefNannyFinishContext();
06021 return NULL;
06022 __pyx_L4_argument_unpacking_done:;
06023 __pyx_r = __pyx_pf_8PyClical_2compare(__pyx_self, __pyx_v_lhs, __pyx_v_rhs);
06024
06025 /* function exit code */
06026 __Pyx_RefNannyFinishContext();
06027 return __pyx_r;
06028 }
06029
06030 static PyObject *__pyx_pf_8PyClical_2compare(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_lhs, PyObject *__pyx_v_rhs) {
06031     PyObject *__pyx_r = NULL;
06032     __Pyx_RefNannyDeclarations
06033     PyObject *__pyx_t_1 = NULL;
06034     int __pyx_lineno = 0;
06035     const char *__pyx_filename = NULL;
06036     int __pyx_clineno = 0;
06037     __Pyx_RefNannySetupContext("compare", 0);
06038     __Pyx_XDECREF(__pyx_r);
06039     __pyx_t_1 = __pyx_f_8PyClical_compare(__pyx_v_lhs, __pyx_v_rhs, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 492, __pyx_L1_error)
06040     __Pyx_GOTREF(__pyx_t_1);
06041     __pyx_r = __pyx_t_1;
06042     __pyx_t_1 = 0;
06043     goto __pyx_L0;
06044
06045 /* function exit code */
06046 __pyx_L1_error:;
06047 __Pyx_XDECREF(__pyx_t_1);
06048 __Pyx_AddTraceback("PyClical.compare", __pyx_clineno, __pyx_lineno, __pyx_filename);
06049 __pyx_r = NULL;
06050 __pyx_L0:;
06051 __Pyx_XGIVEREF(__pyx_r);
06052 __Pyx_RefNannyFinishContext();
06053 return __pyx_r;
06054 }
06055
06056 /* "PyClical.pyx":504
06057 *     return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
06058 *
06059 * cpdef inline min_neg(obj): # ««««««««
06060 *     """
06061 *     Minimum negative index, or 0 if none.
06062 */
06063
06064 static PyObject *__pyx_pw_8PyClical_5min_neg(PyObject *__pyx_self, PyObject *__pyx_v_obj);
/*proto*/
06065 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_min_neg(PyObject *__pyx_v_obj, CYTHON_UNUSED
int __pyx_skip_dispatch) {
06066     PyObject *__pyx_r = NULL;
06067     __Pyx_RefNannyDeclarations
06068     PyObject *__pyx_t_1 = NULL;
06069     int __pyx_lineno = 0;
06070     const char *__pyx_filename = NULL;
06071     int __pyx_clineno = 0;
06072     __Pyx_RefNannySetupContext("min_neg", 0);
06073
06074 /* "PyClical.pyx":511
06075 *     0
06076 *     """
06077 *     return glucat.min_neg( toIndexSet(obj) ) # ««««««««
06078 *
06079 * cpdef inline max_pos(obj):
06080 */
06081     __Pyx_XDECREF(__pyx_r);
06082     __pyx_t_1 = __Pyx_PyInt_From_int(min_neg(__pyx_f_8PyClical_toIndexSet(__pyx_v_obj))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 511, __pyx_L1_error)
06083     __Pyx_GOTREF(__pyx_t_1);

```

```

06084     __pyx_r = __pyx_t_1;
06085     __pyx_t_1 = 0;
06086     goto __pyx_L0;
06087
06088     /* "PyClical.pyx":504
06089  *      return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
06090  *
06091  * cpdef inline min_neg(obj):          # ««««««««
06092  *      """
06093  *      Minimum negative index, or 0 if none.
06094  */
06095
06096     /* function exit code */
06097     __pyx_L1_error:;
06098     __Pyx_XDECREF(__pyx_t_1);
06099     __Pyx_AddTraceback("PyClical.min_neg", __pyx_clineno, __pyx_lineno, __pyx_filename);
06100     __pyx_r = 0;
06101     __pyx_L0:;
06102     __Pyx_XGIVEREF(__pyx_r);
06103     __Pyx_RefNannyFinishContext();
06104     return __pyx_r;
06105 }
06106
06107     /* Python wrapper */
06108     static PyObject *__pyx_pw_8PyClical_5min_neg(PyObject *__pyx_self, PyObject *__pyx_v_obj);
06109 /*proto*/
06110     static char __pyx_doc_8PyClical_4min_neg[] = "\n      Minimum negative index, or 0 if none.\n\n
06111 >> min_neg(index_set({1,2}))\n      0\n      ";
06112     static PyObject *__pyx_pw_8PyClical_5min_neg(PyObject *__pyx_self, PyObject *__pyx_v_obj) {
06113         PyObject *__pyx_r = 0;
06114         __Pyx_RefNannyDeclarations
06115         __Pyx_RefNannySetupContext("min_neg (wrapper)", 0);
06116         __pyx_r = __pyx_pf_8PyClical_4min_neg(__pyx_self, ((PyObject *)__pyx_v_obj));
06117
06118         /* function exit code */
06119         __Pyx_RefNannyFinishContext();
06120         return __pyx_r;
06121     }
06122
06123     static PyObject *__pyx_pf_8PyClical_4min_neg(CYTHON_UNUSED PyObject *__pyx_self, PyObject
06124 *__pyx_v_obj) {
06125         PyObject *__pyx_r = NULL;
06126         __Pyx_RefNannyDeclarations
06127         PyObject *__pyx_t_1 = NULL;
06128         int __pyx_lineno = 0;
06129         const char *__pyx_filename = NULL;
06130         int __pyx_clineno = 0;
06131         __Pyx_RefNannySetupContext("min_neg", 0);
06132         __Pyx_XDECREF(__pyx_r);
06133         __pyx_t_1 = __pyx_f_8PyClical_min_neg(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
06134         __PYX_ERR(0, 504, __pyx_L1_error)
06135         __Pyx_GOTREF(__pyx_t_1);
06136         __pyx_r = __pyx_t_1;
06137         __pyx_t_1 = 0;
06138         goto __pyx_L0;
06139
06140         /* function exit code */
06141         __pyx_L1_error:;
06142         __Pyx_XDECREF(__pyx_t_1);
06143         __Pyx_AddTraceback("PyClical.min_neg", __pyx_clineno, __pyx_lineno, __pyx_filename);
06144         __pyx_r = NULL;
06145         __pyx_L0:;
06146         __Pyx_XGIVEREF(__pyx_r);
06147         __Pyx_RefNannyFinishContext();
06148         return __pyx_r;
06149     }
06150
06151     /* "PyClical.pyx":513
06152  *      return glucat.min_neg( toIndexSet(obj) )
06153  *
06154  * cpdef inline max_pos(obj):          # ««««««««
06155  *      """
06156  *      Maximum positive index, or 0 if none.
06157  */
06158
06159     static PyObject *__pyx_pw_8PyClical_7max_pos(PyObject *__pyx_self, PyObject *__pyx_v_obj);
06160 /*proto*/
06161     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_max_pos(PyObject *__pyx_v_obj, CYTHON_UNUSED
06162 int __pyx_skip_dispatch) {
06163         PyObject *__pyx_r = NULL;
06164         __Pyx_RefNannyDeclarations
06165         PyObject *__pyx_t_1 = NULL;
06166         int __pyx_lineno = 0;
06167         const char *__pyx_filename = NULL;
06168         int __pyx_clineno = 0;
06169         __Pyx_RefNannySetupContext("max_pos", 0);

```

```

06165         /* "PyClical.pyx":520
06166         *      2
06167         *      """
06168         *      return glucat.max_pos( toIndexSet(obj) )          # ««««««««
06169         *
06170         * cdef inline vector[scalar_t] list_to_vector(lst):
06171         */
06172         __Pyx_XDECREF(__pyx_r);
06173         __pyx_t_1 = __Pyx_PyInt_From_int(max_pos(__pyx_f_8PyClical_toIndexSet(__pyx_v_obj))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 520, __pyx_L1_error)
06174         __Pyx_GOTREF(__pyx_t_1);
06175         __pyx_r = __pyx_t_1;
06176         __pyx_t_1 = 0;
06177         goto __pyx_L0;
06178
06179         /* "PyClical.pyx":513
06180         *      return glucat.min_neg( toIndexSet(obj) )
06181         *
06182         * cpdef inline max_pos(obj):          # ««««««««
06183         *      """
06184         *      Maximum positive index, or 0 if none.
06185         */
06186
06187         /* function exit code */
06188         __pyx_L1_error:;
06189         __Pyx_XDECREF(__pyx_t_1);
06190         __Pyx_AddTraceback("PyClical.max_pos", __pyx_clineno, __pyx_lineno, __pyx_filename);
06191         __pyx_r = 0;
06192         __pyx_L0:;
06193         __Pyx_XGIVEREF(__pyx_r);
06194         __Pyx_RefNannyFinishContext();
06195         return __pyx_r;
06196     }
06197
06198     /* Python wrapper */
06199     static PyObject *__pyx_pw_8PyClical_7max_pos(PyObject *__pyx_self, PyObject *__pyx_v_obj);
/*proto*/
06200     static char __pyx_doc_8PyClical_6max_pos[] = "\n      Maximum positive index, or 0 if none.\n\n
>> max_pos(index_set({1,2}))\n      2\n      ";
06201     static PyObject *__pyx_pw_8PyClical_7max_pos(PyObject *__pyx_self, PyObject *__pyx_v_obj) {
06202         PyObject *__pyx_r = 0;
06203         __Pyx_RefNannyDeclarations
06204         __Pyx_RefNannySetupContext("max_pos (wrapper)", 0);
06205         __pyx_r = __pyx_pf_8PyClical_6max_pos(__pyx_self, ((PyObject *)__pyx_v_obj));
06206
06207         /* function exit code */
06208         __Pyx_RefNannyFinishContext();
06209         return __pyx_r;
06210     }
06211
06212     static PyObject *__pyx_pf_8PyClical_6max_pos(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
06213         PyObject *__pyx_r = NULL;
06214         __Pyx_RefNannyDeclarations
06215         PyObject *__pyx_t_1 = NULL;
06216         int __pyx_lineno = 0;
06217         const char *__pyx_filename = NULL;
06218         int __pyx_clineno = 0;
06219         __Pyx_RefNannySetupContext("max_pos", 0);
06220         __Pyx_XDECREF(__pyx_r);
06221         __pyx_t_1 = __pyx_f_8PyClical_max_pos(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 513, __pyx_L1_error)
06222         __Pyx_GOTREF(__pyx_t_1);
06223         __pyx_r = __pyx_t_1;
06224         __pyx_t_1 = 0;
06225         goto __pyx_L0;
06226
06227         /* function exit code */
06228         __pyx_L1_error:;
06229         __Pyx_XDECREF(__pyx_t_1);
06230         __Pyx_AddTraceback("PyClical.max_pos", __pyx_clineno, __pyx_lineno, __pyx_filename);
06231         __pyx_r = NULL;
06232         __pyx_L0:;
06233         __Pyx_XGIVEREF(__pyx_r);
06234         __Pyx_RefNannyFinishContext();
06235         return __pyx_r;
06236     }
06237
06238     /* "PyClical.pyx":522
06239     *      return glucat.max_pos( toIndexSet(obj) )
06240     *
06241     * cdef inline vector[scalar_t] list_to_vector(lst):          # ««««««««
06242     *      """
06243     *      Create a C++ std::vector[scalar_t] from an iterable Python object.
06244     */
06245
06246     static CYTHON_INLINE std::vector<scalar_t> __pyx_f_8PyClical_list_to_vector(PyObject

```

```

__pyx_v_lst) {
06247     std::vector<scalar_t> __pyx_v_v;
06248     PyObject *__pyx_v_s = NULL;
06249     std::vector<scalar_t> __pyx_r;
06250     __Pyx_RefNannyDeclarations
06251     PyObject *__pyx_t_1 = NULL;
06252     Py_ssize_t __pyx_t_2;
06253     PyObject *(*__pyx_t_3)(PyObject *);
06254     PyObject *__pyx_t_4 = NULL;
06255     scalar_t __pyx_t_5;
06256     int __pyx_lineno = 0;
06257     const char *__pyx_filename = NULL;
06258     int __pyx_clineno = 0;
06259     __Pyx_RefNannySetupContext("list_to_vector", 0);
06260
06261     /* "PyClical.pyx":527
06262     *
06263     * cdef vector[scalar_t] v
06264     * for s in lst:          # ««««««««
06265     *     v.push_back(<scalar_t>s)
06266     * return v
06267     */
06268     if (likely(PyList_CheckExact(__pyx_v_lst)) || PyTuple_CheckExact(__pyx_v_lst)) {
06269         __pyx_t_1 = __pyx_v_lst; __Pyx_INCREF(__pyx_t_1); __pyx_t_2 = 0;
06270         __pyx_t_3 = NULL;
06271     } else {
06272         __pyx_t_2 = -1; __pyx_t_1 = PyObject_GetIter(__pyx_v_lst); if (unlikely(!__pyx_t_1))
06273         __PYX_ERR(0, 527, __pyx_L1_error)
06274         __Pyx_GOTREF(__pyx_t_1);
06275         __pyx_t_3 = Py_TYPE(__pyx_t_1)->tp_iternext; if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 527,
06276         __pyx_L1_error)
06277     }
06278     for (;;) {
06279         if (likely(!__pyx_t_3)) {
06280             if (likely(PyList_CheckExact(__pyx_t_1))) {
06281                 if (__pyx_t_2 >= PyList_GET_SIZE(__pyx_t_1)) break;
06282                 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
06283                 __pyx_t_4 = PyList_GET_ITEM(__pyx_t_1, __pyx_t_2); __Pyx_INCREF(__pyx_t_4);
06284                 __pyx_t_2++; if (unlikely(0 < 0)) __PYX_ERR(0, 527, __pyx_L1_error)
06285                 #else
06286                 __pyx_t_4 = PySequence_ITEM(__pyx_t_1, __pyx_t_2); __pyx_t_2++; if
06287                 (unlikely(!__pyx_t_4)) __PYX_ERR(0, 527, __pyx_L1_error)
06288                 __Pyx_GOTREF(__pyx_t_4);
06289                 #endif
06290             } else {
06291                 if (__pyx_t_2 >= PyTuple_GET_SIZE(__pyx_t_1)) break;
06292                 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
06293                 __pyx_t_4 = PyTuple_GET_ITEM(__pyx_t_1, __pyx_t_2); __Pyx_INCREF(__pyx_t_4);
06294                 __pyx_t_2++; if (unlikely(0 < 0)) __PYX_ERR(0, 527, __pyx_L1_error)
06295                 #else
06296                 __pyx_t_4 = PySequence_ITEM(__pyx_t_1, __pyx_t_2); __pyx_t_2++; if
06297                 (unlikely(!__pyx_t_4)) __PYX_ERR(0, 527, __pyx_L1_error)
06298                 __Pyx_GOTREF(__pyx_t_4);
06299                 #endif
06300             } else {
06301                 __pyx_t_4 = __pyx_t_3(__pyx_t_1);
06302                 if (unlikely(!__pyx_t_4)) {
06303                     PyObject* exc_type = PyErr_Occurred();
06304                     if (exc_type) {
06305                         if (likely(__Pyx_PyErr_GivenExceptionMatches(exc_type, PyExc_StopIteration)))
06306                             PyErr_Clear();
06307                         else __PYX_ERR(0, 527, __pyx_L1_error)
06308                     }
06309                     break;
06310                 }
06311                 __Pyx_GOTREF(__pyx_t_4);
06312             }
06313             __Pyx_XDECREF_SET(__pyx_v_s, __pyx_t_4);
06314             __pyx_t_4 = 0;
06315
06316             /* "PyClical.pyx":528
06317             * cdef vector[scalar_t] v
06318             * for s in lst:
06319             *     v.push_back(<scalar_t>s)          # ««««««««
06320             * return v
06321             */
06322             __pyx_t_5 = __pyx_PyFloat_AsDouble(__pyx_v_s); if (unlikely((__pyx_t_5 == ((scalar_t)-1))
06323             && PyErr_Occurred())) __PYX_ERR(0, 528, __pyx_L1_error)
06324             try {
06325                 __pyx_v_v.push_back(((scalar_t)__pyx_t_5));
06326             } catch (...) {
06327                 __Pyx_CppExn2PyErr();
06328                 __PYX_ERR(0, 528, __pyx_L1_error)
06329             }
06330         }
06331     }
06332 }

```

```

06325         /* "PyClicl.pyx":527
06326         *      """
06327         *      cdef vector[scalar_t] v
06328         *      for s in lst:          # ««««««««
06329         *          v.push_back(<scalar_t>s)
06330         *      return v
06331         */
06332         }
06333         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06334
06335         /* "PyClicl.pyx":529
06336         *      for s in lst:
06337         *          v.push_back(<scalar_t>s)
06338         *      return v          # ««««««««
06339         *
06340         * # Forward reference.
06341         */
06342         __pyx_r = __pyx_v_v;
06343         goto __pyx_L0;
06344
06345         /* "PyClicl.pyx":522
06346         *      return glucat.max_pos( toIndexSet(obj) )
06347         *
06348         * cdef inline vector[scalar_t] list_to_vector(lst):          # ««««««««
06349         *      """
06350         *      Create a C++ std::vector[scalar_t] from an iterable Python object.
06351         */
06352
06353         /* function exit code */
06354         __pyx_L1_error++;
06355         __Pyx_XDECREF(__pyx_t_1);
06356         __Pyx_XDECREF(__pyx_t_4);
06357         __Pyx_WriteUnraisable("PyClicl.list_to_vector", __pyx_clineno, __pyx_lineno,
__pyx_filename, 1, 0);
06358         __Pyx_pretend_to_initialize(&__pyx_r);
06359         __pyx_L0;
06360         __Pyx_XDECREF(__pyx_v_s);
06361         __Pyx_RefNannyFinishContext();
06362         return __pyx_r;
06363     }
06364
06365     /* "PyClicl.pyx":534
06366     * cdef class clifford
06367     *
06368     * cdef inline Clifford toClifford(obj):          # ««««««««
06369     *     return clifford(obj).instance[0]
06370     *
06371     */
06372
06373     static CYTHON_INLINE Clifford __pyx_f_8PyClicl_toClifford(PyObject *__pyx_v_obj) {
06374         Clifford __pyx_r;
06375         __Pyx_RefNannyDeclarations
06376         PyObject *__pyx_t_1 = NULL;
06377         int __pyx_lineno = 0;
06378         const char *__pyx_filename = NULL;
06379         int __pyx_clineno = 0;
06380         __Pyx_RefNannySetupContext("toClifford", 0);
06381
06382         /* "PyClicl.pyx":535
06383         *
06384         * cdef inline Clifford toClifford(obj):
06385         *     return clifford(obj).instance[0]          # ««««««««
06386         *
06387         * cdef class clifford:
06388         */
06389         __pyx_t_1 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClicl_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 535, __pyx_L1_error)
06390         __Pyx_GOTREF(__pyx_t_1);
06391         __pyx_r = (((struct __pyx_obj_8PyClicl_clifford *)__pyx_t_1)->instance[0]);
06392         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06393         goto __pyx_L0;
06394
06395         /* "PyClicl.pyx":534
06396         * cdef class clifford
06397         *
06398         * cdef inline Clifford toClifford(obj):          # ««««««««
06399         *     return clifford(obj).instance[0]
06400         *
06401         */
06402
06403         /* function exit code */
06404         __pyx_L1_error++;
06405         __Pyx_XDECREF(__pyx_t_1);
06406         __Pyx_WriteUnraisable("PyClicl.toClifford", __pyx_clineno, __pyx_lineno, __pyx_filename, 1,
0);
06407         __Pyx_pretend_to_initialize(&__pyx_r);
06408         __pyx_L0;

```



```

06409     __Pyx_RefNannyFinishContext();
06410     return __pyx_r;
06411 }
06412
06413     /* "PyClical.pyx":543
06414     * cdef Clifford *instance # Wrapped instance of C++ class Clifford.
06415     *
06416     * cdef inline wrap(clifford self, Clifford other):                # ««««««««
06417     *     """
06418     *     Wrap an instance of the C++ class Clifford.
06419     */
06420
06421     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_8clifford_wrap(struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, Clifford __pyx_v_other) {
06422     PyObject *__pyx_r = NULL;
06423     __Pyx_RefNannyDeclarations
06424     __Pyx_RefNannySetupContext("wrap", 0);
06425
06426     /* "PyClical.pyx":547
06427     *     Wrap an instance of the C++ class Clifford.
06428     *     """
06429     *     self.instance[0] = other                # ««««««««
06430     *     return self
06431     */
06432     /*
06433     ( __pyx_v_self->instance[0] ) = __pyx_v_other;
06434
06435     /* "PyClical.pyx":548
06436     *     """
06437     *     self.instance[0] = other
06438     *     return self                # ««««««««
06439     */
06440     cdef inline Clifford unwrap(clifford self):
06441     /*
06442     __Pyx_XDECREF(__pyx_r);
06443     __Pyx_INCREF((PyObject *)__pyx_v_self));
06444     __pyx_r = (PyObject *)__pyx_v_self;
06445     goto __pyx_L0;
06446
06447     /* "PyClical.pyx":543
06448     * cdef Clifford *instance # Wrapped instance of C++ class Clifford.
06449     *
06450     * cdef inline wrap(clifford self, Clifford other):                # ««««««««
06451     *     """
06452     *     Wrap an instance of the C++ class Clifford.
06453     */
06454
06455     /* function exit code */
06456     __pyx_L0:;
06457     __Pyx_XGIVEREF(__pyx_r);
06458     __Pyx_RefNannyFinishContext();
06459     return __pyx_r;
06460 }
06461
06462     /* "PyClical.pyx":550
06463     *     return self
06464     */
06465     cdef inline Clifford unwrap(clifford self):                # ««««««««
06466     *     """
06467     *     Return the wrapped C++ Clifford instance.
06468     */
06469
06470     static CYTHON_INLINE Clifford __pyx_f_8PyClical_8clifford_unwrap(struct
__pyx_obj_8PyClical_clifford *__pyx_v_self) {
06471     Clifford __pyx_r;
06472     __Pyx_RefNannyDeclarations
06473     __Pyx_RefNannySetupContext("unwrap", 0);
06474
06475     /* "PyClical.pyx":554
06476     *     Return the wrapped C++ Clifford instance.
06477     *     """
06478     *     return self.instance[0]                # ««««««««
06479     */
06480     cpdef copy(clifford self):
06481     /*
06482     __pyx_r = ( __pyx_v_self->instance[0] );
06483     goto __pyx_L0;
06484
06485     /* "PyClical.pyx":550
06486     *     return self
06487     */
06488     cdef inline Clifford unwrap(clifford self):                # ««««««««
06489     *     """
06490     *     Return the wrapped C++ Clifford instance.
06491     */
06492
06493     /* function exit code */

```

```

06494     __pyx_L0;;
06495     __Pyx_RefNannyFinishContext();
06496     return __pyx_r;
06497 }
06498
06499 /* "PyClical.pyx":556
06500 *     return self.instance[0]
06501 *
06502 * cpdef copy(clifford self):          # ««««««««
06503 *     """
06504 *     Copy this clifford object.
06505 */
06506
06507     static PyObject *__pyx_pw_8PyClical_8clifford_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
06508     static PyObject *__pyx_f_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, int __pyx_skip_dispatch) {
06509         PyObject *__pyx_r = NULL;
06510         __Pyx_RefNannyDeclarations
06511         PyObject *__pyx_t_1 = NULL;
06512         PyObject *__pyx_t_2 = NULL;
06513         PyObject *__pyx_t_3 = NULL;
06514         PyObject *__pyx_t_4 = NULL;
06515         int __pyx_lineno = 0;
06516         const char *__pyx_filename = NULL;
06517         int __pyx_clineno = 0;
06518         __Pyx_RefNannySetupContext("copy", 0);
06519         /* Check if called by wrapper */
06520         if (unlikely(__pyx_skip_dispatch)) ;
06521         /* Check if overridden in Python */
06522         else if (unlikely((Py_TYPE((PyObject *)__pyx_v_self))->tp_dictoffset != 0) ||
(Py_TYPE((PyObject *)__pyx_v_self))->tp_flags & (Py_TPFLAGS_IS_ABSTRACT | Py_TPFLAGS_HEAPTYPE)))) {
06523             #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
06524                 static PY_UINT64_T __pyx_tp_dict_version = __PYX_DICT_VERSION_INIT, __pyx_obj_dict_version
= __PYX_DICT_VERSION_INIT;
06525                 if (unlikely(!__Pyx_object_dict_version_matches((PyObject *)__pyx_v_self),
__pyx_tp_dict_version, __pyx_obj_dict_version)) {
06526                     PY_UINT64_T __pyx_type_dict_guard = __Pyx_get_tp_dict_version((PyObject
*)__pyx_v_self));
06527                     #endif
06528                     __pyx_t_1 = __Pyx_PyObject_GetAttrStr((PyObject *)__pyx_v_self, __pyx_n_s_copy); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 556, __pyx_L1_error)
06529                     __Pyx_GOTREF(__pyx_t_1);
06530                     if (!PyCFunction_Check(__pyx_t_1) || (PyCFunction_GET_FUNCTION(__pyx_t_1) !=
(PyCFunction)(void*)__pyx_pw_8PyClical_8clifford_1copy)) {
06531                         __Pyx_XDECREF(__pyx_r);
06532                         __Pyx_INCREF(__pyx_t_1);
06533                         __pyx_t_3 = __pyx_t_1; __pyx_t_4 = NULL;
06534                         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_3))) {
06535                             __pyx_t_4 = PyMethod_GET_SELF(__pyx_t_3);
06536                             if (likely(__pyx_t_4)) {
06537                                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
06538                                 __Pyx_INCREF(__pyx_t_4);
06539                                 __Pyx_INCREF(function);
06540                                 __Pyx_DECREF_SET(__pyx_t_3, function);
06541                             }
06542                         }
06543                         __pyx_t_2 = (__pyx_t_4) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_4) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
06544                         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
06545                         if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 556, __pyx_L1_error)
06546                         __Pyx_GOTREF(__pyx_t_2);
06547                         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
06548                         __pyx_r = __pyx_t_2;
06549                         __pyx_t_2 = 0;
06550                         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06551                         goto __pyx_L0;
06552                     }
06553                     #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
06554                     __pyx_tp_dict_version = __Pyx_get_tp_dict_version((PyObject *)__pyx_v_self);
06555                     __pyx_obj_dict_version = __Pyx_get_obj_dict_version((PyObject *)__pyx_v_self);
06556                     if (unlikely(__pyx_type_dict_guard != __pyx_tp_dict_version)) {
06557                         __pyx_tp_dict_version = __pyx_obj_dict_version = __PYX_DICT_VERSION_INIT;
06558                     }
06559                     #endif
06560                     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06561                     #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
06562                 }
06563             #endif
06564         }
06565
06566         /* "PyClical.pyx":563
06567         *     {2}
06568         *     """
06569         *     return clifford(self)          # ««««««««
06570         *
06571         * def __cinit__(self, other = 0, ixt = None):

```

```

06572 */
06573     __Pyx_XDECREF(__pyx_r);
06574     __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
((PyObject *)__pyx_v_self)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 563, __pyx_L1_error)
06575     __Pyx_GOTREF(__pyx_t_1);
06576     __pyx_r = __pyx_t_1;
06577     __pyx_t_1 = 0;
06578     goto __pyx_L0;
06579
06580     /* "PyClical.pyx":556
06581     *     return self.instance[0]
06582     *
06583     * cpdef copy(clifford self):          # ««««««««
06584     *     """
06585     *     Copy this clifford object.
06586     */
06587
06588     /* function exit code */
06589     __pyx_L1_error:;
06590     __Pyx_XDECREF(__pyx_t_1);
06591     __Pyx_XDECREF(__pyx_t_2);
06592     __Pyx_XDECREF(__pyx_t_3);
06593     __Pyx_XDECREF(__pyx_t_4);
06594     __Pyx_AddTraceback("PyClical.clifford.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
06595     __pyx_r = 0;
06596     __pyx_L0:;
06597     __Pyx_XGIVEREF(__pyx_r);
06598     __Pyx_RefNannyFinishContext();
06599     return __pyx_r;
06600 }
06601
06602     /* Python wrapper */
06603     static PyObject *__pyx_pw_8PyClical_8clifford_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
06604     static char __pyx_doc_8PyClical_8clifford_copy[] = "\n          Copy this clifford object.\n\n
>> x=clifford(\"1{2}\"); y=x.copy(); print(y)\n          {2}\n          ";
06605     static PyObject *__pyx_pw_8PyClical_8clifford_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
06606         PyObject *__pyx_r = 0;
06607         __Pyx_RefNannyDeclarations
06608         __Pyx_RefNannySetupContext("copy (wrapper)", 0);
06609         __pyx_r = __pyx_pf_8PyClical_8clifford_copy(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
06610
06611         /* function exit code */
06612         __Pyx_RefNannyFinishContext();
06613         return __pyx_r;
06614     }
06615
06616     static PyObject *__pyx_pf_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
06617         PyObject *__pyx_r = NULL;
06618         __Pyx_RefNannyDeclarations
06619         PyObject *__pyx_t_1 = NULL;
06620         int __pyx_lineno = 0;
06621         const char *__pyx_filename = NULL;
06622         int __pyx_clineno = 0;
06623         __Pyx_RefNannySetupContext("copy", 0);
06624         __Pyx_XDECREF(__pyx_r);
06625         __pyx_t_1 = __pyx_f_8PyClical_8clifford_copy(__pyx_v_self, 1); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 556, __pyx_L1_error)
06626         __Pyx_GOTREF(__pyx_t_1);
06627         __pyx_r = __pyx_t_1;
06628         __pyx_t_1 = 0;
06629         goto __pyx_L0;
06630
06631         /* function exit code */
06632         __pyx_L1_error:;
06633         __Pyx_XDECREF(__pyx_t_1);
06634         __Pyx_AddTraceback("PyClical.clifford.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
06635         __pyx_r = NULL;
06636         __pyx_L0:;
06637         __Pyx_XGIVEREF(__pyx_r);
06638         __Pyx_RefNannyFinishContext();
06639         return __pyx_r;
06640     }
06641
06642     /* "PyClical.pyx":565
06643     *     return clifford(self)
06644     *
06645     * def __cinit__(self, other = 0, ixt = None):          # ««««««««
06646     *     """
06647     *     Construct an object of type clifford.
06648     */
06649
06650     /* Python wrapper */
06651     static int __pyx_pw_8PyClical_8clifford_3__cinit__(PyObject *__pyx_v_self, PyObject

```

```

    *__pyx_args, PyObject *__pyx_kwds); /*proto*/
06652     static int __pyx_pw_8PyClical_8clifford_3__cinit__(PyObject *__pyx_v_self, PyObject
    *__pyx_args, PyObject *__pyx_kwds) {
06653         PyObject *__pyx_v_other = 0;
06654         PyObject *__pyx_v_ixt = 0;
06655         int __pyx_lineno = 0;
06656         const char *__pyx_filename = NULL;
06657         int __pyx_clineno = 0;
06658         int __pyx_r;
06659         __Pyx_RefNannyDeclarations
06660         __Pyx_RefNannySetupContext("__cinit__ (wrapper)", 0);
06661         {
06662             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_other,&__pyx_n_s_ixt,0};
06663             PyObject* values[2] = {0,0};
06664             values[0] = ((PyObject *)__pyx_int_0);
06665             values[1] = ((PyObject *)Py_None);
06666             if (unlikely(__pyx_kwds)) {
06667                 Py_ssize_t kw_args;
06668                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
06669                 switch (pos_args) {
06670                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
06671                         CYTHON_FALLTHROUGH;
06672                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
06673                         CYTHON_FALLTHROUGH;
06674                     case 0: break;
06675                     default: goto __pyx_L5_argtuple_error;
06676                 }
06677                 kw_args = PyDict_Size(__pyx_kwds);
06678                 switch (pos_args) {
06679                     case 0:
06680                         if (kw_args > 0) {
06681                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_other);
06682                             if (value) { values[0] = value; kw_args--; }
06683                         }
06684                         CYTHON_FALLTHROUGH;
06685                     case 1:
06686                         if (kw_args > 0) {
06687                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_ixt);
06688                             if (value) { values[1] = value; kw_args--; }
06689                         }
06690                 }
06691                 if (unlikely(kw_args > 0)) {
06692                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
pos_args, "__cinit__") < 0)) __PYX_ERR(0, 565, __pyx_L3_error)
06693                 }
06694                 } else {
06695                     switch (PyTuple_GET_SIZE(__pyx_args)) {
06696                         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
06697                             CYTHON_FALLTHROUGH;
06698                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
06699                             CYTHON_FALLTHROUGH;
06700                         case 0: break;
06701                         default: goto __pyx_L5_argtuple_error;
06702                     }
06703                 }
06704                 __pyx_v_other = values[0];
06705                 __pyx_v_ixt = values[1];
06706             }
06707             goto __pyx_L4_argument_unpacking_done;
06708             __pyx_L5_argtuple_error:;
06709             __Pyx_RaiseArgtupleInvalid("__cinit__", 0, 0, 2, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0,
565, __pyx_L3_error)
06710             __pyx_L3_error:;
06711             __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
06712             __Pyx_RefNannyFinishContext();
06713             return -1;
06714             __pyx_L4_argument_unpacking_done:;
06715             __pyx_r = __pyx_pf_8PyClical_8clifford_2__cinit__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), __pyx_v_other, __pyx_v_ixt);
06716
06717             /* function exit code */
06718             __Pyx_RefNannyFinishContext();
06719             return __pyx_r;
06720         }
06721
06722         static int __pyx_pf_8PyClical_8clifford_2__cinit__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_other, PyObject *__pyx_v_ixt) {
06723             PyObject *__pyx_v_error_msg_prefix = NULL;
06724             PyObject *__pyx_v_bother = NULL;
06725             PyObject *__pyx_v_err = NULL;
06726             int __pyx_r;
06727             __Pyx_RefNannyDeclarations
06728             int __pyx_t_1;
06729             int __pyx_t_2;
06730             PyObject *__pyx_t_3 = NULL;
06731             PyObject *__pyx_t_4 = NULL;

```

```

06732     PyObject *__pyx_t_5 = NULL;
06733     Clifford *__pyx_t_6;
06734     PyObject *__pyx_t_7 = NULL;
06735     PyObject *__pyx_t_8 = NULL;
06736     scalar_t __pyx_t_9;
06737     PyObject *__pyx_t_10 = NULL;
06738     PyObject *__pyx_t_11 = NULL;
06739     PyObject *__pyx_t_12 = NULL;
06740     PyObject *__pyx_t_13 = NULL;
06741     char *__pyx_t_14;
06742     int __pyx_t_15;
06743     PyObject *__pyx_t_16 = NULL;
06744     PyObject *__pyx_t_17 = NULL;
06745     PyObject *__pyx_t_18 = NULL;
06746     int __pyx_t_19;
06747     char const *__pyx_t_20;
06748     PyObject *__pyx_t_21 = NULL;
06749     PyObject *__pyx_t_22 = NULL;
06750     PyObject *__pyx_t_23 = NULL;
06751     int __pyx_lineno = 0;
06752     const char *__pyx_filename = NULL;
06753     int __pyx_clineno = 0;
06754     __Pyx_RefNannySetupContext("__cinit__", 0);
06755
06756     /* "PyClical.pyx":588
06757     * 2{1}+3{2}
06758     * """
06759     *     error_msg_prefix = "Cannot initialize clifford object from"          # ««««««««
06760     *     if ixt is None:
06761     *         try:
06762     */
06763     __Pyx_INCREF(__pyx_kp_u_Cannot_initialize_clifford_objec);
06764     __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_initialize_clifford_objec;
06765
06766     /* "PyClical.pyx":589
06767     * """
06768     *     error_msg_prefix = "Cannot initialize clifford object from"
06769     *     if ixt is None:          # ««««««««
06770     *         try:
06771     *             if isinstance(other, clifford):
06772     */
06773     __pyx_t_1 = (__pyx_v_ixt == Py_None);
06774     __pyx_t_2 = (__pyx_t_1 != 0);
06775     if (__pyx_t_2) {
06776
06777         /* "PyClical.pyx":590
06778         *     error_msg_prefix = "Cannot initialize clifford object from"
06779         *     if ixt is None:
06780         *         try:          # ««««««««
06781         *             if isinstance(other, clifford):
06782         *                 self.instance = new Clifford((<clifford>other).unwrap())
06783         */
06784         {
06785             __Pyx_PyThreadState_declare
06786             __Pyx_PyThreadState_assign
06787             __Pyx_ExceptionSave(&__pyx_t_3, &__pyx_t_4, &__pyx_t_5);
06788             __Pyx_XGOTREF(__pyx_t_3);
06789             __Pyx_XGOTREF(__pyx_t_4);
06790             __Pyx_XGOTREF(__pyx_t_5);
06791             /*try:*/ {
06792
06793                 /* "PyClical.pyx":591
06794                 *     if ixt is None:
06795                 *         try:
06796                 *             if isinstance(other, clifford):          # ««««««««
06797                 *                 self.instance = new Clifford((<clifford>other).unwrap())
06798                 *             elif isinstance(other, index_set):
06799                 */
06800                 __pyx_t_2 = __Pyx_TypeCheck(__pyx_v_other, __pyx_ptype_8PyClical_clifford);
06801                 __pyx_t_1 = (__pyx_t_2 != 0);
06802                 if (__pyx_t_1) {
06803
06804                     /* "PyClical.pyx":592
06805                     *     try:
06806                     *         if isinstance(other, clifford):
06807                     *             self.instance = new Clifford((<clifford>other).unwrap())          # ««««««««
06808                     *         elif isinstance(other, index_set):
06809                     *             self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06810                     */
06811                     try {
06812                         __pyx_t_6 = new Clifford(__pyx_f_8PyClical_8clifford_unwrap(((struct
06813                         __pyx_obj_8PyClical_clifford *)__pyx_v_other)));
06814                         } catch(...) {
06815                             __Pyx_CppExn2PyErr();
06816                             __PYX_ERR(0, 592, __pyx_L4_error)
06817                         }
06818                         __pyx_v_self->instance = __pyx_t_6;

```

```

06818
06819      /* "PyClical.pyx":591
06820      *      if ixt is None:
06821      *          try:
06822      *              if isinstance(other, clifford):          # ««««««««
06823      *                  self.instance = new Clifford((<clifford>other).unwrap())
06824      *                  elif isinstance(other, index_set):
06825      */
06826      *          goto __pyx_L10;
06827      *      }
06828
06829      /* "PyClical.pyx":593
06830      *      if isinstance(other, clifford):
06831      *          self.instance = new Clifford((<clifford>other).unwrap())
06832      *          elif isinstance(other, index_set):          # ««««««««
06833      *              self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06834      *          elif isinstance(other, numbers.Real):
06835      */
06836      __pyx_t_1 = __Pyx_TypeCheck(__pyx_v_other, __pyx_ptype_8PyClical_index_set);
06837      __pyx_t_2 = (__pyx_t_1 != 0);
06838      if (__pyx_t_2) {
06839
06840          /* "PyClical.pyx":594
06841          *          self.instance = new Clifford((<clifford>other).unwrap())
06842          *          elif isinstance(other, index_set):
06843          *              self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06844          *          # ««««««««
06845          *          elif isinstance(other, numbers.Real):
06846          *              self.instance = new Clifford(<scalar_t>other)
06847          */
06848          try {
06849              __pyx_t_6 = new Clifford(__pyx_f_8PyClical_9index_set_unwrap(((struct
06850      __pyx_obj_8PyClical_index_set *)__pyx_v_other)), ((scalar_t)1.0));
06851          } catch(...) {
06852              __Pyx_CppExn2PyErr();
06853              __PYX_ERR(0, 594, __pyx_L4_error)
06854          }
06855          __pyx_v_self->instance = __pyx_t_6;
06856
06857          /* "PyClical.pyx":593
06858          *      if isinstance(other, clifford):
06859          *          self.instance = new Clifford((<clifford>other).unwrap())
06860          *          elif isinstance(other, index_set):          # ««««««««
06861          *              self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06862          *          elif isinstance(other, numbers.Real):
06863          */
06864          goto __pyx_L10;
06865      }
06866
06867      /* "PyClical.pyx":595
06868      *      elif isinstance(other, index_set):
06869      *          self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06870      *      elif isinstance(other, numbers.Real):          # ««««««««
06871      *          self.instance = new Clifford(<scalar_t>other)
06872      *      elif isinstance(other, str):
06873      */
06874      __Pyx_GetModuleGlobalName(__pyx_t_7, __pyx_n_s_numbers); if (unlikely(!__pyx_t_7))
06875      __PYX_ERR(0, 595, __pyx_L4_error)
06876      __Pyx_GOTREF(__pyx_t_7);
06877      __pyx_t_8 = __Pyx_PyObject_GetAttrStr(__pyx_t_7, __pyx_n_s_Real); if
06878      (unlikely(!__pyx_t_8)) __PYX_ERR(0, 595, __pyx_L4_error)
06879      __Pyx_GOTREF(__pyx_t_8);
06880      __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
06881      __pyx_t_2 = PyObject_IsInstance(__pyx_v_other, __pyx_t_8); if (unlikely(__pyx_t_2 ==
06882      ((int)-1))) __PYX_ERR(0, 595, __pyx_L4_error)
06883      __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
06884      __pyx_t_1 = (__pyx_t_2 != 0);
06885      if (__pyx_t_1) {
06886
06887          /* "PyClical.pyx":596
06888          *          self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06889          *          elif isinstance(other, numbers.Real):
06890          *              self.instance = new Clifford(<scalar_t>other)          # ««««««««
06891          *          elif isinstance(other, str):
06892          *              try:
06893          */
06894          __pyx_t_9 = __pyx_PyFloat_AsDouble(__pyx_v_other); if (unlikely((__pyx_t_9 ==
06895      ((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 596, __pyx_L4_error)
06896          try {
06897              __pyx_t_6 = new Clifford(((scalar_t)__pyx_t_9));
06898          } catch(...) {
06899              __Pyx_CppExn2PyErr();
06900              __PYX_ERR(0, 596, __pyx_L4_error)
06901          }
06902          __pyx_v_self->instance = __pyx_t_6;
06903
06904          /* "PyClical.pyx":595

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06899 *         elif isinstance(other, index_set):
06900 *             self.instance = new Clifford(<index_set>other).unwrap(), <scalar_t>1.0)
06901 *         elif isinstance(other, numbers.Real):
06902 *             self.instance = new Clifford(<scalar_t>other)
06903 *         elif isinstance(other, str):
06904 */
06905         goto __pyx_L10;
06906     }
06907
06908     /* "PyClicl.pyx":597
06909 *         elif isinstance(other, numbers.Real):
06910 *             self.instance = new Clifford(<scalar_t>other)
06911 *         elif isinstance(other, str):
06912 *             try:
06913 *                 bother = other.encode("UTF-8")
06914 */
06915     __pyx_t_1 = PyUnicode_Check(__pyx_v_other);
06916     __pyx_t_2 = (__pyx_t_1 != 0);
06917     if (likely(__pyx_t_2)) {
06918
06919         /* "PyClicl.pyx":598
06920 *             self.instance = new Clifford(<scalar_t>other)
06921 *         elif isinstance(other, str):
06922 *             try:
06923 *                 bother = other.encode("UTF-8")
06924 *             self.instance = new Clifford(<char *>bother)
06925 */
06926         {
06927             __Pyx_PyThreadState_declare
06928             __Pyx_PyThreadState_assign
06929             __Pyx_ExceptionSave(&__pyx_t_10, &__pyx_t_11, &__pyx_t_12);
06930             __Pyx_XGOTREF(__pyx_t_10);
06931             __Pyx_XGOTREF(__pyx_t_11);
06932             __Pyx_XGOTREF(__pyx_t_12);
06933             /*try:*/ {
06934
06935                 /* "PyClicl.pyx":599
06936 *                 elif isinstance(other, str):
06937 *                 try:
06938 *                     bother = other.encode("UTF-8")
06939 *                     self.instance = new Clifford(<char *>bother)
06940 *                 except RuntimeError:
06941 */
06942                 __pyx_t_7 = __Pyx_PyObject_GetAttrStr(__pyx_v_other, __pyx_n_s_encode); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 599, __pyx_L11_error)
06943                 __Pyx_GOTREF(__pyx_t_7);
06944                 __pyx_t_13 = NULL;
06945                 if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_7))) {
06946                     __pyx_t_13 = PyMethod_GET_SELF(__pyx_t_7);
06947                     if (likely(__pyx_t_13)) {
06948                         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_7);
06949                         __Pyx_INCREF(__pyx_t_13);
06950                         __Pyx_INCREF(function);
06951                         __Pyx_DECREF_SET(__pyx_t_7, function);
06952                     }
06953                 }
06954                 __pyx_t_8 = (__pyx_t_13) ? __Pyx_PyObject_Call2Args(__pyx_t_7, __pyx_t_13,
__pyx_kp_u_UTF_8) : __Pyx_PyObject_CallOneArg(__pyx_t_7, __pyx_kp_u_UTF_8);
06955                 __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
06956                 if (unlikely(!__pyx_t_8)) __PYX_ERR(0, 599, __pyx_L11_error)
06957                 __Pyx_GOTREF(__pyx_t_8);
06958                 __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
06959                 __pyx_v_bother = __pyx_t_8;
06960                 __pyx_t_8 = 0;
06961
06962                 /* "PyClicl.pyx":600
06963 *                 try:
06964 *                     bother = other.encode("UTF-8")
06965 *                     self.instance = new Clifford(<char *>bother)
06966 *                 except RuntimeError:
06967 *                     raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
06968 */
06969                 __pyx_t_14 = __Pyx_PyObject_AsWritableString(__pyx_v_bother); if
(unlikely(!__pyx_t_14) && PyErr_Occurred()) __PYX_ERR(0, 600, __pyx_L11_error)
06970                 try {
06971                     __pyx_t_6 = new Clifford(((char *)__pyx_t_14));
06972                 } catch (...) {
06973                     __Pyx_CppExn2PyErr();
06974                     __PYX_ERR(0, 600, __pyx_L11_error)
06975                 }
06976                 __pyx_v_self->instance = __pyx_t_6;
06977
06978                 /* "PyClicl.pyx":598
06979 *                 self.instance = new Clifford(<scalar_t>other)
06980 *             elif isinstance(other, str):
06981 *                 try:
06982 *                     bother = other.encode("UTF-8")

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06983 *                 self.instance = new Clifford(<char *>bother)
06984 */
06985     }
06986     __Pyx_XDECREF(__pyx_t_10); __pyx_t_10 = 0;
06987     __Pyx_XDECREF(__pyx_t_11); __pyx_t_11 = 0;
06988     __Pyx_XDECREF(__pyx_t_12); __pyx_t_12 = 0;
06989     goto __pyx_L16_try_end;
06990     __pyx_L11_error:;
06991     __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
06992     __Pyx_XDECREF(__pyx_t_7); __pyx_t_7 = 0;
06993     __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
06994
06995     /* "PyClical.pyx":601
06996 *         bother = other.encode("UTF-8")
06997 *         self.instance = new Clifford(<char *>bother)
06998 *         except RuntimeError:
06999 *             # ««««««
07000 *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
07001 */
07002     else:
07003         __pyx_t_15 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
07004         if (__pyx_t_15) {
07005             __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
07006 __pyx_filename);
07007             if (__Pyx_GetException(&__pyx_t_8, &__pyx_t_7, &__pyx_t_13) < 0) __PYX_ERR(0,
07008 601, __pyx_L13_except_error)
07009             __Pyx_GOTREF(__pyx_t_8);
07010             __Pyx_GOTREF(__pyx_t_7);
07011             __Pyx_GOTREF(__pyx_t_13);
07012
07013             /* "PyClical.pyx":602
07014 *                 self.instance = new Clifford(<char *>bother)
07015 *                 except RuntimeError:
07016 *                     raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
07017 *             # ««««««
07018 *             else:
07019 *                 raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07020 */
07021             __pyx_t_16 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix,
07022 __pyx_kp_u_invalid_string); if (unlikely(!__pyx_t_16)) __PYX_ERR(0, 602, __pyx_L13_except_error)
07023             __Pyx_GOTREF(__pyx_t_16);
07024             __pyx_t_17 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_17))
07025 __PYX_ERR(0, 602, __pyx_L13_except_error)
07026             __Pyx_GOTREF(__pyx_t_17);
07027             __pyx_t_18 = PyNumber_Add(__pyx_t_16, __pyx_t_17); if (unlikely(!__pyx_t_18))
07028 __PYX_ERR(0, 602, __pyx_L13_except_error)
07029             __Pyx_GOTREF(__pyx_t_18);
07030             __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07031             __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07032             __pyx_t_17 = PyNumber_Add(__pyx_t_18, __pyx_kp_u_); if (unlikely(!__pyx_t_17))
07033 __PYX_ERR(0, 602, __pyx_L13_except_error)
07034             __Pyx_GOTREF(__pyx_t_17);
07035             __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07036             __pyx_t_18 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_17); if
07037 (unlikely(!__pyx_t_18)) __PYX_ERR(0, 602, __pyx_L13_except_error)
07038             __Pyx_GOTREF(__pyx_t_18);
07039             __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07040             Pyx_Raise(__pyx_t_18, 0, 0, 0);
07041             __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07042             __PYX_ERR(0, 602, __pyx_L13_except_error)
07043         }
07044         goto __pyx_L13_except_error;
07045         __pyx_L13_except_error:;
07046
07047         /* "PyClical.pyx":598
07048 *         self.instance = new Clifford(<scalar_t>other)
07049 *         elif isinstance(other, str):
07050 *             try:
07051 *                 # ««««««
07052 *                 bother = other.encode("UTF-8")
07053 *                 self.instance = new Clifford(<char *>bother)
07054 */
07055         __Pyx_XGIVeref(__pyx_t_10);
07056         __Pyx_XGIVeref(__pyx_t_11);
07057         __Pyx_XGIVeref(__pyx_t_12);
07058         __Pyx_ExceptionReset(__pyx_t_10, __pyx_t_11, __pyx_t_12);
07059         goto __pyx_L4_error;
07060         __pyx_L16_try_end:;
07061     }
07062
07063     /* "PyClical.pyx":597
07064 *         elif isinstance(other, numbers.Real):
07065 *             self.instance = new Clifford(<scalar_t>other)
07066 *         elif isinstance(other, str):
07067 *             # ««««««
07068 *             try:
07069 *                 bother = other.encode("UTF-8")
07070 */
07071     goto __pyx_L10;
07072 }

```



```

07062
07063         /* "PyClical.pyx":604
07064         *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
07065         *             else:
07066         *                 raise TypeError(error_msg_prefix + " " + str(type(other)) + ".") #
07067         *             ««««««
07068         *             except RuntimeError as err:
07069         *                 raise ValueError(error_msg_prefix + " " + str(type(other))
07070         */
07071         /*else*/ {
07072             __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_2); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 604, __pyx_L4_error)
07072             __Pyx_GOTREF(__pyx_t_13);
07073             __pyx_t_7 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 604, __pyx_L4_error)
07074             __Pyx_GOTREF(__pyx_t_7);
07075             __pyx_t_8 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_t_7); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 604, __pyx_L4_error)
07076             __Pyx_GOTREF(__pyx_t_8);
07077             __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07078             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07079             __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_t_8, __pyx_kp_u_); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 604, __pyx_L4_error)
07080             __Pyx_GOTREF(__pyx_t_7);
07081             __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07082             __pyx_t_8 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_7); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 604, __pyx_L4_error)
07083             __Pyx_GOTREF(__pyx_t_8);
07084             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07085             __Pyx_Raise(__pyx_t_8, 0, 0, 0);
07086             __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07087             __PYX_ERR(0, 604, __pyx_L4_error)
07088         }
07089         __pyx_L10:;
07090
07091         /* "PyClical.pyx":590
07092         * error_msg_prefix = "Cannot initialize clifford object from"
07093         * if ixt is None:
07094         *     try: # ««««««
07095         *         if isinstance(other, clifford):
07096         *             self.instance = new Clifford(<clifford>other).unwrap()
07097         */
07098         }
07099         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
07100         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
07101         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
07102         goto __pyx_L9_try_end;
07103         __pyx_L4_error:;
07104         __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
07105         __Pyx_XDECREF(__pyx_t_16); __pyx_t_16 = 0;
07106         __Pyx_XDECREF(__pyx_t_17); __pyx_t_17 = 0;
07107         __Pyx_XDECREF(__pyx_t_18); __pyx_t_18 = 0;
07108         __Pyx_XDECREF(__pyx_t_7); __pyx_t_7 = 0;
07109         __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
07110
07111         /* "PyClical.pyx":605
07112         *             else:
07113         *                 raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07114         *             except RuntimeError as err: # ««««««
07115         *                 raise ValueError(error_msg_prefix + " " + str(type(other))
07116         *                     + " value " + repr(other) + ":",
07117         *                     + "\n\t" + str(err))
07118         */
07118         __pyx_t_15 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
07119         if (__pyx_t_15) {
07120             __Pyx_AddTraceback("PyClical.Clifford.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07121             if (__Pyx_GetException(&__pyx_t_8, &__pyx_t_7, &__pyx_t_13) < 0) __PYX_ERR(0, 605,
__pyx_L6_except_error)
07122             __Pyx_GOTREF(__pyx_t_8);
07123             __Pyx_GOTREF(__pyx_t_7);
07124             __Pyx_GOTREF(__pyx_t_13);
07125             __Pyx_INCREF(__pyx_t_7);
07126             __pyx_v_err = __pyx_t_7;
07127             /*try:*/ {
07128
07129             /* "PyClical.pyx":606
07130             *                 raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07131             *             except RuntimeError as err:
07132             *                 raise ValueError(error_msg_prefix + " " + str(type(other)) # ««««««
07133             *                     + " value " + repr(other) + ":",
07134             *                     + "\n\t" + str(err))
07135             */
07136             __pyx_t_18 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_2); if
(unlikely(!__pyx_t_18)) __PYX_ERR(0, 606, __pyx_L24_error)
07137             __Pyx_GOTREF(__pyx_t_18);
07138             __pyx_t_17 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other)); if (unlikely(!__pyx_t_17)) __PYX_ERR(0, 606, __pyx_L24_error)

```



```

07214         if ((PY_MAJOR_VERSION < 3) || unlikely(__Pyx_GetException(&__pyx_t_12,
&__pyx_t_11, &__pyx_t_10) < 0)) __Pyx_ErrFetch(&__pyx_t_12, &__pyx_t_11, &__pyx_t_10);
07215         __Pyx_XGOTREF(__pyx_t_12);
07216         __Pyx_XGOTREF(__pyx_t_11);
07217         __Pyx_XGOTREF(__pyx_t_10);
07218         __Pyx_XGOTREF(__pyx_t_21);
07219         __Pyx_XGOTREF(__pyx_t_22);
07220         __Pyx_XGOTREF(__pyx_t_23);
07221         __pyx_t_15 = __pyx_lineno; __pyx_t_19 = __pyx_clineno; __pyx_t_20 =
__pyx_filename;
07222         {
07223             __Pyx_DECREF(__pyx_v_err);
07224             __pyx_v_err = NULL;
07225         }
07226         if (PY_MAJOR_VERSION >= 3) {
07227             __Pyx_XGIVEREF(__pyx_t_21);
07228             __Pyx_XGIVEREF(__pyx_t_22);
07229             __Pyx_XGIVEREF(__pyx_t_23);
07230             __Pyx_ExceptionReset(__pyx_t_21, __pyx_t_22, __pyx_t_23);
07231         }
07232         __Pyx_XGIVEREF(__pyx_t_12);
07233         __Pyx_XGIVEREF(__pyx_t_11);
07234         __Pyx_XGIVEREF(__pyx_t_10);
07235         __Pyx_ErrRestore(__pyx_t_12, __pyx_t_11, __pyx_t_10);
07236         __pyx_t_12 = 0; __pyx_t_11 = 0; __pyx_t_10 = 0; __pyx_t_21 = 0; __pyx_t_22 = 0;
__pyx_t_23 = 0;
07237         __pyx_lineno = __pyx_t_15; __pyx_clineno = __pyx_t_19; __pyx_filename =
__pyx_t_20;
07238         goto __pyx_L6_except_error;
07239     }
07240 }
07241 }
07242 goto __pyx_L6_except_error;
07243 __pyx_L6_except_error;;
07244
07245 /* "PyClical.pyx":590
07246 * error_msg_prefix = "Cannot initialize clifford object from"
07247 * if ixt is None:
07248 *     try:
07249 *         if isinstance(other, clifford):
07250 *             self.instance = new Clifford((<clifford>other).unwrap())
07251 */
07252     __Pyx_XGIVEREF(__pyx_t_3);
07253     __Pyx_XGIVEREF(__pyx_t_4);
07254     __Pyx_XGIVEREF(__pyx_t_5);
07255     __Pyx_ExceptionReset(__pyx_t_3, __pyx_t_4, __pyx_t_5);
07256     goto __pyx_L1_error;
07257     __pyx_L9_try_end;;
07258 }
07259
07260 /* "PyClical.pyx":589
07261 * """
07262 * error_msg_prefix = "Cannot initialize clifford object from"
07263 * if ixt is None:
07264 *     try:
07265 *         if isinstance(other, clifford):
07266 */
07267     goto __pyx_L3;
07268 }
07269
07270 /* "PyClical.pyx":609
07271 *
07272 *         + " value " + repr(other) + ":"
07273 *         + "\n\t" + str(err))
07274 *     elif isinstance(ixt, index_set):
07275 *         if isinstance(other, numbers.Real):
07276 *             self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07277 */
07278 __pyx_t_2 = __Pyx_TypeCheck(__pyx_v_ixt, __pyx_ptype_8PyClical_index_set);
07279 __pyx_t_1 = (__pyx_t_2 != 0);
07280 if (likely(__pyx_t_1)) {
07281     /* "PyClical.pyx":610
07282 *
07283 *         + "\n\t" + str(err))
07284 *     elif isinstance(ixt, index_set):
07285 *         if isinstance(other, numbers.Real):
07286 *             self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07287 *         elif isinstance(other, collections.abc.Sequence):
07288 */
07289     __Pyx_GetModuleGlobalName(__pyx_t_13, __pyx_n_s_numbers); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 610, __pyx_L1_error)
07290     __Pyx_GOTREF(__pyx_t_13);
07291     __pyx_t_7 = __Pyx_PyObject_GetAttrStr(__pyx_t_13, __pyx_n_s_Real); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 610, __pyx_L1_error)
07292     __Pyx_GOTREF(__pyx_t_7);
07293     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07294     __pyx_t_1 = PyObject_IsInstance(__pyx_v_other, __pyx_t_7); if (unlikely(__pyx_t_1 ==
((int)-1))) __PYX_ERR(0, 610, __pyx_L1_error)

```

```

07294     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07295     __pyx_t_2 = (__pyx_t_1 != 0);
07296     if (__pyx_t_2) {
07297
07298         /* "PyClical.pyx":611
07299  *         elif isinstance(ixt, index_set):
07300  *             if isinstance(other, numbers.Real):
07301  *                 self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07302  *
07303  *             elif isinstance(other, collections.abc.Sequence):
07304  *                 self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07305  */
07306         __pyx_t_9 = __pyx_PyFloat_AsDouble(__pyx_v_other); if (unlikely((__pyx_t_9 ==
07307 ((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 611, __pyx_L1_error)
07308         try {
07309             __pyx_t_6 = new Clifford(__pyx_f_8PyClical_9index_set_unwrap(((struct
07310 __pyx_obj_8PyClical_index_set *)__pyx_v_ixt)), ((scalar_t) __pyx_t_9));
07311         } catch(...) {
07312             __Pyx_CppExn2PyErr();
07313             __PYX_ERR(0, 611, __pyx_L1_error)
07314         }
07315         __pyx_v_self->instance = __pyx_t_6;
07316
07317         /* "PyClical.pyx":610
07318  *
07319  *         elif isinstance(ixt, index_set):
07320  *             if isinstance(other, numbers.Real):
07321  *                 self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07322  *             elif isinstance(other, collections.abc.Sequence):
07323  *                 self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07324  *             else:
07325  *                 goto __pyx_L30;
07326  */
07327         goto __pyx_L30;
07328     }
07329
07330     /* "PyClical.pyx":612
07331  *         if isinstance(other, numbers.Real):
07332  *             self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07333  *         elif isinstance(other, collections.abc.Sequence):
07334  *             self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07335  *         else:
07336  *             __Pyx_GetModuleGlobalName(__pyx_t_7, __pyx_n_s_collections); if (unlikely(!__pyx_t_7))
07337  *             __PYX_ERR(0, 612, __pyx_L1_error)
07338  *             __Pyx_GOTREF(__pyx_t_7);
07339  *             __pyx_t_13 = __Pyx_PyObject_GetAttrStr(__pyx_t_7, __pyx_n_s_abc); if
07340  *             (unlikely(!__pyx_t_13)) __PYX_ERR(0, 612, __pyx_L1_error)
07341  *             __Pyx_GOTREF(__pyx_t_13);
07342  *             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07343  *             __pyx_t_7 = __Pyx_PyObject_GetAttrStr(__pyx_t_13, __pyx_n_s_Sequence); if
07344  *             (unlikely(!__pyx_t_7)) __PYX_ERR(0, 612, __pyx_L1_error)
07345  *             __Pyx_GOTREF(__pyx_t_7);
07346  *             __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07347  *             __pyx_t_2 = PyObject_IsInstance(__pyx_v_other, __pyx_t_7); if (unlikely(__pyx_t_2 ==
07348  *             ((int)-1))) __PYX_ERR(0, 612, __pyx_L1_error)
07349  *             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07350  *             __pyx_t_1 = (__pyx_t_2 != 0);
07351  *             if (likely(__pyx_t_1)) {
07352  *
07353  *                 /* "PyClical.pyx":613
07354  *                 *                 self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07355  *                 *                 elif isinstance(other, collections.abc.Sequence):
07356  *                 *                 self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07357  *                 *
07358  *                 *                 else:
07359  *                 *                 raise TypeError(error_msg_prefix + " (" + str(type(other))
07360  *                 */
07361  *                 try {
07362  *                     __pyx_t_6 = new Clifford(__pyx_f_8PyClical_list_to_vector(__pyx_v_other),
07363  *                     __pyx_f_8PyClical_9index_set_unwrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_v_ixt)));
07364  *                 } catch(...) {
07365  *                     __Pyx_CppExn2PyErr();
07366  *                     __PYX_ERR(0, 613, __pyx_L1_error)
07367  *                 }
07368  *                 __pyx_v_self->instance = __pyx_t_6;
07369  *
07370  *                 /* "PyClical.pyx":612
07371  *                 *                 if isinstance(other, numbers.Real):
07372  *                 *                 self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07373  *                 *                 elif isinstance(other, collections.abc.Sequence):
07374  *                 *                 self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07375  *                 *                 else:
07376  *                 *                 goto __pyx_L30;
07377  *                 */
07378  *                 goto __pyx_L30;
07379  *             }
07380
07381     /* "PyClical.pyx":615
07382  *         self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07383  *         else:

```

```

07372 *             raise TypeError(error_msg_prefix + " (" + str(type(other))          # ««««««««
07373 *                 + ", " + repr(ixt) + ").")
07374 *         else:
07375 */
07376         /*else*/ {
07377             __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_7); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 615, __pyx_L1_error)
07378             __Pyx_GOTREF(__pyx_t_7);
07379             __pyx_t_13 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other)); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 615, __pyx_L1_error)
07380             __Pyx_GOTREF(__pyx_t_13);
07381             __pyx_t_8 = __Pyx_PyUnicode_Concat(__pyx_t_7, __pyx_t_13); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 615, __pyx_L1_error)
07382             __Pyx_GOTREF(__pyx_t_8);
07383             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07384             __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07385
07386             /* "PyClical.pyx":616
07387 *             else:
07388 *                 raise TypeError(error_msg_prefix + " (" + str(type(other))          # ««««««««
07389 *                     + ", " + repr(ixt) + ").")
07390 *             else:
07391 *                 raise TypeError(error_msg_prefix + " (" + str(type(other))
07392 */
07393             __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_8, __pyx_kp_u_8); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 616, __pyx_L1_error)
07394             __Pyx_GOTREF(__pyx_t_13);
07395             __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07396             __pyx_t_8 = PyObject_Repr(__pyx_v_ixt); if (unlikely(!__pyx_t_8)) __PYX_ERR(0, 616,
__pyx_L1_error)
07397             __Pyx_GOTREF(__pyx_t_8);
07398             __pyx_t_7 = PyNumber_Add(__pyx_t_13, __pyx_t_8); if (unlikely(!__pyx_t_7)) __PYX_ERR(0,
616, __pyx_L1_error)
07399             __Pyx_GOTREF(__pyx_t_7);
07400             __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07401             __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07402             __pyx_t_8 = PyNumber_Add(__pyx_t_7, __pyx_kp_u_9); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 616, __pyx_L1_error)
07403             __Pyx_GOTREF(__pyx_t_8);
07404             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07405
07406             /* "PyClical.pyx":615
07407 *             self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07408 *             else:
07409 *                 raise TypeError(error_msg_prefix + " (" + str(type(other))          # ««««««««
07410 *                     + ", " + repr(ixt) + ").")
07411 *             else:
07412 */
07413             __pyx_t_7 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_8); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 615, __pyx_L1_error)
07414             __Pyx_GOTREF(__pyx_t_7);
07415             __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07416             __Pyx_Raise(__pyx_t_7, 0, 0, 0);
07417             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07418             __PYX_ERR(0, 615, __pyx_L1_error)
07419         }
07420         __pyx_L30::
07421
07422         /* "PyClical.pyx":609
07423 *                                     + " value " + repr(other) + ":"
07424 *                                     + "\n\t" + str(err)
07425 *             elif isinstance(ixt, index_set):          # ««««««««
07426 *                 if isinstance(other, numbers.Real):
07427 *                     self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07428 */
07429             goto __pyx_L3;
07430         }
07431
07432         /* "PyClical.pyx":618
07433 *                                     + ", " + repr(ixt) + ").")
07434 *             else:
07435 *                 raise TypeError(error_msg_prefix + " (" + str(type(other))          # ««««««««
07436 *                     + ", " + str(type(ixt)) + ").")
07437 *             */
07438         /*else*/ {
07439             __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_7); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 618, __pyx_L1_error)
07440             __Pyx_GOTREF(__pyx_t_7);
07441             __pyx_t_8 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other)); if (unlikely(!__pyx_t_8)) __PYX_ERR(0, 618, __pyx_L1_error)
07442             __Pyx_GOTREF(__pyx_t_8);
07443             __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_7, __pyx_t_8); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 618, __pyx_L1_error)
07444             __Pyx_GOTREF(__pyx_t_13);
07445             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07446             __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;

```

```

07448
07449     /* "PyClical.pyx":619
07450     *
07451     *         raise TypeError(error_msg_prefix + " (" + str(type(other))
07452     *             + ", " + str(type(ixt)) + ").") # ««««««««
07453     *
07454     *     def __dealloc__(self):
07455     */
07456         __pyx_t_8 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_kp_u_8); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 619, __pyx_L1_error)
07457         __Pyx_GOTREF(__pyx_t_8);
07458         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07459         __pyx_t_13 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_ixt))); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 619, __pyx_L1_error)
07460         __Pyx_GOTREF(__pyx_t_13);
07461         __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_t_8, __pyx_t_13); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 619, __pyx_L1_error)
07462         __Pyx_GOTREF(__pyx_t_7);
07463         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07464         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07465         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_7, __pyx_kp_u_9); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 619, __pyx_L1_error)
07466         __Pyx_GOTREF(__pyx_t_13);
07467         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07468
07469         /* "PyClical.pyx":618
07470         *
07471         *         + ", " + repr(ixt) + ").")
07472         *
07473         *         raise TypeError(error_msg_prefix + " (" + str(type(other)) # ««««««««
07474         *             + ", " + str(type(ixt)) + ").")
07475         *
07476         */
07477         __pyx_t_7 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_13); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 618, __pyx_L1_error)
07478         __Pyx_GOTREF(__pyx_t_7);
07479         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07480         __Pyx_Raise(__pyx_t_7, 0, 0, 0);
07481         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07482         __PYX_ERR(0, 618, __pyx_L1_error)
07483     }
07484     __pyx_L3:;
07485
07486     /* "PyClical.pyx":565
07487     *
07488     *     return clifford(self)
07489     *
07490     *     def __cinit__(self, other = 0, ixt = None): # ««««««««
07491     *         """
07492     *         Construct an object of type clifford.
07493     */
07494
07495     /* function exit code */
07496     __pyx_r = 0;
07497     goto __pyx_L0;
07498     __pyx_L1_error:;
07499     __Pyx_XDECREF(__pyx_t_7);
07500     __Pyx_XDECREF(__pyx_t_8);
07501     __Pyx_XDECREF(__pyx_t_13);
07502     __Pyx_XDECREF(__pyx_t_16);
07503     __Pyx_XDECREF(__pyx_t_17);
07504     __Pyx_XDECREF(__pyx_t_18);
07505     __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07506     __pyx_r = -1;
07507     __pyx_L0:;
07508     __Pyx_XDECREF(__pyx_v_error_msg_prefix);
07509     __Pyx_XDECREF(__pyx_v_bother);
07510     __Pyx_XDECREF(__pyx_v_err);
07511     __Pyx_RefNannyFinishContext();
07512     return __pyx_r;
07513 }
07514
07515     /* "PyClical.pyx":621
07516     *
07517     *         + ", " + str(type(ixt)) + ").")
07518     *
07519     *     def __dealloc__(self): # ««««««««
07520     *         """
07521     *         Clean up by deallocating the instance of C++ class Clifford.
07522     */
07523
07524     /* Python wrapper */
07525     static void __pyx_pw_8PyClical_8clifford_5__dealloc__(PyObject *__pyx_v_self); /*proto*/
07526     static void __pyx_pw_8PyClical_8clifford_5__dealloc__(PyObject *__pyx_v_self) {
07527         __Pyx_RefNannyDeclarations
07528         __Pyx_RefNannySetupContext("__dealloc__ (wrapper)", 0);
07529         __pyx_pf_8PyClical_8clifford_4__dealloc__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
07527

```

```

7528         /* function exit code */
7529         __Pyx_RefNannyFinishContext();
7530     }
7531
7532     static void __pyx_pf_8PyClical_8clifford_4__dealloc__(struct __pyx_obj_8PyClical_clifford
7533 *__pyx_v_self) {
7534         __Pyx_RefNannyDeclarations
7535         __Pyx_RefNannySetupContext("__dealloc__", 0);
7536
7537         /* "PyClical.pyx":625
7538         *
7539         *     del self.instance          # ««««««««
7540         *
7541         * def __contains__(self, x):
7542         */
7543         delete __pyx_v_self->instance;
7544
7545         /* "PyClical.pyx":621
7546         *
7547         *
7548         * def __dealloc__(self):          # ««««««««
7549         *
7550         *     Clean up by deallocating the instance of C++ class Clifford.
7551         */
7552
7553         /* function exit code */
7554         __Pyx_RefNannyFinishContext();
7555     }
7556
7557     /* "PyClical.pyx":627
7558     *
7559     *
7560     * def __contains__(self, x):          # ««««««««
7561     *
7562     *     Not applicable.
7563     */
7564
7565     /* Python wrapper */
7566     static int __pyx_pw_8PyClical_8clifford_7__contains__(PyObject *__pyx_v_self, PyObject
7567 *__pyx_v_x); /*proto*/
7568     static char __pyx_doc_8PyClical_8clifford_6__contains__[] = "\n        Not applicable.\n\n
7569 >> x=clifford(index_set({-3,4,7})); -3 in x\n        Traceback (most recent call last):\n
7570 ... \n        TypeError: Not applicable.\n        ";
7571     #if CYTHON_COMPILING_IN_CPYTHON
7572     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_6__contains__;
7573     #endif
7574     static int __pyx_pw_8PyClical_8clifford_7__contains__(PyObject *__pyx_v_self, PyObject
7575 *__pyx_v_x) {
7576         int __pyx_r;
7577         __Pyx_RefNannyDeclarations
7578         __Pyx_RefNannySetupContext("__contains__ (wrapper)", 0);
7579         __pyx_r = __pyx_pf_8PyClical_8clifford_6__contains__(((struct __pyx_obj_8PyClical_clifford
7580 *)__pyx_v_self), ((PyObject *)__pyx_v_x));
7581
7582         /* function exit code */
7583         __Pyx_RefNannyFinishContext();
7584         return __pyx_r;
7585     }
7586
7587     static int __pyx_pf_8PyClical_8clifford_6__contains__(CYTHON_UNUSED struct
7588 __pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v_x) {
7589         int __pyx_r;
7590         __Pyx_RefNannyDeclarations
7591         PyObject *__pyx_t_1 = NULL;
7592         int __pyx_lineno = 0;
7593         const char *__pyx_filename = NULL;
7594         int __pyx_clineno = 0;
7595         __Pyx_RefNannySetupContext("__contains__", 0);
7596
7597         /* "PyClical.pyx":636
7598         *
7599         *     TypeError: Not applicable.
7600         *
7601         *     raise TypeError("Not applicable.")          # ««««««««
7602         *
7603         * def __iter__(self):
7604         */
7605         __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__10, NULL); if
7606 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 636, __pyx_L1_error)
7607         __Pyx_GOTREF(__pyx_t_1);
7608         __Pyx_Raise(__pyx_t_1, 0, 0, 0);
7609         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
7610         __PYX_ERR(0, 636, __pyx_L1_error)
7611
7612         /* "PyClical.pyx":627
7613         *
7614         *     del self.instance
7615         */

```

```

07607 *      def __contains__(self, x):          # ««««««««
07608 *          """
07609 *              Not applicable.
07610 */
07611
07612         /* function exit code */
07613         __pyx_L1_error++;
07614         __Pyx_XDECREF(__pyx_t_1);
07615         __Pyx_AddTraceback("PyClical.clifford.__contains__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07616         __pyx_r = -1;
07617         __Pyx_RefNannyFinishContext();
07618         return __pyx_r;
07619     }
07620
07621     /* "PyClical.pyx":638
07622 *         raise TypeError("Not applicable.")
07623 *
07624 *      def __iter__(self):                # ««««««««
07625 *          """
07626 *              Not applicable.
07627 */
07628
07629     /* Python wrapper */
07630     static PyObject *__pyx_pw_8PyClical_8clifford_9__iter__(PyObject *__pyx_v_self); /*proto*/
07631     static char __pyx_doc_8PyClical_8clifford_8__iter__[] = "\n        Not applicable.\n\n
>> for a in clifford(index_set({-3,4,7})):print(a, end=\",\\n\")\n        Traceback (most recent call
last):\n          ...\\n          TypeError: Not applicable.\n          ";
07632     #if CYTHON_COMPILING_IN_CPYTHON
07633     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_8__iter__;
07634     #endif
07635     static PyObject *__pyx_pw_8PyClical_8clifford_9__iter__(PyObject *__pyx_v_self) {
07636         PyObject *__pyx_r = 0;
07637         __Pyx_RefNannyDeclarations
07638         __Pyx_RefNannySetupContext("__iter__ (wrapper)", 0);
07639         __pyx_r = __pyx_pf_8PyClical_8clifford_8__iter__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
07640
07641         /* function exit code */
07642         __Pyx_RefNannyFinishContext();
07643         return __pyx_r;
07644     }
07645
07646     static PyObject *__pyx_pf_8PyClical_8clifford_8__iter__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self) {
07647         PyObject *__pyx_r = NULL;
07648         __Pyx_RefNannyDeclarations
07649         PyObject *__pyx_t_1 = NULL;
07650         int __pyx_lineno = 0;
07651         const char *__pyx_filename = NULL;
07652         int __pyx_clineno = 0;
07653         __Pyx_RefNannySetupContext("__iter__", 0);
07654
07655         /* "PyClical.pyx":647
07656 *         TypeError: Not applicable.
07657 *         """
07658 *         raise TypeError("Not applicable.")          # ««««««««
07659 *
07660 *      def reframe(self, ixt):
07661 */
07662         __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__10, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 647, __pyx_L1_error)
07663         __Pyx_GOTREF(__pyx_t_1);
07664         __Pyx_Raise(__pyx_t_1, 0, 0, 0);
07665         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
07666         __PYX_ERR(0, 647, __pyx_L1_error)
07667
07668         /* "PyClical.pyx":638
07669 *         raise TypeError("Not applicable.")
07670 *
07671 *      def __iter__(self):                # ««««««««
07672 *          """
07673 *              Not applicable.
07674 */
07675
07676         /* function exit code */
07677         __pyx_L1_error++;
07678         __Pyx_XDECREF(__pyx_t_1);
07679         __Pyx_AddTraceback("PyClical.clifford.__iter__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07680         __pyx_r = NULL;
07681         __Pyx_XGIVEREF(__pyx_r);
07682         __Pyx_RefNannyFinishContext();
07683         return __pyx_r;
07684     }
07685
07686     /* "PyClical.pyx":649

```



```

07687 *         raise TypeError("Not applicable.")
07688 *
07689 *     def reframe(self, ixt):                                # ««««««««
07690 *         """
07691 *         Put self into a larger frame, containing the union of self.frame() and index set ixt.
07692 */
07693
07694     /* Python wrapper */
07695     static PyObject *__pyx_pw_8PyClical_8clifford_11reframe(PyObject *__pyx_v_self, PyObject
07696 *__pyx_v_ixt); /*proto*/
07697     static char __pyx_doc_8PyClical_8clifford_10reframe[] = "\n        Put self into a larger
07698 frame, containing the union of self.frame() and index set ixt.\n        This can be used to make
07699 multiplication faster, by multiplying within a common frame.\n\n        >>
07700 clifford(\"2+3{1}\").reframe(index_set({1,2,3}))\n        clifford(\"2+3{1}\")\n        >>
07701 s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() == (s|t);\n
07702 True\n";
07703     static PyObject *__pyx_pw_8PyClical_8clifford_11reframe(PyObject *__pyx_v_self, PyObject
07704 *__pyx_v_ixt) {
07705     PyObject *__pyx_r = 0;
07706     __Pyx_RefNannyDeclarations
07707     __Pyx_RefNannySetupContext("reframe (wrapper)", 0);
07708     __pyx_r = __pyx_pf_8PyClical_8clifford_10reframe(((struct __pyx_obj_8PyClical_clifford
07709 *)__pyx_v_self), ((PyObject *)__pyx_v_ixt));
07710
07711     /* function exit code */
07712     __Pyx_RefNannyFinishContext();
07713     return __pyx_r;
07714 }
07715
07716     static PyObject *__pyx_pf_8PyClical_8clifford_10reframe(struct __pyx_obj_8PyClical_clifford
07717 *__pyx_v_self, PyObject *__pyx_v_ixt) {
07718     PyObject *__pyx_v_error_msg_prefix = NULL;
07719     struct __pyx_obj_8PyClical_clifford *__pyx_v_result = NULL;
07720     PyObject *__pyx_v_err = NULL;
07721     PyObject *__pyx_r = NULL;
07722     __Pyx_RefNannyDeclarations
07723     int __pyx_t_1;
07724     int __pyx_t_2;
07725     PyObject *__pyx_t_3 = NULL;
07726     PyObject *__pyx_t_4 = NULL;
07727     PyObject *__pyx_t_5 = NULL;
07728     PyObject *__pyx_t_6 = NULL;
07729     Clifford *__pyx_t_7;
07730     int __pyx_t_8;
07731     PyObject *__pyx_t_9 = NULL;
07732     PyObject *__pyx_t_10 = NULL;
07733     PyObject *__pyx_t_11 = NULL;
07734     PyObject *__pyx_t_12 = NULL;
07735     PyObject *__pyx_t_13 = NULL;
07736     int __pyx_t_14;
07737     char const *__pyx_t_15;
07738     PyObject *__pyx_t_16 = NULL;
07739     PyObject *__pyx_t_17 = NULL;
07740     PyObject *__pyx_t_18 = NULL;
07741     PyObject *__pyx_t_19 = NULL;
07742     PyObject *__pyx_t_20 = NULL;
07743     PyObject *__pyx_t_21 = NULL;
07744     int __pyx_lineno = 0;
07745     const char *__pyx_filename = NULL;
07746     int __pyx_clineno = 0;
07747     __Pyx_RefNannySetupContext("reframe", 0);
07748
07749     /* "PyClical.pyx":659
07750 *     True
07751 *     """
07752 *     error_msg_prefix = "Cannot reframe"                                # ««««««««
07753 *     if isinstance(ixt, index_set):
07754 *         try:
07755 */
07756     __Pyx_INCREF(__pyx_kp_u_Cannot_reframe);
07757     __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_reframe;
07758
07759     /* "PyClical.pyx":660
07760 *     """
07761 *     error_msg_prefix = "Cannot reframe"
07762 *     if isinstance(ixt, index_set):                                # ««««««««
07763 *         try:
07764 *             result = clifford()
07765 */
07766     __pyx_t_1 = __Pyx_TypeCheck(__pyx_v_ixt, __pyx_ptype_8PyClical_index_set);
07767     __pyx_t_2 = (__pyx_t_1 != 0);
07768     if (likely(__pyx_t_2)) {
07769
07770         /* "PyClical.pyx":661
07771 *         error_msg_prefix = "Cannot reframe"
07772 *         if isinstance(ixt, index_set):
07773 *             try:
07774 *                 # ««««««««

```

```

07765 *             result = clifford()
07766 *             result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07767 */
07768 {
07769     __Pyx_PyThreadState_declare
07770     __Pyx_PyThreadState_assign
07771     __Pyx_ExceptionSave(&__pyx_t_3, &__pyx_t_4, &__pyx_t_5);
07772     __Pyx_XGOTREF(__pyx_t_3);
07773     __Pyx_XGOTREF(__pyx_t_4);
07774     __Pyx_XGOTREF(__pyx_t_5);
07775     /*try:*/ {
07776
07777         /* "PyClicl.pyx":662
07778 *         if isinstance(ixt, index_set):
07779 *             try:
07780 *                 result = clifford()
07781 *                 result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07782 *             except RuntimeError as err:
07783 */
07784         __pyx_t_6 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 662, __pyx_L4_error)
07785         __Pyx_GOTREF(__pyx_t_6);
07786         __pyx_v_result = ((struct __pyx_obj_8PyClicl_clifford *)__pyx_t_6);
07787         __pyx_t_6 = 0;
07788
07789         /* "PyClicl.pyx":663
07790 *         try:
07791 *             result = clifford()
07792 *             result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07793 # ««««««««
07794 *             except RuntimeError as err:
07795 *                 raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07796 */
07797         try {
07798             __pyx_t_7 = new Clifford(__pyx_f_8PyClicl_8clifford_unwrap(__pyx_v_self),
__pyx_f_8PyClicl_9index_set_unwrap(((struct __pyx_obj_8PyClicl_index_set *)__pyx_v_ixt)));
07799         } catch (...) {
07800             __Pyx_CppExn2PyErr();
07801             __PYX_ERR(0, 663, __pyx_L4_error)
07802         }
07803         __pyx_v_result->instance = __pyx_t_7;
07804
07805         /* "PyClicl.pyx":661
07806 *         error_msg_prefix = "Cannot reframe"
07807 *         if isinstance(ixt, index_set):
07808 *             try:
07809 *                 # ««««««««
07810 *                 result = clifford()
07811 *                 result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07812 */
07813         }
07814         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
07815         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
07816         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
07817         goto __pyx_L9_try_end;
07818         __pyx_L4_error:;
07819         __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
07820
07821         /* "PyClicl.pyx":664
07822 *         result = clifford()
07823 *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07824 *         except RuntimeError as err:
07825 *             # ««««««««
07826 *             raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07827 *                             + str(ixt) + ":",
07828 */
07829         __pyx_t_8 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
07830         if (__pyx_t_8) {
07831             __Pyx_AddTraceback("PyClicl.clifford.reframe", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07832             if (__Pyx_GetException(&__pyx_t_6, &__pyx_t_9, &__pyx_t_10) < 0) __PYX_ERR(0, 664,
__pyx_L6_except_error)
07833             __Pyx_GOTREF(__pyx_t_6);
07834             __Pyx_GOTREF(__pyx_t_9);
07835             __Pyx_GOTREF(__pyx_t_10);
07836             __Pyx_INCREF(__pyx_t_9);
07837             __pyx_v_err = __pyx_t_9;
07838             /*try:*/ {
07839
07840                 /* "PyClicl.pyx":665
07841 *                 result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07842 *                 except RuntimeError as err:
07843 *                     raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07844 # ««««««««
07845 *                                     + str(ixt) + ":"
07846 *                                     + "\n\t" + str(err))
07847 */
07848             __pyx_t_11 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_from); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 665, __pyx_L15_error)

```

```

07845         __Pyx_GOTREF(__pyx_t_11);
07846         __pyx_t_12 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)), ((PyObject
*)__pyx_v_self)); if (unlikely(!__pyx_t_12)) __PYX_ERR(0, 665, __pyx_L15_error)
07847         __Pyx_GOTREF(__pyx_t_12);
07848         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_11, __pyx_t_12); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 665, __pyx_L15_error)
07849         __Pyx_GOTREF(__pyx_t_13);
07850         __Pyx_DECREF(__pyx_t_11); __pyx_t_11 = 0;
07851         __Pyx_DECREF(__pyx_t_12); __pyx_t_12 = 0;
07852         __pyx_t_12 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_kp_u_to_frame); if
(unlikely(!__pyx_t_12)) __PYX_ERR(0, 665, __pyx_L15_error)
07853         __Pyx_GOTREF(__pyx_t_12);
07854         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07855
07856         /* "PyClical.pyx":666
07857         *         except RuntimeError as err:
07858         *             raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07859         *                 + str(ixt) + ":"
07860         *                 + "\n\t" + str(err))
07861         *         else:
07862         */
07863         __pyx_t_13 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)),
__pyx_v_ixt); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 666, __pyx_L15_error)
07864         __Pyx_GOTREF(__pyx_t_13);
07865         __pyx_t_11 = __Pyx_PyUnicode_Concat(__pyx_t_12, __pyx_t_13); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 666, __pyx_L15_error)
07866         __Pyx_GOTREF(__pyx_t_11);
07867         __Pyx_DECREF(__pyx_t_12); __pyx_t_12 = 0;
07868         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07869         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_11, __pyx_kp_u_5); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 666, __pyx_L15_error)
07870         __Pyx_GOTREF(__pyx_t_13);
07871         __Pyx_DECREF(__pyx_t_11); __pyx_t_11 = 0;
07872
07873         /* "PyClical.pyx":667
07874         *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07875         *             + str(ixt) + ":"
07876         *             + "\n\t" + str(err))
07877         *         else:
07878         *             raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
07879         */
07880         __pyx_t_11 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_kp_u_6); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 667, __pyx_L15_error)
07881         __Pyx_GOTREF(__pyx_t_11);
07882         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07883         __pyx_t_13 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)),
__pyx_v_err); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 667, __pyx_L15_error)
07884         __Pyx_GOTREF(__pyx_t_13);
07885         __pyx_t_12 = __Pyx_PyUnicode_Concat(__pyx_t_11, __pyx_t_13); if
(unlikely(!__pyx_t_12)) __PYX_ERR(0, 667, __pyx_L15_error)
07886         __Pyx_GOTREF(__pyx_t_12);
07887         __Pyx_DECREF(__pyx_t_11); __pyx_t_11 = 0;
07888         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07889
07890         /* "PyClical.pyx":665
07891         *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07892         *         except RuntimeError as err:
07893         *             raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07894         *                 + str(ixt) + ":"
07895         *                 + "\n\t" + str(err))
07896         */
07897         __pyx_t_13 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_12); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 665, __pyx_L15_error)
07898         __Pyx_GOTREF(__pyx_t_13);
07899         __Pyx_DECREF(__pyx_t_12); __pyx_t_12 = 0;
07900         __Pyx_Raise(__pyx_t_13, 0, 0, 0);
07901         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07902         __PYX_ERR(0, 665, __pyx_L15_error)
07903     }
07904
07905     /* "PyClical.pyx":664
07906     *     result = clifford()
07907     *     result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07908     *     except RuntimeError as err:
07909     *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07910     *             + str(ixt) + ":"
07911     */
07912     /*finally:*/ {
07913         __pyx_L15_error;
07914         /*exception exit:*/{
07915             __Pyx_PyThreadState_declare
07916             __Pyx_PyThreadState_assign
07917             __pyx_t_16 = 0; __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0;
__pyx_t_21 = 0;
07918             __Pyx_XDECREF(__pyx_t_11); __pyx_t_11 = 0;

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07919         __Pyx_XDECREF(__pyx_t_12); __pyx_t_12 = 0;
07920         __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
07921         if (PY_MAJOR_VERSION >= 3) __Pyx_ExceptionSwap(&__pyx_t_19, &__pyx_t_20,
&__pyx_t_21);
07922         if ((PY_MAJOR_VERSION < 3) || unlikely(__Pyx_GetException(&__pyx_t_16,
&__pyx_t_17, &__pyx_t_18) < 0)) __Pyx_ErrFetch(&__pyx_t_16, &__pyx_t_17, &__pyx_t_18);
07923         __Pyx_XGOTREF(__pyx_t_16);
07924         __Pyx_XGOTREF(__pyx_t_17);
07925         __Pyx_XGOTREF(__pyx_t_18);
07926         __Pyx_XGOTREF(__pyx_t_19);
07927         __Pyx_XGOTREF(__pyx_t_20);
07928         __Pyx_XGOTREF(__pyx_t_21);
07929         __pyx_t_8 = __pyx_lineno; __pyx_t_14 = __pyx_clineno; __pyx_t_15 = __pyx_filename;
07930         {
07931             __Pyx_DECREF(__pyx_v_err);
07932             __pyx_v_err = NULL;
07933         }
07934         if (PY_MAJOR_VERSION >= 3) {
07935             __Pyx_XGIVEREF(__pyx_t_19);
07936             __Pyx_XGIVEREF(__pyx_t_20);
07937             __Pyx_XGIVEREF(__pyx_t_21);
07938             __Pyx_ExceptionReset(__pyx_t_19, __pyx_t_20, __pyx_t_21);
07939         }
07940         __Pyx_XGIVEREF(__pyx_t_16);
07941         __Pyx_XGIVEREF(__pyx_t_17);
07942         __Pyx_XGIVEREF(__pyx_t_18);
07943         __Pyx_ErrRestore(__pyx_t_16, __pyx_t_17, __pyx_t_18);
07944         __pyx_t_16 = 0; __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0;
__pyx_t_21 = 0;
07945         __pyx_lineno = __pyx_t_8; __pyx_clineno = __pyx_t_14; __pyx_filename = __pyx_t_15;
07946         goto __pyx_L6_except_error;
07947     }
07948 }
07949 }
07950 goto __pyx_L6_except_error;
07951 __pyx_L6_except_error;
07952
07953 /* "PyClical.pyx":661
07954 * error_msg_prefix = "Cannot reframe"
07955 * if isinstance(ixt, index_set):
07956 *     try: # ««««««««
07957 *         result = clifford()
07958 *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07959 */
07960         __Pyx_XGIVEREF(__pyx_t_3);
07961         __Pyx_XGIVEREF(__pyx_t_4);
07962         __Pyx_XGIVEREF(__pyx_t_5);
07963         __Pyx_ExceptionReset(__pyx_t_3, __pyx_t_4, __pyx_t_5);
07964         goto __pyx_L1_error;
07965         __pyx_L9_try_end;
07966     }
07967
07968 /* "PyClical.pyx":660
07969 * """
07970 * error_msg_prefix = "Cannot reframe"
07971 * if isinstance(ixt, index_set): # ««««««««
07972 *     try:
07973 *         result = clifford()
07974 */
07975         goto __pyx_L3;
07976     }
07977
07978 /* "PyClical.pyx":669
07979 *
07980 *         + "\n\t" + str(err))
07981 *         raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").") #
««««««««
07982 *         return result
07983 *
07984 */
07985 /*else*/ {
07986     __pyx_t_10 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_using); if
(unlikely(!__pyx_t_10)) __PYX_ERR(0, 669, __pyx_L1_error)
07987     __Pyx_GOTREF(__pyx_t_10);
07988     __pyx_t_9 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), ((PyObject
*)Py_TYPE(__pyx_v_ixt))); if (unlikely(!__pyx_t_9)) __PYX_ERR(0, 669, __pyx_L1_error)
07989     __Pyx_GOTREF(__pyx_t_9);
07990     __pyx_t_6 = __Pyx_PyUnicode_Concat(__pyx_t_10, __pyx_t_9); if (unlikely(!__pyx_t_6))
__PYX_ERR(0, 669, __pyx_L1_error)
07991     __Pyx_GOTREF(__pyx_t_6);
07992     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
07993     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
07994     __pyx_t_9 = __Pyx_PyUnicode_Concat(__pyx_t_6, __pyx_kp_u_9); if (unlikely(!__pyx_t_9))
__PYX_ERR(0, 669, __pyx_L1_error)
07995     __Pyx_GOTREF(__pyx_t_9);
07996     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
07997     __pyx_t_6 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_9); if

```

```

(unlikely(!__pyx_t_6)) __PYX_ERR(0, 669, __pyx_L1_error)
07998     __Pyx_GOTREF(__pyx_t_6);
07999     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
08000     __Pyx_Raise(__pyx_t_6, 0, 0, 0);
08001     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08002     __PYX_ERR(0, 669, __pyx_L1_error)
08003 }
08004 __pyx_L3:;
08005
08006 /* "PyClical.pyx":670
08007 *     else:
08008 *         raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
08009 *         return result # ««««««««
08010 *
08011 * def __richcmp__(lhs, rhs, int op):
08012 */
08013     __Pyx_XDECREF(__pyx_r);
08014     __Pyx_INCREF((PyObject *)__pyx_v_result);
08015     __pyx_r = (PyObject *)__pyx_v_result;
08016     goto __pyx_L0;
08017
08018 /* "PyClical.pyx":649
08019 *     raise TypeError("Not applicable.")
08020 *
08021 * def reframe(self, ixt): # ««««««««
08022 *     """
08023 *     Put self into a larger frame, containing the union of self.frame() and index set ixt.
08024 */
08025
08026 /* function exit code */
08027     __pyx_L1_error:;
08028     __Pyx_XDECREF(__pyx_t_6);
08029     __Pyx_XDECREF(__pyx_t_9);
08030     __Pyx_XDECREF(__pyx_t_10);
08031     __Pyx_XDECREF(__pyx_t_11);
08032     __Pyx_XDECREF(__pyx_t_12);
08033     __Pyx_XDECREF(__pyx_t_13);
08034     __Pyx_AddTraceback("PyClical.clifford.reframe", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08035     __pyx_r = NULL;
08036     __pyx_L0:;
08037     __Pyx_XDECREF(__pyx_v_error_msg_prefix);
08038     __Pyx_XDECREF((PyObject *)__pyx_v_result);
08039     __Pyx_XDECREF(__pyx_v_err);
08040     __Pyx_XGIVEREF(__pyx_r);
08041     __Pyx_RefNannyFinishContext();
08042     return __pyx_r;
08043 }
08044
08045 /* "PyClical.pyx":672
08046 *     return result
08047 *
08048 * def __richcmp__(lhs, rhs, int op): # ««««««««
08049 *     """
08050 *     Compare objects of type clifford.
08051 */
08052
08053 /* Python wrapper */
08054 static PyObject *__pyx_pw_8PyClical_8clifford_13__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op); /*proto*/
08055 static PyObject *__pyx_pw_8PyClical_8clifford_13__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op) {
08056     PyObject *__pyx_r = 0;
08057     __Pyx_RefNannyDeclarations
08058     __Pyx_RefNannySetupContext("__richcmp__ (wrapper)", 0);
08059     __pyx_r = __pyx_pf_8PyClical_8clifford_12__richcmp__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_lhs), ((PyObject *)__pyx_v_rhs), ((int)__pyx_v_op));
08060
08061 /* function exit code */
08062     __Pyx_RefNannyFinishContext();
08063     return __pyx_r;
08064 }
08065
08066 static PyObject *__pyx_pf_8PyClical_8clifford_12__richcmp__(struct
__pyx_obj_8PyClical_clifford *__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op) {
08067     PyObject *__pyx_r = NULL;
08068     __Pyx_RefNannyDeclarations
08069     int __pyx_t_1;
08070     int __pyx_t_2;
08071     int __pyx_t_3;
08072     PyObject *__pyx_t_4 = NULL;
08073     PyObject *__pyx_t_5 = NULL;
08074     PyObject *__pyx_t_6 = NULL;
08075     int __pyx_lineno = 0;
08076     const char *__pyx_filename = NULL;
08077     int __pyx_clineno = 0;
08078     __Pyx_RefNannySetupContext("__richcmp__", 0);

```

```

08079
08080     /* "PyCliclcal.pyx":691
08081     *     True
08082     *     """
08083     *     if op == 2: # ==                # ««««««««
08084     *         if (lhs is None) or (rhs is None):
08085     *             return bool(lhs is rhs)
08086     */
08087     __pyx_t_1 = ((__pyx_v_op == 2) != 0);
08088     if (__pyx_t_1) {
08089
08090         /* "PyCliclcal.pyx":692
08091         *     """
08092         *     if op == 2: # ==
08093         *         if (lhs is None) or (rhs is None):                # ««««««««
08094         *             return bool(lhs is rhs)
08095         *         else:
08096         */
08097         __pyx_t_2 = (((PyObject *) __pyx_v_lhs) == Py_None);
08098         __pyx_t_3 = (__pyx_t_2 != 0);
08099         if (!__pyx_t_3) {
08100         } else {
08101             __pyx_t_1 = __pyx_t_3;
08102             goto __pyx_L5_bool_binop_done;
08103         }
08104         __pyx_t_3 = (__pyx_v_rhs == Py_None);
08105         __pyx_t_2 = (__pyx_t_3 != 0);
08106         __pyx_t_1 = __pyx_t_2;
08107         __pyx_L5_bool_binop_done;
08108         if (__pyx_t_1) {
08109
08110             /* "PyCliclcal.pyx":693
08111             *     if op == 2: # ==
08112             *         if (lhs is None) or (rhs is None):
08113             *             return bool(lhs is rhs)                # ««««««««
08114             *         else:
08115             *             return bool( toClifford(lhs) == toClifford(rhs) )
08116             */
08117             __Pyx_XDECREF(__pyx_r);
08118             __pyx_t_1 = (((PyObject *) __pyx_v_lhs) == __pyx_v_rhs);
08119             __pyx_t_4 = __Pyx_PyBool_FromLong((!(__pyx_t_1))); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 693, __pyx_L1_error)
08120             __Pyx_GOTREF(__pyx_t_4);
08121             __pyx_r = __pyx_t_4;
08122             __pyx_t_4 = 0;
08123             goto __pyx_L0;
08124
08125             /* "PyCliclcal.pyx":692
08126             *     """
08127             *     if op == 2: # ==
08128             *         if (lhs is None) or (rhs is None):                # ««««««««
08129             *             return bool(lhs is rhs)
08130             *         else:
08131             */
08132             }
08133
08134             /* "PyCliclcal.pyx":695
08135             *         return bool(lhs is rhs)
08136             *         else:
08137             *             return bool( toClifford(lhs) == toClifford(rhs) )                # ««««««««
08138             *     elif op == 3: # !=
08139             *         if (lhs is None) or (rhs is None):
08140             */
08141             /*else*/ {
08142                 __Pyx_XDECREF(__pyx_r);
08143                 __pyx_t_1 = (__pyx_f_8PyCliclcal_toClifford(((PyObject *) __pyx_v_lhs)) ==
__pyx_f_8PyCliclcal_toClifford(__pyx_v_rhs));
08144                 __pyx_t_4 = __Pyx_PyBool_FromLong((!(__pyx_t_1))); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 695, __pyx_L1_error)
08145                 __Pyx_GOTREF(__pyx_t_4);
08146                 __pyx_r = __pyx_t_4;
08147                 __pyx_t_4 = 0;
08148                 goto __pyx_L0;
08149             }
08150
08151             /* "PyCliclcal.pyx":691
08152             *     True
08153             *     """
08154             *     if op == 2: # ==                # ««««««««
08155             *         if (lhs is None) or (rhs is None):
08156             *             return bool(lhs is rhs)
08157             */
08158             }
08159
08160             /* "PyCliclcal.pyx":696
08161             *         else:
08162             *             return bool( toClifford(lhs) == toClifford(rhs) )

```

```

08163 *         elif op == 3: # !=             # ««««««««
08164 *             if (lhs is None) or (rhs is None):
08165 *                 return not bool(lhs is rhs)
08166 */
08167 __pyx_t_1 = (__pyx_v_op == 3) != 0;
08168 if (__pyx_t_1) {
08169     /* "PyClical.pyx":697
08170     return bool( toClifford(lhs) == toClifford(rhs) )
08171 *
08172 *     elif op == 3: # !=
08173 *         if (lhs is None) or (rhs is None):             # ««««««««
08174 *             return not bool(lhs is rhs)
08175 *         else:
08176 */
08177     __pyx_t_2 = (((PyObject *)__pyx_v_lhs) == Py_None);
08178     __pyx_t_3 = (__pyx_t_2 != 0);
08179     if (!__pyx_t_3) {
08180     } else {
08181         __pyx_t_1 = __pyx_t_3;
08182         goto __pyx_L8_bool_binop_done;
08183     }
08184     __pyx_t_3 = (__pyx_v_rhs == Py_None);
08185     __pyx_t_2 = (__pyx_t_3 != 0);
08186     __pyx_t_1 = __pyx_t_2;
08187     __pyx_L8_bool_binop_done;
08188     if (__pyx_t_1) {
08189     /* "PyClical.pyx":698
08190     elif op == 3: # !=
08191 *         if (lhs is None) or (rhs is None):
08192 *             return not bool(lhs is rhs)             # ««««««««
08193 *         else:
08194 *             return bool( toClifford(lhs) != toClifford(rhs) )
08195 */
08196     __Pyx_XDECREF(__pyx_r);
08197     __pyx_t_1 = (((PyObject *)__pyx_v_lhs) == __pyx_v_rhs);
08198     __pyx_t_4 = __Pyx_PyBool_FromLong((((!(!__pyx_t_1)) != 0))); if (unlikely(!__pyx_t_4))
08199 __PYX_ERR(0, 698, __pyx_L1_error)
08200     __Pyx_GOTREF(__pyx_t_4);
08201     __pyx_r = __pyx_t_4;
08202     __pyx_t_4 = 0;
08203     goto __pyx_L0;
08204
08205     /* "PyClical.pyx":697
08206     return bool( toClifford(lhs) == toClifford(rhs) )
08207 *
08208 *     elif op == 3: # !=
08209 *         if (lhs is None) or (rhs is None):             # ««««««««
08210 *             return not bool(lhs is rhs)
08211 *         else:
08212 */
08213     }
08214     /* "PyClical.pyx":700
08215     return not bool(lhs is rhs)
08216 *
08217 *         else:
08218 *             return bool( toClifford(lhs) != toClifford(rhs) )             # ««««««««
08219 *         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08220 *             raise TypeError("This comparison operator is not implemented for "
08221 */
08222     /*else*/ {
08223     __Pyx_XDECREF(__pyx_r);
08224     __pyx_t_1 = (__pyx_f_8PyClical_toClifford(((PyObject *)__pyx_v_lhs)) !=
08225 __pyx_f_8PyClical_toClifford(__pyx_v_rhs));
08226     __pyx_t_4 = __Pyx_PyBool_FromLong((((!(!__pyx_t_1)) != 0))); if (unlikely(!__pyx_t_4))
08227 __PYX_ERR(0, 700, __pyx_L1_error)
08228     __Pyx_GOTREF(__pyx_t_4);
08229     __pyx_r = __pyx_t_4;
08230     __pyx_t_4 = 0;
08231     goto __pyx_L0;
08232
08233     /* "PyClical.pyx":696
08234     else:
08235     return bool( toClifford(lhs) == toClifford(rhs) )
08236 *
08237 *     elif op == 3: # !=             # ««««««««
08238 *         if (lhs is None) or (rhs is None):
08239 *             return not bool(lhs is rhs)
08240 */
08241     }
08242     /* "PyClical.pyx":701
08243     else:
08244     return bool( toClifford(lhs) != toClifford(rhs) )
08245 *
08246 *         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):             # ««««««««
08247 *             raise TypeError("This comparison operator is not implemented for "
08248 *                 + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08249 */

```

```

08247     __pyx_t_2 = __Pyx_TypeCheck(((PyObject *) __pyx_v_lhs), __pyx_ptype_8PyClical_clifford);
08248     __pyx_t_3 = (__pyx_t_2 != 0);
08249     if (!__pyx_t_3) {
08250     } else {
08251         __pyx_t_1 = __pyx_t_3;
08252         goto __pyx_L10_bool_binop_done;
08253     }
08254     __pyx_t_3 = __Pyx_TypeCheck(__pyx_v_rhs, __pyx_ptype_8PyClical_clifford);
08255     __pyx_t_2 = (__pyx_t_3 != 0);
08256     __pyx_t_1 = __pyx_t_2;
08257     __pyx_L10_bool_binop_done;
08258     if (unlikely(__pyx_t_1)) {
08259
08260         /* "PyClical.pyx":703
08261         *     elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08262         *         raise TypeError("This comparison operator is not implemented for "
08263         *             + str(type(lhs)) + ", " + str(type(rhs)) + ".")           # ««««««««
08264         *     else:
08265         *         return NotImplemented
08266         */
08267         __pyx_t_4 = __Pyx_PyObject_CallOneArg(((PyObject *) (&PyUnicode_Type)), ((PyObject
08268 *)Py_TYPE(((PyObject *) __pyx_v_lhs)))); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 703, __pyx_L1_error)
08269         __Pyx_GOTREF(__pyx_t_4);
08270         __pyx_t_5 = __Pyx_PyUnicode_Concat(__pyx_kp_u_This_comparison_operator_is_not, __pyx_t_4);
08271         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 703, __pyx_L1_error)
08272         __Pyx_GOTREF(__pyx_t_5);
08273         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
08274         __pyx_t_4 = __Pyx_PyUnicode_Concat(__pyx_t_5, __pyx_kp_u_8); if (unlikely(!__pyx_t_4))
08275         __PYX_ERR(0, 703, __pyx_L1_error)
08276         __Pyx_GOTREF(__pyx_t_4);
08277         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
08278         __pyx_t_5 = __Pyx_PyObject_CallOneArg(((PyObject *) (&PyUnicode_Type)), ((PyObject
08279 *)Py_TYPE(__pyx_v_rhs)); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 703, __pyx_L1_error)
08280         __Pyx_GOTREF(__pyx_t_5);
08281         __pyx_t_6 = __Pyx_PyUnicode_Concat(__pyx_t_4, __pyx_t_5); if (unlikely(!__pyx_t_6))
08282         __PYX_ERR(0, 703, __pyx_L1_error)
08283         __Pyx_GOTREF(__pyx_t_6);
08284         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
08285         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
08286         __pyx_t_5 = __Pyx_PyUnicode_Concat(__pyx_t_6, __pyx_kp_u_); if (unlikely(!__pyx_t_5))
08287         __PYX_ERR(0, 703, __pyx_L1_error)
08288         __Pyx_GOTREF(__pyx_t_5);
08289         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08290
08291         /* "PyClical.pyx":702
08292         *         return bool( toClifford(lhs) != toClifford(rhs) )
08293         *     elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08294         *         raise TypeError("This comparison operator is not implemented for "           #
08295         *             + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08296         *     else:
08297         *         __pyx_t_6 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_5); if
08298         *         (unlikely(!__pyx_t_6)) __PYX_ERR(0, 702, __pyx_L1_error)
08299         *         __Pyx_GOTREF(__pyx_t_6);
08300         *         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
08301         *         __Pyx_Raise(__pyx_t_6, 0, 0, 0);
08302         *         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08303         *         __PYX_ERR(0, 702, __pyx_L1_error)
08304         *
08305         *         /* "PyClical.pyx":701
08306         *         *     else:
08307         *         *         return bool( toClifford(lhs) != toClifford(rhs) )
08308         *         *     elif isinstance(lhs, clifford) or isinstance(rhs, clifford):           # ««««««««
08309         *         *         raise TypeError("This comparison operator is not implemented for "
08310         *         *             + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08311         *         *     */
08312         *     }
08313         *
08314         *     /* "PyClical.pyx":705
08315         *         + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08316         *     else:
08317         *         return NotImplemented           # ««««««««
08318         *
08319         *     def __getitem__(self, ixt):
08320         *     */
08321         *     /*else*/ {
08322         *         __Pyx_XDECREF(__pyx_r);
08323         *         __Pyx_INCREF(__pyx_builtin_NotImplemented);
08324         *         __pyx_r = __pyx_builtin_NotImplemented;
08325         *         goto __pyx_L0;
08326         *     }
08327
08328         *     /* "PyClical.pyx":672
08329         *         return result
08330         *
08331         *     def __richcmp__(lhs, rhs, int op):           # ««««««««

```



```

08326 *          """
08327 *          Compare objects of type clifford.
08328 */
08329
08330 /* function exit code */
08331 __pyx_L1_error:;
08332 __Pyx_XDECREF(__pyx_t_4);
08333 __Pyx_XDECREF(__pyx_t_5);
08334 __Pyx_XDECREF(__pyx_t_6);
08335 __Pyx_AddTraceback("PyClicl.clifford.__richcmp__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08336 __pyx_r = NULL;
08337 __pyx_L0:;
08338 __Pyx_XGIVEREF(__pyx_r);
08339 __Pyx_RefNannyFinishContext();
08340 return __pyx_r;
08341 }
08342
08343 /* "PyClicl.pyx":707
08344 *          return NotImplemented
08345 *
08346 *          def __getitem__(self, ixt):          # ««««««««
08347 *          """
08348 *          Subscripting: map from index set to scalar coordinate.
08349 */
08350
08351 /* Python wrapper */
08352 static PyObject *__pyx_pw_8PyClicl_8clifford_15_getitem__(PyObject *__pyx_v_self, PyObject
*__pyx_v_ixt); /*proto*/
08353 static char __pyx_doc_8PyClicl_8clifford_14_getitem__[] = "\n          Subscripting: map from
index set to scalar coordinate.\n\n          >> clifford(\"{1}\") [index_set(1)]\n          1.0\n          >> clifford(\"{1}\") [index_set({1})]\n          1.0\n          >> clifford(\"{1}\") [index_set({1,2})]\n          0.0\n          >> clifford(\"2{1,2}\") [index_set({1,2})]\n          2.0\n          ";
08354 #if CYTHON_COMPILING_IN_CPYTHON
08355 struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_14_getitem__;
08356 #endif
08357 static PyObject *__pyx_pw_8PyClicl_8clifford_15_getitem__(PyObject *__pyx_v_self, PyObject
*__pyx_v_ixt) {
08358     PyObject *__pyx_r = 0;
08359     __Pyx_RefNannyDeclarations
08360     __Pyx_RefNannySetupContext("__getitem__ (wrapper)", 0);
08361     __pyx_r = __pyx_pf_8PyClicl_8clifford_14_getitem__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_ixt));
08362
08363 /* function exit code */
08364 __Pyx_RefNannyFinishContext();
08365 return __pyx_r;
08366 }
08367
08368 static PyObject *__pyx_pf_8PyClicl_8clifford_14_getitem__(struct
__pyx_obj_8PyClicl_clifford *__pyx_v_self, PyObject *__pyx_v_ixt) {
08369     PyObject *__pyx_r = NULL;
08370     __Pyx_RefNannyDeclarations
08371     PyObject *__pyx_t_1 = NULL;
08372     int __pyx_lineno = 0;
08373     const char *__pyx_filename = NULL;
08374     int __pyx_clineno = 0;
08375     __Pyx_RefNannySetupContext("__getitem__", 0);
08376
08377 /* "PyClicl.pyx":720
08378 *          2.0
08379 *          """
08380 *          return self.instance.getitem(toIndexSet(ixt))          # ««««««««
08381 *
08382 *          def __neg__(self):
08383 */
08384     __Pyx_XDECREF(__pyx_r);
08385     __pyx_t_1 =
PyFloat_FromDouble(__pyx_v_self->instance->operator[](__pyx_f_8PyClicl_toIndexSet(__pyx_v_ixt))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 720, __pyx_L1_error)
08386     __Pyx_GOTREF(__pyx_t_1);
08387     __pyx_r = __pyx_t_1;
08388     __pyx_t_1 = 0;
08389     goto __pyx_L0;
08390
08391 /* "PyClicl.pyx":707
08392 *          return NotImplemented
08393 *
08394 *          def __getitem__(self, ixt):          # ««««««««
08395 *          """
08396 *          Subscripting: map from index set to scalar coordinate.
08397 */
08398
08399 /* function exit code */
08400 __pyx_L1_error:;
08401 __Pyx_XDECREF(__pyx_t_1);
08402 __Pyx_AddTraceback("PyClicl.clifford.__getitem__", __pyx_clineno, __pyx_lineno,

```

```

__pyx_filename);
08403     __pyx_r = NULL;
08404     __pyx_L0;;
08405     __Pyx_XGIVEREF(__pyx_r);
08406     __Pyx_RefNannyFinishContext();
08407     return __pyx_r;
08408 }
08409
08410     /* "PyClical.pyx":722
08411     *         return self.instance.getitem(toIndexSet(ixt))
08412     *
08413     * def __neg__(self):          # ««««««««
08414     *     """
08415     *     Unary -.
08416     */
08417
08418     /* Python wrapper */
08419     static PyObject *__pyx_pw_8PyClical_8clifford_17__neg__(PyObject *__pyx_v_self); /*proto*/
08420     static char __pyx_doc_8PyClical_8clifford_16__neg__[] = "\n        Unary -. \n\n        >>
08421 print(-clifford(\">{1}\")\n        -{1}\n        ";
08422 #if CYTHON_COMPILING_IN_CPYTHON
08423 struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_16__neg__;
08424 #endif
08425 static PyObject *__pyx_pw_8PyClical_8clifford_17__neg__(PyObject *__pyx_v_self) {
08426     PyObject *__pyx_r = 0;
08427     __Pyx_RefNannyDeclarations
08428     __Pyx_RefNannySetupContext("__neg__ (wrapper)", 0);
08429     __pyx_r = __pyx_pf_8PyClical_8clifford_16__neg__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
08430
08431     /* function exit code */
08432     __Pyx_RefNannyFinishContext();
08433     return __pyx_r;
08434 }
08435
08436 static PyObject *__pyx_pf_8PyClical_8clifford_16__neg__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
08437     PyObject *__pyx_r = NULL;
08438     __Pyx_RefNannyDeclarations
08439     PyObject *__pyx_t_1 = NULL;
08440     PyObject *__pyx_t_2 = NULL;
08441     int __pyx_lineno = 0;
08442     const char *__pyx_filename = NULL;
08443     int __pyx_clineno = 0;
08444     __Pyx_RefNannySetupContext("__neg__", 0);
08445
08446     /* "PyClical.pyx":729
08447     *     -{1}
08448     *     """
08449     *     return clifford().wrap( self.instance.neg() )          # ««««««««
08450     *
08451     * def __pos__(self):
08452     */
08453     __Pyx_XDECREF(__pyx_r);
08454     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 729, __pyx_L1_error)
08455     __Pyx_GOTREF(__pyx_t_1);
08456     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->operator-()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 729,
__pyx_L1_error)
08457     __Pyx_GOTREF(__pyx_t_2);
08458     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08459     __pyx_r = __pyx_t_2;
08460     __pyx_t_2 = 0;
08461     goto __pyx_L0;
08462
08463     /* "PyClical.pyx":722
08464     *         return self.instance.getitem(toIndexSet(ixt))
08465     *
08466     * def __neg__(self):          # ««««««««
08467     *     """
08468     *     Unary -.
08469     */
08470
08471     /* function exit code */
08472     __pyx_L1_error:;
08473     __Pyx_XDECREF(__pyx_t_1);
08474     __Pyx_XDECREF(__pyx_t_2);
08475     __Pyx_AddTraceback("PyClical.clifford.__neg__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08476     __pyx_r = NULL;
08477     __pyx_L0;;
08478     __Pyx_XGIVEREF(__pyx_r);
08479     __Pyx_RefNannyFinishContext();
08480     return __pyx_r;
08481 }

```

```

08482         /* "PyClical.pyx":731
08483         *         return clifford().wrap( self.instance.neg() )
08484         *
08485         *     def __pos__(self):                # ««««««««
08486         *         """
08487         *         Unary +.
08488         */
08489
08490         /* Python wrapper */
08491         static PyObject *__pyx_pw_8PyClical_8clifford_19__pos__(PyObject *__pyx_v_self); /*proto*/
08492         static char __pyx_doc_8PyClical_8clifford_18__pos__[] = "\n        Unary +.\n\n    >>
print(+clifford(\{1\}\n\n        {1}\n\n        ";
08493         #if CYTHON_COMPILING_IN_CPYTHON
08494         struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_18__pos__;
08495         #endif
08496         static PyObject *__pyx_pw_8PyClical_8clifford_19__pos__(PyObject *__pyx_v_self) {
08497             PyObject *__pyx_r = 0;
08498             __Pyx_RefNannyDeclarations
08499             __Pyx_RefNannySetupContext("__pos__ (wrapper)", 0);
08500             __pyx_r = __pyx_pf_8PyClical_8clifford_18__pos__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
08501
08502             /* function exit code */
08503             __Pyx_RefNannyFinishContext();
08504             return __pyx_r;
08505         }
08506
08507         static PyObject *__pyx_pf_8PyClical_8clifford_18__pos__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
08508             PyObject *__pyx_r = NULL;
08509             __Pyx_RefNannyDeclarations
08510             PyObject *__pyx_t_1 = NULL;
08511             int __pyx_lineno = 0;
08512             const char *__pyx_filename = NULL;
08513             int __pyx_clineno = 0;
08514             __Pyx_RefNannySetupContext("__pos__", 0);
08515
08516             /* "PyClical.pyx":738
08517             *         {1}
08518             *         """
08519             *         return clifford(self)                # ««««««««
08520             *
08521             *     def __add__(lhs, rhs):
08522             */
08523             __Pyx_XDECREF(__pyx_r);
08524             __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
((PyObject *)__pyx_v_self)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 738, __pyx_L1_error)
08525             __Pyx_GOTREF(__pyx_t_1);
08526             __pyx_r = __pyx_t_1;
08527             __pyx_t_1 = 0;
08528             goto __pyx_L0;
08529
08530             /* "PyClical.pyx":731
08531             *         return clifford().wrap( self.instance.neg() )
08532             *
08533             *     def __pos__(self):                # ««««««««
08534             *         """
08535             *         Unary +.
08536             */
08537
08538             /* function exit code */
08539             __pyx_L1_error:;
08540             __Pyx_XDECREF(__pyx_t_1);
08541             __Pyx_AddTraceback("PyClical.clifford.__pos__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08542             __pyx_r = NULL;
08543             __pyx_L0:;
08544             __Pyx_XGIVEREF(__pyx_r);
08545             __Pyx_RefNannyFinishContext();
08546             return __pyx_r;
08547         }
08548
08549         /* "PyClical.pyx":740
08550         *         return clifford(self)
08551         *
08552         *     def __add__(lhs, rhs):                # ««««««««
08553         *         """
08554         *         Geometric sum.
08555         */
08556
08557         /* Python wrapper */
08558         static PyObject *__pyx_pw_8PyClical_8clifford_21__add__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
08559         static char __pyx_doc_8PyClical_8clifford_20__add__[] = "\n        Geometric sum.\n\n
>> print(clifford(1) + clifford(\{2\}\n\n        1+{2}\n\n        >> print(clifford(\{1\}\n\n
clifford(\{2\}\n\n        {1}+{2}\n\n        ";
08560         #if CYTHON_COMPILING_IN_CPYTHON

```

```

08561     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_20__add__
08562     #endif
08563     static PyObject *__pyx_pw_8PyClical_8clifford_21__add__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
08564         PyObject *__pyx_r = 0;
08565         __Pyx_RefNannyDeclarations
08566         __Pyx_RefNannySetupContext("__add__ (wrapper)", 0);
08567         __pyx_r = __pyx_pf_8PyClical_8clifford_20__add__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
08568
08569         /* function exit code */
08570         __Pyx_RefNannyFinishContext();
08571         return __pyx_r;
08572     }
08573
08574     static PyObject *__pyx_pf_8PyClical_8clifford_20__add__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
08575         PyObject *__pyx_r = NULL;
08576         __Pyx_RefNannyDeclarations
08577         PyObject *__pyx_t_1 = NULL;
08578         PyObject *__pyx_t_2 = NULL;
08579         int __pyx_lineno = 0;
08580         const char *__pyx_filename = NULL;
08581         int __pyx_clineno = 0;
08582         __Pyx_RefNannySetupContext("__add__", 0);
08583
08584         /* "PyClical.pyx":749
08585         *     {1}+{2}
08586         *     """
08587         *     return clifford().wrap( toClifford(lhs) + toClifford(rhs) )           # ««««««««
08588         *
08589         *     def __iadd__(self, rhs):
08590         */
08591         __Pyx_XDECREF(__pyx_r);
08592         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 749, __pyx_L1_error)
08593         __Pyx_GOTREF(__pyx_t_1);
08594         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) +
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 749,
__pyx_L1_error)
08595         __Pyx_GOTREF(__pyx_t_2);
08596         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08597         __pyx_r = __pyx_t_2;
08598         __pyx_t_2 = 0;
08599         goto __pyx_L0;
08600
08601         /* "PyClical.pyx":740
08602         *     return clifford(self)
08603         *
08604         *     def __add__(lhs, rhs):           # ««««««««
08605         *         """
08606         *         Geometric sum.
08607         */
08608
08609         /* function exit code */
08610         __pyx_L1_error:;
08611         __Pyx_XDECREF(__pyx_t_1);
08612         __Pyx_XDECREF(__pyx_t_2);
08613         __Pyx_AddTraceback("PyClical.clifford.__add__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08614         __pyx_r = NULL;
08615         __pyx_L0:;
08616         __Pyx_XGIVEREF(__pyx_r);
08617         __Pyx_RefNannyFinishContext();
08618         return __pyx_r;
08619     }
08620
08621         /* "PyClical.pyx":751
08622         *     return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
08623         *
08624         *     def __iadd__(self, rhs):           # ««««««««
08625         *         """
08626         *         Geometric sum.
08627         */
08628
08629         /* Python wrapper */
08630         static PyObject *__pyx_pw_8PyClical_8clifford_23__iadd__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
08631         static char __pyx_doc_8PyClical_8clifford_22__iadd__[] = "\n          Geometric sum.\n\n
>> x = clifford(1); x += clifford(\"{2}\"); print(x)\n          1+{2}\n          ";
08632         #if CYTHON_COMPILING_IN_CPYTHON
08633         struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_22__iadd__
08634         #endif
08635         static PyObject *__pyx_pw_8PyClical_8clifford_23__iadd__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
08636             PyObject *__pyx_r = 0;

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```

08637     __Pyx_RefNannyDeclarations
08638     __Pyx_RefNannySetupContext("__iadd__ (wrapper)", 0);
08639     __pyx_r = __pyx_pf_8PyClical_8clifford_22__iadd__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
08640
08641     /* function exit code */
08642     __Pyx_RefNannyFinishContext();
08643     return __pyx_r;
08644 }
08645
08646 static PyObject *__pyx_pf_8PyClical_8clifford_22__iadd__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
08647     PyObject *__pyx_r = NULL;
08648     __Pyx_RefNannyDeclarations
08649     PyObject *__pyx_t_1 = NULL;
08650     int __pyx_lineno = 0;
08651     const char *__pyx_filename = NULL;
08652     int __pyx_clineno = 0;
08653     __Pyx_RefNannySetupContext("__iadd__", 0);
08654
08655     /* "PyClical.pyx":758
08656  *      1+{2}
08657  *      """
08658  *      return self.wrap( self.unwrap() + toClifford(rhs) )          # ««««««««
08659  *
08660  *      def __sub__(lhs, rhs):
08661  */
08662     __Pyx_XDECREF(__pyx_r);
08663     __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
((__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) + __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 758, __pyx_L1_error)
08664     __Pyx_GOTREF(__pyx_t_1);
08665     __pyx_r = __pyx_t_1;
08666     __pyx_t_1 = 0;
08667     goto __pyx_L0;
08668
08669     /* "PyClical.pyx":751
08670  *      return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
08671  *
08672  *      def __iadd__(self, rhs):          # ««««««««
08673  *      """
08674  *      Geometric sum.
08675  */
08676
08677     /* function exit code */
08678     __pyx_L1_error:;
08679     __Pyx_XDECREF(__pyx_t_1);
08680     __Pyx_AddTraceback("PyClical.clifford.__iadd__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08681     __pyx_r = NULL;
08682     __pyx_L0:;
08683     __Pyx_XGIVEREF(__pyx_r);
08684     __Pyx_RefNannyFinishContext();
08685     return __pyx_r;
08686 }
08687
08688     /* "PyClical.pyx":760
08689  *      return self.wrap( self.unwrap() + toClifford(rhs) )
08690  *
08691  *      def __sub__(lhs, rhs):          # ««««««««
08692  *      """
08693  *      Geometric difference.
08694  */
08695
08696     /* Python wrapper */
08697     static PyObject *__pyx_pw_8PyClical_8clifford_25__sub__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
08698     static char __pyx_doc_8PyClical_8clifford_24__sub__[] = "\n          Geometric difference.\n\n
>> print(clifford(1) - clifford(\"{2}\"))\n          1-{2}\n          >> print(clifford(\"{1}\") -
clifford(\"{2}\"))\n          {1}-{2}\n          ";
08699     #if CYTHON_COMPILING_IN_CPYTHON
08700     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_24__sub__;
08701     #endif
08702     static PyObject *__pyx_pw_8PyClical_8clifford_25__sub__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
08703         PyObject *__pyx_r = 0;
08704         __Pyx_RefNannyDeclarations
08705         __Pyx_RefNannySetupContext("__sub__ (wrapper)", 0);
08706         __pyx_r = __pyx_pf_8PyClical_8clifford_24__sub__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
08707
08708         /* function exit code */
08709         __Pyx_RefNannyFinishContext();
08710         return __pyx_r;
08711     }
08712
08713     static PyObject *__pyx_pf_8PyClical_8clifford_24__sub__(PyObject *__pyx_v_lhs, PyObject

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```

    *__pyx_v_rhs) {
08714         PyObject *__pyx_r = NULL;
08715         __Pyx_RefNannyDeclarations
08716         PyObject *__pyx_t_1 = NULL;
08717         PyObject *__pyx_t_2 = NULL;
08718         int __pyx_lineno = 0;
08719         const char *__pyx_filename = NULL;
08720         int __pyx_clineno = 0;
08721         __Pyx_RefNannySetupContext("__sub__", 0);
08722
08723         /* "PyClicl.pyx":769
08724          *
08725          *
08726          *      return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
08727          *
08728          *      def __isub__(self, rhs):
08729          */
08730         __Pyx_XDECREF(__pyx_r);
08731         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 769, __pyx_L1_error)
08732         __Pyx_GOTREF(__pyx_t_1);
08733         __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), (__pyx_f_8PyClicl_toClifford(__pyx_v_lhs) -
__pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 769,
__pyx_L1_error)
08734         __Pyx_GOTREF(__pyx_t_2);
08735         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08736         __pyx_r = __pyx_t_2;
08737         __pyx_t_2 = 0;
08738         goto __pyx_L0;
08739
08740         /* "PyClicl.pyx":760
08741          *
08742          *
08743          *      def __sub__(lhs, rhs):
08744          *
08745          *      Geometric difference.
08746          */
08747
08748         /* function exit code */
08749         __pyx_L1_error:;
08750         __Pyx_XDECREF(__pyx_t_1);
08751         __Pyx_XDECREF(__pyx_t_2);
08752         __Pyx_AddTraceback("PyClicl.clifford.__sub__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08753         __pyx_r = NULL;
08754         __pyx_L0:;
08755         __Pyx_XGIVEREF(__pyx_r);
08756         __Pyx_RefNannyFinishContext();
08757         return __pyx_r;
08758     }
08759
08760     /* "PyClicl.pyx":771
08761      *
08762      *
08763      *      def __isub__(self, rhs):
08764      *
08765      *      Geometric difference.
08766      */
08767
08768     /* Python wrapper */
08769     static PyObject *__pyx_pw_8PyClicl_8clifford_27__isub__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
08770     static char __pyx_doc_8PyClicl_8clifford_26__isub__[] = "\n      Geometric difference.\n\n
>> x = clifford(1); x -= clifford('{2}'); print(x)\n      1-{2}\n      ";
08771     #if CYTHON_COMPILING_IN_CPYTHON
08772     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_26__isub__;
08773     #endif
08774     static PyObject *__pyx_pw_8PyClicl_8clifford_27__isub__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
08775         PyObject *__pyx_r = 0;
08776         __Pyx_RefNannyDeclarations
08777         __Pyx_RefNannySetupContext("__isub__ (wrapper)", 0);
08778         __pyx_r = __pyx_pf_8PyClicl_8clifford_26__isub__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
08779
08780         /* function exit code */
08781         __Pyx_RefNannyFinishContext();
08782         return __pyx_r;
08783     }
08784
08785     static PyObject *__pyx_pf_8PyClicl_8clifford_26__isub__(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
08786         PyObject *__pyx_r = NULL;
08787         __Pyx_RefNannyDeclarations
08788         PyObject *__pyx_t_1 = NULL;
08789         int __pyx_lineno = 0;

```

```

08790         const char *__pyx_filename = NULL;
08791         int __pyx_clineno = 0;
08792         __Pyx_RefNannySetupContext("__isub__", 0);
08793
08794         /* "PyClical.pyx":778
08795         *
08796         * """
08797         *     return self.wrap( self.unwrap() - toClifford(rhs) )
08798         *                                     # ««««««««
08799         *     def __mul__(lhs, rhs):
08800         */
08801         __Pyx_XDECREF(__pyx_r);
08802         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
08803         (__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) - __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
08804         (unlikely(!__pyx_t_1)) __PYX_ERR(0, 778, __pyx_L1_error)
08805         __Pyx_GOTREF(__pyx_t_1);
08806         __pyx_r = __pyx_t_1;
08807         __pyx_t_1 = 0;
08808         goto __pyx_L0;
08809
08810         /* "PyClical.pyx":771
08811         *     return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
08812         *
08813         *     def __isub__(self, rhs):
08814         *                                     # ««««««««
08815         *     """
08816         *     Geometric difference.
08817         */
08818         /* function exit code */
08819         __Pyx_XDECREF(__pyx_t_1);
08820         __Pyx_AddTraceback("PyClical.clifford.__isub__", __pyx_clineno, __pyx_lineno,
08821         __pyx_filename);
08822         __pyx_r = NULL;
08823         __pyx_L0:;
08824         __Pyx_XGIVEREF(__pyx_r);
08825         __Pyx_RefNannyFinishContext();
08826         return __pyx_r;
08827     }
08828
08829         /* "PyClical.pyx":780
08830         *     return self.wrap( self.unwrap() - toClifford(rhs) )
08831         *
08832         *     def __mul__(lhs, rhs):
08833         *                                     # ««««««««
08834         *     """
08835         *     Geometric product.
08836         */
08837         /* Python wrapper */
08838         static PyObject *__pyx_pw_8PyClical_8clifford_29__mul__(PyObject *__pyx_v_lhs, PyObject
08839         *__pyx_v_rhs); /*proto*/
08840         static char __pyx_doc_8PyClical_8clifford_28__mul__[] = "\n          Geometric product.\n\n
08841         >> print(clifford(\'{1}\') * clifford(\'{2}\'))\n          {1,2}\n          >> print(clifford(2) *
08842         clifford(\'{2}\'))\n          2{2}\n          >> print(clifford(\'{1}\') * clifford(\'{1,2}\'))\n
08843         {2}\n          ";
08844         #if CYTHON_COMPILING_IN_CPYTHON
08845         struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_28__mul__;
08846         #endif
08847         static PyObject *__pyx_pw_8PyClical_8clifford_29__mul__(PyObject *__pyx_v_lhs, PyObject
08848         *__pyx_v_rhs) {
08849             PyObject *__pyx_r = 0;
08850             __Pyx_RefNannyDeclarations
08851             __Pyx_RefNannySetupContext("__mul__ (wrapper)", 0);
08852             __pyx_r = __pyx_pf_8PyClical_8clifford_28__mul__((PyObject *)__pyx_v_lhs), ((PyObject
08853             *)__pyx_v_rhs));
08854
08855             /* function exit code */
08856             __Pyx_RefNannyFinishContext();
08857             return __pyx_r;
08858         }
08859
08860         static PyObject *__pyx_pf_8PyClical_8clifford_28__mul__(PyObject *__pyx_v_lhs, PyObject
08861         *__pyx_v_rhs) {
08862             PyObject *__pyx_r = NULL;
08863             __Pyx_RefNannyDeclarations
08864             PyObject *__pyx_t_1 = NULL;
08865             PyObject *__pyx_t_2 = NULL;
08866             int __pyx_lineno = 0;
08867             const char *__pyx_filename = NULL;
08868             int __pyx_clineno = 0;
08869             __Pyx_RefNannySetupContext("__mul__", 0);
08870
08871         /* "PyClical.pyx":791
08872         *     {2}
08873         *     """
08874         *     return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
08875         *                                     # ««««««««

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```

08867 *      def __imul__(self, rhs):
08868 */
08869     __Pyx_XDECREF(__pyx_r);
08870     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 791, __pyx_L1_error)
08871     __Pyx_GOTREF(__pyx_t_1);
08872     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford)(__pyx_v_lhs) *
__pyx_f_8PyClical_toClifford(__pyx_v_rhs)); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 791,
__pyx_L1_error)
08873     __Pyx_GOTREF(__pyx_t_2);
08874     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08875     __pyx_r = __pyx_t_2;
08876     __pyx_t_2 = 0;
08877     goto __pyx_L0;
08878
08879     /* "PyClical.pyx":780
08880 *      return self.wrap( self.unwrap() - toClifford(rhs) )
08881 *
08882 *      def __mul__(lhs, rhs):                # ««««««««
08883 *      """
08884 *      Geometric product.
08885 */
08886
08887     /* function exit code */
08888     __pyx_L1_error:;
08889     __Pyx_XDECREF(__pyx_t_1);
08890     __Pyx_XDECREF(__pyx_t_2);
08891     __Pyx_AddTraceback("PyClical.clifford.__mul__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08892     __pyx_r = NULL;
08893     __pyx_L0:;
08894     __Pyx_XGIVEREF(__pyx_r);
08895     __Pyx_RefNannyFinishContext();
08896     return __pyx_r;
08897 }
08898
08899     /* "PyClical.pyx":793
08900 *      return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
08901 *
08902 *      def __imul__(self, rhs):                # ««««««««
08903 *      """
08904 *      Geometric product.
08905 */
08906
08907     /* Python wrapper */
08908     static PyObject *__pyx_pw_8PyClical_8clifford_31__imul__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
08909     static char __pyx_doc_8PyClical_8clifford_30__imul__[] = "\n      Geometric product.\n\n
>> x = clifford(2); x *= clifford('{2}'); print(x)\n      2{2}\n      >> x = clifford('{1}');
x *= clifford('{2}'); print(x)\n      {1,2}\n      >> x = clifford('{1}'); x *=
clifford('{1,2}'); print(x)\n      {2}\n      ";
08910     #if CYTHON_COMPILING_IN_CPYTHON
08911     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_30__imul__;
08912     #endif
08913     static PyObject *__pyx_pw_8PyClical_8clifford_31__imul__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
08914         PyObject *__pyx_r = 0;
08915         __Pyx_RefNannyDeclarations
08916         __Pyx_RefNannySetupContext("__imul__ (wrapper)", 0);
08917         __pyx_r = __pyx_pf_8PyClical_8clifford_30__imul__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
08918
08919         /* function exit code */
08920         __Pyx_RefNannyFinishContext();
08921         return __pyx_r;
08922     }
08923
08924     static PyObject *__pyx_pf_8PyClical_8clifford_30__imul__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
08925         PyObject *__pyx_r = NULL;
08926         __Pyx_RefNannyDeclarations
08927         PyObject *__pyx_t_1 = NULL;
08928         int __pyx_lineno = 0;
08929         const char *__pyx_filename = NULL;
08930         int __pyx_clineno = 0;
08931         __Pyx_RefNannySetupContext("__imul__", 0);
08932
08933         /* "PyClical.pyx":804
08934 *      {2}
08935 *      """
08936 *      return self.wrap( self.unwrap() * toClifford(rhs) )                # ««««««««
08937 *
08938 *      def __mod__(lhs, rhs):
08939 */
08940         __Pyx_XDECREF(__pyx_r);
08941         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,

```



```

        (__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) * __pyx_f_8PyClical_toClifford(__pyx_v_rhs)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 804, __pyx_L1_error)
08942     __Pyx_GOTREF(__pyx_t_1);
08943     __pyx_r = __pyx_t_1;
08944     __pyx_t_1 = 0;
08945     goto __pyx_L0;
08946
08947     /* "PyClical.pyx":793
08948     *     return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
08949     *
08950     *     def __imul__(self, rhs):                # ««««««««
08951     *         """
08952     *         Geometric product.
08953     */
08954
08955     /* function exit code */
08956     __pyx_L1_error:;
08957     __Pyx_XDECREF(__pyx_t_1);
08958     __Pyx_AddTraceback("PyClical.clifford.__imul__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08959     __pyx_r = NULL;
08960     __pyx_L0:;
08961     __Pyx_XGIVEREF(__pyx_r);
08962     __Pyx_RefNannyFinishContext();
08963     return __pyx_r;
08964 }
08965
08966     /* "PyClical.pyx":806
08967     *     return self.wrap( self.unwrap() * toClifford(rhs) )
08968     *
08969     *     def __mod__(lhs, rhs):                # ««««««««
08970     *         """
08971     *         Contraction.
08972     */
08973
08974     /* Python wrapper */
08975     static PyObject *__pyx_pw_8PyClical_8clifford_33__mod__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
08976     static char __pyx_doc_8PyClical_8clifford_32__mod__[] = "\n        Contraction.\n\n" >>
print(clifford("\{1}\") % clifford("\{2}\"))\n        0\n        >> print(clifford(2) %
clifford("\{2}\")\n        2{2}\n        >> print(clifford("\{1}\") % clifford("\{1}\")\n        1\n
>> print(clifford("\{1}\") % clifford("\{1,2}\"))\n        {2}\n        ";
08977     #if CYTHON_COMPILING_IN_CPYTHON
08978     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_32__mod__;
08979     #endif
08980     static PyObject *__pyx_pw_8PyClical_8clifford_33__mod__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
08981         PyObject *__pyx_r = 0;
08982         __Pyx_RefNannyDeclarations
08983         __Pyx_RefNannySetupContext("__mod__ (wrapper)", 0);
08984         __pyx_r = __pyx_pf_8PyClical_8clifford_32__mod__(((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
08985
08986         /* function exit code */
08987         __Pyx_RefNannyFinishContext();
08988         return __pyx_r;
08989     }
08990
08991     static PyObject *__pyx_pf_8PyClical_8clifford_32__mod__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
08992         PyObject *__pyx_r = NULL;
08993         __Pyx_RefNannyDeclarations
08994         PyObject *__pyx_t_1 = NULL;
08995         PyObject *__pyx_t_2 = NULL;
08996         int __pyx_lineno = 0;
08997         const char *__pyx_filename = NULL;
08998         int __pyx_clineno = 0;
08999         __Pyx_RefNannySetupContext("__mod__", 0);
09000
09001         /* "PyClical.pyx":819
09002         *     {2}
09003         *     """
09004         *     return clifford().wrap( toClifford(lhs) % toClifford(rhs) )                # ««««««««
09005         *
09006         *     def __imod__(self, rhs):
09007         */
09008         __Pyx_XDECREF(__pyx_r);
09009         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 819, __pyx_L1_error)
09010         __Pyx_GOTREF(__pyx_t_1);
09011         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) %
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 819,
__pyx_L1_error)
09012         __Pyx_GOTREF(__pyx_t_2);
09013         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09014         __pyx_r = __pyx_t_2;

```

```

09015         __pyx_t_2 = 0;
09016         goto __pyx_L0;
09017
09018         /* "PyClicl.pyx":806
09019  *         return self.wrap( self.unwrap() * toClifford(rhs) )
09020  *
09021  *     def __mod__(lhs, rhs):
09022  *         """
09023  *         Contraction.
09024  */
09025
09026         /* function exit code */
09027         __pyx_L1_error++;
09028         __Pyx_XDECREF(__pyx_t_1);
09029         __Pyx_XDECREF(__pyx_t_2);
09030         __Pyx_AddTraceback("PyClicl.clifford.__mod__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09031         __pyx_r = NULL;
09032         __pyx_L0;
09033         __Pyx_XGIVEREF(__pyx_r);
09034         __Pyx_RefNannyFinishContext();
09035         return __pyx_r;
09036     }
09037
09038     /* "PyClicl.pyx":821
09039  *     return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
09040  *
09041  *     def __imod__(self, rhs):
09042  *         """
09043  *         Contraction.
09044  */
09045
09046     /* Python wrapper */
09047     static PyObject *__pyx_pw_8PyClicl_8clifford_35__imod__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09048     static char __pyx_doc_8PyClicl_8clifford_34__imod__[] = "\n        Contraction.\n\n    >>
x = clifford(\">{1}\"); x %= clifford(\">{2}\"); print(x)\n        0\n    >> x = clifford(2); x %=
clifford(\">{2}\"); print(x)\n        2{2}\n    >> x = clifford(\">{1}\"); x %= clifford(\">{1}\");
print(x)\n        1\n    >> x = clifford(\">{1}\"); x %= clifford(\">{1,2}\"); print(x)\n
{2}\n    ";
09049     #if CYTHON_COMPILING_IN_CPYTHON
09050     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_34__imod__;
09051     #endif
09052     static PyObject *__pyx_pw_8PyClicl_8clifford_35__imod__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
09053         PyObject *__pyx_r = 0;
09054         __Pyx_RefNannyDeclarations
09055         __Pyx_RefNannySetupContext("__imod__ (wrapper)", 0);
09056         __pyx_r = __pyx_pf_8PyClicl_8clifford_34__imod__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09057
09058         /* function exit code */
09059         __Pyx_RefNannyFinishContext();
09060         return __pyx_r;
09061     }
09062
09063     static PyObject *__pyx_pf_8PyClicl_8clifford_34__imod__(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
09064         PyObject *__pyx_r = NULL;
09065         __Pyx_RefNannyDeclarations
09066         PyObject *__pyx_t_1 = NULL;
09067         int __pyx_lineno = 0;
09068         const char *__pyx_filename = NULL;
09069         int __pyx_clineno = 0;
09070         __Pyx_RefNannySetupContext("__imod__", 0);
09071
09072         /* "PyClicl.pyx":834
09073  *     {2}
09074  *     """
09075  *     return self.wrap( self.unwrap() % toClifford(rhs) )
09076  *
09077  *     def __and__(lhs, rhs):
09078  */
09079         __Pyx_XDECREF(__pyx_r);
09080         __pyx_t_1 = __pyx_f_8PyClicl_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClicl_8clifford_unwrap(__pyx_v_self) % __pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 834, __pyx_L1_error)
09081         __Pyx_GOTREF(__pyx_t_1);
09082         __pyx_r = __pyx_t_1;
09083         __pyx_t_1 = 0;
09084         goto __pyx_L0;
09085
09086     /* "PyClicl.pyx":821
09087  *     return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
09088  *
09089  *     def __imod__(self, rhs):
09090  *         """

```

```

09091 *          Contraction.
09092 */
09093
09094     /* function exit code */
09095     __pyx_L1_error++;
09096     __Pyx_XDECREF(__pyx_t_1);
09097     __Pyx_AddTraceback("PyClical.clifford.__imod__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09098     __pyx_r = NULL;
09099     __pyx_L0++;
09100     __Pyx_XGIVEREF(__pyx_r);
09101     __Pyx_RefNannyFinishContext();
09102     return __pyx_r;
09103 }
09104
09105     /* "PyClical.pyx":836
09106 *         return self.wrap( self.unwrap() % toClifford(rhs) )
09107 *
09108 *     def __and__(lhs, rhs):
09109 *         """
09110 *         Inner product.
09111 */
09112
09113     /* Python wrapper */
09114     static PyObject *__pyx_pw_8PyClical_8clifford_37__and__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
09115     static char __pyx_doc_8PyClical_8clifford_36__and__[] = "\n        Inner product.\n\n
>> print(clifford(\"{1}\") & clifford(\"{2}\"))\n        0\n        >> print(clifford(2) &
clifford(\"{2}\"))\n        0\n        >> print(clifford(\"{1}\") & clifford(\"{1}\"))\n        1\n
>> print(clifford(\"{1}\") & clifford(\"{1,2}\"))\n        {2}\n        ";
09116     #if CYTHON_COMPILING_IN_CPYTHON
09117     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_36__and__;
09118     #endif
09119     static PyObject *__pyx_pw_8PyClical_8clifford_37__and__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09120         PyObject *__pyx_r = 0;
09121         __Pyx_RefNannyDeclarations
09122         __Pyx_RefNannySetupContext("__and__ (wrapper)", 0);
09123         __pyx_r = __pyx_pf_8PyClical_8clifford_36__and__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
09124
09125     /* function exit code */
09126     __Pyx_RefNannyFinishContext();
09127     return __pyx_r;
09128 }
09129
09130     static PyObject *__pyx_pf_8PyClical_8clifford_36__and__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09131         PyObject *__pyx_r = NULL;
09132         __Pyx_RefNannyDeclarations
09133         PyObject *__pyx_t_1 = NULL;
09134         PyObject *__pyx_t_2 = NULL;
09135         int __pyx_lineno = 0;
09136         const char *__pyx_filename = NULL;
09137         int __pyx_clineno = 0;
09138         __Pyx_RefNannySetupContext("__and__", 0);
09139
09140     /* "PyClical.pyx":849
09141 *         {2}
09142 *         """
09143 *         return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
09144 *
09145 *     def __iand__(self, rhs):
09146 */
09147         __Pyx_XDECREF(__pyx_r);
09148         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 849, __pyx_L1_error)
09149         __Pyx_GOTREF(__pyx_t_1);
09150         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) &
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 849,
__pyx_L1_error)
09151         __Pyx_GOTREF(__pyx_t_2);
09152         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09153         __pyx_r = __pyx_t_2;
09154         __pyx_t_2 = 0;
09155         goto __pyx_L0;
09156
09157     /* "PyClical.pyx":836
09158 *         return self.wrap( self.unwrap() % toClifford(rhs) )
09159 *
09160 *     def __and__(lhs, rhs):
09161 *         """
09162 *         Inner product.
09163 */
09164
09165     /* function exit code */

```

```

09166         __pyx_L1_error;;
09167         __Pyx_XDECREFF(__pyx_t_1);
09168         __Pyx_XDECREFF(__pyx_t_2);
09169         __Pyx_AddTraceback("PyClicl.clifford.__and__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09170         __pyx_r = NULL;
09171         __pyx_L0;;
09172         __Pyx_XGIVEREF(__pyx_r);
09173         __Pyx_RefNannyFinishContext();
09174         return __pyx_r;
09175     }
09176
09177     /* "PyClicl.pyx":851
09178     *     return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
09179     *
09180     *     def __iand__(self, rhs):                # ««««««««
09181     *         """
09182     *         Inner product.
09183     */
09184
09185     /* Python wrapper */
09186     static PyObject *__pyx_pw_8PyClicl_8clifford_39__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09187     static char __pyx_doc_8PyClicl_8clifford_38__iand__[] = "\n        Inner product.\n\n
>> x = clifford(\"{1}\"); x &= clifford(\"{2}\"); print(x)\n        0\n        >> x = clifford(2); x
&= clifford(\"{2}\"); print(x)\n        0\n        >> x = clifford(\"{1}\"); x &= clifford(\"{1}\");
print(x)\n        1\n        >> x = clifford(\"{1}\"); x &= clifford(\"{1,2}\"); print(x)\n
{2}\n        ";
09188     #if CYTHON_COMPILING_IN_CPYTHON
09189     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_38__iand__;
09190     #endif
09191     static PyObject *__pyx_pw_8PyClicl_8clifford_39__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
09192         PyObject *__pyx_r = 0;
09193         __Pyx_RefNannyDeclarations
09194         __Pyx_RefNannySetupContext("__iand__ (wrapper)", 0);
09195         __pyx_r = __pyx_pf_8PyClicl_8clifford_38__iand__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09196
09197         /* function exit code */
09198         __Pyx_RefNannyFinishContext();
09199         return __pyx_r;
09200     }
09201
09202     static PyObject *__pyx_pf_8PyClicl_8clifford_38__iand__(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
09203         PyObject *__pyx_r = NULL;
09204         __Pyx_RefNannyDeclarations
09205         PyObject *__pyx_t_1 = NULL;
09206         int __pyx_lineno = 0;
09207         const char *__pyx_filename = NULL;
09208         int __pyx_clineno = 0;
09209         __Pyx_RefNannySetupContext("__iand__", 0);
09210
09211         /* "PyClicl.pyx":864
09212     *     {2}
09213     *     """
09214     *     return self.wrap( self.unwrap() & toClifford(rhs) )                # ««««««««
09215     *
09216     *     def __xor__(lhs, rhs):
09217     */
09218         __Pyx_XDECREFF(__pyx_r);
09219         __pyx_t_1 = __pyx_f_8PyClicl_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClicl_8clifford_unwrap(__pyx_v_self) & __pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 864, __pyx_L1_error)
09220         __Pyx_GOTREF(__pyx_t_1);
09221         __pyx_r = __pyx_t_1;
09222         __pyx_t_1 = 0;
09223         goto __pyx_L0;
09224
09225         /* "PyClicl.pyx":851
09226     *     return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
09227     *
09228     *     def __iand__(self, rhs):                # ««««««««
09229     *         """
09230     *         Inner product.
09231     */
09232
09233         /* function exit code */
09234         __pyx_L1_error;;
09235         __Pyx_XDECREFF(__pyx_t_1);
09236         __Pyx_AddTraceback("PyClicl.clifford.__iand__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09237         __pyx_r = NULL;
09238         __pyx_L0;;
09239         __Pyx_XGIVEREF(__pyx_r);
09240         __Pyx_RefNannyFinishContext();

```

```

09241         return __pyx_r;
09242     }
09243
09244     /* "PyClical.pyx":866
09245     *         return self.wrap( self.unwrap() & toClifford(rhs) )
09246     *
09247     *     def __xor__(lhs, rhs):
09248     *         """
09249     *         Outer product.
09250     */
09251
09252     /* Python wrapper */
09253     static PyObject *__pyx_pw_8PyClical_8clifford_41__xor__(PyObject *__pyx_v_lhs, PyObject
09254 *__pyx_v_rhs); /*proto*/
09255     static char __pyx_doc_8PyClical_8clifford_40__xor__[] = "\n        Outer product.\n\n
>> print(clifford(\"{1}\") ^ clifford(\"{2}\"))\n        {1,2}\n        >> print(clifford(2) ^
clifford(\"{2}\"))\n        2{2}\n        >> print(clifford(\"{1}\") ^ clifford(\"{1}\"))\n        0\n
>> print(clifford(\"{1}\") ^ clifford(\"{1,2}\"))\n        0\n        ";
09256     #if CYTHON_COMPILING_IN_CPYTHON
09257     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_40__xor__;
09258     #endif
09259     static PyObject *__pyx_pw_8PyClical_8clifford_41__xor__(PyObject *__pyx_v_lhs, PyObject
09260 *__pyx_v_rhs) {
09261         PyObject *__pyx_r = 0;
09262         __Pyx_RefNannyDeclarations
09263         __Pyx_RefNannySetupContext("__xor__ (wrapper)", 0);
09264         __pyx_r = __pyx_pf_8PyClical_8clifford_40__xor__((PyObject *)__pyx_v_lhs), ((PyObject
09265 *)__pyx_v_rhs));
09266
09267         /* function exit code */
09268         __Pyx_RefNannyFinishContext();
09269         return __pyx_r;
09270     }
09271
09272     static PyObject *__pyx_pf_8PyClical_8clifford_40__xor__(PyObject *__pyx_v_lhs, PyObject
09273 *__pyx_v_rhs) {
09274         PyObject *__pyx_r = NULL;
09275         __Pyx_RefNannyDeclarations
09276         PyObject *__pyx_t_1 = NULL;
09277         PyObject *__pyx_t_2 = NULL;
09278         int __pyx_lineno = 0;
09279         const char *__pyx_filename = NULL;
09280         int __pyx_clineno = 0;
09281         __Pyx_RefNannySetupContext("__xor__", 0);
09282
09283         /* "PyClical.pyx":879
09284         *         0
09285         *         """
09286         *         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
09287         *         # ««««««««
09288         *
09289         *     def __ixor__(self, rhs):
09290         *
09291         *         __Pyx_XDECREF(__pyx_r);
09292         *         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
09293         *         (unlikely(!__pyx_t_1)) __PYX_ERR(0, 879, __pyx_L1_error)
09294         *         __Pyx_GOTREF(__pyx_t_1);
09295         *         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
09296         *         *)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) ^
09297         *         __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 879,
09298         *         __pyx_L1_error)
09299         *         __Pyx_GOTREF(__pyx_t_2);
09300         *         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09301         *         __pyx_r = __pyx_t_2;
09302         *         __pyx_t_2 = 0;
09303         *         goto __pyx_L0;
09304         *
09305         *         /* "PyClical.pyx":866
09306         *         *         return self.wrap( self.unwrap() & toClifford(rhs) )
09307         *         *
09308         *         def __xor__(lhs, rhs):
09309         *             """
09310         *             Outer product.
09311         */
09312
09313         /* function exit code */
09314         __pyx_L1_error:;
09315         __Pyx_XDECREF(__pyx_t_1);
09316         __Pyx_XDECREF(__pyx_t_2);
09317         __Pyx_AddTraceback("PyClical.clifford.__xor__", __pyx_clineno, __pyx_lineno,
09318 __pyx_filename);
09319         __pyx_r = NULL;
09320         __pyx_L0:;
09321         __Pyx_XGIVEREF(__pyx_r);
09322         __Pyx_RefNannyFinishContext();
09323         return __pyx_r;
09324     }
09325

```

```

09316         /* "PyClical.pyx":881
09317         *         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
09318         *
09319         *     def __ixor__(self, rhs):                # ««««««««
09320         *         """
09321         *         Outer product.
09322         */
09323
09324         /* Python wrapper */
09325         static PyObject *__pyx_pw_8PyClical_8clifford_43__ixor__(PyObject *__pyx_v_self, PyObject
09326 *__pyx_v_rhs); /*proto*/
09326         static char __pyx_doc_8PyClical_8clifford_42__ixor__[] = "\n        Outer product.\n\n
>> x = clifford(\"{1}\"); x ^= clifford(\"{2}\"); print(x)\n        {1,2}\n        >> x = clifford(2);
x ^= clifford(\"{2}\"); print(x)\n        2{2}\n        >> x = clifford(\"{1}\"); x ^=
clifford(\"{1}\"); print(x)\n        0\n        >> x = clifford(\"{1}\"); x ^= clifford(\"{1,2}\");
print(x)\n        0\n        ";
09327         #if CYTHON_COMPILING_IN_CPYTHON
09328         struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_42__ixor__;
09329         #endif
09330         static PyObject *__pyx_pw_8PyClical_8clifford_43__ixor__(PyObject *__pyx_v_self, PyObject
09331 *__pyx_v_rhs) {
09331         PyObject *__pyx_r = 0;
09332         __Pyx_RefNannyDeclarations
09333         __Pyx_RefNannySetupContext("__ixor__ (wrapper)", 0);
09334         __pyx_r = __pyx_pf_8PyClical_8clifford_42__ixor__(((struct __pyx_obj_8PyClical_clifford
09335 *)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09336
09336         /* function exit code */
09337         __Pyx_RefNannyFinishContext();
09338         return __pyx_r;
09339     }
09340
09341     static PyObject *__pyx_pf_8PyClical_8clifford_42__ixor__(struct __pyx_obj_8PyClical_clifford
09342 *__pyx_v_self, PyObject *__pyx_v_rhs) {
09342         PyObject *__pyx_r = NULL;
09343         __Pyx_RefNannyDeclarations
09344         PyObject *__pyx_t_1 = NULL;
09345         int __pyx_lineno = 0;
09346         const char *__pyx_filename = NULL;
09347         int __pyx_clineno = 0;
09348         __Pyx_RefNannySetupContext("__ixor__", 0);
09349
09350         /* "PyClical.pyx":894
09351         *         0
09352         *         """
09353         *         return self.wrap( self.unwrap() ^ toClifford(rhs) )                # ««««««««
09354         *
09355         *     def __truediv__(lhs, rhs):
09356         */
09357         __Pyx_XDECREF(__pyx_r);
09358         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
09359 (__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) ^ __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
09359 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 894, __pyx_L1_error)
09360         __Pyx_GOTREF(__pyx_t_1);
09361         __pyx_r = __pyx_t_1;
09362         __pyx_t_1 = 0;
09363         goto __pyx_L0;
09364
09364         /* "PyClical.pyx":881
09365         *         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
09366         *
09367         *     def __ixor__(self, rhs):                # ««««««««
09368         *         """
09369         *         Outer product.
09370         */
09371
09372         /* function exit code */
09373         __pyx_L1_error:;
09374         __Pyx_XDECREF(__pyx_t_1);
09375         __Pyx_AddTraceback("PyClical.clifford.__ixor__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09376         __pyx_r = NULL;
09377         __pyx_L0:;
09378         __Pyx_XGIVEREF(__pyx_r);
09379         __Pyx_RefNannyFinishContext();
09380         return __pyx_r;
09381     }
09382
09383         /* "PyClical.pyx":896
09384         *         return self.wrap( self.unwrap() ^ toClifford(rhs) )
09385         *
09386         *     def __truediv__(lhs, rhs):                # ««««««««
09387         *         """
09388         *         Geometric quotient.
09389         */
09390
09391         /* Python wrapper */

```

```

09392     static PyObject *__pyx_pw_8PyClical_8clifford_45__truediv__(PyObject *__pyx_v_lhs, PyObject
09393 *__pyx_v_rhs); /*proto*/
09393     static char __pyx_doc_8PyClical_8clifford_44__truediv__[] = "\n          Geometric quotient.\n\n
>> print(clifford("{1}\n") / clifford("{2}\n"))\n          {1,2}\n          >> print(clifford(2) /
clifford("{2}\n"))\n          2{2}\n          >> print(clifford("{1}\n") / clifford("{1}\n"))\n          1\n
>> print(clifford("{1}\n") / clifford("{1,2}\n"))\n          -{2}\n          ";
09394     #if CYTHON_COMPILING_IN_CPYTHON
09395     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_44__truediv__;
09396     #endif
09397     static PyObject *__pyx_pw_8PyClical_8clifford_45__truediv__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09398         PyObject *__pyx_r = 0;
09399         __Pyx_RefNannyDeclarations
09400         __Pyx_RefNannySetupContext("__truediv__ (wrapper)", 0);
09401         __pyx_r = __pyx_pf_8PyClical_8clifford_44__truediv__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
09402
09403         /* function exit code */
09404         __Pyx_RefNannyFinishContext();
09405         return __pyx_r;
09406     }
09407
09408     static PyObject *__pyx_pf_8PyClical_8clifford_44__truediv__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09409         PyObject *__pyx_r = NULL;
09410         __Pyx_RefNannyDeclarations
09411         PyObject *__pyx_t_1 = NULL;
09412         PyObject *__pyx_t_2 = NULL;
09413         int __pyx_lineno = 0;
09414         const char *__pyx_filename = NULL;
09415         int __pyx_clineno = 0;
09416         __Pyx_RefNannySetupContext("__truediv__", 0);
09417
09418         /* "PyClical.pyx":909
09419 *         -{2}
09420 *         """
09421 *         return clifford().wrap( toClifford(lhs) / toClifford(rhs) )          # ««««««««
09422 *
09423 *         def __idiv__(self, rhs):
09424 */
09425         __Pyx_XDECREF(__pyx_r);
09426         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 909, __pyx_L1_error)
09427         __Pyx_GOTREF(__pyx_t_1);
09428         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) /
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 909,
__pyx_L1_error)
09429         __Pyx_GOTREF(__pyx_t_2);
09430         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09431         __pyx_r = __pyx_t_2;
09432         __pyx_t_2 = 0;
09433         goto __pyx_L0;
09434
09435         /* "PyClical.pyx":896
09436 *         return self.wrap( self.unwrap() ^ toClifford(rhs) )
09437 *
09438 *         def __truediv__(lhs, rhs):          # ««««««««
09439 *         """
09440 *         Geometric quotient.
09441 */
09442
09443         /* function exit code */
09444         __pyx_L1_error:;
09445         __Pyx_XDECREF(__pyx_t_1);
09446         __Pyx_XDECREF(__pyx_t_2);
09447         __Pyx_AddTraceback("PyClical.clifford.__truediv__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09448         __pyx_r = NULL;
09449         __pyx_L0:;
09450         __Pyx_XGIVEREF(__pyx_r);
09451         __Pyx_RefNannyFinishContext();
09452         return __pyx_r;
09453     }
09454
09455     /* "PyClical.pyx":911
09456 *         return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
09457 *
09458 *         def __idiv__(self, rhs):          # ««««««««
09459 *         """
09460 *         Geometric quotient.
09461 */
09462
09463         /* Python wrapper */
09464         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
09465         static PyObject *__pyx_pw_8PyClical_8clifford_47__idiv__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/

```

```

09466         static char __pyx_doc_8PyClical_8clifford_46__idiv__[] = "\n          Geometric quotient.\n\n
>> x = clifford(\"{1}\"); x /= clifford(\"{2}\"); print(x)\n          {1,2}\n          >> x = clifford(2);
x /= clifford(\"{2}\"); print(x)\n          2{2}\n          >> x = clifford(\"{1}\"); x /=
clifford(\"{1}\"); print(x)\n          1\n          >> x = clifford(\"{1}\"); x /= clifford(\"{1,2}\");
print(x)\n          -{2}\n          ";
09467     #if CYTHON_COMPILING_IN_CPYTHON
09468     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_46__idiv__;
09469     #endif
09470     static PyObject * __pyx_pw_8PyClical_8clifford_47__idiv__(PyObject * __pyx_v_self, PyObject
* __pyx_v_rhs) {
09471         PyObject * __pyx_r = 0;
09472         __Pyx_RefNannyDeclarations
09473         __Pyx_RefNannySetupContext("__idiv__ (wrapper)", 0);
09474         __pyx_r = __pyx_pf_8PyClical_8clifford_46__idiv__(((struct __pyx_obj_8PyClical_clifford
*) __pyx_v_self), ((PyObject *) __pyx_v_rhs));
09475
09476         /* function exit code */
09477         __Pyx_RefNannyFinishContext();
09478         return __pyx_r;
09479     }
09480     #endif
09481
09482     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
09483     static PyObject * __pyx_pf_8PyClical_8clifford_46__idiv__(struct __pyx_obj_8PyClical_clifford
* __pyx_v_self, PyObject * __pyx_v_rhs) {
09484         PyObject * __pyx_r = NULL;
09485         __Pyx_RefNannyDeclarations
09486         PyObject * __pyx_t_1 = NULL;
09487         int __pyx_lineno = 0;
09488         const char * __pyx_filename = NULL;
09489         int __pyx_clineno = 0;
09490         __Pyx_RefNannySetupContext("__idiv__", 0);
09491
09492         /* "PyClical.pyx":924
09493         *
09494         *
09495         *     return self.wrap( self.unwrap() / toClifford(rhs) )           # ««««««««
09496         *
09497         *     def inv(self):
09498         */
09499         __Pyx_XDECREF(__pyx_r);
09500         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) / __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 924, __pyx_L1_error)
09501         __Pyx_GOTREF(__pyx_t_1);
09502         __pyx_r = __pyx_t_1;
09503         __pyx_t_1 = 0;
09504         goto __pyx_L0;
09505
09506         /* "PyClical.pyx":911
09507         *     return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
09508         *
09509         *     def __idiv__(self, rhs):           # ««««««««
09510         *         """
09511         *         Geometric quotient.
09512         */
09513
09514         /* function exit code */
09515         __pyx_L1_error:;
09516         __Pyx_XDECREF(__pyx_t_1);
09517         __Pyx_AddTraceback("PyClical.clifford.__idiv__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09518         __pyx_r = NULL;
09519         __pyx_L0:;
09520         __Pyx_XGIVEREF(__pyx_r);
09521         __Pyx_RefNannyFinishContext();
09522         return __pyx_r;
09523     }
09524     #endif
09525
09526     /* "PyClical.pyx":926
09527     *     return self.wrap( self.unwrap() / toClifford(rhs) )
09528     *
09529     *     def inv(self):           # ««««««««
09530     *         """
09531     *         Geometric multiplicative inverse.
09532     */
09533
09534     /* Python wrapper */
09535     static PyObject * __pyx_pw_8PyClical_8clifford_49inv(PyObject * __pyx_v_self, CYTHON_UNUSED
PyObject * unused); /*proto*/
09536     static char __pyx_doc_8PyClical_8clifford_48inv[] = "\n          Geometric multiplicative
inverse.\n\n          >> x = clifford(\"{1}\"); print(x.inv())\n          {1}\n          >> x = clifford(2);
print(x.inv())\n          0.5\n          >> x = clifford(\"{1,2}\"); print(x.inv())\n          -{1,2}\n
          ";
09537     static PyObject * __pyx_pw_8PyClical_8clifford_49inv(PyObject * __pyx_v_self, CYTHON_UNUSED
PyObject * unused) {

```



```

09538     PyObject *__pyx_r = 0;
09539     __Pyx_RefNannyDeclarations
09540     __Pyx_RefNannySetupContext("inv (wrapper)", 0);
09541     __pyx_r = __pyx_pf_8PyClical_8clifford_48inv(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
09542
09543     /* function exit code */
09544     __Pyx_RefNannyFinishContext();
09545     return __pyx_r;
09546 }
09547
09548 static PyObject *__pyx_pf_8PyClical_8clifford_48inv(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
09549     PyObject *__pyx_r = NULL;
09550     __Pyx_RefNannyDeclarations
09551     PyObject *__pyx_t_1 = NULL;
09552     PyObject *__pyx_t_2 = NULL;
09553     int __pyx_lineno = 0;
09554     const char *__pyx_filename = NULL;
09555     int __pyx_clineno = 0;
09556     __Pyx_RefNannySetupContext("inv", 0);
09557
09558     /* "PyClical.pyx":937
09559     *     -{1,2}
09560     *     """
09561     *     return clifford().wrap( self.instance.inv() )           # ««««««««
09562     *
09563     *     def __or__(lhs, rhs):
09564     */
09565     __Pyx_XDECREF(__pyx_r);
09566     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 937, __pyx_L1_error)
09567     __Pyx_GOTREF(__pyx_t_1);
09568     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->inv()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 937,
__pyx_L1_error)
09569     __Pyx_GOTREF(__pyx_t_2);
09570     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09571     __pyx_r = __pyx_t_2;
09572     __pyx_t_2 = 0;
09573     goto __pyx_L0;
09574
09575     /* "PyClical.pyx":926
09576     *     return self.wrap( self.unwrap() / toClifford(rhs) )
09577     *
09578     *     def inv(self):           # ««««««««
09579     *     """
09580     *     Geometric multiplicative inverse.
09581     */
09582
09583     /* function exit code */
09584     __pyx_L1_error:;
09585     __Pyx_XDECREF(__pyx_t_1);
09586     __Pyx_XDECREF(__pyx_t_2);
09587     __Pyx_AddTraceback("PyClical.clifford.inv", __pyx_clineno, __pyx_lineno, __pyx_filename);
09588     __pyx_r = NULL;
09589     __pyx_L0:;
09590     __Pyx_XGIVEREF(__pyx_r);
09591     __Pyx_RefNannyFinishContext();
09592     return __pyx_r;
09593 }
09594
09595     /* "PyClical.pyx":939
09596     *     return clifford().wrap( self.instance.inv() )
09597     *
09598     *     def __or__(lhs, rhs):           # ««««««««
09599     *     """
09600     *     Transform left hand side, using right hand side as a transformation.
09601     */
09602
09603     /* Python wrapper */
09604     static PyObject *__pyx_pw_8PyClical_8clifford_51__or__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
09605     static char __pyx_doc_8PyClical_8clifford_50__or__[] = "\n        Transform left hand side,
using right hand side as a transformation.\n\n        >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); print(y|x)\n        -{1}\n        >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); print(y|exp(x))\n        -{1}\n        ";
09606     #if CYTHON_COMPILING_IN_CPYTHON
09607     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_50__or__;
09608     #endif
09609     static PyObject *__pyx_pw_8PyClical_8clifford_51__or__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09610         PyObject *__pyx_r = 0;
09611         __Pyx_RefNannyDeclarations
09612         __Pyx_RefNannySetupContext("__or__ (wrapper)", 0);
09613         __pyx_r = __pyx_pf_8PyClical_8clifford_50__or__(((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));

```

```

09614
09615     /* function exit code */
09616     __Pyx_RefNannyFinishContext();
09617     return __pyx_r;
09618 }
09619
09620 static PyObject *__pyx_pf_8PyClical_8clifford_50__or__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09621     PyObject *__pyx_r = NULL;
09622     __Pyx_RefNannyDeclarations
09623     PyObject *__pyx_t_1 = NULL;
09624     PyObject *__pyx_t_2 = NULL;
09625     int __pyx_lineno = 0;
09626     const char *__pyx_filename = NULL;
09627     int __pyx_clineno = 0;
09628     __Pyx_RefNannySetupContext("__or__", 0);
09629
09630     /* "PyClical.pyx":948
09631     *
09632     *
09633     *     return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
09634     *
09635     *     def __ior__(self, rhs):
09636     */
09637     __Pyx_XDECREF(__pyx_r);
09638     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 948, __pyx_L1_error)
09639     __Pyx_GOTREF(__pyx_t_1);
09640     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) |
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 948,
__pyx_L1_error)
09641     __Pyx_GOTREF(__pyx_t_2);
09642     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09643     __pyx_r = __pyx_t_2;
09644     __pyx_t_2 = 0;
09645     goto __pyx_L0;
09646
09647     /* "PyClical.pyx":939
09648     *
09649     *     return clifford().wrap( self.instance.inv() )
09650     *
09651     *     def __or__(lhs, rhs):
09652     *
09653     *         Transform left hand side, using right hand side as a transformation.
09654     */
09655     /* function exit code */
09656     __pyx_L1_error:;
09657     __Pyx_XDECREF(__pyx_t_1);
09658     __Pyx_XDECREF(__pyx_t_2);
09659     __Pyx_AddTraceback("PyClical.clifford.__or__", __pyx_clineno, __pyx_lineno, __pyx_filename);
09660     __pyx_r = NULL;
09661     __pyx_L0:;
09662     __Pyx_XGIVEREF(__pyx_r);
09663     __Pyx_RefNannyFinishContext();
09664     return __pyx_r;
09665 }
09666
09667     /* "PyClical.pyx":950
09668     *
09669     *     return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
09670     *
09671     *     def __ior__(self, rhs):
09672     *
09673     *         Transform left hand side, using right hand side as a transformation.
09674     */
09675     /* Python wrapper */
09676     static PyObject *__pyx_pw_8PyClical_8clifford_53__ior__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09677     static char __pyx_doc_8PyClical_8clifford_52__ior__[] = "\n
Transform left hand side,
using right hand side as a transformation.\n\n
>> x=clifford("{1,2}") * pi/2;
y=clifford("{1}") * pi/2; print(y)\n
- {1}\n
>> x=clifford("{1,2}") * pi/2;
y=clifford("{1}") * pi/2; print(y)\n
- {1}\n
";
09678     #if CYTHON_COMPILING_IN_CPYTHON
09679     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_52__ior__;
09680     #endif
09681     static PyObject *__pyx_pw_8PyClical_8clifford_53__ior__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
09682     PyObject *__pyx_r = 0;
09683     __Pyx_RefNannyDeclarations
09684     __Pyx_RefNannySetupContext("__ior__ (wrapper)", 0);
09685     __pyx_r = __pyx_pf_8PyClical_8clifford_52__ior__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09686
09687     /* function exit code */
09688     __Pyx_RefNannyFinishContext();
09689     return __pyx_r;

```

```

09690     }
09691
09692     static PyObject *__pyx_pf_8PyClical_8clifford_52__ior__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_rhs) {
09693         PyObject *__pyx_r = NULL;
09694         __Pyx_RefNannyDeclarations
09695         PyObject *__pyx_t_1 = NULL;
09696         int __pyx_lineno = 0;
09697         const char *__pyx_filename = NULL;
09698         int __pyx_clineno = 0;
09699         __Pyx_RefNannySetupContext("__ior__", 0);
09700
09701         /* "PyClical.pyx":959
09702         *     -{1}
09703         *     """
09704         *     return self.wrap( self.unwrap() | toClifford(rhs) )           # ««««««««
09705         *
09706         *     def __pow__(self, m, dummy):
09707         */
09708         __Pyx_XDECREF(__pyx_r);
09709         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) | __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 959, __pyx_L1_error)
09710         __Pyx_GOTREF(__pyx_t_1);
09711         __pyx_r = __pyx_t_1;
09712         __pyx_t_1 = 0;
09713         goto __pyx_L0;
09714
09715         /* "PyClical.pyx":950
09716         *     return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
09717         *
09718         *     def __ior__(self, rhs):           # ««««««««
09719         *     """
09720         *     Transform left hand side, using right hand side as a transformation.
09721         */
09722
09723         /* function exit code */
09724         __pyx_L1_error:;
09725         __Pyx_XDECREF(__pyx_t_1);
09726         __Pyx_AddTraceback("PyClical.clifford.__ior__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09727         __pyx_r = NULL;
09728         __pyx_L0:;
09729         __Pyx_XGIVEREF(__pyx_r);
09730         __Pyx_RefNannyFinishContext();
09731         return __pyx_r;
09732     }
09733
09734     /* "PyClical.pyx":961
09735     *     return self.wrap( self.unwrap() | toClifford(rhs) )
09736     *
09737     *     def __pow__(self, m, dummy):           # ««««««««
09738     *     """
09739     *     Power: self to the m.
09740     */
09741
09742     /* Python wrapper */
09743     static PyObject *__pyx_pw_8PyClical_8clifford_55__pow__(PyObject *__pyx_v_self, PyObject
__pyx_v_m, PyObject *__pyx_v_dummy); /*proto*/
09744     static char __pyx_doc_8PyClical_8clifford_54__pow__[] = "\n        Power: self to the m.\n\n
>> x=clifford(\"{1}\"); print(x ** 2)\n                1\n                >> x=clifford(\"2\"); print(x ** 2)\n
4\n                >> x=clifford(\"2+{1}\"); print(x ** 0)\n                1\n                >> x=clifford(\"2+{1}\");
print(x ** 1)\n                2+{1}\n                >> x=clifford(\"2+{1}\"); print(x ** 2)\n                5+4{1}\n
>> i=clifford(\"{1,2}\"); print(exp(pi/2) * (i ** i))\n                1\n                ";
09745     #if CYTHON_COMPILING_IN_CPYTHON
09746     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_54__pow__;
09747     #endif
09748     static PyObject *__pyx_pw_8PyClical_8clifford_55__pow__(PyObject *__pyx_v_self, PyObject
__pyx_v_m, PyObject *__pyx_v_dummy) {
09749         PyObject *__pyx_r = 0;
09750         __Pyx_RefNannyDeclarations
09751         __Pyx_RefNannySetupContext("__pow__ (wrapper)", 0);
09752         __pyx_r = __pyx_pf_8PyClical_8clifford_54__pow__((PyObject *)__pyx_v_self), ((PyObject
*)__pyx_v_m), ((PyObject *)__pyx_v_dummy));
09753
09754         /* function exit code */
09755         __Pyx_RefNannyFinishContext();
09756         return __pyx_r;
09757     }
09758
09759     static PyObject *__pyx_pf_8PyClical_8clifford_54__pow__(PyObject *__pyx_v_self, PyObject
__pyx_v_m, CYTHON_UNUSED PyObject *__pyx_v_dummy) {
09760         PyObject *__pyx_r = NULL;
09761         __Pyx_RefNannyDeclarations
09762         PyObject *__pyx_t_1 = NULL;
09763         int __pyx_lineno = 0;
09764         const char *__pyx_filename = NULL;

```

```

09765         int __pyx_clineno = 0;
09766         __Pyx_RefNannySetupContext("__pow__", 0);
09767
09768         /* "PyClicl.pyx":978
09769  *         1
09770  *         """
09771  *         return pow(self, m)                # ««««««««
09772  *
09773  *         def pow(self, m):
09774  */
09775         __Pyx_XDECREF(__pyx_r);
09776         __pyx_t_1 = __pyx_f_8PyClicl_pow(__pyx_v_self, __pyx_v_m, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 978, __pyx_L1_error)
09777         __Pyx_GOTREF(__pyx_t_1);
09778         __pyx_r = __pyx_t_1;
09779         __pyx_t_1 = 0;
09780         goto __pyx_L0;
09781
09782         /* "PyClicl.pyx":961
09783  *         return self.wrap( self.unwrap() | toClifford(rhs) )
09784  *
09785  *         def __pow__(self, m, dummy):                # ««««««««
09786  *         """
09787  *         Power: self to the m.
09788  */
09789
09790         /* function exit code */
09791         __pyx_L1_error:;
09792         __Pyx_XDECREF(__pyx_t_1);
09793         __Pyx_AddTraceback("PyClicl.clifford.__pow__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09794         __pyx_r = NULL;
09795         __pyx_L0:;
09796         __Pyx_XGIVEREF(__pyx_r);
09797         __Pyx_RefNannyFinishContext();
09798         return __pyx_r;
09799     }
09800
09801     /* "PyClicl.pyx":980
09802  *         return pow(self, m)
09803  *
09804  *         def pow(self, m):                # ««««««««
09805  *         """
09806  *         Power: self to the m.
09807  */
09808
09809     /* Python wrapper */
09810     static PyObject *__pyx_pw_8PyClicl_8clifford_57pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m); /*proto*/
09811     static char __pyx_doc_8PyClicl_8clifford_56pow[] = "\n        Power: self to the m.\n\n
>> x=clifford(\"{1}\"); print(x.pow(2))\n                1\n                >> x=clifford(\"2\"); print(x.pow(2))\n
4\n                >> x=clifford(\"2+{1}\"); print(x.pow(0))\n                1\n                >> x=clifford(\"2+{1}\");
print(x.pow(1))\n                2+{1}\n                >> x=clifford(\"2+{1}\"); print(x.pow(2))\n                5+4{1}\n
>> print(clifford(\"1+{1}+{1,2}\").pow(3))\n                1+3{1}+3{1,2}\n                >> i=clifford(\"{1,2}\");
print(exp(pi/2) * i.pow(i))\n                1\n                ";
09812     static PyObject *__pyx_pw_8PyClicl_8clifford_57pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m) {
09813         PyObject *__pyx_r = 0;
09814         __Pyx_RefNannyDeclarations
09815         __Pyx_RefNannySetupContext("pow (wrapper)", 0);
09816         __pyx_r = __pyx_pf_8PyClicl_8clifford_56pow(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_m));
09817
09818         /* function exit code */
09819         __Pyx_RefNannyFinishContext();
09820         return __pyx_r;
09821     }
09822
09823     static PyObject *__pyx_pf_8PyClicl_8clifford_56pow(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject *__pyx_v_m) {
09824         PyObject *__pyx_r = NULL;
09825         __Pyx_RefNannyDeclarations
09826         PyObject *__pyx_t_1 = NULL;
09827         PyObject *__pyx_t_2 = NULL;
09828         int __pyx_t_3;
09829         int __pyx_t_4;
09830         int __pyx_t_5;
09831         int __pyx_lineno = 0;
09832         const char *__pyx_filename = NULL;
09833         int __pyx_clineno = 0;
09834         __Pyx_RefNannySetupContext("pow", 0);
09835
09836         /* "PyClicl.pyx":999
09837  *         1
09838  *         """
09839  *         if isinstance(m, numbers.Integral):                # ««««««««
09840  *         return clifford().wrap( self.instance.pow(m) )

```

```

09841 *           else:
09842 */
09843     __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_numbers); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 999, __pyx_L1_error)
09844     __Pyx_GOTREF(__pyx_t_1);
09845     __pyx_t_2 = __Pyx_PyObject_GetAttrStr(__pyx_t_1, __pyx_n_s_Integral); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 999, __pyx_L1_error)
09846     __Pyx_GOTREF(__pyx_t_2);
09847     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09848     __pyx_t_3 = PyObject_IsInstance(__pyx_v_m, __pyx_t_2); if (unlikely(__pyx_t_3 == ((int)-1)))
__PYX_ERR(0, 999, __pyx_L1_error)
09849     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
09850     __pyx_t_4 = (__pyx_t_3 != 0);
09851     if (__pyx_t_4) {
09852
09853         /* "PyClical.pyx":1000
09854 *         """
09855 *         if isinstance(m, numbers.Integral):
09856 *             return clifford().wrap( self.instance.pow(m) )           # ««««««««
09857 *         else:
09858 *             return exp(m * log(self))
09859 */
09860         __Pyx_XDECREF(__pyx_r);
09861         __pyx_t_2 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 1000, __pyx_L1_error)
09862         __Pyx_GOTREF(__pyx_t_2);
09863         __pyx_t_5 = __Pyx_PyInt_As_int(__pyx_v_m); if (unlikely((__pyx_t_5 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 1000, __pyx_L1_error)
09864         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_2), __pyx_v_self->instance->pow(__pyx_t_5)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1000,
__pyx_L1_error)
09865         __Pyx_GOTREF(__pyx_t_1);
09866         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
09867         __pyx_r = __pyx_t_1;
09868         __pyx_t_1 = 0;
09869         goto __pyx_L0;
09870
09871         /* "PyClical.pyx":999
09872 *         1
09873 *         """
09874 *         if isinstance(m, numbers.Integral):           # ««««««««
09875 *             return clifford().wrap( self.instance.pow(m) )
09876 *         else:
09877 */
09878     }
09879
09880     /* "PyClical.pyx":1002
09881 *         return clifford().wrap( self.instance.pow(m) )
09882 *     else:
09883 *         return exp(m * log(self))           # ««««««««
09884 *
09885 *     def outer_pow(self, m):
09886 */
09887     /*else*/ {
09888         __Pyx_XDECREF(__pyx_r);
09889         __pyx_t_1 = __pyx_f_8PyClical_log(((PyObject *)__pyx_v_self), 0, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1002, __pyx_L1_error)
09890         __Pyx_GOTREF(__pyx_t_1);
09891         __pyx_t_2 = PyNumber_Multiply(__pyx_v_m, __pyx_t_1); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1002, __pyx_L1_error)
09892         __Pyx_GOTREF(__pyx_t_2);
09893         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09894         __pyx_t_1 = __pyx_f_8PyClical_exp(__pyx_t_2, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1002, __pyx_L1_error)
09895         __Pyx_GOTREF(__pyx_t_1);
09896         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
09897         __pyx_r = __pyx_t_1;
09898         __pyx_t_1 = 0;
09899         goto __pyx_L0;
09900     }
09901
09902     /* "PyClical.pyx":980
09903 *         return pow(self, m)
09904 *
09905 *     def pow(self, m):           # ««««««««
09906 *         """
09907 *         Power: self to the m.
09908 */
09909
09910     /* function exit code */
09911     __pyx_L1_error:;
09912     __Pyx_XDECREF(__pyx_t_1);
09913     __Pyx_XDECREF(__pyx_t_2);
09914     __Pyx_AddTraceback("PyClical.clifford.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
09915     __pyx_r = NULL;
09916     __pyx_L0:;
09917     __Pyx_XGIVEREF(__pyx_r);

```

```

09918     __Pyx_RefNannyFinishContext();
09919     return __pyx_r;
09920 }
09921
09922     /* "PyClicl.pyx":1004
09923     *         return exp(m * log(self))
09924     *
09925     * def outer_pow(self, m):          # ««««««««
09926     *     """
09927     *     Outer product power.
09928     */
09929
09930     /* Python wrapper */
09931     static PyObject *__pyx_pw_8PyClicl_8clifford_59outer_pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m); /*proto*/
09932     static char __pyx_doc_8PyClicl_8clifford_58outer_pow[] = "\n        Outer product power.\n\n
>> x=clifford(\"2+{1}\") ; print(x.outer_pow(0))\n        1\n        >> x=clifford(\"2+{1}\");
print(x.outer_pow(1))\n        2+{1}\n        >> x=clifford(\"2+{1}\"); print(x.outer_pow(2))\n
4+4{1}\n        >> print (clifford(\"1+{1}+{1,2}\").outer_pow(3))\n        1+3{1}+3{1,2}\n\n
";
09933     static PyObject *__pyx_pw_8PyClicl_8clifford_59outer_pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m) {
09934         PyObject *__pyx_r = 0;
09935         __Pyx_RefNannyDeclarations
09936         __Pyx_RefNannySetupContext("outer_pow (wrapper)", 0);
09937         __pyx_r = __pyx_pf_8PyClicl_8clifford_58outer_pow(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_m));
09938
09939         /* function exit code */
09940         __Pyx_RefNannyFinishContext();
09941         return __pyx_r;
09942     }
09943
09944     static PyObject *__pyx_pf_8PyClicl_8clifford_58outer_pow(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject *__pyx_v_m) {
09945         PyObject *__pyx_r = NULL;
09946         __Pyx_RefNannyDeclarations
09947         PyObject *__pyx_t_1 = NULL;
09948         int __pyx_t_2;
09949         PyObject *__pyx_t_3 = NULL;
09950         int __pyx_lineno = 0;
09951         const char *__pyx_filename = NULL;
09952         int __pyx_clineno = 0;
09953         __Pyx_RefNannySetupContext("outer_pow", 0);
09954
09955         /* "PyClicl.pyx":1018
09956     *
09957     *     """
09958     *     return clifford().wrap( self.instance.outer_pow(m) )          # ««««««««
09959     *
09960     * def __call__(self, grade):
09961     */
09962         __Pyx_XDECREF(__pyx_r);
09963         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1018, __pyx_L1_error)
09964         __Pyx_GOTREF(__pyx_t_1);
09965         __pyx_t_2 = __Pyx_PyInt_As_int(__pyx_v_m); if (unlikely((__pyx_t_2 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 1018, __pyx_L1_error)
09966         __pyx_t_3 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), __pyx_v_self->instance->outer_pow(__pyx_t_2)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0,
1018, __pyx_L1_error)
09967         __Pyx_GOTREF(__pyx_t_3);
09968         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09969         __pyx_r = __pyx_t_3;
09970         __pyx_t_3 = 0;
09971         goto __pyx_L0;
09972
09973         /* "PyClicl.pyx":1004
09974     *         return exp(m * log(self))
09975     *
09976     * def outer_pow(self, m):          # ««««««««
09977     *     """
09978     *     Outer product power.
09979     */
09980
09981         /* function exit code */
09982         __pyx_L1_error:;
09983         __Pyx_XDECREF(__pyx_t_1);
09984         __Pyx_XDECREF(__pyx_t_3);
09985         __Pyx_AddTraceback("PyClicl.clifford.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09986         __pyx_r = NULL;
09987         __pyx_L0:;
09988         __Pyx_XGIVEREF(__pyx_r);
09989         __Pyx_RefNannyFinishContext();
09990         return __pyx_r;
09991     }
09992

```

```

09993         /* "PyClicl.pyx":1020
09994         *         return clifford().wrap( self.instance.outer_pow(m) )
09995         *
09996         *     def __call__(self, grade):                # ««««««««
09997         *         """
09998         *         Pure grade-vector part.
09999         */
10000
10001         /* Python wrapper */
10002         static PyObject *__pyx_pw_8PyClicl_8clifford_61__call__(PyObject *__pyx_v_self, PyObject
10003         *__pyx_args, PyObject *__pyx_kwds); /*proto*/
10004         static char __pyx_doc_8PyClicl_8clifford_60__call__[] = "\n        Pure grade-vector
10005         part.\n\n        >> print(clifford(\"{1}\") (1))\n                {1}\n                >> print(clifford(\"{1}\") (0))\n0\n        >> print(clifford(\"1+{1}+{1,2}\") (0))\n                1\n        >> print(clifford(\"1+{1}+{1,2}\") (1))\n                {1}\n        >> print(clifford(\"1+{1}+{1,2}\") (2))\n        {1,2}\n        >> print(clifford(\"1+{1}+{1,2}\") (3))\n                0\n        ";
10006         #if CYTHON_COMPILING_IN_CPYTHON
10007         struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_60__call__;
10008         #endif
10009         static PyObject *__pyx_pw_8PyClicl_8clifford_61__call__(PyObject *__pyx_v_self, PyObject
10010         *__pyx_args, PyObject *__pyx_kwds) {
10011             PyObject *__pyx_v_grade = 0;
10012             int __pyx_lineno = 0;
10013             const char *__pyx_filename = NULL;
10014             int __pyx_clineno = 0;
10015             PyObject *__pyx_r = 0;
10016             __Pyx_RefNannyDeclarations
10017             __Pyx_RefNannySetupContext("__call__ (wrapper)", 0);
10018             {
10019                 static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_grade,0};
10020                 PyObject* values[1] = {0};
10021                 if (unlikely(__pyx_kwds)) {
10022                     Py_ssize_t kw_args;
10023                     const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
10024                     switch (pos_args) {
10025                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10026                         CYTHON_FALLTHROUGH;
10027                         case 0: break;
10028                         default: goto __pyx_L5_argtuple_error;
10029                     }
10030                     kw_args = PyDict_Size(__pyx_kwds);
10031                     switch (pos_args) {
10032                         case 0:
10033                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_grade)) != 0))
10034                             kw_args--;
10035                         else goto __pyx_L5_argtuple_error;
10036                     }
10037                     if (unlikely(kw_args > 0)) {
10038                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
10039                             pos_args, "__call__") < 0)) __PYX_ERR(0, 1020, __pyx_L3_error)
10040                     }
10041                     } else if (PyTuple_GET_SIZE(__pyx_args) != 1) {
10042                         goto __pyx_L5_argtuple_error;
10043                     } else {
10044                         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10045                     }
10046                     __pyx_v_grade = values[0];
10047                 }
10048                 goto __pyx_L4_argument_unpacking_done;
10049                 __pyx_L5_argtuple_error:;
10050                 __Pyx_RaiseArgtupleInvalid("__call__", 1, 1, 1, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0,
10051                 1020, __pyx_L3_error)
10052                 __pyx_L3_error:;
10053                 __Pyx_AddTraceback("PyClicl.clifford.__call__", __pyx_clineno, __pyx_lineno,
10054                 __pyx_filename);
10055                 __Pyx_RefNannyFinishContext();
10056                 return NULL;
10057                 __pyx_L4_argument_unpacking_done:;
10058                 __pyx_r = __pyx_pf_8PyClicl_8clifford_60__call__(((struct __pyx_obj_8PyClicl_clifford
10059                 *)__pyx_v_self), __pyx_v_grade);
10060
10061                 /* function exit code */
10062                 __Pyx_RefNannyFinishContext();
10063                 return __pyx_r;
10064             }
10065
10066             static PyObject *__pyx_pf_8PyClicl_8clifford_60__call__(struct __pyx_obj_8PyClicl_clifford
10067             *__pyx_v_self, PyObject *__pyx_v_grade) {
10068                 PyObject *__pyx_r = NULL;
10069                 __Pyx_RefNannyDeclarations
10070                 PyObject *__pyx_t_1 = NULL;
10071                 int __pyx_t_2;
10072                 PyObject *__pyx_t_3 = NULL;
10073                 int __pyx_lineno = 0;
10074                 const char *__pyx_filename = NULL;
10075                 int __pyx_clineno = 0;
10076                 __Pyx_RefNannySetupContext("__call__", 0);

```

```

10068
10069      /* "PyClicl.pyx":1037
10070      *      0
10071      *      """
10072      *      return clifford().wrap( self.instance.call(grade) )      # ««««««««
10073      *
10074      *      def scalar(self):
10075      */
10076      __Pyx_XDECREF(__pyx_r);
10077      __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1037, __pyx_L1_error)
10078      __Pyx_GOTREF(__pyx_t_1);
10079      __pyx_t_2 = __Pyx_PyInt_As_int(__pyx_v_grade); if (unlikely((__pyx_t_2 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 1037, __pyx_L1_error)
10080      __pyx_t_3 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), __pyx_v_self->instance->operator() (__pyx_t_2)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0,
1037, __pyx_L1_error)
10081      __Pyx_GOTREF(__pyx_t_3);
10082      __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10083      __pyx_r = __pyx_t_3;
10084      __pyx_t_3 = 0;
10085      goto __pyx_L0;
10086
10087      /* "PyClicl.pyx":1020
10088      *      return clifford().wrap( self.instance.outer_pow(m) )
10089      *
10090      *      def __call__(self, grade):      # ««««««««
10091      *      """
10092      *      Pure grade-vector part.
10093      */
10094
10095      /* function exit code */
10096      __pyx_L1_error:;
10097      __Pyx_XDECREF(__pyx_t_1);
10098      __Pyx_XDECREF(__pyx_t_3);
10099      __Pyx_AddTraceback("PyClicl.clifford.__call__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10100      __pyx_r = NULL;
10101      __pyx_L0:;
10102      __Pyx_XGIVEREF(__pyx_r);
10103      __Pyx_RefNannyFinishContext();
10104      return __pyx_r;
10105  }
10106
10107      /* "PyClicl.pyx":1039
10108      *      return clifford().wrap( self.instance.call(grade) )
10109      *
10110      *      def scalar(self):      # ««««««««
10111      *      """
10112      *      Scalar part.
10113      */
10114
10115      /* Python wrapper */
10116      static PyObject *__pyx_pw_8PyClicl_8clifford_63scalar(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10117      static char __pyx_doc_8PyClicl_8clifford_62scalar[] = "\n      Scalar part.\n\n      >>
clifford(\"1+{1}+{3,2}\").scalar()\n      1.0\n      >> clifford(\"{1,2}\").scalar()\n
0.0\n      ";
10118      static PyObject *__pyx_pw_8PyClicl_8clifford_63scalar(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10119      PyObject *__pyx_r = 0;
10120      __Pyx_RefNannyDeclarations
10121      __Pyx_RefNannySetupContext("scalar (wrapper)", 0);
10122      __pyx_r = __pyx_pf_8PyClicl_8clifford_62scalar(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self));
10123
10124      /* function exit code */
10125      __Pyx_RefNannyFinishContext();
10126      return __pyx_r;
10127  }
10128
10129      static PyObject *__pyx_pf_8PyClicl_8clifford_62scalar(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self) {
10130      PyObject *__pyx_r = NULL;
10131      __Pyx_RefNannyDeclarations
10132      PyObject *__pyx_t_1 = NULL;
10133      int __pyx_lineno = 0;
10134      const char *__pyx_filename = NULL;
10135      int __pyx_clineno = 0;
10136      __Pyx_RefNannySetupContext("scalar", 0);
10137
10138      /* "PyClicl.pyx":1048
10139      *      0.0
10140      *      """
10141      *      return self.instance.scalar()      # ««««««««
10142      *
10143      *      def pure(self):

```



```

10144  */
10145      __Pyx_XDECREF(__pyx_r);
10146      __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->scalar()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1048, __pyx_L1_error)
10147      __Pyx_GOTREF(__pyx_t_1);
10148      __pyx_r = __pyx_t_1;
10149      __pyx_t_1 = 0;
10150      goto __pyx_L0;
10151
10152      /* "PyClical.pyx":1039
10153      *      return clifford().wrap( self.instance.call(grade) )
10154      *
10155      *      def scalar(self):
10156      *          """
10157      *          Scalar part.
10158      */
10159
10160      /* function exit code */
10161      __pyx_L1_error:;
10162      __Pyx_XDECREF(__pyx_t_1);
10163      __Pyx_AddTraceback("PyClical.clifford.scalar", __pyx_clineno, __pyx_lineno, __pyx_filename);
10164      __pyx_r = NULL;
10165      __pyx_L0:;
10166      __Pyx_XGIVEREF(__pyx_r);
10167      __Pyx_RefNannyFinishContext();
10168      return __pyx_r;
10169  }
10170
10171      /* "PyClical.pyx":1050
10172      *      return self.instance.scalar()
10173      *
10174      *      def pure(self):
10175      *          """
10176      *          Pure part.
10177      */
10178
10179      /* Python wrapper */
10180      static PyObject *__pyx_pw_8PyClical_8clifford_65pure(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10181      static char __pyx_doc_8PyClical_8clifford_64pure[] = "\n      Pure part.\n\n      >>
print(clifford(\"{1}+{1}+{1,2}\").pure())\n      {1}+{1,2}\n      >>
print(clifford(\"{1,2}\").pure())\n      {1,2}\n      ";
10182      static PyObject *__pyx_pw_8PyClical_8clifford_65pure(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10183          PyObject *__pyx_r = 0;
10184          __Pyx_RefNannyDeclarations
10185          __Pyx_RefNannySetupContext("pure (wrapper)", 0);
10186          __pyx_r = __pyx_pf_8PyClical_8clifford_64pure(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10187
10188      /* function exit code */
10189      __Pyx_RefNannyFinishContext();
10190      return __pyx_r;
10191  }
10192
10193      static PyObject *__pyx_pf_8PyClical_8clifford_64pure(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
10194          PyObject *__pyx_r = NULL;
10195          __Pyx_RefNannyDeclarations
10196          PyObject *__pyx_t_1 = NULL;
10197          PyObject *__pyx_t_2 = NULL;
10198          int __pyx_lineno = 0;
10199          const char *__pyx_filename = NULL;
10200          int __pyx_clineno = 0;
10201          __Pyx_RefNannySetupContext("pure", 0);
10202
10203      /* "PyClical.pyx":1059
10204      *      {1,2}
10205      *      """
10206      *      return clifford().wrap( self.instance.pure() )
10207      *
10208      *      def even(self):
10209      */
10210      __Pyx_XDECREF(__pyx_r);
10211      __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1059, __pyx_L1_error)
10212      __Pyx_GOTREF(__pyx_t_1);
10213      __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->pure()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1059,
__pyx_L1_error)
10214      __Pyx_GOTREF(__pyx_t_2);
10215      __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10216      __pyx_r = __pyx_t_2;
10217      __pyx_t_2 = 0;
10218      goto __pyx_L0;
10219
10220      /* "PyClical.pyx":1050

```

```

10221 *         return self.instance.scalar()
10222 *
10223 *     def pure(self):             # ««««««««
10224 *         """
10225 *         Pure part.
10226 */
10227
10228     /* function exit code */
10229     __pyx_L1_error++;
10230     __Pyx_XDECREF(__pyx_t_1);
10231     __Pyx_XDECREF(__pyx_t_2);
10232     __Pyx_AddTraceback("PyClicl.clifford.pure", __pyx_clineno, __pyx_lineno, __pyx_filename);
10233     __pyx_r = NULL;
10234     __pyx_L0;
10235     __Pyx_XGIVEREF(__pyx_r);
10236     __Pyx_RefNannyFinishContext();
10237     return __pyx_r;
10238 }
10239
10240     /* "PyClicl.pyx":1061
10241 *     return clifford().wrap( self.instance.pure() )
10242 *
10243 *     def even(self):             # ««««««««
10244 *         """
10245 *         Even part of multivector, sum of even grade terms.
10246 */
10247
10248     /* Python wrapper */
10249     static PyObject* __pyx_pw_8PyClicl_8clifford_67even(PyObject* __pyx_v_self, CYTHON_UNUSED
PyObject* *unused); /*proto*/
10250     static char __pyx_doc_8PyClicl_8clifford_66even[] = "\n        Even part of multivector, sum
of even grade terms.\n\n        »> print(clifford(\"1+{1}+{1,2}\").even())\n        1+{1,2}\n
";
10251     static PyObject* __pyx_pw_8PyClicl_8clifford_67even(PyObject* __pyx_v_self, CYTHON_UNUSED
PyObject* *unused) {
10252         PyObject* __pyx_r = 0;
10253         __Pyx_RefNannyDeclarations
10254         __Pyx_RefNannySetupContext("even (wrapper)", 0);
10255         __pyx_r = __pyx_pf_8PyClicl_8clifford_66even(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self));
10256
10257     /* function exit code */
10258     __Pyx_RefNannyFinishContext();
10259     return __pyx_r;
10260 }
10261
10262     static PyObject* __pyx_pf_8PyClicl_8clifford_66even(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self) {
10263         PyObject* __pyx_r = NULL;
10264         __Pyx_RefNannyDeclarations
10265         PyObject* __pyx_t_1 = NULL;
10266         PyObject* __pyx_t_2 = NULL;
10267         int __pyx_lineno = 0;
10268         const char* __pyx_filename = NULL;
10269         int __pyx_clineno = 0;
10270         __Pyx_RefNannySetupContext("even", 0);
10271
10272     /* "PyClicl.pyx":1068
10273 *         1+{1,2}
10274 *         """
10275 *         return clifford().wrap( self.instance.even() )             # ««««««««
10276 *
10277 *     def odd(self):
10278 */
10279     __Pyx_XDECREF(__pyx_r);
10280     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject*)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1068, __pyx_L1_error)
10281     __Pyx_GOTREF(__pyx_t_1);
10282     __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), __pyx_v_self->instance->even()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1068,
__pyx_L1_error)
10283     __Pyx_GOTREF(__pyx_t_2);
10284     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10285     __pyx_r = __pyx_t_2;
10286     __pyx_t_2 = 0;
10287     goto __pyx_L0;
10288
10289     /* "PyClicl.pyx":1061
10290 *     return clifford().wrap( self.instance.pure() )
10291 *
10292 *     def even(self):             # ««««««««
10293 *         """
10294 *         Even part of multivector, sum of even grade terms.
10295 */
10296
10297     /* function exit code */
10298     __pyx_L1_error++;

```

```

10299         __Pyx_XDECREF(__pyx_t_1);
10300         __Pyx_XDECREF(__pyx_t_2);
10301         __Pyx_AddTraceback("PyClical.clifford.even", __pyx_clineno, __pyx_lineno, __pyx_filename);
10302         __pyx_r = NULL;
10303         __pyx_L0:;
10304         __Pyx_XGIVEREF(__pyx_r);
10305         __Pyx_RefNannyFinishContext();
10306         return __pyx_r;
10307     }
10308
10309     /* "PyClical.pyx":1070
10310     *         return clifford().wrap( self.instance.even() )
10311     *
10312     *     def odd(self):
10313     *         """
10314     *         Odd part of multivector, sum of odd grade terms.
10315     */
10316
10317     /* Python wrapper */
10318     static PyObject *__pyx_pw_8PyClical_8clifford_69odd(PyObject *__pyx_v_self, CYTHON_UNUSED
10319 PyObject *unused); /*proto*/
10319     static char __pyx_doc_8PyClical_8clifford_68odd[] = "\n        Odd part of multivector, sum of
10320 odd grade terms.\n\n        >> print(clifford(\"1+{1}+{1,2}\").odd())\n        {1}\n        ";
10320     static PyObject *__pyx_pw_8PyClical_8clifford_69odd(PyObject *__pyx_v_self, CYTHON_UNUSED
10321 PyObject *unused) {
10321         PyObject *__pyx_r = 0;
10322         __Pyx_RefNannyDeclarations
10323         __Pyx_RefNannySetupContext("odd (wrapper)", 0);
10324         __pyx_r = __pyx_pf_8PyClical_8clifford_68odd(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10325
10326         /* function exit code */
10327         __Pyx_RefNannyFinishContext();
10328         return __pyx_r;
10329     }
10330
10331     static PyObject *__pyx_pf_8PyClical_8clifford_68odd(struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self) {
10332         PyObject *__pyx_r = NULL;
10333         __Pyx_RefNannyDeclarations
10334         PyObject *__pyx_t_1 = NULL;
10335         PyObject *__pyx_t_2 = NULL;
10336         int __pyx_lineno = 0;
10337         const char *__pyx_filename = NULL;
10338         int __pyx_clineno = 0;
10339         __Pyx_RefNannySetupContext("odd", 0);
10340
10341         /* "PyClical.pyx":1077
10342     *         {1}
10343     *         """
10344     *         return clifford().wrap( self.instance.odd() )
10345     *
10346     *     def vector_part(self, frm = None):
10347     */
10348         __Pyx_XDECREF(__pyx_r);
10349         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1077, __pyx_L1_error)
10350         __Pyx_GOTREF(__pyx_t_1);
10351         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->odd()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1077,
__pyx_L1_error)
10352         __Pyx_GOTREF(__pyx_t_2);
10353         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10354         __pyx_r = __pyx_t_2;
10355         __pyx_t_2 = 0;
10356         goto __pyx_L0;
10357
10358         /* "PyClical.pyx":1070
10359     *         return clifford().wrap( self.instance.even() )
10360     *
10361     *     def odd(self):
10362     *         """
10363     *         Odd part of multivector, sum of odd grade terms.
10364     */
10365
10366         /* function exit code */
10367         __pyx_L1_error:;
10368         __Pyx_XDECREF(__pyx_t_1);
10369         __Pyx_XDECREF(__pyx_t_2);
10370         __Pyx_AddTraceback("PyClical.clifford.odd", __pyx_clineno, __pyx_lineno, __pyx_filename);
10371         __pyx_r = NULL;
10372         __pyx_L0:;
10373         __Pyx_XGIVEREF(__pyx_r);
10374         __Pyx_RefNannyFinishContext();
10375         return __pyx_r;
10376     }
10377

```

```

10378         /* "PyClicl.pyx":1079
10379         *         return clifford().wrap( self.instance.odd() )
10380         *
10381         *         def vector_part(self, frm = None):           # ««««««««
10382         *         """
10383         *             Vector part of multivector, as a Python list, with respect to frm.
10384         */
10385
10386         /* Python wrapper */
10387         static PyObject *__pyx_pw_8PyClicl_8clifford_71vector_part(PyObject *__pyx_v_self, PyObject
10388 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
10389         static char __pyx_doc_8PyClicl_8clifford_70vector_part[] = "\n          Vector part of
multivector, as a Python list, with respect to frm.\n\n          >>
print(clifford(\"1+2{1}+3{2}+4{1,2}\").vector_part())\n          [2.0, 3.0]\n          >>
print(clifford(\"1+2{1}+3{2}+4{1,2}\").vector_part(index_set({-1,1,2})))\n          [0.0, 2.0, 3.0]\n
\";
10389         static PyObject *__pyx_pw_8PyClicl_8clifford_71vector_part(PyObject *__pyx_v_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
10390             PyObject *__pyx_v_frm = 0;
10391             int __pyx_lineno = 0;
10392             const char *__pyx_filename = NULL;
10393             int __pyx_clineno = 0;
10394             PyObject *__pyx_r = 0;
10395             __Pyx_RefNannyDeclarations
10396             __Pyx_RefNannySetupContext("vector_part (wrapper)", 0);
10397             {
10398                 static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_frm,0};
10399                 PyObject* values[1] = {0};
10400                 values[0] = ((PyObject *)Py_None);
10401                 if (unlikely(__pyx_kwds)) {
10402                     Py_ssize_t kw_args;
10403                     const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
10404                     switch (pos_args) {
10405                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10406                         CYTHON_FALLTHROUGH;
10407                         case 0: break;
10408                         default: goto __pyx_L5_argtuple_error;
10409                     }
10410                     kw_args = PyDict_Size(__pyx_kwds);
10411                     switch (pos_args) {
10412                         case 0:
10413                             if (kw_args > 0) {
10414                                 PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_frm);
10415                                 if (value) { values[0] = value; kw_args--; }
10416                             }
10417                         if (unlikely(kw_args > 0)) {
10418                             if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
10419 pos_args, "vector_part") < 0)) __PYX_ERR(0, 1079, __pyx_L3_error)
10420                         }
10421                     } else {
10422                         switch (PyTuple_GET_SIZE(__pyx_args)) {
10423                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10424                             CYTHON_FALLTHROUGH;
10425                             case 0: break;
10426                             default: goto __pyx_L5_argtuple_error;
10427                         }
10428                     }
10429                     __pyx_v_frm = values[0];
10430                 }
10431                 goto __pyx_L4_argument_unpacking_done;
10432                 __pyx_L5_argtuple_error:;
10433                 __Pyx_RaiseArgtupleInvalid("vector_part", 0, 0, 1, PyTuple_GET_SIZE(__pyx_args));
10434                 __PYX_ERR(0, 1079, __pyx_L3_error)
10435                 __pyx_L3_error:;
10436                 __Pyx_AddTraceback("PyClicl.clifford.vector_part", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10437                 __Pyx_RefNannyFinishContext();
10438                 return NULL;
10439                 __pyx_L4_argument_unpacking_done:;
10440                 __pyx_r = __pyx_pf_8PyClicl_8clifford_70vector_part(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), __pyx_v_frm);
10441
10442                 /* function exit code */
10443                 __Pyx_RefNannyFinishContext();
10444                 return __pyx_r;
10445             }
10446             static PyObject *__pyx_pf_8PyClicl_8clifford_70vector_part(struct
__pyx_obj_8PyClicl_clifford *__pyx_v_self, PyObject *__pyx_v_frm) {
10447                 PyObject *__pyx_v_error_msg_prefix = NULL;
10448                 std::vector<scalar_t> __pyx_v_vec;
10449                 int __pyx_v_n;
10450                 int __pyx_v_i;
10451                 PyObject *__pyx_v_lst = NULL;
10452                 PyObject *__pyx_v_err = NULL;
10453                 PyObject *__pyx_r = NULL;

```

```

10454     __Pyx_RefNannyDeclarations
10455     PyObject *__pyx_t_1 = NULL;
10456     PyObject *__pyx_t_2 = NULL;
10457     PyObject *__pyx_t_3 = NULL;
10458     int __pyx_t_4;
10459     int __pyx_t_5;
10460     std::vector<scalar_t> __pyx_t_6;
10461     PyObject *__pyx_t_7 = NULL;
10462     int __pyx_t_8;
10463     int __pyx_t_9;
10464     int __pyx_t_10;
10465     PyObject *__pyx_t_11 = NULL;
10466     PyObject *__pyx_t_12 = NULL;
10467     PyObject *__pyx_t_13 = NULL;
10468     PyObject *__pyx_t_14 = NULL;
10469     PyObject *__pyx_t_15 = NULL;
10470     char const *__pyx_t_16;
10471     PyObject *__pyx_t_17 = NULL;
10472     PyObject *__pyx_t_18 = NULL;
10473     PyObject *__pyx_t_19 = NULL;
10474     PyObject *__pyx_t_20 = NULL;
10475     PyObject *__pyx_t_21 = NULL;
10476     PyObject *__pyx_t_22 = NULL;
10477     int __pyx_lineno = 0;
10478     const char *__pyx_filename = NULL;
10479     int __pyx_clineno = 0;
10480     __Pyx_RefNannySetupContext("vector_part", 0);
10481
10482     /* "PyClical.pyx":1088
10483     *
10484     * """
10485     * error_msg_prefix = "Cannot take vector part of " # ««««««««
10486     * cdef vector[scalar_t] vec
10487     * cdef int n
10488     */
10489     __Pyx_INCREF(__pyx_kp_u_Cannot_take_vector_part_of);
10490     __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_take_vector_part_of;
10491
10492     /* "PyClical.pyx":1092
10493     * cdef int n
10494     * cdef int i
10495     * try: # ««««««««
10496     *     if frm is None:
10497     *         vec = self.instance.vector_part()
10498     */
10499     {
10500         __Pyx_PyThreadState_declare
10501         __Pyx_PyThreadState_assign
10502         __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
10503         __Pyx_XGOTREF(__pyx_t_1);
10504         __Pyx_XGOTREF(__pyx_t_2);
10505         __Pyx_XGOTREF(__pyx_t_3);
10506         /*try:*/ {
10507
10508             /* "PyClical.pyx":1093
10509             * cdef int i
10510             * try:
10511             *     if frm is None: # ««««««««
10512             *         vec = self.instance.vector_part()
10513             *     else:
10514             */
10515             __pyx_t_4 = (__pyx_v_frm == Py_None);
10516             __pyx_t_5 = (__pyx_t_4 != 0);
10517             if (__pyx_t_5) {
10518
10519                 /* "PyClical.pyx":1094
10520                 * try:
10521                 *     if frm is None:
10522                 *         vec = self.instance.vector_part() # ««««««««
10523                 *     else:
10524                 *         vec = self.instance.vector_part((<index_set>frm).unwrap())
10525                 */
10526                 __pyx_t_6 = __pyx_v_self->instance->vector_part();
10527                 __pyx_v_vec = __pyx_t_6;
10528
10529                 /* "PyClical.pyx":1093
10530                 * cdef int i
10531                 * try:
10532                 *     if frm is None: # ««««««««
10533                 *         vec = self.instance.vector_part()
10534                 *     else:
10535                 */
10536                 goto __pyx_L9;
10537             }
10538
10539             /* "PyClical.pyx":1096
10540             * vec = self.instance.vector_part()

```

```

10541 *         else:
10542 *             vec = self.instance.vector_part((<index_set>frm).unwrap()) # ««««««««
10543 *             n = vec.size()
10544 *             lst = [0.0]*n
10545 */
10546         /*else*/ {
10547             try {
10548                 __pyx_t_6 =
__pyx_v_self->instance->vector_part(__pyx_f_8PyClical_9index_set_unwrap(((struct
__pyx_obj_8PyClical_index_set *)__pyx_v_frm)));
10549             } catch (...) {
10550                 __Pyx_CppExn2PyErr();
10551                 __PYX_ERR(0, 1096, __pyx_L3_error)
10552             }
10553             __pyx_v_vec = __pyx_t_6;
10554         }
10555         __pyx_L9;;
10556
10557         /* "PyClical.pyx":1097
10558 *         else:
10559 *             vec = self.instance.vector_part((<index_set>frm).unwrap())
10560 *             n = vec.size() # ««««««««
10561 *             lst = [0.0]*n
10562 *             for i in xrange(n):
10563 */
10564         __pyx_v_n = __pyx_v_vec.size();
10565
10566         /* "PyClical.pyx":1098
10567 *             vec = self.instance.vector_part((<index_set>frm).unwrap())
10568 *             n = vec.size()
10569 *             lst = [0.0]*n # ««««««««
10570 *             for i in xrange(n):
10571 *                 lst[i] = vec[i]
10572 */
10573         __pyx_t_7 = PyList_New(1 * ((__pyx_v_n<0) ? 0:__pyx_v_n)); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 1098, __pyx_L3_error)
10574         __Pyx_GOTREF(__pyx_t_7);
10575         { Py_ssize_t __pyx_temp;
10576             for (__pyx_temp=0; __pyx_temp < __pyx_v_n; __pyx_temp++) {
10577                 __Pyx_INCREF(__pyx_float_0_0);
10578                 __Pyx_GIVEREF(__pyx_float_0_0);
10579                 PyList_SET_ITEM(__pyx_t_7, __pyx_temp, __pyx_float_0_0);
10580             }
10581         }
10582         __pyx_v_lst = ((PyObject*)__pyx_t_7);
10583         __pyx_t_7 = 0;
10584
10585         /* "PyClical.pyx":1099
10586 *             n = vec.size()
10587 *             lst = [0.0]*n
10588 *             for i in xrange(n): # ««««««««
10589 *                 lst[i] = vec[i]
10590 *             return lst
10591 */
10592         __pyx_t_8 = __pyx_v_n;
10593         __pyx_t_9 = __pyx_t_8;
10594         for (__pyx_t_10 = 0; __pyx_t_10 < __pyx_t_9; __pyx_t_10+=1) {
10595             __pyx_v_i = __pyx_t_10;
10596
10597             /* "PyClical.pyx":1100
10598 *                 lst = [0.0]*n
10599 *                 for i in xrange(n):
10600 *                     lst[i] = vec[i] # ««««««««
10601 *                 return lst
10602 *             except RuntimeError as err:
10603 */
10604             __pyx_t_7 = PyFloat_FromDouble((__pyx_v_vec[__pyx_v_i])); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 1100, __pyx_L3_error)
10605             __Pyx_GOTREF(__pyx_t_7);
10606             if (unlikely(__Pyx_SetItemInt(__pyx_v_lst, __pyx_v_i, __pyx_t_7, int, 1,
__Pyx_PyInt_From_int, 1, 1, 1) < 0)) __PYX_ERR(0, 1100, __pyx_L3_error)
10607             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
10608         }
10609
10610         /* "PyClical.pyx":1101
10611 *                 for i in xrange(n):
10612 *                     lst[i] = vec[i]
10613 *                 return lst # ««««««««
10614 *             except RuntimeError as err:
10615 *                 raise ValueError(error_msg_prefix + str(self) + " using invalid "
10616 */
10617             __Pyx_XDECREF(__pyx_r);
10618             __Pyx_INCREF(__pyx_v_lst);
10619             __pyx_r = __pyx_v_lst;
10620             goto __pyx_L7_try_return;
10621
10622         /* "PyClical.pyx":1092

```

```

10623 *         cdef int n
10624 *         cdef int i
10625 *         try:
10626 *             # ««««««««
10627 *             if frm is None:
10628 *                 vec = self.instance.vector_part()
10629 */
10629     }
10630     __pyx_L3_error:;
10631     __Pyx_XDECREF(__pyx_t_7); __pyx_t_7 = 0;
10632
10633     /* "PyClical.pyx":1102
10634     lst[i] = vec[i]
10635     return lst
10636     except RuntimeError as err:
10637     raise ValueError(error_msg_prefix + str(self) + " using invalid "
10638     + repr(frm) + " as frame:\n\t"
10639     + repr(err))
10640 */
10640     __pyx_t_8 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
10641     if (__pyx_t_8) {
10642         __Pyx_AddTraceback("PyClical.clifford.vector_part", __pyx_clineno, __pyx_lineno,
10643         __pyx_filename);
10644         if (__Pyx_GetException(&__pyx_t_7, &__pyx_t_11, &__pyx_t_12) < 0) __PYX_ERR(0, 1102,
10645         __pyx_L5_except_error)
10646         __Pyx_GOTREF(__pyx_t_7);
10647         __Pyx_GOTREF(__pyx_t_11);
10648         __Pyx_GOTREF(__pyx_t_12);
10649         __Pyx_INCREF(__pyx_t_11);
10650         __pyx_v_err = __pyx_t_11;
10651         /*try:*/ {
10652             /* "PyClical.pyx":1103
10653             return lst
10654             except RuntimeError as err:
10655             raise ValueError(error_msg_prefix + str(self) + " using invalid "
10656             + repr(frm) + " as frame:\n\t"
10657             + str(err))
10658             # ««««««««
10659             __pyx_t_13 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
10660             *)__pyx_v_self)); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 1103, __pyx_L17_error)
10661             __Pyx_GOTREF(__pyx_t_13);
10662             __pyx_t_14 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_t_13); if
10663             (unlikely(!__pyx_t_14)) __PYX_ERR(0, 1103, __pyx_L17_error)
10664             __Pyx_GOTREF(__pyx_t_14);
10665             __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
10666             __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_14, __pyx_kp_u_using_invalid); if
10667             (unlikely(!__pyx_t_13)) __PYX_ERR(0, 1103, __pyx_L17_error)
10668             __Pyx_GOTREF(__pyx_t_13);
10669             __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
10670             /* "PyClical.pyx":1104
10671             except RuntimeError as err:
10672             raise ValueError(error_msg_prefix + str(self) + " using invalid "
10673             + repr(frm) + " as frame:\n\t"
10674             + str(err))
10675             # ««««««««
10676             __pyx_t_14 = PyObject_Repr(__pyx_v_frm); if (unlikely(!__pyx_t_14)) __PYX_ERR(0, 1104,
10677             __pyx_L17_error)
10678             __Pyx_GOTREF(__pyx_t_14);
10679             __pyx_t_15 = PyNumber_Add(__pyx_t_13, __pyx_t_14); if (unlikely(!__pyx_t_15))
10680             __PYX_ERR(0, 1104, __pyx_L17_error)
10681             __Pyx_GOTREF(__pyx_t_15);
10682             __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
10683             __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
10684             __pyx_t_14 = PyNumber_Add(__pyx_t_15, __pyx_kp_u_as_frame); if (unlikely(!__pyx_t_14))
10685             __PYX_ERR(0, 1104, __pyx_L17_error)
10686             __Pyx_GOTREF(__pyx_t_14);
10687             __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
10688             /* "PyClical.pyx":1105
10689             raise ValueError(error_msg_prefix + str(self) + " using invalid "
10690             + repr(frm) + " as frame:\n\t"
10691             + str(err))
10692             # ««««««««
10693             def involute(self):
10694             __pyx_t_15 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), __pyx_v_err);
10695             if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 1105, __pyx_L17_error)
10696             __Pyx_GOTREF(__pyx_t_15);
10697             __pyx_t_13 = PyNumber_Add(__pyx_t_14, __pyx_t_15); if (unlikely(!__pyx_t_13))
10698             __PYX_ERR(0, 1105, __pyx_L17_error)
10699             __Pyx_GOTREF(__pyx_t_13);
10700             __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
10701             __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
10702             /* "PyClical.pyx":1103
10703             return lst

```

```

10700 *         except RuntimeError as err:
10701 *             raise ValueError(error_msg_prefix + str(self) + " using invalid " # ««««««««
10702 *                 + repr(frm) + " as frame:\n\t"
10703 *                 + str(err))
10704 */
10705         __pyx_t_15 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_13); if
(unlikely(!__pyx_t_15)) __PYX_ERR(0, 1103, __pyx_L17_error)
10706         __Pyx_GOTREF(__pyx_t_15);
10707         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
10708         __Pyx_Raise(__pyx_t_15, 0, 0, 0);
10709         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
10710         __PYX_ERR(0, 1103, __pyx_L17_error)
10711     }
10712
10713     /* "PyClical.pyx":1102
10714 *         lst[i] = vec[i]
10715 *         return lst
10716 *     except RuntimeError as err: # ««««««««
10717 *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10718 *                 + repr(frm) + " as frame:\n\t"
10719 *
10720     /*finally:*/ {
10721         __pyx_L17_error:;
10722         /*exception exit:*/{
10723             __Pyx_PyThreadState_declare
10724             __Pyx_PyThreadState_assign
10725             __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0; __pyx_t_21 = 0;
__pyx_t_22 = 0;
10726             __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
10727             __Pyx_XDECREF(__pyx_t_14); __pyx_t_14 = 0;
10728             __Pyx_XDECREF(__pyx_t_15); __pyx_t_15 = 0;
10729             if (PY_MAJOR_VERSION >= 3) __Pyx_ExceptionSwap(&__pyx_t_20, &__pyx_t_21,
&__pyx_t_22);
10730             if ((PY_MAJOR_VERSION < 3) || unlikely(__Pyx_GetException(&__pyx_t_17, &__pyx_t_18,
&__pyx_t_19) < 0)) __Pyx_ErrFetch(&__pyx_t_17, &__pyx_t_18, &__pyx_t_19);
10731             __Pyx_XGOTREF(__pyx_t_17);
10732             __Pyx_XGOTREF(__pyx_t_18);
10733             __Pyx_XGOTREF(__pyx_t_19);
10734             __Pyx_XGOTREF(__pyx_t_20);
10735             __Pyx_XGOTREF(__pyx_t_21);
10736             __Pyx_XGOTREF(__pyx_t_22);
10737             __pyx_t_8 = __pyx_lineno; __pyx_t_9 = __pyx_clineno; __pyx_t_16 = __pyx_filename;
10738             {
10739                 __Pyx_DECREF(__pyx_v_err);
10740                 __pyx_v_err = NULL;
10741             }
10742             if (PY_MAJOR_VERSION >= 3) {
10743                 __Pyx_XGIVEREF(__pyx_t_20);
10744                 __Pyx_XGIVEREF(__pyx_t_21);
10745                 __Pyx_XGIVEREF(__pyx_t_22);
10746                 __Pyx_ExceptionReset(__pyx_t_20, __pyx_t_21, __pyx_t_22);
10747             }
10748             __Pyx_XGIVEREF(__pyx_t_17);
10749             __Pyx_XGIVEREF(__pyx_t_18);
10750             __Pyx_XGIVEREF(__pyx_t_19);
10751             __Pyx_ErrRestore(__pyx_t_17, __pyx_t_18, __pyx_t_19);
10752             __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0; __pyx_t_21 = 0;
__pyx_t_22 = 0;
10753             __pyx_lineno = __pyx_t_8; __pyx_clineno = __pyx_t_9; __pyx_filename = __pyx_t_16;
10754             goto __pyx_L5_except_error;
10755         }
10756     }
10757 }
10758 goto __pyx_L5_except_error;
10759 __pyx_L5_except_error:;
10760
10761     /* "PyClical.pyx":1092
10762 *     cdef int n
10763 *     cdef int i
10764 *     try: # ««««««««
10765 *         if frm is None:
10766 *             vec = self.instance.vector_part()
10767 */
10768         __Pyx_XGIVEREF(__pyx_t_1);
10769         __Pyx_XGIVEREF(__pyx_t_2);
10770         __Pyx_XGIVEREF(__pyx_t_3);
10771         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
10772         goto __pyx_L1_error;
10773         __pyx_L7_try_return:;
10774         __Pyx_XGIVEREF(__pyx_t_1);
10775         __Pyx_XGIVEREF(__pyx_t_2);
10776         __Pyx_XGIVEREF(__pyx_t_3);
10777         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
10778         goto __pyx_L0;
10779     }
10780
10781     /* "PyClical.pyx":1079

```



```

10782 *         return clifford().wrap( self.instance.odd() )
10783 *
10784 *     def vector_part(self, frm = None):          # ««««««««
10785 *         """
10786 *         Vector part of multivector, as a Python list, with respect to frm.
10787 */
10788
10789         /* function exit code */
10790         __pyx_L1_error++;
10791         __Pyx_XDECREF(__pyx_t_7);
10792         __Pyx_XDECREF(__pyx_t_11);
10793         __Pyx_XDECREF(__pyx_t_12);
10794         __Pyx_XDECREF(__pyx_t_13);
10795         __Pyx_XDECREF(__pyx_t_14);
10796         __Pyx_XDECREF(__pyx_t_15);
10797         __Pyx_AddTraceback("PyClical.clifford.vector_part", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10798         __pyx_r = NULL;
10799         __pyx_L0++;
10800         __Pyx_XDECREF(__pyx_v_error_msg_prefix);
10801         __Pyx_XDECREF(__pyx_v_lst);
10802         __Pyx_XDECREF(__pyx_v_err);
10803         __Pyx_XGIVEREF(__pyx_r);
10804         __Pyx_RefNannyFinishContext();
10805         return __pyx_r;
10806     }
10807
10808     /* "PyClical.pyx":1107
10809 *                                     + str(err))
10810 *
10811 *     def involute(self):                      # ««««««««
10812 *         """
10813 *         Main involution, each {i} is replaced by -{i} in each term,
10814 */
10815
10816         /* Python wrapper */
10817         static PyObject * __pyx_pw_8PyClical_8clifford_73involute(PyObject * __pyx_v_self, CYTHON_UNUSED
PyObject * unused); /*proto*/
10818         static char __pyx_doc_8PyClical_8clifford_72involute[] = "\n        Main involution, each {i}
is replaced by -{i} in each term,\n        eg. clifford(\"{1}\") -> -clifford(\"{1}\").\n\n    >>
print(clifford(\"{1}\").involute())\n        -{1}\n    >> print((clifford(\"{2}\") *
clifford(\"{1}\")).involute())\n        -(1,2)\n    >> print((clifford(\"{1}\") *
clifford(\"{2}\")).involute())\n        {1,2}\n    >>
print(clifford(\"1+{1}+{1,2}\").involute())\n        1-{1}+{1,2}\n    ";
10819         static PyObject * __pyx_pw_8PyClical_8clifford_73involute(PyObject * __pyx_v_self, CYTHON_UNUSED
PyObject * unused) {
10820             PyObject * __pyx_r = 0;
10821             __Pyx_RefNannyDeclarations
10822             __Pyx_RefNannySetupContext("involute (wrapper)", 0);
10823             __pyx_r = __pyx_pf_8PyClical_8clifford_72involute(((struct __pyx_obj_8PyClical_clifford
*) __pyx_v_self));
10824
10825             /* function exit code */
10826             __Pyx_RefNannyFinishContext();
10827             return __pyx_r;
10828         }
10829
10830         static PyObject * __pyx_pf_8PyClical_8clifford_72involute(struct __pyx_obj_8PyClical_clifford
* __pyx_v_self) {
10831             PyObject * __pyx_r = NULL;
10832             __Pyx_RefNannyDeclarations
10833             PyObject * __pyx_t_1 = NULL;
10834             PyObject * __pyx_t_2 = NULL;
10835             int __pyx_lineno = 0;
10836             const char * __pyx_filename = NULL;
10837             int __pyx_clineno = 0;
10838             __Pyx_RefNannySetupContext("involute", 0);
10839
10840             /* "PyClical.pyx":1121
10841 *         1-{1}+{1,2}
10842 *         """
10843 *         return clifford().wrap( self.instance.involute() )          # ««««««««
10844 *
10845 *     def reverse(self):
10846 */
10847             __Pyx_XDECREF(__pyx_r);
10848             __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1121, __pyx_L1_error)
10849             __Pyx_GOTREF(__pyx_t_1);
10850             __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*) __pyx_t_1), __pyx_v_self->instance->involute()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1121,
__pyx_L1_error)
10851             __Pyx_GOTREF(__pyx_t_2);
10852             __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10853             __pyx_r = __pyx_t_2;
10854             __pyx_t_2 = 0;
10855             goto __pyx_L0;

```

```

10856
10857     /* "PyClicl.pyx":1107
10858     *
10859     *
10860     * def involute(self):
10861     *     """
10862     *     Main involution, each {i} is replaced by -{i} in each term,
10863     */
10864
10865     /* function exit code */
10866     __pyx_L1_error:;
10867     __Pyx_XDECREF(__pyx_t_1);
10868     __Pyx_XDECREF(__pyx_t_2);
10869     __Pyx_AddTraceback("PyClicl.clifford.involute", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10870     __pyx_r = NULL;
10871     __pyx_L0:;
10872     __Pyx_XGIVEREF(__pyx_r);
10873     __Pyx_RefNannyFinishContext();
10874     return __pyx_r;
10875 }
10876
10877     /* "PyClicl.pyx":1123
10878     * return clifford().wrap( self.instance.involute() )
10879     *
10880     * def reverse(self):
10881     *     """
10882     *     Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").
10883     */
10884
10885     /* Python wrapper */
10886     static PyObject *__pyx_pw_8PyClicl_8clifford_75reverse(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10887     static char __pyx_doc_8PyClicl_8clifford_74reverse[] = "\n      Reversion, eg.
clifford(\"{1}\")*clifford(\"{2}\") -> clifford(\"{2}\")*clifford(\"{1}\").\n\n      >>
print(clifford(\"{1}\").reverse())\n      {1}\n      >> print((clifford(\"{2}\") *
clifford(\"{1}\")).reverse())\n      {1,2}\n      >> print((clifford(\"{1}\") *
clifford(\"{2}\")).reverse())\n      -{1,2}\n      >> print(clifford(\"1+{1}+{1,2}\").reverse())\n
1+{1}-{1,2}\n      ";
10888     static PyObject *__pyx_pw_8PyClicl_8clifford_75reverse(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10889     PyObject *__pyx_r = 0;
10890     __Pyx_RefNannyDeclarations
10891     __Pyx_RefNannySetupContext("reverse (wrapper)", 0);
10892     __pyx_r = __pyx_pf_8PyClicl_8clifford_74reverse(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self));
10893
10894     /* function exit code */
10895     __Pyx_RefNannyFinishContext();
10896     return __pyx_r;
10897 }
10898
10899     static PyObject *__pyx_pf_8PyClicl_8clifford_74reverse(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self) {
10900     PyObject *__pyx_r = NULL;
10901     __Pyx_RefNannyDeclarations
10902     PyObject *__pyx_t_1 = NULL;
10903     PyObject *__pyx_t_2 = NULL;
10904     int __pyx_lineno = 0;
10905     const char *__pyx_filename = NULL;
10906     int __pyx_clineno = 0;
10907     __Pyx_RefNannySetupContext("reverse", 0);
10908
10909     /* "PyClicl.pyx":1136
10910     * 1+{1}-{1,2}
10911     *
10912     * return clifford().wrap( self.instance.reverse() )
10913     *
10914     * def conj(self):
10915     */
10916     __Pyx_XDECREF(__pyx_r);
10917     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1136, __pyx_L1_error)
10918     __Pyx_GOTREF(__pyx_t_1);
10919     __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1, __pyx_v_self->instance->reverse()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1136,
__pyx_L1_error)
10920     __Pyx_GOTREF(__pyx_t_2);
10921     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10922     __pyx_r = __pyx_t_2;
10923     __pyx_t_2 = 0;
10924     goto __pyx_L0;
10925
10926     /* "PyClicl.pyx":1123
10927     * return clifford().wrap( self.instance.involute() )
10928     *
10929     * def reverse(self):

```

```

10930 *          """
10931 *          Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").
10932 */
10933
10934 /* function exit code */
10935 __pyx_L1_error++;
10936 __Pyx_XDECREF(__pyx_t_1);
10937 __Pyx_XDECREF(__pyx_t_2);
10938 __Pyx_AddTraceback("PyClical.clifford.reverse", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10939 __pyx_r = NULL;
10940 __pyx_L0++;
10941 __Pyx_XGIVEREF(__pyx_r);
10942 __Pyx_RefNannyFinishContext();
10943 return __pyx_r;
10944 }
10945
10946 /* "PyClical.pyx":1138
10947 *      return clifford().wrap( self.instance.reverse() )
10948 *
10949 *      def conj(self):          # ««««««««
10950 *          """
10951 *          Conjugation, reverse o involute == involute o reverse.
10952 */
10953
10954 /* Python wrapper */
10955 static PyObject *__pyx_pw_8PyClical_8clifford_77conj(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10956 static char __pyx_doc_8PyClical_8clifford_76conj[] = "\n      Conjugation, reverse o
involute == involute o reverse.\n\n      >> print((clifford("{1}\n\n      -{1}\n
>> print((clifford("{2}\n\n      {1,2}\n      >>
print((clifford("{1}\n\n      -{1,2}\n      >>
print(clifford("{1+{1}+{1,2}\n\n      1-{1}-{1,2}\n
";
10957 static PyObject *__pyx_pw_8PyClical_8clifford_77conj(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10958     PyObject *__pyx_r = 0;
10959     __Pyx_RefNannyDeclarations
10960     __Pyx_RefNannySetupContext("conj (wrapper)", 0);
10961     __pyx_r = __pyx_pf_8PyClical_8clifford_76conj(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10962
10963 /* function exit code */
10964 __Pyx_RefNannyFinishContext();
10965 return __pyx_r;
10966 }
10967
10968 static PyObject *__pyx_pf_8PyClical_8clifford_76conj(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
10969     PyObject *__pyx_r = NULL;
10970     __Pyx_RefNannyDeclarations
10971     PyObject *__pyx_t_1 = NULL;
10972     PyObject *__pyx_t_2 = NULL;
10973     int __pyx_lineno = 0;
10974     const char *__pyx_filename = NULL;
10975     int __pyx_clineno = 0;
10976     __Pyx_RefNannySetupContext("conj", 0);
10977
10978 /* "PyClical.pyx":1151
10979 *      1-{1}-{1,2}
10980 *      """
10981 *      return clifford().wrap( self.instance.conj() )          # ««««««««
10982 *
10983 *      def quad(self):
10984 */
10985     __Pyx_XDECREF(__pyx_r);
10986     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1151, __pyx_L1_error)
10987     __Pyx_GOTREF(__pyx_t_1);
10988     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1, __pyx_v_self->instance->conj()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1151,
__pyx_L1_error)
10989     __Pyx_GOTREF(__pyx_t_2);
10990     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10991     __pyx_r = __pyx_t_2;
10992     __pyx_t_2 = 0;
10993     goto __pyx_L0;
10994
10995 /* "PyClical.pyx":1138
10996 *      return clifford().wrap( self.instance.reverse() )
10997 *
10998 *      def conj(self):          # ««««««««
10999 *          """
11000 *          Conjugation, reverse o involute == involute o reverse.
11001 */
11002
11003 /* function exit code */
11004 __pyx_L1_error++;

```

```

11005         __Pyx_XDECREF(__pyx_t_1);
11006         __Pyx_XDECREF(__pyx_t_2);
11007         __Pyx_AddTraceback("PyClical.clifford.conj", __pyx_clineno, __pyx_lineno, __pyx_filename);
11008         __pyx_r = NULL;
11009         __pyx_L0;
11010         __Pyx_XGIVEREF(__pyx_r);
11011         __Pyx_RefNannyFinishContext();
11012         return __pyx_r;
11013     }
11014
11015     /* "PyClical.pyx":1153
11016     *     return clifford().wrap( self.instance.conj() )
11017     *
11018     *     def quad(self):
11019     *         """
11020     *         Quadratic form == (rev(x)*x)(0).
11021     */
11022
11023     /* Python wrapper */
11024     static PyObject *__pyx_pw_8PyClical_8clifford_79quad(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11025     static char __pyx_doc_8PyClical_8clifford_78quad[] = "\n        Quadratic form ==
(rev(x)*x)(0).\n        >> print(clifford(\"1+{1}+{1,2}\").quad())\n        3.0\n        >>
print(clifford(\"1+{-1}+{1,2}+{1,2,3}\").quad())\n        2.0\n        ";
11026     static PyObject *__pyx_pw_8PyClical_8clifford_79quad(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11027         PyObject *__pyx_r = 0;
11028         __Pyx_RefNannyDeclarations
11029         __Pyx_RefNannySetupContext("quad (wrapper)", 0);
11030         __pyx_r = __pyx_pf_8PyClical_8clifford_78quad(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11031
11032         /* function exit code */
11033         __Pyx_RefNannyFinishContext();
11034         return __pyx_r;
11035     }
11036
11037     static PyObject *__pyx_pf_8PyClical_8clifford_78quad(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11038         PyObject *__pyx_r = NULL;
11039         __Pyx_RefNannyDeclarations
11040         PyObject *__pyx_t_1 = NULL;
11041         int __pyx_lineno = 0;
11042         const char *__pyx_filename = NULL;
11043         int __pyx_clineno = 0;
11044         __Pyx_RefNannySetupContext("quad", 0);
11045
11046         /* "PyClical.pyx":1162
11047     *     2.0
11048     *     """
11049     *     return self.instance.quad()
11050     *
11051     *     def norm(self):
11052     */
11053         __Pyx_XDECREF(__pyx_r);
11054         __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->quad()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1162, __pyx_L1_error)
11055         __Pyx_GOTREF(__pyx_t_1);
11056         __pyx_r = __pyx_t_1;
11057         __pyx_t_1 = 0;
11058         goto __pyx_L0;
11059
11060         /* "PyClical.pyx":1153
11061     *     return clifford().wrap( self.instance.conj() )
11062     *
11063     *     def quad(self):
11064     *         """
11065     *         Quadratic form == (rev(x)*x)(0).
11066     */
11067
11068         /* function exit code */
11069         __pyx_L1_error;
11070         __Pyx_XDECREF(__pyx_t_1);
11071         __Pyx_AddTraceback("PyClical.clifford.quad", __pyx_clineno, __pyx_lineno, __pyx_filename);
11072         __pyx_r = NULL;
11073         __pyx_L0;
11074         __Pyx_XGIVEREF(__pyx_r);
11075         __Pyx_RefNannyFinishContext();
11076         return __pyx_r;
11077     }
11078
11079     /* "PyClical.pyx":1164
11080     *     return self.instance.quad()
11081     *
11082     *     def norm(self):
11083     *         """
11084     *         Norm == sum of squares of coordinates.

```

```

11085 */
11086
11087     /* Python wrapper */
11088     static PyObject *__pyx_pw_8PyClical_8clifford_8lnorm(PyObject *__pyx_v_self, CYTHON_UNUSED
11089     PyObject *unused); /*proto*/
11089     static char __pyx_doc_8PyClical_8clifford_80norm[] = "\n        Norm == sum of squares of
11089 coordinates.\n\n        >> clifford(\"1+{1}+{1,2}\").norm()\n        3.0\n        >>
11089 clifford(\"1+{-1}+{1,2}+{1,2,3}\").norm()\n        4.0\n        ";
11090     static PyObject *__pyx_pw_8PyClical_8clifford_8lnorm(PyObject *__pyx_v_self, CYTHON_UNUSED
11090     PyObject *unused) {
11091         PyObject *__pyx_r = 0;
11092         __Pyx_RefNannyDeclarations
11093         __Pyx_RefNannySetupContext("norm (wrapper)", 0);
11094         __pyx_r = __pyx_pf_8PyClical_8clifford_80norm(((struct __pyx_obj_8PyClical_clifford
11094 *)__pyx_v_self));
11095
11096         /* function exit code */
11097         __Pyx_RefNannyFinishContext();
11098         return __pyx_r;
11099     }
11100
11101     static PyObject *__pyx_pf_8PyClical_8clifford_80norm(struct __pyx_obj_8PyClical_clifford
11101     *__pyx_v_self) {
11102         PyObject *__pyx_r = NULL;
11103         __Pyx_RefNannyDeclarations
11104         PyObject *__pyx_t_1 = NULL;
11105         int __pyx_lineno = 0;
11106         const char *__pyx_filename = NULL;
11107         int __pyx_clineno = 0;
11108         __Pyx_RefNannySetupContext("norm", 0);
11109
11110         /* "PyClical.pyx":1173
11111         *
11112         *
11113         *     return self.instance.norm()
11114         *
11115         *     def abs(self):
11116         */
11117         __Pyx_XDECREF(__pyx_r);
11118         __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->norm()); if (unlikely(!__pyx_t_1))
11118         __PYX_ERR(0, 1173, __pyx_L1_error)
11119         __Pyx_GOTREF(__pyx_t_1);
11120         __pyx_r = __pyx_t_1;
11121         __pyx_t_1 = 0;
11122         goto __pyx_L0;
11123
11124         /* "PyClical.pyx":1164
11125         *
11126         *
11127         *     def norm(self):
11128         *
11129         *         Norm == sum of squares of coordinates.
11130         */
11131
11132         /* function exit code */
11133         __pyx_L1_error:;
11134         __Pyx_XDECREF(__pyx_t_1);
11135         __Pyx_AddTraceback("PyClical.clifford.norm", __pyx_clineno, __pyx_lineno, __pyx_filename);
11136         __pyx_r = NULL;
11137         __pyx_L0:;
11138         __Pyx_XGIVEREF(__pyx_r);
11139         __Pyx_RefNannyFinishContext();
11140         return __pyx_r;
11141     }
11142
11143     /* "PyClical.pyx":1175
11144     *
11145     *
11146     *     def abs(self):
11147     *
11148     *         Absolute value: square root of norm.
11149     */
11150
11151     /* Python wrapper */
11152     static PyObject *__pyx_pw_8PyClical_8clifford_83abs(PyObject *__pyx_v_self, CYTHON_UNUSED
11152     PyObject *unused); /*proto*/
11153     static char __pyx_doc_8PyClical_8clifford_82abs[] = "\n        Absolute value: square root of
11153 norm.\n\n        >> clifford(\"1+{-1}+{1,2}+{1,2,3}\").abs()\n        2.0\n        ";
11154     static PyObject *__pyx_pw_8PyClical_8clifford_83abs(PyObject *__pyx_v_self, CYTHON_UNUSED
11154     PyObject *unused) {
11155         PyObject *__pyx_r = 0;
11156         __Pyx_RefNannyDeclarations
11157         __Pyx_RefNannySetupContext("abs (wrapper)", 0);
11158         __pyx_r = __pyx_pf_8PyClical_8clifford_82abs(((struct __pyx_obj_8PyClical_clifford
11158 *)__pyx_v_self));
11159
11160         /* function exit code */

```

```

11161         __Pyx_RefNannyFinishContext();
11162         return __pyx_r;
11163     }
11164
11165     static PyObject *__pyx_pf_8PyClical_8clifford_82abs(struct __pyx_obj_8PyClical_clifford
11166 *__pyx_v_self) {
11167         PyObject *__pyx_r = NULL;
11168         __Pyx_RefNannyDeclarations
11169         PyObject *__pyx_t_1 = NULL;
11170         int __pyx_lineno = 0;
11171         const char *__pyx_filename = NULL;
11172         int __pyx_clineno = 0;
11173         __Pyx_RefNannySetupContext("abs", 0);
11174
11175         /* "PyClical.pyx":1182
11176         *
11177         *     return glucat.abs( self.unwrap() )
11178         *
11179         *     def max_abs(self):
11180         */
11181         __Pyx_XDECREF(__pyx_r);
11182         __pyx_t_1 = PyFloat_FromDouble(abs(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1182, __pyx_L1_error)
11183         __Pyx_GOTREF(__pyx_t_1);
11184         __pyx_r = __pyx_t_1;
11185         __pyx_t_1 = 0;
11186         goto __pyx_L0;
11187
11188         /* "PyClical.pyx":1175
11189         *
11190         *     return self.instance.norm()
11191         *
11192         *     def abs(self):
11193         *         """
11194         *         Absolute value: square root of norm.
11195         */
11196         /* function exit code */
11197         __pyx_L1_error++;
11198         __Pyx_XDECREF(__pyx_t_1);
11199         __Pyx_AddTraceback("PyClical.clifford.abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
11200         __pyx_r = NULL;
11201         __pyx_L0:;
11202         __Pyx_XGIVEREF(__pyx_r);
11203         __Pyx_RefNannyFinishContext();
11204         return __pyx_r;
11205     }
11206
11207     /* "PyClical.pyx":1184
11208     *
11209     *     return glucat.abs( self.unwrap() )
11210     *
11211     *     def max_abs(self):
11212     *         """
11213     *         Maximum of absolute values of components of multivector: multivector infinity norm.
11214     */
11215     /* Python wrapper */
11216     static PyObject *__pyx_pw_8PyClical_8clifford_85max_abs(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11217     static char __pyx_doc_8PyClical_8clifford_84max_abs[] = "\n        Maximum of absolute values
of components of multivector: multivector infinity norm.\n\n        >>
clifford(\\"1+{-1}+{1,2}+{1,2,3}\\").max_abs()\n        1.0\n        >>
clifford(\\"3+2{1}+{1,2}\\").max_abs()\n        3.0\n        ";
11218     static PyObject *__pyx_pw_8PyClical_8clifford_85max_abs(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11219         PyObject *__pyx_r = 0;
11220         __Pyx_RefNannyDeclarations
11221         __Pyx_RefNannySetupContext("max_abs (wrapper)", 0);
11222         __pyx_r = __pyx_pf_8PyClical_8clifford_84max_abs(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11223
11224         /* function exit code */
11225         __Pyx_RefNannyFinishContext();
11226         return __pyx_r;
11227     }
11228
11229     static PyObject *__pyx_pf_8PyClical_8clifford_84max_abs(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11230         PyObject *__pyx_r = NULL;
11231         __Pyx_RefNannyDeclarations
11232         PyObject *__pyx_t_1 = NULL;
11233         int __pyx_lineno = 0;
11234         const char *__pyx_filename = NULL;
11235         int __pyx_clineno = 0;
11236         __Pyx_RefNannySetupContext("max_abs", 0);
11237
11238         /* "PyClical.pyx":1193

```

```

11239 *          3.0
11240 *          """
11241 *          return self.instance.max_abs()          # ««««««««
11242 *
11243 *          def truncated(self, limit):
11244 */
11245         __Pyx_XDECREF(__pyx_r);
11246         __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->max_abs()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1193, __pyx_L1_error)
11247         __Pyx_GOTREF(__pyx_t_1);
11248         __pyx_r = __pyx_t_1;
11249         __pyx_t_1 = 0;
11250         goto __pyx_L0;
11251
11252         /* "PyClical.pyx":1184
11253 *          return glucat.abs( self.unwrap() )
11254 *
11255 *          def max_abs(self):          # ««««««««
11256 *          """
11257 *          Maximum of absolute values of components of multivector: multivector infinity norm.
11258 */
11259
11260         /* function exit code */
11261         __pyx_L1_error:;
11262         __Pyx_XDECREF(__pyx_t_1);
11263         __Pyx_AddTraceback("PyClical.clifford.max_abs", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11264         __pyx_r = NULL;
11265         __pyx_L0:;
11266         __Pyx_XGIVEREF(__pyx_r);
11267         __Pyx_RefNannyFinishContext();
11268         return __pyx_r;
11269     }
11270
11271     /* "PyClical.pyx":1195
11272 *          return self.instance.max_abs()
11273 *
11274 *          def truncated(self, limit):          # ««««««««
11275 *          """
11276 *          Remove all terms of self with relative size smaller than limit.
11277 */
11278
11279         /* Python wrapper */
11280         static PyObject *__pyx_pw_8PyClical_8clifford_87truncated(PyObject *__pyx_v_self, PyObject
*__pyx_v_limit); /*proto*/
11281         static char __pyx_doc_8PyClical_8clifford_86truncated[] = "\n          Remove all terms of self
with relative size smaller than limit.\n\n          >>
clifford(\"1e8+{1}+1e-8{1,2}\").truncated(1.0e-6)\n          clifford(\"100000000\")\n          >>
clifford(\"1e4+{1}+1e-4{1,2}\").truncated(1.0e-6)\n          clifford(\"10000+{1}\")\n          ";
11282         static PyObject *__pyx_pw_8PyClical_8clifford_87truncated(PyObject *__pyx_v_self, PyObject
*__pyx_v_limit) {
11283             PyObject *__pyx_r = 0;
11284             __Pyx_RefNannyDeclarations
11285             __Pyx_RefNannySetupContext("truncated (wrapper)", 0);
11286             __pyx_r = __pyx_pf_8PyClical_8clifford_86truncated(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_limit));
11287
11288             /* function exit code */
11289             __Pyx_RefNannyFinishContext();
11290             return __pyx_r;
11291         }
11292
11293         static PyObject *__pyx_pf_8PyClical_8clifford_86truncated(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_limit) {
11294             PyObject *__pyx_r = NULL;
11295             __Pyx_RefNannyDeclarations
11296             PyObject *__pyx_t_1 = NULL;
11297             scalar_t __pyx_t_2;
11298             PyObject *__pyx_t_3 = NULL;
11299             int __pyx_lineno = 0;
11300             const char *__pyx_filename = NULL;
11301             int __pyx_clineno = 0;
11302             __Pyx_RefNannySetupContext("truncated", 0);
11303
11304             /* "PyClical.pyx":1204
11305 *          clifford("10000+{1}")
11306 *          """
11307 *          return clifford().wrap( self.instance.truncated(limit) )          # ««««««««
11308 *
11309 *          def isinf(self):
11310 */
11311             __Pyx_XDECREF(__pyx_r);
11312             __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1204, __pyx_L1_error)
11313             __Pyx_GOTREF(__pyx_t_1);
11314             __pyx_t_2 = __pyx_PyFloat_AsDouble(__pyx_v_limit); if (unlikely((__pyx_t_2 ==
((scalar_t)-1) && PyErr_Occurred()))) __PYX_ERR(0, 1204, __pyx_L1_error)

```

```

11315     __pyx_t_3 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->truncated(__pyx_t_2)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0,
1204, __pyx_L1_error)
11316     __Pyx_GOTREF(__pyx_t_3);
11317     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11318     __pyx_r = __pyx_t_3;
11319     __pyx_t_3 = 0;
11320     goto __pyx_L0;
11321
11322     /* "PyClical.pyx":1195
11323     *     return self.instance.max_abs()
11324     *
11325     *     def truncated(self, limit):          # ««««««««
11326     *         """
11327     *         Remove all terms of self with relative size smaller than limit.
11328     */
11329
11330     /* function exit code */
11331     __pyx_L1_error:;
11332     __Pyx_XDECREF(__pyx_t_1);
11333     __Pyx_XDECREF(__pyx_t_3);
11334     __Pyx_AddTraceback("PyClical.clifford.truncated", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11335     __pyx_r = NULL;
11336     __pyx_L0:;
11337     __Pyx_XGIVEREF(__pyx_r);
11338     __Pyx_RefNannyFinishContext();
11339     return __pyx_r;
11340 }
11341
11342     /* "PyClical.pyx":1206
11343     *     return clifford().wrap( self.instance.truncated(limit) )
11344     *
11345     *     def isinf(self):          # ««««««««
11346     *         """
11347     *         Check if a multivector contains any infinite values.
11348     */
11349
11350     /* Python wrapper */
11351     static PyObject *__pyx_pw_8PyClical_8clifford_89isinf(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11352     static char __pyx_doc_8PyClical_8clifford_88isinf[] = "\n          Check if a multivector
contains any infinite values.\n\n          >> clifford().isinf()\n          False\n          ";
11353     static PyObject *__pyx_pw_8PyClical_8clifford_89isinf(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11354         PyObject *__pyx_r = 0;
11355         __Pyx_RefNannyDeclarations
11356         __Pyx_RefNannySetupContext("isinf (wrapper)", 0);
11357         __pyx_r = __pyx_pf_8PyClical_8clifford_88isinf(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11358
11359         /* function exit code */
11360         __Pyx_RefNannyFinishContext();
11361         return __pyx_r;
11362     }
11363
11364     static PyObject *__pyx_pf_8PyClical_8clifford_88isinf(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11365         PyObject *__pyx_r = NULL;
11366         __Pyx_RefNannyDeclarations
11367         PyObject *__pyx_t_1 = NULL;
11368         int __pyx_lineno = 0;
11369         const char *__pyx_filename = NULL;
11370         int __pyx_clineno = 0;
11371         __Pyx_RefNannySetupContext("isinf", 0);
11372
11373         /* "PyClical.pyx":1213
11374         *     False
11375         *     """
11376         *     return self.instance.isnan()          # ««««««««
11377         *
11378         *     def isnan(self):
11379         */
11380         __Pyx_XDECREF(__pyx_r);
11381         __pyx_t_1 = __Pyx_PyBool_FromLong(__pyx_v_self->instance->isnan()); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1213, __pyx_L1_error)
11382         __Pyx_GOTREF(__pyx_t_1);
11383         __pyx_r = __pyx_t_1;
11384         __pyx_t_1 = 0;
11385         goto __pyx_L0;
11386
11387         /* "PyClical.pyx":1206
11388         *     return clifford().wrap( self.instance.truncated(limit) )
11389         *
11390         *     def isinf(self):          # ««««««««
11391         *         """
11392         *         Check if a multivector contains any infinite values.

```



```

11393  */
11394
11395      /* function exit code */
11396      __pyx_L1_error++;
11397      __Pyx_XDECREF(__pyx_t_1);
11398      __Pyx_AddTraceback("PyClical.clifford.isinf", __pyx_clineno, __pyx_lineno, __pyx_filename);
11399      __pyx_r = NULL;
11400      __pyx_L0++;
11401      __Pyx_XGIVEREF(__pyx_r);
11402      __Pyx_RefNannyFinishContext();
11403      return __pyx_r;
11404  }
11405
11406      /* "PyClical.pyx":1215
11407      *      return self.instance.isnan()
11408      *
11409      *      def isnan(self):
11410      *          """
11411      *          Check if a multivector contains any IEEE NaN values.
11412      */
11413
11414      /* Python wrapper */
11415      static PyObject *__pyx_pw_8PyClical_8clifford_9isnan(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11416      static char __pyx_doc_8PyClical_8clifford_90isnan[] = "\n      Check if a multivector
contains any IEEE NaN values.\n\n      >> clifford().isnan()\n      False\n      ";
11417      static PyObject *__pyx_pw_8PyClical_8clifford_9isnan(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11418          PyObject *__pyx_r = 0;
11419          __Pyx_RefNannyDeclarations
11420          __Pyx_RefNannySetupContext("isnan (wrapper)", 0);
11421          __pyx_r = __pyx_pf_8PyClical_8clifford_90isnan(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11422
11423      /* function exit code */
11424      __Pyx_RefNannyFinishContext();
11425      return __pyx_r;
11426  }
11427
11428      static PyObject *__pyx_pf_8PyClical_8clifford_90isnan(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11429          PyObject *__pyx_r = NULL;
11430          __Pyx_RefNannyDeclarations
11431          PyObject *__pyx_t_1 = NULL;
11432          int __pyx_lineno = 0;
11433          const char *__pyx_filename = NULL;
11434          int __pyx_clineno = 0;
11435          __Pyx_RefNannySetupContext("isnan", 0);
11436
11437      /* "PyClical.pyx":1222
11438      *      False
11439      *
11440      *      return self.instance.isnan()
11441      *
11442      *      def frame(self):
11443      */
11444          __Pyx_XDECREF(__pyx_r);
11445          __pyx_t_1 = __Pyx_PyBool_FromLong(__pyx_v_self->instance->isnan()); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1222, __pyx_L1_error)
11446          __Pyx_GOTREF(__pyx_t_1);
11447          __pyx_r = __pyx_t_1;
11448          __pyx_t_1 = 0;
11449          goto __pyx_L0;
11450
11451      /* "PyClical.pyx":1215
11452      *      return self.instance.isnan()
11453      *
11454      *      def isnan(self):
11455      *          """
11456      *          Check if a multivector contains any IEEE NaN values.
11457      */
11458
11459      /* function exit code */
11460      __pyx_L1_error++;
11461      __Pyx_XDECREF(__pyx_t_1);
11462      __Pyx_AddTraceback("PyClical.clifford.isnan", __pyx_clineno, __pyx_lineno, __pyx_filename);
11463      __pyx_r = NULL;
11464      __pyx_L0++;
11465      __Pyx_XGIVEREF(__pyx_r);
11466      __Pyx_RefNannyFinishContext();
11467      return __pyx_r;
11468  }
11469
11470      /* "PyClical.pyx":1224
11471      *      return self.instance.isnan()
11472      *
11473      *      def frame(self):
11474      */

```

```

11474 *          """
11475 *          Subalgebra generated by all generators of terms of given multivector.
11476 */
11477
11478 /* Python wrapper */
11479 static PyObject *__pyx_pw_8PyClical_8clifford_93frame(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11480 static char __pyx_doc_8PyClical_8clifford_92frame[] = "\n          Subalgebra generated by all
generators of terms of given multivector.\n\n          >>
print(clifford(\"1+3{-1}+2{1,2}+4{-2,7}\").frame())\n          {-2,-1,1,2,7}\n          >>
s=clifford(\"1+3{-1}+2{1,2}+4{-2,7}\").frame(); type(s)\n          <class 'PyClical.index_set'>\n
";
11481 static PyObject *__pyx_pw_8PyClical_8clifford_93frame(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11482     PyObject *__pyx_r = 0;
11483     __Pyx_RefNannyDeclarations
11484     __Pyx_RefNannySetupContext("frame (wrapper)", 0);
11485     __pyx_r = __pyx_pf_8PyClical_8clifford_92frame(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11486
11487     /* function exit code */
11488     __Pyx_RefNannyFinishContext();
11489     return __pyx_r;
11490 }
11491
11492 static PyObject *__pyx_pf_8PyClical_8clifford_92frame(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11493     PyObject *__pyx_r = NULL;
11494     __Pyx_RefNannyDeclarations
11495     PyObject *__pyx_t_1 = NULL;
11496     PyObject *__pyx_t_2 = NULL;
11497     int __pyx_lineno = 0;
11498     const char *__pyx_filename = NULL;
11499     int __pyx_clineno = 0;
11500     __Pyx_RefNannySetupContext("frame", 0);
11501
11502     /* "PyClical.pyx":1233
11503 *          <class 'PyClical.index_set'>
11504 *          """
11505 *          return index_set().wrap( self.instance.frame() )          # ««««««««
11506 *
11507 *          def __repr__(self):
11508 */
11509     __Pyx_XDECREF(__pyx_r);
11510     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1233, __pyx_l1_error)
11511     __Pyx_GOTREF(__pyx_t_1);
11512     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set
*)__pyx_t_1), __pyx_v_self->instance->frame()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1233,
__pyx_l1_error)
11513     __Pyx_GOTREF(__pyx_t_2);
11514     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11515     __pyx_r = __pyx_t_2;
11516     __pyx_t_2 = 0;
11517     goto __pyx_L0;
11518
11519     /* "PyClical.pyx":1224
11520 *          return self.instance.isnan()
11521 *
11522 *          def frame(self):          # ««««««««
11523 *          """
11524 *          Subalgebra generated by all generators of terms of given multivector.
11525 */
11526
11527     /* function exit code */
11528     __pyx_l1_error;
11529     __Pyx_XDECREF(__pyx_t_1);
11530     __Pyx_XDECREF(__pyx_t_2);
11531     __Pyx_AddTraceback("PyClical.clifford.frame", __pyx_clineno, __pyx_lineno, __pyx_filename);
11532     __pyx_r = NULL;
11533     __pyx_L0:;
11534     __Pyx_XGIVEREF(__pyx_r);
11535     __Pyx_RefNannyFinishContext();
11536     return __pyx_r;
11537 }
11538
11539 /* "PyClical.pyx":1235
11540 *          return index_set().wrap( self.instance.frame() )
11541 *
11542 *          def __repr__(self):          # ««««««««
11543 *          """
11544 *          The official string representation of self.
11545 */
11546
11547 /* Python wrapper */
11548 static PyObject *__pyx_pw_8PyClical_8clifford_95__repr__(PyObject *__pyx_v_self); /*proto*/
11549 static char __pyx_doc_8PyClical_8clifford_94__repr__[] = "\n          The

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\342\200\234official\342\200\235 string representation of self.\n\n        >>
clifford("\1+3{-1}+2{1,2}+4{-2,7}\").__repr__()\n        'clifford("\1+3{-1}+2{1,2}+4{-2,7}\")'\n
";
11550     #if CYTHON_COMPILING_IN_CPYTHON
11551     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_94__repr__;
11552     #endif
11553     static PyObject *__pyx_pw_8PyClical_8clifford_95__repr__(PyObject *__pyx_v_self) {
11554         PyObject *__pyx_r = 0;
11555         __Pyx_RefNannyDeclarations
11556         __Pyx_RefNannySetupContext("__repr__ (wrapper)", 0);
11557         __pyx_r = __pyx_pf_8PyClical_8clifford_94__repr__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11558
11559         /* function exit code */
11560         __Pyx_RefNannyFinishContext();
11561         return __pyx_r;
11562     }
11563
11564     static PyObject *__pyx_pf_8PyClical_8clifford_94__repr__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11565         PyObject *__pyx_r = NULL;
11566         __Pyx_RefNannyDeclarations
11567         PyObject *__pyx_t_1 = NULL;
11568         int __pyx_lineno = 0;
11569         const char *__pyx_filename = NULL;
11570         int __pyx_clineno = 0;
11571         __Pyx_RefNannySetupContext("__repr__", 0);
11572
11573         /* "PyClical.pyx":1242
11574 *         'clifford("\1+3{-1}+2{1,2}+4{-2,7}")'
11575 *         """
11576 *         return clifford_to_repr( self.unwrap() ).decode()           # ««««««««
11577 *
11578 *     def __str__(self):
11579 */
11580         __Pyx_XDECREF(__pyx_r);
11581         __pyx_t_1 =
__Pyx_decode_cpp_string(clifford_to_repr(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self)), 0,
PY_SSIZE_T_MAX, NULL, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1242, __pyx_L1_error)
11582         __Pyx_GOTREF(__pyx_t_1);
11583         __pyx_r = __pyx_t_1;
11584         __pyx_t_1 = 0;
11585         goto __pyx_L0;
11586
11587         /* "PyClical.pyx":1235
11588 *         return index_set().wrap( self.instance.frame() )
11589 *
11590 *     def __repr__(self):           # ««««««««
11591 *         """
11592 *         The official string representation of self.
11593 */
11594
11595         /* function exit code */
11596         __pyx_L1_error:;
11597         __Pyx_XDECREF(__pyx_t_1);
11598         __Pyx_AddTraceback("PyClical.clifford.__repr__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11599         __pyx_r = NULL;
11600         __pyx_L0:;
11601         __Pyx_XGIVEREF(__pyx_r);
11602         __Pyx_RefNannyFinishContext();
11603         return __pyx_r;
11604     }
11605
11606     /* "PyClical.pyx":1244
11607 *         return clifford_to_repr( self.unwrap() ).decode()
11608 *
11609 *     def __str__(self):           # ««««««««
11610 *         """
11611 *         The informal string representation of self.
11612 */
11613
11614     /* Python wrapper */
11615     static PyObject *__pyx_pw_8PyClical_8clifford_97__str__(PyObject *__pyx_v_self); /*proto*/
11616     static char __pyx_doc_8PyClical_8clifford_96__str__[] = "\n        The
\342\200\234informal\342\200\235 string representation of self.\n\n        >>
clifford("\1+3{-1}+2{1,2}+4{-2,7}\").__str__()\n        '1+3{-1}+2{1,2}+4{-2,7}'\n
";
11617     #if CYTHON_COMPILING_IN_CPYTHON
11618     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_96__str__;
11619     #endif
11620     static PyObject *__pyx_pw_8PyClical_8clifford_97__str__(PyObject *__pyx_v_self) {
11621         PyObject *__pyx_r = 0;
11622         __Pyx_RefNannyDeclarations
11623         __Pyx_RefNannySetupContext("__str__ (wrapper)", 0);
11624         __pyx_r = __pyx_pf_8PyClical_8clifford_96__str__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11625

```

```

11626         /* function exit code */
11627         __Pyx_RefNannyFinishContext();
11628         return __pyx_r;
11629     }
11630
11631     static PyObject *__pyx_pf_8PyClical_8clifford_96__str__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self) {
11632         PyObject *__pyx_r = NULL;
11633         __Pyx_RefNannyDeclarations
11634         PyObject *__pyx_t_1 = NULL;
11635         int __pyx_lineno = 0;
11636         const char *__pyx_filename = NULL;
11637         int __pyx_clineno = 0;
11638         __Pyx_RefNannySetupContext("__str__", 0);
11639
11640         /* "PyClical.pyx":1251
11641         *      '1+3{-1}+2{1,2}+4{-2,7}'
11642         *      """
11643         *      return clifford_to_str( self.unwrap() ).decode()
11644         *      # ««««««««
11645         *  def clifford_hidden_doctests():
11646         */
11647         __Pyx_XDECREF(__pyx_r);
11648         __pyx_t_1 =
__Pyx_decode_cpp_string(clifford_to_str(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self)), 0,
PY_SSIZE_T_MAX, NULL, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1251, __pyx_L1_error)
11649         __Pyx_GOTREF(__pyx_t_1);
11650         __pyx_r = __pyx_t_1;
11651         __pyx_t_1 = 0;
11652         goto __pyx_L0;
11653
11654         /* "PyClical.pyx":1244
11655         *      return clifford_to_repr( self.unwrap() ).decode()
11656         *
11657         *  def __str__(self):
11658         *      """
11659         *      The informal string representation of self.
11660         */
11661
11662         /* function exit code */
11663         __pyx_L1_error:;
11664         __Pyx_XDECREF(__pyx_t_1);
11665         __Pyx_AddTraceback("PyClical.clifford.__str__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11666         __pyx_r = NULL;
11667         __pyx_L0:;
11668         __Pyx_XGIVEREF(__pyx_r);
11669         __Pyx_RefNannyFinishContext();
11670         return __pyx_r;
11671     }
11672
11673     /* "(tree fragment)":1
11674     *  def __reduce_cython__(self):
11675     *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11676     *  def __setstate_cython__(self, __pyx_state):
11677     */
11678
11679     /* Python wrapper */
11680     static PyObject *__pyx_pw_8PyClical_8clifford_99__reduce_cython__(PyObject *__pyx_v_self,
CYTHON_UNUSED PyObject *unused); /*proto*/
11681     static PyObject *__pyx_pw_8PyClical_8clifford_99__reduce_cython__(PyObject *__pyx_v_self,
CYTHON_UNUSED PyObject *unused) {
11682         PyObject *__pyx_r = 0;
11683         __Pyx_RefNannyDeclarations
11684         __Pyx_RefNannySetupContext("__reduce_cython__ (wrapper)", 0);
11685         __pyx_r = __pyx_pf_8PyClical_8clifford_98__reduce_cython__(((struct
__pyx_obj_8PyClical_clifford *)__pyx_v_self));
11686
11687         /* function exit code */
11688         __Pyx_RefNannyFinishContext();
11689         return __pyx_r;
11690     }
11691
11692     static PyObject *__pyx_pf_8PyClical_8clifford_98__reduce_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self) {
11693         PyObject *__pyx_r = NULL;
11694         __Pyx_RefNannyDeclarations
11695         PyObject *__pyx_t_1 = NULL;
11696         int __pyx_lineno = 0;
11697         const char *__pyx_filename = NULL;
11698         int __pyx_clineno = 0;
11699         __Pyx_RefNannySetupContext("__reduce_cython__", 0);
11700
11701         /* "(tree fragment)":2
11702         *  def __reduce_cython__(self):
11703         *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11704         *  def __setstate_cython__(self, __pyx_state):
11705         *      """
11706         *      The informal string representation of self.
11707         */

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11705 *      raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11706 */
11707     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__11, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 2, __pyx_L1_error)
11708     __Pyx_GOTREF(__pyx_t_1);
11709     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
11710     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11711     __PYX_ERR(1, 2, __pyx_L1_error)
11712
11713     /* "(tree fragment)":1
11714     * def __reduce_cython__(self): # ««««««««
11715     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11716     * def __setstate_cython__(self, __pyx_state):
11717     */
11718
11719     /* function exit code */
11720     __pyx_L1_error:;
11721     __Pyx_XDECREF(__pyx_t_1);
11722     __Pyx_AddTraceback("PyClical.clifford.__reduce_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11723     __pyx_r = NULL;
11724     __Pyx_XGIVEREF(__pyx_r);
11725     __Pyx_RefNannyFinishContext();
11726     return __pyx_r;
11727 }
11728
11729     /* "(tree fragment)":3
11730     * def __reduce_cython__(self):
11731     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11732     * def __setstate_cython__(self, __pyx_state): # ««««««««
11733     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11734     */
11735
11736     /* Python wrapper */
11737     static PyObject *__pyx_pw_8PyClical_8clifford_101__setstate_cython__(PyObject *__pyx_v_self,
PyObject *__pyx_v__pyx_state); /*proto*/
11738     static PyObject *__pyx_pw_8PyClical_8clifford_101__setstate_cython__(PyObject *__pyx_v_self,
PyObject *__pyx_v__pyx_state) {
11739         PyObject *__pyx_r = 0;
11740         __Pyx_RefNannyDeclarations
11741         __Pyx_RefNannySetupContext("__setstate_cython__ (wrapper)", 0);
11742         __pyx_r = __pyx_pf_8PyClical_8clifford_100__setstate_cython__(((struct
__pyx_obj_8PyClical_clifford *)__pyx_v_self), ((PyObject *)__pyx_v__pyx_state));
11743
11744         /* function exit code */
11745         __Pyx_RefNannyFinishContext();
11746         return __pyx_r;
11747     }
11748
11749     static PyObject *__pyx_pf_8PyClical_8clifford_100__setstate_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state) {
11750         PyObject *__pyx_r = NULL;
11751         __Pyx_RefNannyDeclarations
11752         PyObject *__pyx_t_1 = NULL;
11753         int __pyx_lineno = 0;
11754         const char *__pyx_filename = NULL;
11755         int __pyx_clineno = 0;
11756         __Pyx_RefNannySetupContext("__setstate_cython__", 0);
11757
11758         /* "(tree fragment)":4
11759         *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11760         * def __setstate_cython__(self, __pyx_state):
11761         *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
11762         */
11763         __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__12, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 4, __pyx_L1_error)
11764         __Pyx_GOTREF(__pyx_t_1);
11765         __Pyx_Raise(__pyx_t_1, 0, 0, 0);
11766         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11767         __PYX_ERR(1, 4, __pyx_L1_error)
11768
11769         /* "(tree fragment)":3
11770         * def __reduce_cython__(self):
11771         *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11772         * def __setstate_cython__(self, __pyx_state): # ««««««««
11773         *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11774         */
11775
11776         /* function exit code */
11777         __pyx_L1_error:;
11778         __Pyx_XDECREF(__pyx_t_1);
11779         __Pyx_AddTraceback("PyClical.clifford.__setstate_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11780         __pyx_r = NULL;
11781         __Pyx_XGIVEREF(__pyx_r);
11782         __Pyx_RefNannyFinishContext();
11783         return __pyx_r;

```

```

11784     }
11785
11786     /* "PyClicl.pyx":1253
11787     *         return clifford_to_str( self.unwrap() ).decode()
11788     *
11789     * def clifford_hidden_doctests():          # ««««««««
11790     *     """
11791     *     Tests for functions that Doctest cannot see.
11792     */
11793
11794     /* Python wrapper */
11795     static PyObject *__pyx_pw_8PyClicl_9clifford_hidden_doctests(PyObject *__pyx_self,
11796     CYTHON_UNUSED PyObject *unused); /*proto*/
11797     static char __pyx_doc_8PyClicl_8clifford_hidden_doctests[] = "\n    Tests for functions that
11798     Doctest cannot see.\n\n    For clifford.__cinit__: Construct an object of type clifford.\n\n    >>
11799     print(clifford(2))\n    2\n    >> print(clifford(2.0))\n    2\n    >> print(clifford(1.0e-1))\n
11800     0.1\n    >> print(clifford(\"2\"))\n    2\n    >> print(clifford(\"2{1,2,3}\"))\n    2{1,2,3}\n
11801     >> print(clifford(clifford(\"2{1,2,3}\")))\n    2{1,2,3}\n    >> print(clifford(\"-1\")\n    -1)\n
11802     >> print(clifford(2,index_set({1,2}))\n    2{1,2}\n    >> print(clifford([2,3],index_set({1,2}))\n
11803     2{1}+3{2}\n    >> print(clifford([1,2]))\n    Traceback (most recent call last):\n    ...\n
11804     TypeError: Cannot initialize clifford object from <class 'list'>.\n    >> print(clifford(None))\n
11805     Traceback (most recent call last):\n    ...\n    TypeError: Cannot initialize clifford object from
11806     <class 'NoneType'>.\n    >> print(clifford(None,[1,2]))\n    Traceback (most recent call last):\n
11807     ...\n    TypeError: Cannot initialize clifford object from ( <class 'NoneType'>, <class 'list'>).\n
11808     >> print(clifford([1,2],[1,2]))\n    Traceback (most recent call last):\n    ...\n    TypeError:
11809     Cannot initialize clifford object from ( <class 'list'>, <class 'list'>).\n    >>
11810     print(clifford(\"\")\n    Traceback (most recent call last):\n    ...\n    ValueError: Cannot
11811     initialize clifford object from invalid string \".\n    >> print(clifford(\"{\")\n    Traceback (most
11812     recent call last):\n    ...\n    ValueError: Cannot initialize clifford object from invalid string
11813     '{'.\n    >> print(clifford(\"{1}\")\n    Traceback (most recent call last):\n    ...\n
11814     ValueError: Cannot initialize clifford object from invalid string '{1'.\n    >>
11815     print(clifford(\"{+}\")\n    Traceback (most recent call last):\n    ...\n    ValueError: Cannot
11816     initialize clifford object from invalid string '{+'.\n    >> print(clifford(\"{-}\")\n    Traceback
11817     (most recent call last):\n    ...\n    ValueError: Cannot initialize clifford object fro\"m invalid
11818     string '{-'.\n    >> print(clifford(\"{1}+\")\n    Traceback (most recent call last):\n    ...\n
11819     ValueError: Cannot initialize clifford object from invalid string '{1}+'.\n    >>
11820     clifford.__richcmp__: Compare objects of type clifford.\n\n    >> clifford(\"{1}\") ==
11821     clifford(\"{1}\")\n    True\n    >> clifford(\"{1}\") != clifford(\"1.0{1}\")\n    False\n
11822     >> clifford(\"{1}\") != clifford(\"1.0\")\n    True\n    >> clifford(\"{1,2}\") == None\n    False\n
11823     >> clifford(\"{1,2}\") != None\n    True\n    >> None == clifford(\"{1,2}\")\n    False\n
11824     >> None
11825     != clifford(\"{1,2}\")\n    True\n    */
11826     static PyMethodDef __pyx_mdef_8PyClicl_9clifford_hidden_doctests =
11827     {\"clifford_hidden_doctests\", (PyCFunction)__pyx_pw_8PyClicl_9clifford_hidden_doctests, METH_NOARGS,
11828     __pyx_doc_8PyClicl_8clifford_hidden_doctests};
11829     static PyObject *__pyx_pw_8PyClicl_9clifford_hidden_doctests(PyObject *__pyx_self,
11830     CYTHON_UNUSED PyObject *unused) {
11831         PyObject *__pyx_r = 0;
11832         __Pyx_RefNannyDeclarations
11833         __Pyx_RefNannySetupContext(\"clifford_hidden_doctests (wrapper)\", 0);
11834         __pyx_r = __pyx_pf_8PyClicl_8clifford_hidden_doctests(__pyx_self);
11835
11836         /* function exit code */
11837         __Pyx_RefNannyFinishContext();
11838         return __pyx_r;
11839     }
11840
11841     static PyObject *__pyx_pf_8PyClicl_8clifford_hidden_doctests(CYTHON_UNUSED PyObject
11842     *__pyx_self) {
11843         PyObject *__pyx_r = NULL;
11844         __Pyx_RefNannyDeclarations
11845         __Pyx_RefNannySetupContext(\"clifford_hidden_doctests\", 0);
11846
11847         /* "PyClicl.pyx":1335
11848         *         True
11849         *         """
11850         *         return          # ««««««««
11851         *
11852         * cpdef inline error_squared_tol(obj):
11853         */
11854         __Pyx_XDECREF(__pyx_r);
11855         __pyx_r = Py_None; __Pyx_INCREF(Py_None);
11856         goto __pyx_L0;
11857
11858         /* "PyClicl.pyx":1253
11859         *         return clifford_to_str( self.unwrap() ).decode()
11860         *
11861         * def clifford_hidden_doctests():          # ««««««««
11862         *         """
11863         *         Tests for functions that Doctest cannot see.
11864         */
11865
11866         /* function exit code */
11867         __pyx_L0;
11868         __Pyx_XGIVEREF(__pyx_r);
11869         __Pyx_RefNannyFinishContext();
11870         return __pyx_r;
11871     }
11872

```

```

11839
11840      /* "PyClical.pyx":1337
11841      *      return
11842      *
11843      * cpdef inline error_squared_tol(obj):          # ««««««««
11844      *      """
11845      *      Quadratic norm error tolerance relative to a specific multivector.
11846      */
11847
11848      static PyObject *__pyx_pw_8PyClical_11error_squared_tol(PyObject *__pyx_self, PyObject
11849      *__pyx_v_obj); /*proto*/
11850      static CYTHON_INLINE PyObject *__pyx_f_8PyClical_error_squared_tol(PyObject
11851      *__pyx_v_obj, CYTHON_UNUSED int __pyx_skip_dispatch) {
11852          PyObject *__pyx_r = NULL;
11853          __Pyx_RefNannyDeclarations
11854          PyObject *__pyx_t_1 = NULL;
11855          int __pyx_lineno = 0;
11856          const char *__pyx_filename = NULL;
11857          int __pyx_clineno = 0;
11858          __Pyx_RefNannySetupContext("error_squared_tol", 0);
11859
11860          /* "PyClical.pyx":1344
11861          *      0.0
11862          *      """
11863          *      return glucat.error_squared_tol(toClifford(obj))          # ««««««««
11864          *
11865          * cpdef inline error_squared(lhs, rhs, threshold):
11866          */
11867          __Pyx_XDECREF(__pyx_r);
11868          __pyx_t_1 =
11869          PyFloat_FromDouble(error_squared_tol(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
11870          (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1344, __pyx_L1_error)
11871          __Pyx_GOTREF(__pyx_t_1);
11872          __pyx_r = __pyx_t_1;
11873          __pyx_t_1 = 0;
11874          goto __pyx_L0;
11875
11876          /* "PyClical.pyx":1337
11877          *      return
11878          *
11879          * cpdef inline error_squared_tol(obj):          # ««««««««
11880          *      """
11881          *      Quadratic norm error tolerance relative to a specific multivector.
11882          */
11883
11884          /* function exit code */
11885          __pyx_L1_error:;
11886          __Pyx_XDECREF(__pyx_t_1);
11887          __Pyx_AddTraceback("PyClical.error_squared_tol", __pyx_clineno, __pyx_lineno,
11888          __pyx_filename);
11889          __pyx_r = 0;
11890          __pyx_L0:;
11891          __Pyx_XGIVEREF(__pyx_r);
11892          __Pyx_RefNannyFinishContext();
11893          return __pyx_r;
11894      }
11895
11896      /* Python wrapper */
11897      static PyObject *__pyx_pw_8PyClical_11error_squared_tol(PyObject *__pyx_self, PyObject
11898      *__pyx_v_obj); /*proto*/
11899      static char __pyx_doc_8PyClical_10error_squared_tol[] = "\n    Quadratic norm error
11900      tolerance relative to a specific multivector.\n\n    >> print(error_squared_tol(clifford(\"{1}\") *
11901      3.0 - error_squared_tol(clifford(\"{1}{1}-2{2}+3{3}\")))\n    0.0\n    ";
11902      static PyObject *__pyx_pw_8PyClical_11error_squared_tol(PyObject *__pyx_self, PyObject
11903      *__pyx_v_obj) {
11904          PyObject *__pyx_r = 0;
11905          __Pyx_RefNannyDeclarations
11906          __Pyx_RefNannySetupContext("error_squared_tol (wrapper)", 0);
11907          __pyx_r = __pyx_pf_8PyClical_10error_squared_tol(__pyx_self, ((PyObject
11908          *)__pyx_v_obj));
11909
11910          /* function exit code */
11911          __Pyx_RefNannyFinishContext();
11912          return __pyx_r;
11913      }
11914
11915      static PyObject *__pyx_pf_8PyClical_10error_squared_tol(CYTHON_UNUSED PyObject
11916      *__pyx_self, PyObject *__pyx_v_obj) {
11917          PyObject *__pyx_r = NULL;
11918          __Pyx_RefNannyDeclarations
11919          PyObject *__pyx_t_1 = NULL;
11920          int __pyx_lineno = 0;
11921          const char *__pyx_filename = NULL;
11922          int __pyx_clineno = 0;
11923          __Pyx_RefNannySetupContext("error_squared_tol", 0);
11924          __Pyx_XDECREF(__pyx_r);
11925          __pyx_t_1 = __pyx_f_8PyClical_error_squared_tol(__pyx_v_obj, 0); if

```

```

(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1337, __pyx_L1_error)
11915     __Pyx_GOTREF(__pyx_t_1);
11916     __pyx_r = __pyx_t_1;
11917     __pyx_t_1 = 0;
11918     goto __pyx_L0;
11919
11920     /* function exit code */
11921     __pyx_L1_error;
11922     __Pyx_XDECREF(__pyx_t_1);
11923     __Pyx_AddTraceback("PyClical.error_squared_tol", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11924     __pyx_r = NULL;
11925     __pyx_L0;
11926     __Pyx_XGIVEREF(__pyx_r);
11927     __Pyx_RefNannyFinishContext();
11928     return __pyx_r;
11929 }
11930
11931     /* "PyClical.pyx":1346
11932 *     return glucat.error_squared_tol(toClifford(obj))
11933 *
11934 * cpdef inline error_squared(lhs, rhs, threshold):          # ««««««««
11935 *     """
11936 *     Relative or absolute error using the quadratic norm.
11937 */
11938
11939     static PyObject * __pyx_pw_8PyClical_13error_squared(PyObject * __pyx_self, PyObject
__pyx_args, PyObject * __pyx_kws); /*proto*/
11940     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_error_squared(PyObject * __pyx_v_lhs,
PyObject * __pyx_v_rhs, PyObject * __pyx_v_threshold, CYTHON_UNUSED int __pyx_skip_dispatch) {
11941         PyObject * __pyx_r = NULL;
11942         __Pyx_RefNannyDeclarations
11943         scalar_t __pyx_t_1;
11944         PyObject * __pyx_t_2 = NULL;
11945         int __pyx_lineno = 0;
11946         const char * __pyx_filename = NULL;
11947         int __pyx_clineno = 0;
11948         __Pyx_RefNannySetupContext("error_squared", 0);
11949
11950         /* "PyClical.pyx":1357
11951 *         25.0
11952 *         """
11953 *         return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
11954 *
11955 * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):
11956 */
11957         __Pyx_XDECREF(__pyx_r);
11958         __pyx_t_1 = __pyx_PyFloat_AsDouble(__pyx_v_threshold); if (unlikely((__pyx_t_1 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1357, __pyx_L1_error)
11959         __pyx_t_2 =
PyFloat_FromDouble(error_squared(__pyx_f_8PyClical_toClifford(__pyx_v_lhs),
__pyx_f_8PyClical_toClifford(__pyx_v_rhs), ((scalar_t) __pyx_t_1))); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1357, __pyx_L1_error)
11960         __Pyx_GOTREF(__pyx_t_2);
11961         __pyx_r = __pyx_t_2;
11962         __pyx_t_2 = 0;
11963         goto __pyx_L0;
11964
11965         /* "PyClical.pyx":1346
11966 *     return glucat.error_squared_tol(toClifford(obj))
11967 *
11968 * cpdef inline error_squared(lhs, rhs, threshold):          # ««««««««
11969 *     """
11970 *     Relative or absolute error using the quadratic norm.
11971 */
11972
11973         /* function exit code */
11974         __pyx_L1_error;
11975         __Pyx_XDECREF(__pyx_t_2);
11976         __Pyx_AddTraceback("PyClical.error_squared", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11977         __pyx_r = 0;
11978         __pyx_L0;
11979         __Pyx_XGIVEREF(__pyx_r);
11980         __Pyx_RefNannyFinishContext();
11981         return __pyx_r;
11982     }
11983
11984     /* Python wrapper */
11985     static PyObject * __pyx_pw_8PyClical_13error_squared(PyObject * __pyx_self, PyObject
__pyx_args, PyObject * __pyx_kws); /*proto*/
11986     static char __pyx_doc_8PyClical_12error_squared[] = "\n    Relative or absolute error
using the quadratic norm.\n\n    >> err2=scalar*epsilon*scalar*epsilon\n\n    >>
print(error_squared(clifford(\"{1}\", clifford(\"{1}\", err2))\n    0.0\n    >>
print(error_squared(clifford(\"{1}-3{2}+4{3}\", clifford(\"{1}\", err2))\n    25.0\n    ";
11987     static PyObject * __pyx_pw_8PyClical_13error_squared(PyObject * __pyx_self, PyObject

```



```

    __pyx_args, PyObject *__pyx_kwds) {
11988         PyObject *__pyx_v_lhs = 0;
11989         PyObject *__pyx_v_rhs = 0;
11990         PyObject *__pyx_v_threshold = 0;
11991         int __pyx_lineno = 0;
11992         const char *__pyx_filename = NULL;
11993         int __pyx_clineno = 0;
11994         PyObject *__pyx_r = 0;
11995         __Pyx_RefNannyDeclarations
11996         __Pyx_RefNannySetupContext("error_squared (wrapper)", 0);
11997         {
11998             static PyObject **__pyx_pyargnames[] =
11999             {&__pyx_n_s_lhs,&__pyx_n_s_rhs,&__pyx_n_s_threshold,0};
12000             PyObject* values[3] = {0,0,0};
12001             if (unlikely(__pyx_kwds)) {
12002                 Py_ssize_t kw_args;
12003                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
12004                 switch (pos_args) {
12005                     case 3: values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
12006                     CYTHON_FALLTHROUGH;
12007                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
12008                     CYTHON_FALLTHROUGH;
12009                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
12010                     CYTHON_FALLTHROUGH;
12011                     case 0: break;
12012                     default: goto __pyx_L5_argtuple_error;
12013                 }
12014                 kw_args = PyDict_Size(__pyx_kwds);
12015                 switch (pos_args) {
12016                     case 0:
12017                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_lhs)) !=
12018 0)) kw_args--;
12019                     else goto __pyx_L5_argtuple_error;
12020                     CYTHON_FALLTHROUGH;
12021                     case 1:
12022                     if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_rhs)) !=
12023 0)) kw_args--;
12024                     else {
12025                         __Pyx_RaiseArgtupleInvalid("error_squared", 1, 3, 3, 1); __PYX_ERR(0, 1346,
12026 __pyx_L3_error)
12027                     }
12028                     CYTHON_FALLTHROUGH;
12029                     case 2:
12030                     if (likely((values[2] = __Pyx_PyDict_GetItemStr(__pyx_kwds,
12031 __pyx_n_s_threshold)) != 0)) kw_args--;
12032                     else {
12033                         __Pyx_RaiseArgtupleInvalid("error_squared", 1, 3, 3, 2); __PYX_ERR(0, 1346,
12034 __pyx_L3_error)
12035                     }
12036                     if (unlikely(kw_args > 0)) {
12037                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
12038 values, pos_args, "error_squared") < 0)) __PYX_ERR(0, 1346, __pyx_L3_error)
12039                     }
12040                     } else if (PyTuple_GET_SIZE(__pyx_args) != 3) {
12041                         goto __pyx_L5_argtuple_error;
12042                     } else {
12043                         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
12044                         values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
12045                         values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
12046                     }
12047                     __pyx_v_lhs = values[0];
12048                     __pyx_v_rhs = values[1];
12049                     __pyx_v_threshold = values[2];
12050                 }
12051                 goto __pyx_L4_argument_unpacking_done;
12052                 __pyx_L5_argtuple_error:;
12053                 __Pyx_RaiseArgtupleInvalid("error_squared", 1, 3, 3, PyTuple_GET_SIZE(__pyx_args));
12054                 __PYX_ERR(0, 1346, __pyx_L3_error)
12055                 __pyx_L3_error:;
12056                 __Pyx_AddTraceback("PyClical.error_squared", __pyx_clineno, __pyx_lineno,
12057 __pyx_filename);
12058                 __Pyx_RefNannyFinishContext();
12059                 return NULL;
12060                 __pyx_L4_argument_unpacking_done:;
12061                 __pyx_r = __pyx_pf_8PyClical_12error_squared(__pyx_self, __pyx_v_lhs, __pyx_v_rhs,
12062 __pyx_v_threshold);
12063             }
12064             /* function exit code */
12065             __Pyx_RefNannyFinishContext();
12066             return __pyx_r;
12067         }
12068     }
12069     static PyObject *__pyx_pf_8PyClical_12error_squared(CYTHON_UNUSED PyObject
12070 *__pyx_self, PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold) {
12071         PyObject *__pyx_r = NULL;
12072         __Pyx_RefNannyDeclarations

```

```

12063         PyObject *__pyx_t_1 = NULL;
12064         int __pyx_lineno = 0;
12065         const char *__pyx_filename = NULL;
12066         int __pyx_clineno = 0;
12067         __Pyx_RefNannySetupContext("error_squared", 0);
12068         __Pyx_XDECREF(__pyx_r);
12069         __pyx_t_1 = __pyx_f_8PyClical_error_squared(__pyx_v_lhs, __pyx_v_rhs,
__pyx_v_threshold, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1346, __pyx_L1_error)
12070         __Pyx_GOTREF(__pyx_t_1);
12071         __pyx_r = __pyx_t_1;
12072         __pyx_t_1 = 0;
12073         goto __pyx_L0;
12074
12075         /* function exit code */
12076         __pyx_L1_error;
12077         __Pyx_XDECREF(__pyx_t_1);
12078         __Pyx_AddTraceback("PyClical.error_squared", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12079         __pyx_r = NULL;
12080         __pyx_L0;
12081         __Pyx_XGIVEREF(__pyx_r);
12082         __Pyx_RefNannyFinishContext();
12083         return __pyx_r;
12084     }
12085
12086     /* "PyClical.pyx":1359
12087     *     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
12088     *
12089     * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):                                # ««««««««
12090     *     """
12091     *     Test for approximate equality of multivectors.
12092     */
12093
12094     static PyObject *__pyx_pw_8PyClical_15approx_equal(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
12095     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_approx_equal(PyObject *__pyx_v_lhs,
PyObject *__pyx_v_rhs, CYTHON_UNUSED int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_approx_equal *__pyx_optional_args) {
12096         PyObject *__pyx_v_threshold = ((PyObject *)Py_None);
12097         PyObject *__pyx_v_tol = ((PyObject *)Py_None);
12098         PyObject *__pyx_r = NULL;
12099         __Pyx_RefNannyDeclarations
12100         PyObject *__pyx_t_1 = NULL;
12101         int __pyx_t_2;
12102         PyObject *__pyx_t_3 = NULL;
12103         scalar_t __pyx_t_4;
12104         scalar_t __pyx_t_5;
12105         int __pyx_lineno = 0;
12106         const char *__pyx_filename = NULL;
12107         int __pyx_clineno = 0;
12108         __Pyx_RefNannySetupContext("approx_equal", 0);
12109         if (__pyx_optional_args) {
12110             if (__pyx_optional_args->__pyx_n > 0) {
12111                 __pyx_v_threshold = __pyx_optional_args->threshold;
12112                 if (__pyx_optional_args->__pyx_n > 1) {
12113                     __pyx_v_tol = __pyx_optional_args->tol;
12114                 }
12115             }
12116         }
12117         __Pyx_INCREF(__pyx_v_threshold);
12118         __Pyx_INCREF(__pyx_v_tol);
12119
12120         /* "PyClical.pyx":1374
12121         *     True
12122         *     """
12123         *     threshold = error_squared_tol(rhs) if threshold is None else threshold                # ««««««««
12124         *     tol = error_squared_tol(rhs) if tol is None else tol
12125         *     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)
12126         */
12127         __pyx_t_2 = (__pyx_v_threshold == Py_None);
12128         if ((__pyx_t_2 != 0)) {
12129             __pyx_t_3 = __pyx_f_8PyClical_error_squared_tol(__pyx_v_rhs, 0); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1374, __pyx_L1_error)
12130             __Pyx_GOTREF(__pyx_t_3);
12131             __pyx_t_1 = __pyx_t_3;
12132             __pyx_t_3 = 0;
12133         } else {
12134             __Pyx_INCREF(__pyx_v_threshold);
12135             __pyx_t_1 = __pyx_v_threshold;
12136         }
12137         __Pyx_DECREF_SET(__pyx_v_threshold, __pyx_t_1);
12138         __pyx_t_1 = 0;
12139
12140         /* "PyClical.pyx":1375
12141         *     """
12142         *     threshold = error_squared_tol(rhs) if threshold is None else threshold

```

```

12143 *      tol      = error_squared_tol(rhs) if tol      is None else tol      # ««««««««
12144 *      return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)
12145 *
12146 */
12147     __pyx_t_2 = (__pyx_v_tol == Py_None);
12148     if ((__pyx_t_2 != 0)) {
12149         __pyx_t_3 = __pyx_f_8PyClical_error_squared_tol(__pyx_v_rhs, 0); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1375, __pyx_L1_error)
12150         __Pyx_GOTREF(__pyx_t_3);
12151         __pyx_t_1 = __pyx_t_3;
12152         __pyx_t_3 = 0;
12153     } else {
12154         __Pyx_INCREF(__pyx_v_tol);
12155         __pyx_t_1 = __pyx_v_tol;
12156     }
12157     __Pyx_DECREF_SET(__pyx_v_tol, __pyx_t_1);
12158     __pyx_t_1 = 0;
12159
12160     /* "PyClical.pyx":1376
12161 *      threshold = error_squared_tol(rhs) if threshold is None else threshold
12162 *      tol      = error_squared_tol(rhs) if tol      is None else tol
12163 *      return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)      # ««««««««
12164 *
12165 * cpdef inline inv(obj):
12166 */
12167     __Pyx_XDECREF(__pyx_r);
12168     __pyx_t_4 = __pyx_PyFloat_AsDouble(__pyx_v_threshold); if (unlikely((__pyx_t_4 ==
((<scalar_t>-1)) && PyErr_Occurred())) __PYX_ERR(0, 1376, __pyx_L1_error)
12169     __pyx_t_5 = __pyx_PyFloat_AsDouble(__pyx_v_tol); if (unlikely((__pyx_t_5 ==
((<scalar_t>-1)) && PyErr_Occurred())) __PYX_ERR(0, 1376, __pyx_L1_error)
12170     __pyx_t_1 =
__Pyx_PyBool_FromLong(approx_equal(__pyx_f_8PyClical_toClifford(__pyx_v_lhs),
__pyx_f_8PyClical_toClifford(__pyx_v_rhs), ((<scalar_t>)__pyx_t_4), ((<scalar_t>)__pyx_t_5))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1376, __pyx_L1_error)
12171     __Pyx_GOTREF(__pyx_t_1);
12172     __pyx_r = __pyx_t_1;
12173     __pyx_t_1 = 0;
12174     goto __pyx_L0;
12175
12176     /* "PyClical.pyx":1359
12177 *      return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
12178 *
12179 * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):      # ««««««««
12180 *      """
12181 *      Test for approximate equality of multivectors.
12182 */
12183
12184     /* function exit code */
12185     __pyx_L1_error;
12186     __Pyx_XDECREF(__pyx_t_1);
12187     __Pyx_XDECREF(__pyx_t_3);
12188     __Pyx_AddTraceback("PyClical.approx_equal", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12189     __pyx_r = 0;
12190     __pyx_L0;
12191     __Pyx_XDECREF(__pyx_v_threshold);
12192     __Pyx_XDECREF(__pyx_v_tol);
12193     __Pyx_XGIVEREF(__pyx_r);
12194     __Pyx_RefNannyFinishContext();
12195     return __pyx_r;
12196 }
12197
12198 /* Python wrapper */
12199 static PyObject *__pyx_pw_8PyClical_15approx_equal(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
12200 static char __pyx_doc_8PyClical_14approx_equal[] = "\n      Test for approximate
equality of multivectors.\n\n      >> err2=scalar_epsilon*scalar_epsilon\n\n      >>
print(approx_equal(clifford(\"1{1}\"), clifford(\"1{1}\")))\n      True\n      >>
print(approx_equal(clifford(\"1{1}-3{2}+4{3}\"), clifford(\"1{1}\")))\n      False\n      >>
print(approx_equal(clifford(\"1{1}-3{2}+4{3}+0.001\"), clifford(\"1{1}-3{2}+4{3}\"), err2, err2))\n
False\n      >> print(approx_equal(clifford(\"1{1}-3{2}+4{3}+1.0e-30\"), clifford(\"1{1}-3{2}+4{3}\"),
err2, err2))\n      True\n      ";
12201 static PyObject *__pyx_pw_8PyClical_15approx_equal(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
12202     PyObject *__pyx_v_lhs = 0;
12203     PyObject *__pyx_v_rhs = 0;
12204     PyObject *__pyx_v_threshold = 0;
12205     PyObject *__pyx_v_tol = 0;
12206     int __pyx_lineno = 0;
12207     const char *__pyx_filename = NULL;
12208     int __pyx_clineno = 0;
12209     PyObject *__pyx_r = 0;
12210     __Pyx_RefNannyDeclarations
12211     __Pyx_RefNannySetupContext("approx_equal (wrapper)", 0);
12212

```

```

12213         static PyObject **__pyx_pyargnames[] =
12214         {&__pyx_n_s_lhs,&__pyx_n_s_rhs,&__pyx_n_s_threshold,&__pyx_n_s_tol,0};
12215         PyObject* values[4] = {0,0,0,0};
12216         values[2] = ((PyObject *)Py_None);
12217         values[3] = ((PyObject *)Py_None);
12218         if (unlikely(__pyx_kwds)) {
12219             Py_ssize_t kw_args;
12220             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
12221             switch (pos_args) {
12222                 case 4: values[3] = PyTuple_GET_ITEM(__pyx_args, 3);
12223                     CYTHON_FALLTHROUGH;
12224                 case 3: values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
12225                     CYTHON_FALLTHROUGH;
12226                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
12227                     CYTHON_FALLTHROUGH;
12228                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
12229                     CYTHON_FALLTHROUGH;
12230                 case 0: break;
12231                 default: goto __pyx_L5_argtuple_error;
12232             }
12233             kw_args = PyDict_Size(__pyx_kwds);
12234             switch (pos_args) {
12235                 case 0:
12236                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_lhs)) !=
12237                     0)) kw_args--;
12238                     else goto __pyx_L5_argtuple_error;
12239                     CYTHON_FALLTHROUGH;
12240                 case 1:
12241                     if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_rhs)) !=
12242                     0)) kw_args--;
12243                     else {
12244                         __Pyx_RaiseArgtupleInvalid("approx_equal", 0, 2, 4, 1); __PYX_ERR(0, 1359,
12245                         __pyx_L3_error)
12246                     }
12247                     CYTHON_FALLTHROUGH;
12248                 case 2:
12249                     if (kw_args > 0) {
12250                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_threshold);
12251                         if (value) { values[2] = value; kw_args--; }
12252                     }
12253                     CYTHON_FALLTHROUGH;
12254                 case 3:
12255                     if (kw_args > 0) {
12256                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_tol);
12257                         if (value) { values[3] = value; kw_args--; }
12258                     }
12259                     if (unlikely(kw_args > 0)) {
12260                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
12261                         values, pos_args, "approx_equal") < 0)) __PYX_ERR(0, 1359, __pyx_L3_error)
12262                     }
12263                     } else {
12264                         switch (PyTuple_GET_SIZE(__pyx_args)) {
12265                             case 4: values[3] = PyTuple_GET_ITEM(__pyx_args, 3);
12266                                 CYTHON_FALLTHROUGH;
12267                             case 3: values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
12268                                 CYTHON_FALLTHROUGH;
12269                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
12270                                 CYTHON_FALLTHROUGH;
12271                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
12272                                 break;
12273                             default: goto __pyx_L5_argtuple_error;
12274                         }
12275                     }
12276                     __pyx_v_lhs = values[0];
12277                     __pyx_v_rhs = values[1];
12278                     __pyx_v_threshold = values[2];
12279                     __pyx_v_tol = values[3];
12280                 }
12281                 goto __pyx_L4_argument_unpacking_done;
12282                 __pyx_L5_argtuple_error:;
12283                 __Pyx_RaiseArgtupleInvalid("approx_equal", 0, 2, 4, PyTuple_GET_SIZE(__pyx_args));
12284                 __PYX_ERR(0, 1359, __pyx_L3_error)
12285                 __pyx_L3_error:;
12286                 __Pyx_AddTraceback("PyClical.approx_equal", __pyx_clineno, __pyx_lineno,
12287                 __pyx_filename);
12288                 __Pyx_RefNannyFinishContext();
12289                 return NULL;
12290                 __pyx_L4_argument_unpacking_done:;
12291                 __pyx_r = __pyx_pf_8PyClical_14approx_equal(__pyx_self, __pyx_v_lhs, __pyx_v_rhs,
12292                 __pyx_v_threshold, __pyx_v_tol);
12293
12294                 /* function exit code */
12295                 __Pyx_RefNannyFinishContext();
12296                 return __pyx_r;
12297             }
12298         }
12299         static PyObject *__pyx_pf_8PyClical_14approx_equal(CYTHON_UNUSED PyObject *__pyx_self,

```

```

PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold, PyObject *__pyx_v_tol) {
12292     PyObject *__pyx_r = NULL;
12293     __Pyx_RefNannyDeclarations
12294     PyObject *__pyx_t_1 = NULL;
12295     struct __pyx_opt_args_8PyClical_approx_equal __pyx_t_2;
12296     int __pyx_lineno = 0;
12297     const char *__pyx_filename = NULL;
12298     int __pyx_clineno = 0;
12299     __Pyx_RefNannySetupContext("approx_equal", 0);
12300     __Pyx_XDECREF(__pyx_r);
12301     __pyx_t_2.__pyx_n = 2;
12302     __pyx_t_2.threshold = __pyx_v_threshold;
12303     __pyx_t_2.tol = __pyx_v_tol;
12304     __pyx_t_1 = __pyx_f_8PyClical_approx_equal(__pyx_v_lhs, __pyx_v_rhs, 0, &__pyx_t_2);
    if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1359, __pyx_L1_error)
12305     __Pyx_GOTREF(__pyx_t_1);
12306     __pyx_r = __pyx_t_1;
12307     __pyx_t_1 = 0;
12308     goto __pyx_L0;
12309
12310     /* function exit code */
12311     __pyx_L1_error;;
12312     __Pyx_XDECREF(__pyx_t_1);
12313     __Pyx_AddTraceback("PyClical.approx_equal", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12314     __pyx_r = NULL;
12315     __pyx_L0;;
12316     __Pyx_XGIVEREF(__pyx_r);
12317     __Pyx_RefNannyFinishContext();
12318     return __pyx_r;
12319 }
12320
12321     /* "PyClical.pyx":1378
12322     *      return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)
12323     *
12324     * cpdef inline inv(obj):          # ««««««««
12325     *     """
12326     *     Geometric multiplicative inverse.
12327     */
12328
12329     static PyObject *__pyx_pw_8PyClical_17inv(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12330     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_inv(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12331     PyObject *__pyx_r = NULL;
12332     __Pyx_RefNannyDeclarations
12333     PyObject *__pyx_t_1 = NULL;
12334     PyObject *__pyx_t_2 = NULL;
12335     PyObject *__pyx_t_3 = NULL;
12336     int __pyx_lineno = 0;
12337     const char *__pyx_filename = NULL;
12338     int __pyx_clineno = 0;
12339     __Pyx_RefNannySetupContext("inv", 0);
12340
12341     /* "PyClical.pyx":1391
12342     *     nan
12343     *     """
12344     *     return clifford(obj).inv()          # ««««««««
12345     *
12346     * cpdef inline scalar(obj):
12347     */
12348     __Pyx_XDECREF(__pyx_r);
12349     __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1391, __pyx_L1_error)
12350     __Pyx_GOTREF(__pyx_t_2);
12351     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_inv); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1391, __pyx_L1_error)
12352     __Pyx_GOTREF(__pyx_t_3);
12353     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12354     __pyx_t_2 = NULL;
12355     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12356         __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12357         if (likely(__pyx_t_2)) {
12358             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12359             __Pyx_INCREF(__pyx_t_2);
12360             __Pyx_INCREF(function);
12361             __Pyx_DECREF_SET(__pyx_t_3, function);
12362         }
12363     }
12364     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12365     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12366     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1391, __pyx_L1_error)
12367     __Pyx_GOTREF(__pyx_t_1);
12368     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12369     __pyx_r = __pyx_t_1;

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12370         __pyx_t_1 = 0;
12371         goto __pyx_L0;
12372
12373         /* "PyClical.pyx":1378
12374 *       return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
12375 <scalar_t>tol)
12376 * cpdef inline inv(obj):                # ««««««««
12377 *     """
12378 *     Geometric multiplicative inverse.
12379 */
12380
12381         /* function exit code */
12382         __pyx_L1_error:;
12383         __Pyx_XDECREF(__pyx_t_1);
12384         __Pyx_XDECREF(__pyx_t_2);
12385         __Pyx_XDECREF(__pyx_t_3);
12386         __Pyx_AddTraceback("PyClical.inv", __pyx_clineno, __pyx_lineno, __pyx_filename);
12387         __pyx_r = 0;
12388         __pyx_L0:;
12389         __Pyx_XGIVEREF(__pyx_r);
12390         __Pyx_RefNannyFinishContext();
12391         return __pyx_r;
12392     }
12393
12394     /* Python wrapper */
12395     static PyObject * __pyx_pw_8PyClical_17inv(PyObject * __pyx_self, PyObject
12396 * __pyx_v_obj); /*proto*/
12397     static char __pyx_doc_8PyClical_16inv[] = "\n    Geometric multiplicative inverse.\n\n
12398 >> print(inv(clifford(\"{1}\")))\n    {1}\n    >> print(inv(clifford(\"{-1}\")))\n    {-1}\n    >>
12399 print(inv(clifford(\"{-2,-1}\")))\n    {-2,-1}\n    >> print(inv(clifford(\"{-1}+{1}\")))\n    nan\n
12400 ";
12401     static PyObject * __pyx_pw_8PyClical_17inv(PyObject * __pyx_self, PyObject * __pyx_v_obj)
12402 {
12403     PyObject * __pyx_r = 0;
12404     __Pyx_RefNannyDeclarations
12405     __Pyx_RefNannySetupContext("inv (wrapper)", 0);
12406     __pyx_r = __pyx_pf_8PyClical_16inv(__pyx_self, ((PyObject *) __pyx_v_obj));
12407
12408     /* function exit code */
12409     __Pyx_RefNannyFinishContext();
12410     return __pyx_r;
12411 }
12412
12413     static PyObject * __pyx_pf_8PyClical_16inv(CYTHON_UNUSED PyObject * __pyx_self, PyObject
12414 * __pyx_v_obj) {
12415     PyObject * __pyx_r = NULL;
12416     __Pyx_RefNannyDeclarations
12417     PyObject * __pyx_t_1 = NULL;
12418     int __pyx_lineno = 0;
12419     const char * __pyx_filename = NULL;
12420     int __pyx_clineno = 0;
12421     __Pyx_RefNannySetupContext("inv", 0);
12422     __Pyx_XDECREF(__pyx_r);
12423     __pyx_t_1 = __pyx_f_8PyClical_inv(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
12424     __PYX_ERR(0, 1378, __pyx_L1_error)
12425     __Pyx_GOTREF(__pyx_t_1);
12426     __pyx_r = __pyx_t_1;
12427     __pyx_t_1 = 0;
12428     goto __pyx_L0;
12429
12430     /* function exit code */
12431     __pyx_L1_error:;
12432     __Pyx_XDECREF(__pyx_t_1);
12433     __Pyx_AddTraceback("PyClical.inv", __pyx_clineno, __pyx_lineno, __pyx_filename);
12434     __pyx_r = NULL;
12435     __pyx_L0:;
12436     __Pyx_XGIVEREF(__pyx_r);
12437     __Pyx_RefNannyFinishContext();
12438     return __pyx_r;
12439 }
12440
12441     /* "PyClical.pyx":1393
12442 *       return clifford(obj).inv()
12443 * cpdef inline scalar(obj):                # ««««««««
12444 *     """
12445 *     Scalar part.
12446 */
12447
12448     static PyObject * __pyx_pw_8PyClical_19scalar(PyObject * __pyx_self, PyObject
12449 * __pyx_v_obj); /*proto*/
12450     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_scalar(PyObject * __pyx_v_obj,
12451 CYTHON_UNUSED int __pyx_skip_dispatch) {
12452     PyObject * __pyx_r = NULL;
12453     __Pyx_RefNannyDeclarations
12454     PyObject * __pyx_t_1 = NULL;

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12447         PyObject *__pyx_t_2 = NULL;
12448         PyObject *__pyx_t_3 = NULL;
12449         int __pyx_lineno = 0;
12450         const char *__pyx_filename = NULL;
12451         int __pyx_clineno = 0;
12452         __Pyx_RefNannySetupContext("scalar", 0);
12453
12454         /* "PyClical.pyx":1402
12455         *      0.0
12456         *      """
12457         *      return clifford(obj).scalar()          # ««««««««
12458         *
12459         * cpdef inline real(obj):
12460         */
12461         __Pyx_XDECREF(__pyx_r);
12462         __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1402, __pyx_L1_error)
12463         __Pyx_GOTREF(__pyx_t_2);
12464         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_scalar); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1402, __pyx_L1_error)
12465         __Pyx_GOTREF(__pyx_t_3);
12466         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12467         __pyx_t_2 = NULL;
12468         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12469             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12470             if (likely(__pyx_t_2)) {
12471                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12472                 __Pyx_INCREF(__pyx_t_2);
12473                 __Pyx_INCREF(function);
12474                 __Pyx_DECREF_SET(__pyx_t_3, function);
12475             }
12476         }
12477         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12478         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12479         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1402, __pyx_L1_error)
12480         __Pyx_GOTREF(__pyx_t_1);
12481         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12482         __pyx_r = __pyx_t_1;
12483         __pyx_t_1 = 0;
12484         goto __pyx_L0;
12485
12486         /* "PyClical.pyx":1393
12487         *      return clifford(obj).inv()
12488         *
12489         * cpdef inline scalar(obj):          # ««««««««
12490         *      """
12491         *      Scalar part.
12492         */
12493
12494         /* function exit code */
12495         __pyx_L1_error;
12496         __Pyx_XDECREF(__pyx_t_1);
12497         __Pyx_XDECREF(__pyx_t_2);
12498         __Pyx_XDECREF(__pyx_t_3);
12499         __Pyx_AddTraceback("PyClical.scalar", __pyx_clineno, __pyx_lineno, __pyx_filename);
12500         __pyx_r = 0;
12501         __pyx_L0;
12502         __Pyx_XGIVEREF(__pyx_r);
12503         __Pyx_RefNannyFinishContext();
12504         return __pyx_r;
12505     }
12506
12507     /* Python wrapper */
12508     static PyObject *__pyx_pw_8PyClical_19scalar(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12509     static char __pyx_doc_8PyClical_18scalar[] = "\n    Scalar part.\n\n    >>
scalar(clifford(\"1+{1}+{1,2}\"))\n    1.0\n    >> scalar(clifford(\"{1,2}\"))\n    0.0\n    ";
12510     static PyObject *__pyx_pw_8PyClical_19scalar(PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
12511         PyObject *__pyx_r = 0;
12512         __Pyx_RefNannyDeclarations
12513         __Pyx_RefNannySetupContext("scalar (wrapper)", 0);
12514         __pyx_r = __pyx_pf_8PyClical_18scalar(__pyx_self, ((PyObject *)__pyx_v_obj));
12515
12516         /* function exit code */
12517         __Pyx_RefNannyFinishContext();
12518         return __pyx_r;
12519     }
12520
12521     static PyObject *__pyx_pf_8PyClical_18scalar(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
12522         PyObject *__pyx_r = NULL;
12523         __Pyx_RefNannyDeclarations
12524         PyObject *__pyx_t_1 = NULL;
12525         int __pyx_lineno = 0;
12526         const char *__pyx_filename = NULL;

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```

12527         int __pyx_clineno = 0;
12528         __Pyx_RefNannySetupContext("scalar", 0);
12529         __Pyx_XDECREF(__pyx_r);
12530         __pyx_t_1 = __pyx_f_8PyClical_scalar(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1393, __pyx_L1_error)
12531         __Pyx_GOTREF(__pyx_t_1);
12532         __pyx_r = __pyx_t_1;
12533         __pyx_t_1 = 0;
12534         goto __pyx_L0;
12535
12536         /* function exit code */
12537         __pyx_L1_error:;
12538         __Pyx_XDECREF(__pyx_t_1);
12539         __Pyx_AddTraceback("PyClical.scalar", __pyx_clineno, __pyx_lineno, __pyx_filename);
12540         __pyx_r = NULL;
12541         __pyx_L0:;
12542         __Pyx_XGIVEREF(__pyx_r);
12543         __Pyx_RefNannyFinishContext();
12544         return __pyx_r;
12545     }
12546
12547     /* "PyClical.pyx":1404
12548     *     return clifford(obj).scalar()
12549     *
12550     * cpdef inline real(obj):
12551     *     """
12552     *     Real part: synonym for scalar part.
12553     */
12554
12555     static PyObject * __pyx_pw_8PyClical_21real(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
12556     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_real(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12557         PyObject * __pyx_r = NULL;
12558         __Pyx_RefNannyDeclarations
12559         PyObject * __pyx_t_1 = NULL;
12560         PyObject * __pyx_t_2 = NULL;
12561         PyObject * __pyx_t_3 = NULL;
12562         int __pyx_lineno = 0;
12563         const char * __pyx_filename = NULL;
12564         int __pyx_clineno = 0;
12565         __Pyx_RefNannySetupContext("real", 0);
12566
12567         /* "PyClical.pyx":1413
12568     *     0.0
12569     *     """
12570     *     return clifford(obj).scalar()
12571     *
12572     * cpdef inline imag(obj):
12573     */
12574         __Pyx_XDECREF(__pyx_r);
12575         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *) __pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1413, __pyx_L1_error)
12576         __Pyx_GOTREF(__pyx_t_2);
12577         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_scalar); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1413, __pyx_L1_error)
12578         __Pyx_GOTREF(__pyx_t_3);
12579         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12580         __pyx_t_2 = NULL;
12581         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12582             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12583             if (likely(__pyx_t_2)) {
12584                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12585                 __Pyx_INCREF(__pyx_t_2);
12586                 __Pyx_INCREF(function);
12587                 __Pyx_DECREF_SET(__pyx_t_3, function);
12588             }
12589         }
12590         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12591         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12592         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1413, __pyx_L1_error)
12593         __Pyx_GOTREF(__pyx_t_1);
12594         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12595         __pyx_r = __pyx_t_1;
12596         __pyx_t_1 = 0;
12597         goto __pyx_L0;
12598
12599         /* "PyClical.pyx":1404
12600     *     return clifford(obj).scalar()
12601     *
12602     * cpdef inline real(obj):
12603     *     """
12604     *     Real part: synonym for scalar part.
12605     */
12606
12607         /* function exit code */

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12608         __pyx_L1_error;;
12609         __Pyx_XDECREF(__pyx_t_1);
12610         __Pyx_XDECREF(__pyx_t_2);
12611         __Pyx_XDECREF(__pyx_t_3);
12612         __Pyx_AddTraceback("PyClical.real", __pyx_clineno, __pyx_lineno, __pyx_filename);
12613         __pyx_r = 0;
12614         __pyx_L0;;
12615         __Pyx_XGIVEREF(__pyx_r);
12616         __Pyx_RefNannyFinishContext();
12617         return __pyx_r;
12618     }
12619
12620     /* Python wrapper */
12621     static PyObject* __pyx_pw_8PyClical_21real(PyObject* __pyx_self, PyObject
12622 *__pyx_v_obj); /*proto*/
12623     static char __pyx_doc_8PyClical_20real[] = "\n    Real part: synonym for scalar
part.\n\n    >> real(clifford(\"1+{1}+{1,2}\"))\n    1.0\n    >> real(clifford(\"{1,2}\"))\n    0.0\n";
12624     static PyObject* __pyx_pw_8PyClical_21real(PyObject* __pyx_self, PyObject
12625 *__pyx_v_obj) {
12626         PyObject* __pyx_r = 0;
12627         __Pyx_RefNannyDeclarations
12628         __Pyx_RefNannySetupContext("real (wrapper)", 0);
12629         __pyx_r = __pyx_pf_8PyClical_20real(__pyx_self, ((PyObject*) __pyx_v_obj));
12630
12631         /* function exit code */
12632         __Pyx_RefNannyFinishContext();
12633         return __pyx_r;
12634     }
12635     static PyObject* __pyx_pf_8PyClical_20real(CYTHON_UNUSED PyObject* __pyx_self,
12636 PyObject* __pyx_v_obj) {
12637         PyObject* __pyx_r = NULL;
12638         __Pyx_RefNannyDeclarations
12639         PyObject* __pyx_t_1 = NULL;
12640         int __pyx_lineno = 0;
12641         const char* __pyx_filename = NULL;
12642         int __pyx_clineno = 0;
12643         __Pyx_RefNannySetupContext("real", 0);
12644         __Pyx_XDECREF(__pyx_r);
12645         __pyx_t_1 = __pyx_f_8PyClical_real(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
12646         __PYX_ERR(0, 1404, __pyx_L1_error)
12647         __Pyx_GOTREF(__pyx_t_1);
12648         __pyx_r = __pyx_t_1;
12649         __pyx_t_1 = 0;
12650         goto __pyx_L0;
12651
12652         /* function exit code */
12653         __Pyx_L1_error;;
12654         __Pyx_XDECREF(__pyx_t_1);
12655         __Pyx_AddTraceback("PyClical.real", __pyx_clineno, __pyx_lineno, __pyx_filename);
12656         __pyx_r = NULL;
12657         __pyx_L0;;
12658         __Pyx_XGIVEREF(__pyx_r);
12659         __Pyx_RefNannyFinishContext();
12660         return __pyx_r;
12661     }
12662
12663     /* "PyClical.pyx":1415
12664 *     return clifford(obj).scalar()
12665 *
12666 * cpdef inline imag(obj): # <<<<<<<<<
12667 *     """
12668 *     Imaginary part: deprecated (always 0).
12669 */
12670     static PyObject* __pyx_pw_8PyClical_23imag(PyObject* __pyx_self, PyObject
12671 *__pyx_v_obj); /*proto*/
12672     static CYTHON_INLINE PyObject* __pyx_f_8PyClical_imag(CYTHON_UNUSED PyObject
12673 *__pyx_v_obj, CYTHON_UNUSED int __pyx_skip_dispatch) {
12674         PyObject* __pyx_r = NULL;
12675         __Pyx_RefNannyDeclarations
12676         __Pyx_RefNannySetupContext("imag", 0);
12677
12678         /* "PyClical.pyx":1424
12679 *     0.0
12680 *     """
12681 *     return 0.0 # <<<<<<<<<
12682 *
12683 * cpdef inline pure(obj):
12684 */
12685         __Pyx_XDECREF(__pyx_r);
12686         __Pyx_INCREF(__pyx_float_0_0);
12687         __pyx_r = __pyx_float_0_0;
12688         goto __pyx_L0;
12689
12690         /* "PyClical.pyx":1415

```

```

12687 *      return clifford(obj).scalar()
12688 *
12689 * cpdef inline imag(obj):          # ««««««««
12690 *      """
12691 *      Imaginary part: deprecated (always 0).
12692 */
12693
12694         /* function exit code */
12695         __pyx_L0:;
12696         __Pyx_XGIVEREF(__pyx_r);
12697         __Pyx_RefNannyFinishContext();
12698         return __pyx_r;
12699     }
12700
12701     /* Python wrapper */
12702     static PyObject * __pyx_pw_8PyClical_23imag(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12703     static char __pyx_doc_8PyClical_22imag[] = "\n      Imaginary part: deprecated (always
0).\n\n      >> imag(clifford(\"1+{1}+{1,2}\"))\n      0.0\n      >> imag(clifford(\"{1,2}\"))\n      0.0\n
";
12704     static PyObject * __pyx_pw_8PyClical_23imag(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
12705         PyObject * __pyx_r = 0;
12706         __Pyx_RefNannyDeclarations
12707         __Pyx_RefNannySetupContext("imag (wrapper)", 0);
12708         __pyx_r = __pyx_pf_8PyClical_22imag(__pyx_self, ((PyObject *) __pyx_v_obj));
12709
12710         /* function exit code */
12711         __Pyx_RefNannyFinishContext();
12712         return __pyx_r;
12713     }
12714
12715     static PyObject * __pyx_pf_8PyClical_22imag(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
12716         PyObject * __pyx_r = NULL;
12717         __Pyx_RefNannyDeclarations
12718         PyObject * __pyx_t_1 = NULL;
12719         int __pyx_lineno = 0;
12720         const char * __pyx_filename = NULL;
12721         int __pyx_clineno = 0;
12722         __Pyx_RefNannySetupContext("imag", 0);
12723         __Pyx_XDECREF(__pyx_r);
12724         __pyx_t_1 = __pyx_f_8PyClical_imag(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1415, __pyx_L1_error)
12725         __Pyx_GOTREF(__pyx_t_1);
12726         __pyx_r = __pyx_t_1;
12727         __pyx_t_1 = 0;
12728         goto __pyx_L0;
12729
12730         /* function exit code */
12731         __pyx_L1_error:;
12732         __Pyx_XDECREF(__pyx_t_1);
12733         __Pyx_AddTraceback("PyClical.imag", __pyx_clineno, __pyx_lineno, __pyx_filename);
12734         __pyx_r = NULL;
12735         __pyx_L0:;
12736         __Pyx_XGIVEREF(__pyx_r);
12737         __Pyx_RefNannyFinishContext();
12738         return __pyx_r;
12739     }
12740
12741     /* "PyClical.pyx":1426
12742 *      return 0.0
12743 *
12744 * cpdef inline pure(obj):          # ««««««««
12745 *      """
12746 *      Pure part
12747 */
12748
12749     static PyObject * __pyx_pw_8PyClical_25pure(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12750     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_pure(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12751         PyObject * __pyx_r = NULL;
12752         __Pyx_RefNannyDeclarations
12753         PyObject * __pyx_t_1 = NULL;
12754         PyObject * __pyx_t_2 = NULL;
12755         PyObject * __pyx_t_3 = NULL;
12756         int __pyx_lineno = 0;
12757         const char * __pyx_filename = NULL;
12758         int __pyx_clineno = 0;
12759         __Pyx_RefNannySetupContext("pure", 0);
12760
12761         /* "PyClical.pyx":1435
12762 *      {1,2}
12763 *      """
12764 *      return clifford(obj).pure()          # ««««««««
12765 *

```

```

12766 * cpdef inline even(obj):
12767 */
12768         __Pyx_XDECREF(__pyx_r);
12769         __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *) __pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1435, __pyx_L1_error)
12770         __Pyx_GOTREF(__pyx_t_2);
12771         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_pure); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1435, __pyx_L1_error)
12772         __Pyx_GOTREF(__pyx_t_3);
12773         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12774         __pyx_t_2 = NULL;
12775         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12776             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12777             if (likely(__pyx_t_2)) {
12778                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12779                 __Pyx_INCREF(__pyx_t_2);
12780                 __Pyx_DECREF(function);
12781                 __Pyx_DECREF_SET(__pyx_t_3, function);
12782             }
12783         }
12784         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12785         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12786         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1435, __pyx_L1_error)
12787         __Pyx_GOTREF(__pyx_t_1);
12788         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12789         __pyx_r = __pyx_t_1;
12790         __pyx_t_1 = 0;
12791         goto __pyx_L0;
12792
12793         /* "PyClical.pyx":1426
12794         *     return 0.0
12795         *
12796         * cpdef inline pure(obj):          # ««««««««
12797         *     """
12798         *     Pure part
12799         */
12800
12801         /* function exit code */
12802         __pyx_L1_error;
12803         __Pyx_XDECREF(__pyx_t_1);
12804         __Pyx_XDECREF(__pyx_t_2);
12805         __Pyx_XDECREF(__pyx_t_3);
12806         __Pyx_AddTraceback("PyClical.pure", __pyx_clineno, __pyx_lineno, __pyx_filename);
12807         __pyx_r = 0;
12808         __pyx_L0;
12809         __Pyx_XGIVEREF(__pyx_r);
12810         __Pyx_RefNannyFinishContext();
12811         return __pyx_r;
12812     }
12813
12814     /* Python wrapper */
12815     static PyObject * __pyx_pw_8PyClical_25pure(PyObject * __pyx_self, PyObject
__pyx_v_obj); /*proto*/
12816     static char __pyx_doc_8PyClical_24pure[] = "\n    Pure part\n\n    »>
print(pure(clifford(\"1+{1}+{1,2}\"))\n    {1}+{1,2}\n    »> print(pure(clifford(\"{1,2}\"))\n
{1,2}\n    ";
12817     static PyObject * __pyx_pw_8PyClical_25pure(PyObject * __pyx_self, PyObject
__pyx_v_obj) {
12818         PyObject * __pyx_r = 0;
12819         __Pyx_RefNannyDeclarations
12820         __Pyx_RefNannySetupContext("pure (wrapper)", 0);
12821         __pyx_r = __pyx_pf_8PyClical_24pure(__pyx_self, ((PyObject *) __pyx_v_obj));
12822
12823         /* function exit code */
12824         __Pyx_RefNannyFinishContext();
12825         return __pyx_r;
12826     }
12827
12828     static PyObject * __pyx_pf_8PyClical_24pure(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
12829         PyObject * __pyx_r = NULL;
12830         __Pyx_RefNannyDeclarations
12831         PyObject * __pyx_t_1 = NULL;
12832         int __pyx_lineno = 0;
12833         const char * __pyx_filename = NULL;
12834         int __pyx_clineno = 0;
12835         __Pyx_RefNannySetupContext("pure", 0);
12836         __Pyx_XDECREF(__pyx_r);
12837         __pyx_t_1 = __pyx_f_8PyClical_pure(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1426, __pyx_L1_error)
12838         __Pyx_GOTREF(__pyx_t_1);
12839         __pyx_r = __pyx_t_1;
12840         __pyx_t_1 = 0;
12841         goto __pyx_L0;
12842
12843         /* function exit code */

```

```

12844         __pyx_L1_error;;
12845         __Pyx_XDECREF(__pyx_t_1);
12846         __Pyx_AddTraceback("PyClical.pure", __pyx_clineno, __pyx_lineno, __pyx_filename);
12847         __pyx_r = NULL;
12848         __pyx_L0;
12849         __Pyx_XGIVEREF(__pyx_r);
12850         __Pyx_RefNannyFinishContext();
12851         return __pyx_r;
12852     }
12853
12854     /* "PyClical.pyx":1437
12855     *     return clifford(obj).pure()
12856     *
12857     * cpdef inline even(obj):
12858     *     """
12859     *     Even part of multivector, sum of even grade terms.
12860     */
12861
12862     static PyObject *__pyx_pw_8PyClical_27even(PyObject *__pyx_self, PyObject
12863 *__pyx_v_obj); /*proto*/
12864     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_even(PyObject *__pyx_v_obj,
12865 CYTHON_UNUSED int __pyx_skip_dispatch) {
12866         PyObject *__pyx_r = NULL;
12867         __Pyx_RefNannyDeclarations
12868         PyObject *__pyx_t_1 = NULL;
12869         PyObject *__pyx_t_2 = NULL;
12870         PyObject *__pyx_t_3 = NULL;
12871         int __pyx_lineno = 0;
12872         const char *__pyx_filename = NULL;
12873         int __pyx_clineno = 0;
12874         __Pyx_RefNannySetupContext("even", 0);
12875
12876         /* "PyClical.pyx":1444
12877         *     1+{1,2}
12878         *     """
12879         *     return clifford(obj).even()
12880         *     # ««««««««
12881         * cpdef inline odd(obj):
12882         */
12883         __Pyx_XDECREF(__pyx_r);
12884         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
12885 __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1444, __pyx_L1_error)
12886         __Pyx_GOTREF(__pyx_t_2);
12887         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_even); if
12888 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1444, __pyx_L1_error)
12889         __Pyx_GOTREF(__pyx_t_3);
12890         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12891         __pyx_t_2 = NULL;
12892         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12893             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12894             if (likely(__pyx_t_2)) {
12895                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12896                 __Pyx_INCREF(__pyx_t_2);
12897                 __Pyx_INCREF(function);
12898                 __Pyx_DECREF_SET(__pyx_t_3, function);
12899             }
12900         }
12901         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
12902 __Pyx_PyObject_CallNoArg(__pyx_t_3);
12903         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12904         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1444, __pyx_L1_error)
12905         __Pyx_GOTREF(__pyx_t_1);
12906         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12907         __pyx_r = __pyx_t_1;
12908         __pyx_t_1 = 0;
12909         goto __pyx_L0;
12910
12911         /* "PyClical.pyx":1437
12912         *     return clifford(obj).pure()
12913         *
12914         * cpdef inline even(obj):
12915         *     """
12916         *     Even part of multivector, sum of even grade terms.
12917         */
12918
12919         /* function exit code */
12920         __pyx_L1_error;;
12921         __Pyx_XDECREF(__pyx_t_1);
12922         __Pyx_XDECREF(__pyx_t_2);
12923         __Pyx_XDECREF(__pyx_t_3);
12924         __Pyx_AddTraceback("PyClical.even", __pyx_clineno, __pyx_lineno, __pyx_filename);
12925         __pyx_r = 0;
12926         __pyx_L0;
12927         __Pyx_XGIVEREF(__pyx_r);
12928         __Pyx_RefNannyFinishContext();
12929         return __pyx_r;
12930     }

```

```

12926
12927         /* Python wrapper */
12928         static PyObject * __pyx_pw_8PyClical_27even(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12929         static char __pyx_doc_8PyClical_26even[] = "\n    Even part of multivector, sum of
even grade terms.\n\n    >> print(even(clifford(\"1+{1}+{1,2}\")))\n    1+{1,2}\n    ";
12930         static PyObject * __pyx_pw_8PyClical_27even(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
12931             PyObject * __pyx_r = 0;
12932             __Pyx_RefNannyDeclarations
12933             __Pyx_RefNannySetupContext("even (wrapper)", 0);
12934             __pyx_r = __pyx_pf_8PyClical_26even(__pyx_self, ((PyObject *) __pyx_v_obj));
12935
12936             /* function exit code */
12937             __Pyx_RefNannyFinishContext();
12938             return __pyx_r;
12939         }
12940
12941         static PyObject * __pyx_pf_8PyClical_26even(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
12942             PyObject * __pyx_r = NULL;
12943             __Pyx_RefNannyDeclarations
12944             PyObject * __pyx_t_1 = NULL;
12945             int __pyx_lineno = 0;
12946             const char * __pyx_filename = NULL;
12947             int __pyx_clineno = 0;
12948             __Pyx_RefNannySetupContext("even", 0);
12949             __Pyx_XDECREF(__pyx_r);
12950             __pyx_t_1 = __pyx_f_8PyClical_even(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1437, __pyx_L1_error)
12951             __Pyx_GOTREF(__pyx_t_1);
12952             __pyx_r = __pyx_t_1;
12953             __pyx_t_1 = 0;
12954             goto __pyx_L0;
12955
12956             /* function exit code */
12957             __pyx_L1_error:;
12958             __Pyx_XDECREF(__pyx_t_1);
12959             __Pyx_AddTraceback("PyClical.even", __pyx_clineno, __pyx_lineno, __pyx_filename);
12960             __pyx_r = NULL;
12961             __pyx_L0:;
12962             __Pyx_XGIVEREF(__pyx_r);
12963             __Pyx_RefNannyFinishContext();
12964             return __pyx_r;
12965         }
12966
12967         /* "PyClical.pyx":1446
12968         *      return clifford(obj).even()
12969         *
12970         * cpdef inline odd(obj):
12971         *     """
12972         *     Odd part of multivector, sum of odd grade terms.
12973         */
12974
12975         static PyObject * __pyx_pw_8PyClical_29odd(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12976         static CYTHON_INLINE PyObject * __pyx_f_8PyClical_odd(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12977             PyObject * __pyx_r = NULL;
12978             __Pyx_RefNannyDeclarations
12979             PyObject * __pyx_t_1 = NULL;
12980             PyObject * __pyx_t_2 = NULL;
12981             PyObject * __pyx_t_3 = NULL;
12982             int __pyx_lineno = 0;
12983             const char * __pyx_filename = NULL;
12984             int __pyx_clineno = 0;
12985             __Pyx_RefNannySetupContext("odd", 0);
12986
12987             /* "PyClical.pyx":1453
12988             *     {1}
12989             *     """
12990             *     return clifford(obj).odd()
12991             *
12992             * cpdef inline involute(obj):
12993             */
12994             __Pyx_XDECREF(__pyx_r);
12995             __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *) __pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1453, __pyx_L1_error)
12996             __Pyx_GOTREF(__pyx_t_2);
12997             __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_odd); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1453, __pyx_L1_error)
12998             __Pyx_GOTREF(__pyx_t_3);
12999             __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13000             __pyx_t_2 = NULL;
13001             if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13002                 __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13003                 if (likely(__pyx_t_2)) {

```

```

13004         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13005         __Pyx_INCREF(__pyx_t_2);
13006         __Pyx_INCREF(function);
13007         __Pyx_DECREF_SET(__pyx_t_3, function);
13008     }
13009 }
13010     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13011     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13012     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1453, __pyx_L1_error)
13013     __Pyx_GOTREF(__pyx_t_1);
13014     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13015     __pyx_r = __pyx_t_1;
13016     __pyx_t_1 = 0;
13017     goto __pyx_L0;
13018
13019     /* "PyClical.pyx":1446
13020 *     return clifford(obj).even()
13021 *
13022 * cpdef inline odd(obj):          # ««««««««
13023 *     """
13024 *     Odd part of multivector, sum of odd grade terms.
13025 */
13026
13027     /* function exit code */
13028     __pyx_L1_error;
13029     __Pyx_XDECREF(__pyx_t_1);
13030     __Pyx_XDECREF(__pyx_t_2);
13031     __Pyx_XDECREF(__pyx_t_3);
13032     __Pyx_AddTraceback("PyClical.odd", __pyx_clineno, __pyx_lineno, __pyx_filename);
13033     __pyx_r = 0;
13034     __pyx_L0;
13035     __Pyx_XGIVEREF(__pyx_r);
13036     __Pyx_RefNannyFinishContext();
13037     return __pyx_r;
13038 }
13039
13040     /* Python wrapper */
13041     static PyObject *__pyx_pw_8PyClical_29odd(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13042     static char __pyx_doc_8PyClical_28odd[] = "\n    Odd part of multivector, sum of odd
grade terms.\n\n    >> print(odd(clifford(\"1+{1}+{1,2}\")))\n    {1}\n    ";
13043     static PyObject *__pyx_pw_8PyClical_29odd(PyObject *__pyx_self, PyObject *__pyx_v_obj)
{
13044         PyObject *__pyx_r = 0;
13045         __Pyx_RefNannyDeclarations
13046         __Pyx_RefNannySetupContext("odd (wrapper)", 0);
13047         __pyx_r = __pyx_pf_8PyClical_28odd(__pyx_self, ((PyObject *)__pyx_v_obj));
13048
13049         /* function exit code */
13050         __Pyx_RefNannyFinishContext();
13051         return __pyx_r;
13052     }
13053
13054     static PyObject *__pyx_pf_8PyClical_28odd(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
13055         PyObject *__pyx_r = NULL;
13056         __Pyx_RefNannyDeclarations
13057         PyObject *__pyx_t_1 = NULL;
13058         int __pyx_lineno = 0;
13059         const char *__pyx_filename = NULL;
13060         int __pyx_clineno = 0;
13061         __Pyx_RefNannySetupContext("odd", 0);
13062         __Pyx_XDECREF(__pyx_r);
13063         __pyx_t_1 = __pyx_f_8PyClical_odd(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1446, __pyx_L1_error)
13064         __Pyx_GOTREF(__pyx_t_1);
13065         __pyx_r = __pyx_t_1;
13066         __pyx_t_1 = 0;
13067         goto __pyx_L0;
13068
13069         /* function exit code */
13070         __pyx_L1_error;
13071         __Pyx_XDECREF(__pyx_t_1);
13072         __Pyx_AddTraceback("PyClical.odd", __pyx_clineno, __pyx_lineno, __pyx_filename);
13073         __pyx_r = NULL;
13074         __pyx_L0;
13075         __Pyx_XGIVEREF(__pyx_r);
13076         __Pyx_RefNannyFinishContext();
13077         return __pyx_r;
13078     }
13079
13080     /* "PyClical.pyx":1455
13081 *     return clifford(obj).odd()
13082 *
13083 * cpdef inline involute(obj):          # ««««««««
13084 *     """

```

```

13085 *      Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
13086 */
13087
13088     static PyObject *__pyx_pw_8PyClical_31involute(PyObject *__pyx_self, PyObject
13089 *__pyx_v_obj); /*proto*/
13089     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_involute(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13090         PyObject *__pyx_r = NULL;
13091         __Pyx_RefNannyDeclarations
13092         PyObject *__pyx_t_1 = NULL;
13093         PyObject *__pyx_t_2 = NULL;
13094         PyObject *__pyx_t_3 = NULL;
13095         int __pyx_lineno = 0;
13096         const char *__pyx_filename = NULL;
13097         int __pyx_clineno = 0;
13098         __Pyx_RefNannySetupContext("involute", 0);
13099
13100         /* "PyClical.pyx":1468
13101 *      1-{1}+{1,2}
13102 *      """
13103 *      return clifford(obj).involute()          # ««««««««
13104 *
13105 * cpdef inline reverse(obj):
13106 */
13107         __Pyx_XDECREF(__pyx_r);
13108         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1468, __pyx_L1_error)
13109         __Pyx_GOTREF(__pyx_t_2);
13110         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_involute); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1468, __pyx_L1_error)
13111         __Pyx_GOTREF(__pyx_t_3);
13112         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13113         __pyx_t_2 = NULL;
13114         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13115             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13116             if (likely(__pyx_t_2)) {
13117                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13118                 __Pyx_INCREF(__pyx_t_2);
13119                 __Pyx_INCREF(function);
13120                 __Pyx_DECREF_SET(__pyx_t_3, function);
13121             }
13122         }
13123         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13124         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13125         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1468, __pyx_L1_error)
13126         __Pyx_GOTREF(__pyx_t_1);
13127         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13128         __pyx_r = __pyx_t_1;
13129         __pyx_t_1 = 0;
13130         goto __pyx_L0;
13131
13132         /* "PyClical.pyx":1455
13133 *      return clifford(obj).odd()
13134 *
13135 * cpdef inline involute(obj):          # ««««««««
13136 *      """
13137 *      Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
13138 */
13139
13140         /* function exit code */
13141         __pyx_L1_error:;
13142         __Pyx_XDECREF(__pyx_t_1);
13143         __Pyx_XDECREF(__pyx_t_2);
13144         __Pyx_XDECREF(__pyx_t_3);
13145         __Pyx_AddTraceback("PyClical.involute", __pyx_clineno, __pyx_lineno,
__pyx_filename);
13146         __pyx_r = 0;
13147         __pyx_L0:;
13148         __Pyx_XGIVEREF(__pyx_r);
13149         __Pyx_RefNannyFinishContext();
13150         return __pyx_r;
13151     }
13152
13153     /* Python wrapper */
13154     static PyObject *__pyx_pw_8PyClical_31involute(PyObject *__pyx_self, PyObject
__pyx_v_obj); /*proto*/
13155     static char __pyx_doc_8PyClical_30involute[] = "\n      Main involution, each {i} is
replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})\n\n      >>
print(involute(clifford(\"{1}\")))\n      -{1}\n      >> print(involute(clifford(\"{2}\")) *
clifford(\"{1}\"))\n      -{1,2}\n      >> print(involute(clifford(\"{1}\")) * clifford(\"{2}\"))\n
{1,2}\n      >> print(involute(clifford(\"1+{1}+{1,2}\")))\n      1-{1}+{1,2}\n      ";
13156     static PyObject *__pyx_pw_8PyClical_31involute(PyObject *__pyx_self, PyObject
__pyx_v_obj) {
13157         PyObject *__pyx_r = 0;
13158         __Pyx_RefNannyDeclarations
13159         __Pyx_RefNannySetupContext("involute (wrapper)", 0);

```

```

13160         __pyx_r = __pyx_pf_8PyClical_30involute(__pyx_self, ((PyObject *)__pyx_v_obj));
13161
13162         /* function exit code */
13163         __Pyx_RefNannyFinishContext();
13164         return __pyx_r;
13165     }
13166
13167     static PyObject *__pyx_pf_8PyClical_30involute(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
13168         PyObject *__pyx_r = NULL;
13169         __Pyx_RefNannyDeclarations
13170         PyObject *__pyx_t_1 = NULL;
13171         int __pyx_lineno = 0;
13172         const char *__pyx_filename = NULL;
13173         int __pyx_clineno = 0;
13174         __Pyx_RefNannySetupContext("involute", 0);
13175         __Pyx_XDECREF(__pyx_r);
13176         __pyx_t_1 = __pyx_f_8PyClical_involute(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1455, __pyx_L1_error)
13177         __Pyx_GOTREF(__pyx_t_1);
13178         __pyx_r = __pyx_t_1;
13179         __pyx_t_1 = 0;
13180         goto __pyx_L0;
13181
13182         /* function exit code */
13183         __Pyx_L1_error;
13184         __Pyx_XDECREF(__pyx_t_1);
13185         __Pyx_AddTraceback("PyClical.involute", __pyx_clineno, __pyx_lineno,
__pyx_filename);
13186         __pyx_r = NULL;
13187         __pyx_L0;
13188         __Pyx_XGIVEREF(__pyx_r);
13189         __Pyx_RefNannyFinishContext();
13190         return __pyx_r;
13191     }
13192
13193     /* "PyClical.pyx":1470
13194     *     return clifford(obj).involute()
13195     *
13196     * cpdef inline reverse(obj):          # ««««««««
13197     *     """
13198     *     Reversion, eg. {1}*{2} -> {2}*{1}
13199     */
13200
13201     static PyObject *__pyx_pw_8PyClical_33reverse(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13202     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_reverse(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13203         PyObject *__pyx_r = NULL;
13204         __Pyx_RefNannyDeclarations
13205         PyObject *__pyx_t_1 = NULL;
13206         PyObject *__pyx_t_2 = NULL;
13207         PyObject *__pyx_t_3 = NULL;
13208         int __pyx_lineno = 0;
13209         const char *__pyx_filename = NULL;
13210         int __pyx_clineno = 0;
13211         __Pyx_RefNannySetupContext("reverse", 0);
13212
13213         /* "PyClical.pyx":1483
13214     *     1+{1}-{1,2}
13215     *     """
13216     *     return clifford(obj).reverse()          # ««««««««
13217     *
13218     * cpdef inline conj(obj):
13219     */
13220         __Pyx_XDECREF(__pyx_r);
13221         __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1483, __pyx_L1_error)
13222         __Pyx_GOTREF(__pyx_t_2);
13223         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_reverse); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1483, __pyx_L1_error)
13224         __Pyx_GOTREF(__pyx_t_3);
13225         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13226         __pyx_t_2 = NULL;
13227         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13228             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13229             if (likely(__pyx_t_2)) {
13230                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13231                 __Pyx_INCREF(__pyx_t_2);
13232                 __Pyx_INCREF(function);
13233                 __Pyx_DECREF_SET(__pyx_t_3, function);
13234             }
13235         }
13236         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13237         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13238         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1483, __pyx_L1_error)

```



```

13239         __Pyx_GOTREF(__pyx_t_1);
13240         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13241         __pyx_r = __pyx_t_1;
13242         __pyx_t_1 = 0;
13243         goto __pyx_L0;
13244
13245         /* "PyClical.pyx":1470
13246  *     return clifford(obj).involute()
13247  *
13248  * cpdef inline reverse(obj):                # ««««««««
13249  *     """
13250  *     Reversion, eg. {1}*{2} -> {2}*{1}
13251  */
13252
13253         /* function exit code */
13254         __pyx_L1_error++;
13255         __Pyx_XDECREF(__pyx_t_1);
13256         __Pyx_XDECREF(__pyx_t_2);
13257         __Pyx_XDECREF(__pyx_t_3);
13258         __Pyx_AddTraceback("PyClical.reverse", __pyx_clineno, __pyx_lineno, __pyx_filename);
13259         __pyx_r = 0;
13260         __pyx_L0++;
13261         __Pyx_XGIVEREF(__pyx_r);
13262         __Pyx_RefNannyFinishContext();
13263         return __pyx_r;
13264     }
13265
13266     /* Python wrapper */
13267     static PyObject * __pyx_pw_8PyClical_33reverse(PyObject * __pyx_self, PyObject
13268 * __pyx_v_obj); /*proto*/
13269     static char __pyx_doc_8PyClical_32reverse[] = "\n    Reversion, eg. {1}*{2} ->
13270 {2}*{1}\n\n    >> print(reverse(clifford(\"{1}\")))\n    {1}\n    >> print(reverse(clifford(\"{2}\")) *
13271 clifford(\"{1}\"))\n    {1,2}\n    >> print(reverse(clifford(\"{1}\") * clifford(\"{2}\"))\n
13272 -{1,2}\n    >> print(reverse(clifford(\"1+{1}+{1,2}\")))\n    1+{1}-{1,2}\n    ";
13273     static PyObject * __pyx_pw_8PyClical_33reverse(PyObject * __pyx_self, PyObject
13274 * __pyx_v_obj) {
13275         PyObject * __pyx_r = 0;
13276         __Pyx_RefNannyDeclarations
13277         __Pyx_RefNannySetupContext("reverse (wrapper)", 0);
13278         __pyx_r = __pyx_pf_8PyClical_32reverse(__pyx_self, ((PyObject *) __pyx_v_obj));
13279
13280         /* function exit code */
13281         __Pyx_RefNannyFinishContext();
13282         return __pyx_r;
13283     }
13284
13285     static PyObject * __pyx_pf_8PyClical_32reverse(CYTHON_UNUSED PyObject * __pyx_self,
13286 PyObject * __pyx_v_obj) {
13287         PyObject * __pyx_r = NULL;
13288         __Pyx_RefNannyDeclarations
13289         PyObject * __pyx_t_1 = NULL;
13290         int __pyx_lineno = 0;
13291         const char * __pyx_filename = NULL;
13292         int __pyx_clineno = 0;
13293         __Pyx_RefNannySetupContext("reverse", 0);
13294         __Pyx_XDECREF(__pyx_r);
13295         __pyx_t_1 = __pyx_f_8PyClical_reverse(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
13296         __PYX_ERR(0, 1470, __pyx_L1_error)
13297         __Pyx_GOTREF(__pyx_t_1);
13298         __pyx_r = __pyx_t_1;
13299         __pyx_t_1 = 0;
13300         goto __pyx_L0;
13301
13302         /* function exit code */
13303         __pyx_L1_error++;
13304         __Pyx_XDECREF(__pyx_t_1);
13305         __Pyx_AddTraceback("PyClical.reverse", __pyx_clineno, __pyx_lineno, __pyx_filename);
13306         __pyx_r = NULL;
13307         __pyx_L0++;
13308         __Pyx_XGIVEREF(__pyx_r);
13309         __Pyx_RefNannyFinishContext();
13310         return __pyx_r;
13311     }
13312
13313     /* "PyClical.pyx":1485
13314  *     return clifford(obj).reverse()
13315  *
13316  * cpdef inline conj(obj):                # ««««««««
13317  *     """
13318  *     Conjugation, reverse o involute == involute o reverse.
13319  */
13320
13321     static PyObject * __pyx_pw_8PyClical_35conj(PyObject * __pyx_self, PyObject
13322 * __pyx_v_obj); /*proto*/
13323     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_conj(PyObject * __pyx_v_obj,
13324 CYTHON_UNUSED int __pyx_skip_dispatch) {
13325         PyObject * __pyx_r = NULL;

```

```

13317         __Pyx_RefNannyDeclarations
13318         PyObject *__pyx_t_1 = NULL;
13319         PyObject *__pyx_t_2 = NULL;
13320         PyObject *__pyx_t_3 = NULL;
13321         int __pyx_lineno = 0;
13322         const char *__pyx_filename = NULL;
13323         int __pyx_clineno = 0;
13324         __Pyx_RefNannySetupContext("conj", 0);
13325
13326         /* "PyCliclcal.pyx":1498
13327         *      1-{1}-{1,2}
13328         *      """
13329         *      return clifford(obj).conj()          # ««««««««
13330         *
13331         * cpdef inline quad(obj):
13332         */
13333         __Pyx_XDECREF(__pyx_r);
13334         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyCliclcal_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1498, __pyx_L1_error)
13335         __Pyx_GOTREF(__pyx_t_2);
13336         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_conj); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1498, __pyx_L1_error)
13337         __Pyx_GOTREF(__pyx_t_3);
13338         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13339         __pyx_t_2 = NULL;
13340         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13341             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13342             if (likely(__pyx_t_2)) {
13343                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13344                 __Pyx_INCREF(__pyx_t_2);
13345                 __Pyx_INCREF(function);
13346                 __Pyx_DECREF_SET(__pyx_t_3, function);
13347             }
13348         }
13349         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13350         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13351         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1498, __pyx_L1_error)
13352         __Pyx_GOTREF(__pyx_t_1);
13353         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13354         __pyx_r = __pyx_t_1;
13355         __pyx_t_1 = 0;
13356         goto __pyx_L0;
13357
13358         /* "PyCliclcal.pyx":1485
13359         *      return clifford(obj).reverse()
13360         *
13361         * cpdef inline conj(obj):          # ««««««««
13362         *      """
13363         *      Conjugation, reverse o involute == involute o reverse.
13364         */
13365
13366         /* function exit code */
13367         __pyx_L1_error:;
13368         __Pyx_XDECREF(__pyx_t_1);
13369         __Pyx_XDECREF(__pyx_t_2);
13370         __Pyx_XDECREF(__pyx_t_3);
13371         __Pyx_AddTraceback("PyCliclcal.conj", __pyx_clineno, __pyx_lineno, __pyx_filename);
13372         __pyx_r = 0;
13373         __pyx_L0:;
13374         __Pyx_XGIVEREF(__pyx_r);
13375         __Pyx_RefNannyFinishContext();
13376         return __pyx_r;
13377     }
13378
13379     /* Python wrapper */
13380     static PyObject *__pyx_pw_8PyCliclcal_35conj(PyObject *__pyx_self, PyObject
__pyx_v_obj); /*proto*/
13381     static char __pyx_doc_8PyCliclcal_34conj[] = "\n      Conjugation, reverse o involute ==
involute o reverse.\n\n      >> print(conj(clifford(\"{1}\")))\n      -{1}\n      >>
print(conj(clifford(\"{2}\")) * clifford(\"{1}\"))\n      {1,2}\n      >> print(conj(clifford(\"{1}\")) *
clifford(\"{2}\"))\n      -{1,2}\n      >> print(conj(clifford(\"1+{1}+{1,2}\")))\n      1-{1}-{1,2}\n
";
13382     static PyObject *__pyx_pw_8PyCliclcal_35conj(PyObject *__pyx_self, PyObject
__pyx_v_obj) {
13383         PyObject *__pyx_r = 0;
13384         __Pyx_RefNannyDeclarations
13385         __Pyx_RefNannySetupContext("conj (wrapper)", 0);
13386         __pyx_r = __pyx_pf_8PyCliclcal_34conj(__pyx_self, ((PyObject *)__pyx_v_obj));
13387
13388         /* function exit code */
13389         __Pyx_RefNannyFinishContext();
13390         return __pyx_r;
13391     }
13392
13393     static PyObject *__pyx_pf_8PyCliclcal_34conj(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {

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```

13394         PyObject *__pyx_r = NULL;
13395         __Pyx_RefNannyDeclarations
13396         PyObject *__pyx_t_1 = NULL;
13397         int __pyx_lineno = 0;
13398         const char *__pyx_filename = NULL;
13399         int __pyx_clineno = 0;
13400         __Pyx_RefNannySetupContext("conj", 0);
13401         __Pyx_XDECREF(__pyx_r);
13402         __pyx_t_1 = __pyx_f_8PyClical_conj(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1485, __pyx_L1_error)
13403         __Pyx_GOTREF(__pyx_t_1);
13404         __pyx_r = __pyx_t_1;
13405         __pyx_t_1 = 0;
13406         goto __pyx_L0;
13407
13408         /* function exit code */
13409         __pyx_L1_error:;
13410         __Pyx_XDECREF(__pyx_t_1);
13411         __Pyx_AddTraceback("PyClical.conj", __pyx_clineno, __pyx_lineno, __pyx_filename);
13412         __pyx_r = NULL;
13413         __pyx_L0:;
13414         __Pyx_XGIVEREF(__pyx_r);
13415         __Pyx_RefNannyFinishContext();
13416         return __pyx_r;
13417     }
13418
13419     /* "PyClical.pyx":1500
13420     *     return clifford(obj).conj()
13421     *
13422     * cpdef inline quad(obj):                # ««««««««
13423     *     """
13424     *     Quadratic form == (rev(x)*x)(0).
13425     */
13426
13427     static PyObject *__pyx_pw_8PyClical_37quad(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13428     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_quad(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13429         PyObject *__pyx_r = NULL;
13430         __Pyx_RefNannyDeclarations
13431         PyObject *__pyx_t_1 = NULL;
13432         PyObject *__pyx_t_2 = NULL;
13433         PyObject *__pyx_t_3 = NULL;
13434         int __pyx_lineno = 0;
13435         const char *__pyx_filename = NULL;
13436         int __pyx_clineno = 0;
13437         __Pyx_RefNannySetupContext("quad", 0);
13438
13439         /* "PyClical.pyx":1509
13440         *     2.0
13441         *     """
13442         *     return clifford(obj).quad()                # ««««««««
13443         *
13444         * cpdef inline norm(obj):
13445         */
13446         __Pyx_XDECREF(__pyx_r);
13447         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1509, __pyx_L1_error)
13448         __Pyx_GOTREF(__pyx_t_2);
13449         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_quad); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1509, __pyx_L1_error)
13450         __Pyx_GOTREF(__pyx_t_3);
13451         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13452         __pyx_t_2 = NULL;
13453         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13454             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13455             if (likely(__pyx_t_2)) {
13456                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13457                 __Pyx_INCREF(__pyx_t_2);
13458                 __Pyx_INCREF(function);
13459                 __Pyx_DECREF_SET(__pyx_t_3, function);
13460             }
13461         }
13462         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13463         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13464         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1509, __pyx_L1_error)
13465         __Pyx_GOTREF(__pyx_t_1);
13466         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13467         __pyx_r = __pyx_t_1;
13468         __pyx_t_1 = 0;
13469         goto __pyx_L0;
13470
13471         /* "PyClical.pyx":1500
13472         *     return clifford(obj).conj()
13473         *
13474         * cpdef inline quad(obj):                # ««««««««

```

```

13475 *      """
13476 *      Quadratic form == (rev(x)*x)(0).
13477 */
13478
13479         /* function exit code */
13480         __pyx_ll_error;
13481         __Pyx_XDECREF(__pyx_t_1);
13482         __Pyx_XDECREF(__pyx_t_2);
13483         __Pyx_XDECREF(__pyx_t_3);
13484         __Pyx_AddTraceback("PyClical.quad", __pyx_clineno, __pyx_lineno, __pyx_filename);
13485         __pyx_r = 0;
13486         __pyx_L0;
13487         __Pyx_XGIVEREF(__pyx_r);
13488         __Pyx_RefNannyFinishContext();
13489         return __pyx_r;
13490     }
13491
13492     /* Python wrapper */
13493     static PyObject * __pyx_pw_8PyClical_37quad(PyObject * __pyx_self, PyObject
13494 * __pyx_v_obj); /*proto*/
13495     static char __pyx_doc_8PyClical_36quad[] = "\n      Quadratic form == (rev(x)*x)(0).\n\n
13496 >> print(quad(clifford(\"1+{1}+{1,2}\"))\n      3.0\n      >>
13497 print(quad(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n      2.0\n      ";
13498     static PyObject * __pyx_pf_8PyClical_37quad(PyObject * __pyx_self, PyObject
13499 * __pyx_v_obj) {
13500         PyObject * __pyx_r = 0;
13501         __Pyx_RefNannyDeclarations
13502         __Pyx_RefNannySetupContext("quad (wrapper)", 0);
13503         __pyx_r = __pyx_pf_8PyClical_36quad(__pyx_self, ((PyObject *) __pyx_v_obj));
13504
13505         /* function exit code */
13506         __Pyx_RefNannyFinishContext();
13507         return __pyx_r;
13508     }
13509
13510     static PyObject * __pyx_pf_8PyClical_36quad(CYTHON_UNUSED PyObject * __pyx_self,
13511 PyObject * __pyx_v_obj) {
13512         PyObject * __pyx_r = NULL;
13513         __Pyx_RefNannyDeclarations
13514         PyObject * __pyx_t_1 = NULL;
13515         int __pyx_lineno = 0;
13516         const char * __pyx_filename = NULL;
13517         int __pyx_clineno = 0;
13518         __Pyx_RefNannySetupContext("quad", 0);
13519         __Pyx_XDECREF(__pyx_r);
13520         __pyx_t_1 = __pyx_f_8PyClical_quad(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
13521 __PYX_ERR(0, 1500, __pyx_ll_error)
13522         __Pyx_GOTREF(__pyx_t_1);
13523         __pyx_r = __pyx_t_1;
13524         __pyx_t_1 = 0;
13525         goto __pyx_L0;
13526
13527         /* function exit code */
13528         __pyx_ll_error;
13529         __Pyx_XDECREF(__pyx_t_1);
13530         __Pyx_AddTraceback("PyClical.quad", __pyx_clineno, __pyx_lineno, __pyx_filename);
13531         __pyx_r = NULL;
13532         __pyx_L0;
13533         __Pyx_XGIVEREF(__pyx_r);
13534         __Pyx_RefNannyFinishContext();
13535         return __pyx_r;
13536     }
13537
13538     /* "PyClical.pyx":1511
13539     return clifford(obj).quad()
13540
13541     * cpdef inline norm(obj): # <<<<<<<<
13542     *      """
13543     *      norm == sum of squares of coordinates.
13544     */
13545
13546     static PyObject * __pyx_pw_8PyClical_39norm(PyObject * __pyx_self, PyObject
13547 * __pyx_v_obj); /*proto*/
13548     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_norm(PyObject * __pyx_v_obj,
13549 CYTHON_UNUSED int __pyx_skip_dispatch) {
13550         PyObject * __pyx_r = NULL;
13551         __Pyx_RefNannyDeclarations
13552         PyObject * __pyx_t_1 = NULL;
13553         PyObject * __pyx_t_2 = NULL;
13554         PyObject * __pyx_t_3 = NULL;
13555         int __pyx_lineno = 0;
13556         const char * __pyx_filename = NULL;
13557         int __pyx_clineno = 0;
13558         __Pyx_RefNannySetupContext("norm", 0);
13559
13560         /* "PyClical.pyx":1520
13561         *      4.0

```

```

13554 *      """
13555 *      return clifford(obj).norm()          # ««««««««
13556 *
13557 * cpdef inline abs(obj):
13558 */
13559         __Pyx_XDECREF(__pyx_r);
13560         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1520, __pyx_L1_error)
13561         __Pyx_GOTREF(__pyx_t_2);
13562         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_norm); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1520, __pyx_L1_error)
13563         __Pyx_GOTREF(__pyx_t_3);
13564         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13565         __pyx_t_2 = NULL;
13566         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13567             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13568             if (likely(__pyx_t_2)) {
13569                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13570                 __Pyx_INCREF(__pyx_t_2);
13571                 __Pyx_INCREF(function);
13572                 __Pyx_DECREF_SET(__pyx_t_3, function);
13573             }
13574         }
13575         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13576         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13577         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1520, __pyx_L1_error)
13578         __Pyx_GOTREF(__pyx_t_1);
13579         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13580         __pyx_r = __pyx_t_1;
13581         __pyx_t_1 = 0;
13582         goto __pyx_L0;
13583
13584         /* "PyClical.pyx":1511
13585 *      return clifford(obj).quad()
13586 *
13587 * cpdef inline norm(obj):          # ««««««««
13588 *      """
13589 *      norm == sum of squares of coordinates.
13590 */
13591
13592         /* function exit code */
13593         __pyx_L1_error;
13594         __Pyx_XDECREF(__pyx_t_1);
13595         __Pyx_XDECREF(__pyx_t_2);
13596         __Pyx_XDECREF(__pyx_t_3);
13597         __Pyx_AddTraceback("PyClical.norm", __pyx_clineno, __pyx_lineno, __pyx_filename);
13598         __pyx_r = 0;
13599         __pyx_L0;
13600         __Pyx_XGIVEREF(__pyx_r);
13601         __Pyx_RefNannyFinishContext();
13602         return __pyx_r;
13603     }
13604
13605     /* Python wrapper */
13606     static PyObject * __pyx_pw_8PyClical_39norm(PyObject * __pyx_self, PyObject
__pyx_v_obj); /*proto*/
13607     static char __pyx_doc_8PyClical_38norm[] = "\n      norm == sum of squares of
coordinates.\n\n      >> norm(clifford(\"1+{1}+{1,2}\")\n      3.0\n      >>
norm(clifford(\"1+{-1}+{1,2}+{1,2,3}\")\n      4.0\n      ";
13608     static PyObject * __pyx_pf_8PyClical_39norm(PyObject * __pyx_self, PyObject
__pyx_v_obj) {
13609         PyObject * __pyx_r = 0;
13610         __Pyx_RefNannyDeclarations
13611         __Pyx_RefNannySetupContext("norm (wrapper)", 0);
13612         __pyx_r = __pyx_pf_8PyClical_38norm(__pyx_self, ((PyObject *)__pyx_v_obj));
13613
13614         /* function exit code */
13615         __Pyx_RefNannyFinishContext();
13616         return __pyx_r;
13617     }
13618
13619     static PyObject * __pyx_pf_8PyClical_38norm(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
13620         PyObject * __pyx_r = NULL;
13621         __Pyx_RefNannyDeclarations
13622         PyObject * __pyx_t_1 = NULL;
13623         int __pyx_lineno = 0;
13624         const char * __pyx_filename = NULL;
13625         int __pyx_clineno = 0;
13626         __Pyx_RefNannySetupContext("norm", 0);
13627         __Pyx_XDECREF(__pyx_r);
13628         __pyx_t_1 = __pyx_f_8PyClical_norm(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1511, __pyx_L1_error)
13629         __Pyx_GOTREF(__pyx_t_1);
13630         __pyx_r = __pyx_t_1;
13631         __pyx_t_1 = 0;

```

```

13632         goto __pyx_L0;
13633
13634         /* function exit code */
13635         __pyx_L1_error++;
13636         __Pyx_XDECREF(__pyx_t_1);
13637         __Pyx_AddTraceback("PyClical.norm", __pyx_clineno, __pyx_lineno, __pyx_filename);
13638         __pyx_r = NULL;
13639         __pyx_L0++;
13640         __Pyx_XGIVEREF(__pyx_r);
13641         __Pyx_RefNannyFinishContext();
13642         return __pyx_r;
13643     }
13644
13645     /* "PyClical.pyx":1522
13646     *     return clifford(obj).norm()
13647     *
13648     * cpdef inline abs(obj):
13649     *     """
13650     *     Absolute value of multivector: multivector 2-norm.
13651     */
13652
13653     static PyObject *__pyx_pw_8PyClical_4labs(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13654     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_abs(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13655         PyObject *__pyx_r = NULL;
13656         __Pyx_RefNannyDeclarations
13657         PyObject *__pyx_t_1 = NULL;
13658         int __pyx_lineno = 0;
13659         const char *__pyx_filename = NULL;
13660         int __pyx_clineno = 0;
13661         __Pyx_RefNannySetupContext("abs", 0);
13662
13663         /* "PyClical.pyx":1529
13664         *     2.0
13665         *     """
13666         *     return glucat.abs(toClifford(obj))
13667         *
13668         * cpdef inline max_abs(obj):
13669         */
13670         __Pyx_XDECREF(__pyx_r);
13671         __pyx_t_1 = PyFloat_FromDouble(abs(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1529, __pyx_L1_error)
13672         __Pyx_GOTREF(__pyx_t_1);
13673         __pyx_r = __pyx_t_1;
13674         __pyx_t_1 = 0;
13675         goto __pyx_L0;
13676
13677         /* "PyClical.pyx":1522
13678         *     return clifford(obj).norm()
13679         *
13680         * cpdef inline abs(obj):
13681         *     """
13682         *     Absolute value of multivector: multivector 2-norm.
13683         */
13684
13685         /* function exit code */
13686         __pyx_L1_error++;
13687         __Pyx_XDECREF(__pyx_t_1);
13688         __Pyx_AddTraceback("PyClical.abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13689         __pyx_r = 0;
13690         __pyx_L0++;
13691         __Pyx_XGIVEREF(__pyx_r);
13692         __Pyx_RefNannyFinishContext();
13693         return __pyx_r;
13694     }
13695
13696     /* Python wrapper */
13697     static PyObject *__pyx_pw_8PyClical_4labs(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13698     static char __pyx_doc_8PyClical_40abs[] = "\n    Absolute value of multivector:
multivector 2-norm.\n\n    >> abs(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n    2.0\n    ";
13699     static PyObject *__pyx_pw_8PyClical_4labs(PyObject *__pyx_self, PyObject *__pyx_v_obj)
{
13700         PyObject *__pyx_r = 0;
13701         __Pyx_RefNannyDeclarations
13702         __Pyx_RefNannySetupContext("abs (wrapper)", 0);
13703         __pyx_r = __pyx_pf_8PyClical_40abs(__pyx_self, ((PyObject *)__pyx_v_obj));
13704
13705         /* function exit code */
13706         __Pyx_RefNannyFinishContext();
13707         return __pyx_r;
13708     }
13709
13710     static PyObject *__pyx_pf_8PyClical_40abs(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
13711         PyObject *__pyx_r = NULL;

```

```

13712         __Pyx_RefNannyDeclarations
13713         PyObject *__pyx_t_1 = NULL;
13714         int __pyx_lineno = 0;
13715         const char *__pyx_filename = NULL;
13716         int __pyx_clineno = 0;
13717         __Pyx_RefNannySetupContext("abs", 0);
13718         __Pyx_XDECREF(__pyx_r);
13719         __pyx_t_1 = __pyx_f_8PyClical_abs(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1522, __pyx_L1_error)
13720         __Pyx_GOTREF(__pyx_t_1);
13721         __pyx_r = __pyx_t_1;
13722         __pyx_t_1 = 0;
13723         goto __pyx_L0;
13724
13725         /* function exit code */
13726         __pyx_L1_error++;
13727         __Pyx_XDECREF(__pyx_t_1);
13728         __Pyx_AddTraceback("PyClical.abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13729         __pyx_r = NULL;
13730         __pyx_L0++;
13731         __Pyx_XGIVEREF(__pyx_r);
13732         __Pyx_RefNannyFinishContext();
13733         return __pyx_r;
13734     }
13735
13736     /* "PyClical.pyx":1531
13737     *     return glucat.abs(toClifford(obj))
13738     *
13739     * cpdef inline max_abs(obj):
13740     *     """
13741     *     Maximum absolute value of coordinates multivector: multivector infinity-norm.
13742     */
13743
13744     static PyObject *__pyx_pw_8PyClical_43max_abs(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13745     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_max_abs(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13746         PyObject *__pyx_r = NULL;
13747         __Pyx_RefNannyDeclarations
13748         PyObject *__pyx_t_1 = NULL;
13749         int __pyx_lineno = 0;
13750         const char *__pyx_filename = NULL;
13751         int __pyx_clineno = 0;
13752         __Pyx_RefNannySetupContext("max_abs", 0);
13753
13754         /* "PyClical.pyx":1541
13755     *
13756     *     """
13757     *     return glucat.max_abs(toClifford(obj))
13758     *
13759     * cpdef inline pow(obj, m):
13760     */
13761         __Pyx_XDECREF(__pyx_r);
13762         __pyx_t_1 = PyFloat_FromDouble(max_abs(__pyx_f_8PyClical_toClifford(__pyx_v_obj)));
13763         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1541, __pyx_L1_error)
13764         __Pyx_GOTREF(__pyx_t_1);
13765         __pyx_r = __pyx_t_1;
13766         __pyx_t_1 = 0;
13767         goto __pyx_L0;
13768
13769         /* "PyClical.pyx":1531
13770     *     return glucat.abs(toClifford(obj))
13771     *
13772     * cpdef inline max_abs(obj):
13773     *     """
13774     *     Maximum absolute value of coordinates multivector: multivector infinity-norm.
13775     */
13776         /* function exit code */
13777         __pyx_L1_error++;
13778         __Pyx_XDECREF(__pyx_t_1);
13779         __Pyx_AddTraceback("PyClical.max_abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13780         __pyx_r = 0;
13781         __pyx_L0++;
13782         __Pyx_XGIVEREF(__pyx_r);
13783         __Pyx_RefNannyFinishContext();
13784         return __pyx_r;
13785     }
13786
13787     /* Python wrapper */
13788     static PyObject *__pyx_pw_8PyClical_43max_abs(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13789     static char __pyx_doc_8PyClical_42max_abs[] = "\n    Maximum absolute value of
coordinates multivector: multivector infinity-norm.\n\n    >>
max_abs(clifford(\"1+{-1}+{1,2}+{1,2,3}\")\n    1.0\n    >> max_abs(clifford(\"3+2{1}+{1,2}\")\n
3.0\n\n    ";
13790     static PyObject *__pyx_pw_8PyClical_43max_abs(PyObject *__pyx_self, PyObject

```

```

    *__pyx_v_obj) {
13791         PyObject *__pyx_r = 0;
13792         __Pyx_RefNannyDeclarations
13793         __Pyx_RefNannySetupContext("max_abs (wrapper)", 0);
13794         __pyx_r = __pyx_pf_8PyClical_42max_abs(__pyx_self, ((PyObject *)__pyx_v_obj));
13795
13796         /* function exit code */
13797         __Pyx_RefNannyFinishContext();
13798         return __pyx_r;
13799     }
13800
13801     static PyObject *__pyx_pf_8PyClical_42max_abs(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
13802         PyObject *__pyx_r = NULL;
13803         __Pyx_RefNannyDeclarations
13804         PyObject *__pyx_t_1 = NULL;
13805         int __pyx_lineno = 0;
13806         const char *__pyx_filename = NULL;
13807         int __pyx_clineno = 0;
13808         __Pyx_RefNannySetupContext("max_abs", 0);
13809         __Pyx_XDECREF(__pyx_r);
13810         __pyx_t_1 = __pyx_f_8PyClical_max_abs(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1531, __pyx_L1_error)
13811         __Pyx_GOTREF(__pyx_t_1);
13812         __pyx_r = __pyx_t_1;
13813         __pyx_t_1 = 0;
13814         goto __pyx_L0;
13815
13816         /* function exit code */
13817         __pyx_L1_error:;
13818         __Pyx_XDECREF(__pyx_t_1);
13819         __Pyx_AddTraceback("PyClical.max_abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13820         __pyx_r = NULL;
13821         __pyx_L0:;
13822         __Pyx_XGIVEREF(__pyx_r);
13823         __Pyx_RefNannyFinishContext();
13824         return __pyx_r;
13825     }
13826
13827     /* "PyClical.pyx":1543
13828     *     return glucat.max_abs(toClifford(obj))
13829     *
13830     * cpdef inline pow(obj, m):
13831     *     """
13832     *     Integer power of multivector: obj to the m.
13833     */
13834
13835     static PyObject *__pyx_pw_8PyClical_45pow(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
13836     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_pow(PyObject *__pyx_v_obj, PyObject
*__pyx_v_m, CYTHON_UNUSED int __pyx_skip_dispatch) {
13837         PyObject *__pyx_r = NULL;
13838         __Pyx_RefNannyDeclarations
13839         PyObject *__pyx_t_1 = NULL;
13840         PyObject *__pyx_t_2 = NULL;
13841         PyObject *__pyx_t_3 = NULL;
13842         PyObject *__pyx_t_4 = NULL;
13843         PyObject *__pyx_t_5 = NULL;
13844         PyObject *__pyx_t_6 = NULL;
13845         int __pyx_t_7;
13846         PyObject *__pyx_t_8 = NULL;
13847         PyObject *__pyx_t_9 = NULL;
13848         PyObject *__pyx_t_10 = NULL;
13849         int __pyx_lineno = 0;
13850         const char *__pyx_filename = NULL;
13851         int __pyx_clineno = 0;
13852         __Pyx_RefNannySetupContext("pow", 0);
13853
13854         /* "PyClical.pyx":1562
13855     *     1
13856     *     """
13857     *     try:
13858     *         math.pow(obj, m)
13859     *     except:
13860     */
13861         {
13862             __Pyx_PyThreadState_declare
13863             __Pyx_PyThreadState_assign
13864             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
13865             __Pyx_XGOTREF(__pyx_t_1);
13866             __Pyx_XGOTREF(__pyx_t_2);
13867             __Pyx_XGOTREF(__pyx_t_3);
13868             /*try:*/ {
13869
13870             /* "PyClical.pyx":1563
13871     *     """
13872     *     try:

```



```

13873 *         math.pow(obj, m)                # ««««««««
13874 *     except:
13875 *         return clifford(obj).pow(m)
13876 */
13877
13878     __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
13879     __PYX_ERR(0, 1563, __pyx_L3_error)
13880     __Pyx_GOTREF(__pyx_t_5);
13881     __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_pow); if
13882     (unlikely(!__pyx_t_6)) __PYX_ERR(0, 1563, __pyx_L3_error)
13883     __Pyx_GOTREF(__pyx_t_6);
13884     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
13885     __pyx_t_5 = NULL;
13886     __pyx_t_7 = 0;
13887     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
13888         __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
13889         if (likely(__pyx_t_5)) {
13890             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
13891             __Pyx_INCREF(__pyx_t_5);
13892             __Pyx_INCREF(function);
13893             __Pyx_DECREF_SET(__pyx_t_6, function);
13894             __pyx_t_7 = 1;
13895         }
13896     }
13897     #if CYTHON_FAST_PYCALL
13898     if (PyFunction_Check(__pyx_t_6)) {
13899         PyObject * __pyx_temp[3] = {__pyx_t_5, __pyx_v_obj, __pyx_v_m};
13900         __pyx_t_4 = __Pyx_PyFunction_FastCall(__pyx_t_6, __pyx_temp+1-__pyx_t_7,
13901         2+__pyx_t_7); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1563, __pyx_L3_error)
13902         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
13903         __Pyx_GOTREF(__pyx_t_4);
13904     } else
13905     #endif
13906     #if CYTHON_FAST_PYCCALL
13907     if (__Pyx_PyFastCFunction_Check(__pyx_t_6)) {
13908         PyObject * __pyx_temp[3] = {__pyx_t_5, __pyx_v_obj, __pyx_v_m};
13909         __pyx_t_4 = __Pyx_PyCFunction_FastCall(__pyx_t_6, __pyx_temp+1-__pyx_t_7,
13910         2+__pyx_t_7); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1563, __pyx_L3_error)
13911         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
13912         __Pyx_GOTREF(__pyx_t_4);
13913     } else
13914     #endif
13915     {
13916         __pyx_t_8 = PyTuple_New(2+__pyx_t_7); if (unlikely(!__pyx_t_8)) __PYX_ERR(0,
13917         1563, __pyx_L3_error)
13918         __Pyx_GOTREF(__pyx_t_8);
13919         if (__pyx_t_5) {
13920             __Pyx_GIVEREF(__pyx_t_5); PyTuple_SET_ITEM(__pyx_t_8, 0, __pyx_t_5);
13921             __pyx_t_5 = NULL;
13922         }
13923         __Pyx_INCREF(__pyx_v_obj);
13924         __Pyx_GIVEREF(__pyx_v_obj);
13925         PyTuple_SET_ITEM(__pyx_t_8, 0+__pyx_t_7, __pyx_v_obj);
13926         __Pyx_INCREF(__pyx_v_m);
13927         __Pyx_GIVEREF(__pyx_v_m);
13928         PyTuple_SET_ITEM(__pyx_t_8, 1+__pyx_t_7, __pyx_v_m);
13929         __pyx_t_4 = __Pyx_PyObject_Call(__pyx_t_6, __pyx_t_8, NULL); if
13930         (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1563, __pyx_L3_error)
13931         __Pyx_GOTREF(__pyx_t_4);
13932         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
13933     }
13934     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
13935     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
13936
13937     /* "PyClical.pyx":1562
13938     *
13939     *     try:
13940     *         math.pow(obj, m)
13941     *     except:
13942     */
13943     }
13944     __Pyx_XDECREF(__pyx_t_1); __pyx_t_1 = 0;
13945     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13946     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
13947     goto __pyx_L8_try_end;
13948     __pyx_L3_error:;
13949     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
13950     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
13951     __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
13952     __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
13953
13954     /* "PyClical.pyx":1564
13955     *
13956     *     try:
13957     *         math.pow(obj, m)
13958     *     except:
13959     *         return clifford(obj).pow(m)
13960     */

```

```

13953 */
13954
13955         /*except:*/ {
13956             __Pyx_AddTraceback("PyCliclal.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
13957             if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_8) < 0) __PYX_ERR(0,
1564, __pyx_L5_except_error)
13957             __Pyx_GOTREF(__pyx_t_4);
13958             __Pyx_GOTREF(__pyx_t_6);
13959             __Pyx_GOTREF(__pyx_t_8);
13960
13961             /* "PyCliclal.pyx":1565
13962 *         math.pow(obj, m)
13963 *     except:
13964 *         return clifford(obj).pow(m)
13965 *         # ««««««««
13966 * cpdef inline outer_pow(obj, m):
13967 */
13968             __Pyx_XDECREF(__pyx_r);
13969             __pyx_t_9 = __Pyx_PyObject_CallOneArg((PyObject
*)__pyx_ptype_8PyCliclal_clifford), __pyx_v_obj); if (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1565,
__pyx_L5_except_error)
13970             __Pyx_GOTREF(__pyx_t_9);
13971             __pyx_t_10 = __Pyx_PyObject_GetAttrStr(__pyx_t_9, __pyx_n_s_pow); if
(unlikely(!__pyx_t_10)) __PYX_ERR(0, 1565, __pyx_L5_except_error)
13972             __Pyx_GOTREF(__pyx_t_10);
13973             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
13974             __pyx_t_9 = NULL;
13975             if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_10))) {
13976                 __pyx_t_9 = PyMethod_GET_SELF(__pyx_t_10);
13977                 if (likely(__pyx_t_9)) {
13978                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_10);
13979                     __Pyx_INCREF(__pyx_t_9);
13980                     __Pyx_INCREF(function);
13981                     __Pyx_DECREF_SET(__pyx_t_10, function);
13982                 }
13983             }
13984             __pyx_t_5 = (__pyx_t_9) ? __Pyx_PyObject_Call2Args(__pyx_t_10, __pyx_t_9,
__pyx_v_m) : __Pyx_PyObject_CallOneArg(__pyx_t_10, __pyx_v_m);
13985             __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
13986             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1565, __pyx_L5_except_error)
13987             __Pyx_GOTREF(__pyx_t_5);
13988             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
13989             __pyx_r = __pyx_t_5;
13990             __pyx_t_5 = 0;
13991             __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
13992             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
13993             __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
13994             goto __pyx_L6_except_return;
13995         }
13996         __pyx_L5_except_error;;
13997
13998         /* "PyCliclal.pyx":1562
13999 *     1
14000 *     """
14001 *     try:
14002 *         math.pow(obj, m)
14003 *     except:
14004 */
14005             __Pyx_XGIVEREF(__pyx_t_1);
14006             __Pyx_XGIVEREF(__pyx_t_2);
14007             __Pyx_XGIVEREF(__pyx_t_3);
14008             __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14009             goto __pyx_L1_error;
14010             __pyx_L6_except_return;;
14011             __Pyx_XGIVEREF(__pyx_t_1);
14012             __Pyx_XGIVEREF(__pyx_t_2);
14013             __Pyx_XGIVEREF(__pyx_t_3);
14014             __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14015             goto __pyx_L0;
14016             __pyx_L8_try_end;;
14017         }
14018
14019         /* "PyCliclal.pyx":1543
14020 *     return glucat.max_abs(toClifford(obj))
14021 *
14022 * cpdef inline pow(obj, m):
14023 *     """
14024 *     Integer power of multivector: obj to the m.
14025 */
14026
14027         /* function exit code */
14028         __pyx_r = Py_None; __Pyx_INCREF(Py_None);
14029         goto __pyx_L0;
14030         __pyx_L1_error;;
14031         __Pyx_XDECREF(__pyx_t_4);
14032         __Pyx_XDECREF(__pyx_t_5);
14033         __Pyx_XDECREF(__pyx_t_6);
14034         __Pyx_XDECREF(__pyx_t_8);

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14035         __Pyx_XDECREF(__pyx_t_9);
14036         __Pyx_XDECREF(__pyx_t_10);
14037         __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
14038         __pyx_r = 0;
14039         __pyx_L0;
14040         __Pyx_XGIVEREF(__pyx_r);
14041         __Pyx_RefNannyFinishContext();
14042         return __pyx_r;
14043     }
14044
14045     /* Python wrapper */
14046     static PyObject *__pyx_pw_8PyClical_45pow(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
14047     static char __pyx_doc_8PyClical_44pow[] = "\n Integer power of multivector: obj to
the m.\n\n >> x=clifford(\"{1}\"); print(pow(x,2))\n 1\n >> x=clifford(\"2\");
print(pow(x,2))\n 4\n >> x=clifford(\"2+{1}\"); print(pow(x,0))\n 1\n >>
x=clifford(\"2+{1}\"); print(pow(x,1))\n 2+{1}\n >> x=clifford(\"2+{1}\"); print(pow(x,2))\n
5+4{1}\n >> print(pow(clifford(\"1+{1}+{1,2}\"),3))\n 1+3{1}+3{1,2}\n >>
i=clifford(\"{1,2}\"); print(exp(pi/2) * pow(i, i))\n 1\n ";
14048     static PyObject *__pyx_pw_8PyClical_45pow(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
14049         PyObject *__pyx_v_obj = 0;
14050         PyObject *__pyx_v_m = 0;
14051         int __pyx_lineno = 0;
14052         const char *__pyx_filename = NULL;
14053         int __pyx_clineno = 0;
14054         PyObject *__pyx_r = 0;
14055         __Pyx_RefNannyDeclarations
14056         __Pyx_RefNannySetupContext("pow (wrapper)", 0);
14057         {
14058             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_m,0};
14059             PyObject* values[2] = {0,0};
14060             if (unlikely(__pyx_kwds)) {
14061                 Py_ssize_t kw_args;
14062                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
14063                 switch (pos_args) {
14064                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
14065                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
14066                     case 0: break;
14067                     default: goto __pyx_L5_argtuple_error;
14068                 }
14069                 kw_args = PyDict_Size(__pyx_kwds);
14070                 switch (pos_args) {
14071                     case 0:
14072                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
14073                         else goto __pyx_L5_argtuple_error;
14074                         CYTHON_FALLTHROUGH;
14075                         case 1:
14076                             if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_m)) !=
0)) kw_args--;
14077                             else {
14078                                 __Pyx_RaiseArgtupleInvalid("pow", 1, 2, 2, 1); __PYX_ERR(0, 1543,
__pyx_L3_error)
14079                             }
14080                             if (unlikely(kw_args > 0)) {
14081                                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "pow") < 0)) __PYX_ERR(0, 1543, __pyx_L3_error)
14082                             }
14083                             else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
14084                                 goto __pyx_L5_argtuple_error;
14085                             } else {
14086                                 values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14087                                 values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14088                             }
14089                             __pyx_v_obj = values[0];
14090                             __pyx_v_m = values[1];
14091                         }
14092                         goto __pyx_L4_argument_unpacking_done;
14093                     __pyx_L5_argtuple_error:;
14094                     __Pyx_RaiseArgtupleInvalid("pow", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args));
14095                     __PYX_ERR(0, 1543, __pyx_L3_error)
14096                     __pyx_L3_error:;
14097                     __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
14098                     __Pyx_RefNannyFinishContext();
14099                     return NULL;
14100                     __pyx_L4_argument_unpacking_done:;
14101                     __pyx_r = __pyx_pf_8PyClical_44pow(__pyx_self, __pyx_v_obj, __pyx_v_m);
14102
14103                     /* function exit code */
14104                     __Pyx_RefNannyFinishContext();
14105                     return __pyx_r;
14106                 }
14107             }
14108         }
14109     }

```

```

14110         static PyObject *__pyx_pf_8PyClical_44pow(CYTHON_UNUSED PyObject *__pyx_self, PyObject
14111 *__pyx_v_obj, PyObject *__pyx_v_m) {
14112         PyObject *__pyx_r = NULL;
14113         __Pyx_RefNannyDeclarations
14114         PyObject *__pyx_t_1 = NULL;
14115         int __pyx_lineno = 0;
14116         const char *__pyx_filename = NULL;
14117         int __pyx_clineno = 0;
14118         __Pyx_RefNannySetupContext("pow", 0);
14119         __Pyx_XDECREF(__pyx_r);
14120         __pyx_t_1 = __pyx_f_8PyClical_pow(__pyx_v_obj, __pyx_v_m, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1543, __pyx_L1_error)
14121         __Pyx_GOTREF(__pyx_t_1);
14122         __pyx_r = __pyx_t_1;
14123         __pyx_t_1 = 0;
14124         goto __pyx_L0;
14125
14126         /* function exit code */
14127         __pyx_L1_error:;
14128         __Pyx_XDECREF(__pyx_t_1);
14129         __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
14130         __pyx_r = NULL;
14131         __pyx_L0:;
14132         __Pyx_XGIVEREF(__pyx_r);
14133         __Pyx_RefNannyFinishContext();
14134         return __pyx_r;
14135     }
14136
14137     /* "PyClical.pyx":1567
14138     *
14139     * cpdef inline outer_pow(obj, m):
14140     *     """
14141     *     Outer product power of multivector.
14142     */
14143
14144     static PyObject *__pyx_pw_8PyClical_47outer_pow(PyObject *__pyx_self, PyObject
14145 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
14146     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_outer_pow(PyObject *__pyx_v_obj,
14147 PyObject *__pyx_v_m, CYTHON_UNUSED int __pyx_skip_dispatch) {
14148         PyObject *__pyx_r = NULL;
14149         __Pyx_RefNannyDeclarations
14150         PyObject *__pyx_t_1 = NULL;
14151         PyObject *__pyx_t_2 = NULL;
14152         PyObject *__pyx_t_3 = NULL;
14153         int __pyx_lineno = 0;
14154         const char *__pyx_filename = NULL;
14155         int __pyx_clineno = 0;
14156         __Pyx_RefNannySetupContext("outer_pow", 0);
14157
14158         /* "PyClical.pyx":1574
14159         *     1+3{1}+3{1,2}
14160         *     """
14161         *     return clifford(obj).outer_pow(m)
14162         *
14163         * cpdef inline complexifier(obj):
14164         */
14165         __Pyx_XDECREF(__pyx_r);
14166         __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1574, __pyx_L1_error)
14167         __Pyx_GOTREF(__pyx_t_2);
14168         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_outer_pow); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1574, __pyx_L1_error)
14169         __Pyx_GOTREF(__pyx_t_3);
14170         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
14171         __pyx_t_2 = NULL;
14172         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
14173             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
14174             if (likely(__pyx_t_2)) {
14175                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
14176                 __Pyx_INCREF(__pyx_t_2);
14177                 __Pyx_INCREF(function);
14178                 __Pyx_DECREF_SET(__pyx_t_3, function);
14179             }
14180         }
14181         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_Call2Args(__pyx_t_3, __pyx_t_2, __pyx_v_m)
: __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_v_m);
14182         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
14183         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1574, __pyx_L1_error)
14184         __Pyx_GOTREF(__pyx_t_1);
14185         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14186         __pyx_r = __pyx_t_1;
14187         __pyx_t_1 = 0;
14188         goto __pyx_L0;
14189
14190         /* "PyClical.pyx":1567
14191         *
14192         * return clifford(obj).pow(m)

```

```

14190 *
14191 * cpdef inline outer_pow(obj, m):          # ««««««««
14192 *     """
14193 *     Outer product power of multivector.
14194 */
14195
14196         /* function exit code */
14197         __pyx_L1_error++;
14198         __Pyx_XDECREF(__pyx_t_1);
14199         __Pyx_XDECREF(__pyx_t_2);
14200         __Pyx_XDECREF(__pyx_t_3);
14201         __Pyx_AddTraceback("PyClical.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14202         __pyx_r = 0;
14203         __pyx_L0++;
14204         __Pyx_XGIVEREF(__pyx_r);
14205         __Pyx_RefNannyFinishContext();
14206         return __pyx_r;
14207     }
14208
14209     /* Python wrapper */
14210     static PyObject *__pyx_pw_8PyClical_47outer_pow(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
14211     static char __pyx_doc_8PyClical_46outer_pow[] = "\n    Outer product power of
multivector.\n\n    >> print(outer_pow(clifford(\"1+{1}+{1,2}\"),3))\n    1+3{1}+3{1,2}\n    ";
14212     static PyObject *__pyx_pw_8PyClical_47outer_pow(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
14213         PyObject *__pyx_v_obj = 0;
14214         PyObject *__pyx_v_m = 0;
14215         int __pyx_lineno = 0;
14216         const char *__pyx_filename = NULL;
14217         int __pyx_clineno = 0;
14218         PyObject *__pyx_r = 0;
14219         __Pyx_RefNannyDeclarations
14220         __Pyx_RefNannySetupContext("outer_pow (wrapper)", 0);
14221         {
14222             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_m,0};
14223             PyObject* values[2] = {0,0};
14224             if (unlikely(__pyx_kwds)) {
14225                 Py_ssize_t kw_args;
14226                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
14227                 switch (pos_args) {
14228                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
14229                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
14230                     case 0: break;
14231                     default: goto __pyx_L5_argtuple_error;
14232                 }
14233                 kw_args = PyDict_Size(__pyx_kwds);
14234                 switch (pos_args) {
14235                     case 0:
14236                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
14237                         else goto __pyx_L5_argtuple_error;
14238                         CYTHON_FALLTHROUGH;
14239                     case 1:
14240                         if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_m)) !=
0)) kw_args--;
14241                         else {
14242                             __Pyx_RaiseArgtupleInvalid("outer_pow", 1, 2, 2, 1); __PYX_ERR(0, 1567,
__pyx_L3_error)
14243                         }
14244                     }
14245                 if (unlikely(kw_args > 0)) {
14246                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "outer_pow") < 0)) __PYX_ERR(0, 1567, __pyx_L3_error)
14247                 }
14248                 } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
14249                     goto __pyx_L5_argtuple_error;
14250                 } else {
14251                     values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14252                     values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14253                 }
14254                 __pyx_v_obj = values[0];
14255                 __pyx_v_m = values[1];
14256             }
14257             goto __pyx_L4_argument_unpacking_done;
14258             __pyx_L5_argtuple_error:;
14259             __Pyx_RaiseArgtupleInvalid("outer_pow", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args));
14260             __PYX_ERR(0, 1567, __pyx_L3_error)
14261             __pyx_L3_error:;
14262             __Pyx_AddTraceback("PyClical.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14263             __Pyx_RefNannyFinishContext();
14264             return NULL;
14265             __pyx_L4_argument_unpacking_done:;

```

```

14267         __pyx_r = __pyx_pf_8PyClical_46outer_pow(__pyx_self, __pyx_v_obj, __pyx_v_m);
14268
14269         /* function exit code */
14270         __Pyx_RefNannyFinishContext();
14271         return __pyx_r;
14272     }
14273
14274     static PyObject *__pyx_pf_8PyClical_46outer_pow(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_m) {
14275         PyObject *__pyx_r = NULL;
14276         __Pyx_RefNannyDeclarations
14277         PyObject *__pyx_t_1 = NULL;
14278         int __pyx_lineno = 0;
14279         const char *__pyx_filename = NULL;
14280         int __pyx_clineno = 0;
14281         __Pyx_RefNannySetupContext("outer_pow", 0);
14282         __Pyx_XDECREF(__pyx_r);
14283         __pyx_t_1 = __pyx_f_8PyClical_outer_pow(__pyx_v_obj, __pyx_v_m, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1567, __pyx_L1_error)
14284         __Pyx_GOTREF(__pyx_t_1);
14285         __pyx_r = __pyx_t_1;
14286         __pyx_t_1 = 0;
14287         goto __pyx_L0;
14288
14289         /* function exit code */
14290         __pyx_L1_error;
14291         __Pyx_XDECREF(__pyx_t_1);
14292         __Pyx_AddTraceback("PyClical.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14293         __pyx_r = NULL;
14294         __pyx_L0;
14295         __Pyx_XGIVEREF(__pyx_r);
14296         __Pyx_RefNannyFinishContext();
14297         return __pyx_r;
14298     }
14299
14300     /* "PyClical.pyx":1576
14301     *     return clifford(obj).outer_pow(m)
14302     *
14303     * cpdef inline complexifier(obj):          # ««««««««
14304     *     """
14305     *     Square root of -1 which commutes with all members of the frame of the given multivector.
14306     */
14307
14308     static PyObject *__pyx_pw_8PyClical_49complexifier(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
14309     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_complexifier(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
14310         PyObject *__pyx_r = NULL;
14311         __Pyx_RefNannyDeclarations
14312         PyObject *__pyx_t_1 = NULL;
14313         PyObject *__pyx_t_2 = NULL;
14314         int __pyx_lineno = 0;
14315         const char *__pyx_filename = NULL;
14316         int __pyx_clineno = 0;
14317         __Pyx_RefNannySetupContext("complexifier", 0);
14318
14319         /* "PyClical.pyx":1589
14320     *     {-1}
14321     *     """
14322     *     return clifford().wrap( glucat.complexifier(toClifford(obj)) )          # ««««««««
14323     *
14324     * cpdef inline sqrt(obj, i = None):
14325     */
14326         __Pyx_XDECREF(__pyx_r);
14327         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford));
14328         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1589, __pyx_L1_error)
14329         __Pyx_GOTREF(__pyx_t_1);
14330         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), complexifier(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1589, __pyx_L1_error)
14331         __Pyx_GOTREF(__pyx_t_2);
14332         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
14333         __pyx_r = __pyx_t_2;
14334         __pyx_t_2 = 0;
14335         goto __pyx_L0;
14336
14337         /* "PyClical.pyx":1576
14338     *     return clifford(obj).outer_pow(m)
14339     *
14340     * cpdef inline complexifier(obj):          # ««««««««
14341     *     """
14342     *     Square root of -1 which commutes with all members of the frame of the given multivector.
14343     */
14344
14345         /* function exit code */
__pyx_L1_error;

```

```

14346         __Pyx_XDECREF(__pyx_t_1);
14347         __Pyx_XDECREF(__pyx_t_2);
14348         __Pyx_AddTraceback("PyClical.complexifier", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14349         __pyx_r = 0;
14350         __pyx_L0;
14351         __Pyx_XGIVEREF(__pyx_r);
14352         __Pyx_RefNannyFinishContext();
14353         return __pyx_r;
14354     }
14355
14356     /* Python wrapper */
14357     static PyObject * __pyx_pw_8PyClical_49complexifier(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
14358     static char __pyx_doc_8PyClical_48complexifier[] = "\n    Square root of -1 which
commutes with all members of the frame of the given multivector.\n\n    >>
print(complexifier(clifford(index_set({1})))\n    {1,2,3}\n    >>
print(complexifier(clifford(index_set({-1})))\n    {-1}\n    >> print(complexifier(index_set({1})))\n
{1,2,3}\n    >> print(complexifier(index_set({-1})))\n    {-1}\n    ";
14359     static PyObject * __pyx_pf_8PyClical_49complexifier(PyObject * __pyx_self, PyObject
* __pyx_v_obj) {
14360         PyObject * __pyx_r = 0;
14361         __Pyx_RefNannyDeclarations
14362         __Pyx_RefNannySetupContext("complexifier (wrapper)", 0);
14363         __pyx_r = __pyx_pf_8PyClical_48complexifier(__pyx_self, ((PyObject *) __pyx_v_obj));
14364
14365         /* function exit code */
14366         __Pyx_RefNannyFinishContext();
14367         return __pyx_r;
14368     }
14369
14370     static PyObject * __pyx_pf_8PyClical_48complexifier(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
14371         PyObject * __pyx_r = NULL;
14372         __Pyx_RefNannyDeclarations
14373         PyObject * __pyx_t_1 = NULL;
14374         int __pyx_lineno = 0;
14375         const char * __pyx_filename = NULL;
14376         int __pyx_clineno = 0;
14377         __Pyx_RefNannySetupContext("complexifier", 0);
14378         __Pyx_XDECREF(__pyx_r);
14379         __pyx_t_1 = __pyx_f_8PyClical_complexifier(__pyx_v_obj, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1576, __pyx_L1_error)
14380         __Pyx_GOTREF(__pyx_t_1);
14381         __pyx_r = __pyx_t_1;
14382         __pyx_t_1 = 0;
14383         goto __pyx_L0;
14384
14385         /* function exit code */
14386         __pyx_L1_error:;
14387         __Pyx_XDECREF(__pyx_t_1);
14388         __Pyx_AddTraceback("PyClical.complexifier", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14389         __pyx_r = NULL;
14390         __pyx_L0;
14391         __Pyx_XGIVEREF(__pyx_r);
14392         __Pyx_RefNannyFinishContext();
14393         return __pyx_r;
14394     }
14395
14396     /* "PyClical.pyx":1591
14397     * return clifford().wrap( glucat.complexifier(toClifford(obj)) )
14398     *
14399     * cpdef inline sqrt(obj, i = None): # ««««««
14400     *     """
14401     *     Square root of multivector with optional complexifier.
14402     */
14403
14404     static PyObject * __pyx_pw_8PyClical_51sqrt(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
14405     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_sqrt(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_sqrt * __pyx_optional_args) {
14406         PyObject * __pyx_v_i = ((PyObject *) Py_None);
14407         PyObject * __pyx_r = NULL;
14408         __Pyx_RefNannyDeclarations
14409         int __pyx_t_1;
14410         int __pyx_t_2;
14411         PyObject * __pyx_t_3 = NULL;
14412         Clifford __pyx_t_4;
14413         PyObject * __pyx_t_5 = NULL;
14414         PyObject * __pyx_t_6 = NULL;
14415         PyObject * __pyx_t_7 = NULL;
14416         PyObject * __pyx_t_8 = NULL;
14417         PyObject * __pyx_t_9 = NULL;
14418         PyObject * __pyx_t_10 = NULL;
14419         PyObject * __pyx_t_11 = NULL;
14420         int __pyx_lineno = 0;

```

```

14421         const char *__pyx_filename = NULL;
14422         int __pyx_clineno = 0;
14423         __Pyx_RefNannySetupContext("sqrt", 0);
14424         if (__pyx_optional_args) {
14425             if (__pyx_optional_args->__pyx_n > 0) {
14426                 __pyx_v_i = __pyx_optional_args->i;
14427             }
14428         }
14429
14430         /* "PyClicl.pyx":1606
14431         *
14432         * """
14433         * if not (i is None): # ««««««««
14434         *     return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14435         * else:
14436         */
14437         __pyx_t_1 = (__pyx_v_i != Py_None);
14438         __pyx_t_2 = (__pyx_t_1 != 0);
14439         if (__pyx_t_2) {
14440
14441             /* "PyClicl.pyx":1607
14442             *
14443             * """
14444             * if not (i is None): # ««««««««
14445             *     return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14446             * else:
14447             *     try:
14448             */
14449             __Pyx_XDECREF(__pyx_r);
14450             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
14451 *)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1607, __pyx_L1_error)
14452             __Pyx_GOTREF(__pyx_t_3);
14453             try {
14454                 __pyx_t_4 = sqrt(__pyx_f_8PyClicl_toClifford(__pyx_v_obj),
14455 __pyx_f_8PyClicl_toClifford(__pyx_v_i));
14456             } catch (...) {
14457                 __Pyx_CppExn2PyErr();
14458                 __PYX_ERR(0, 1607, __pyx_L1_error)
14459             }
14460             __pyx_t_5 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
14461 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1607, __pyx_L1_error)
14462             __Pyx_GOTREF(__pyx_t_5);
14463             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14464             __pyx_r = __pyx_t_5;
14465             __pyx_t_5 = 0;
14466             goto __pyx_L0;
14467
14468             /* "PyClicl.pyx":1606
14469             *
14470             * """
14471             * if not (i is None): # ««««««««
14472             *     return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14473             * else:
14474             */
14475             }
14476
14477             /* "PyClicl.pyx":1609
14478             *
14479             * return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14480             * else:
14481             *     try: # ««««««««
14482             *         return math.sqrt(obj)
14483             *     except:
14484             */
14485             /*else*/ {
14486                 {
14487                     __Pyx_PyThreadState_declare
14488                     __Pyx_PyThreadState_assign
14489                     __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
14490                     __Pyx_XGOTREF(__pyx_t_6);
14491                     __Pyx_XGOTREF(__pyx_t_7);
14492                     __Pyx_XGOTREF(__pyx_t_8);
14493                     /*try*/ {
14494
14495                         /* "PyClicl.pyx":1610
14496                         *
14497                         * else:
14498                         *     try:
14499                         *         return math.sqrt(obj) # ««««««««
14500                         *     except:
14501                         *         return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14502                         */
14503                         __Pyx_XDECREF(__pyx_r);
14504                         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
14505 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1610, __pyx_L4_error)
14506                         __Pyx_GOTREF(__pyx_t_3);
14507                         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_sqrt); if
14508 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1610, __pyx_L4_error)
14509                         __Pyx_GOTREF(__pyx_t_9);
14510                         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;

```



```

14503         __pyx_t_3 = NULL;
14504         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
14505             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
14506             if (likely(__pyx_t_3)) {
14507                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
14508                 __Pyx_INCREF(__pyx_t_3);
14509                 __Pyx_INCREF(function);
14510                 __Pyx_DECREF_SET(__pyx_t_9, function);
14511             }
14512         }
14513         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
14514         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
14515         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1610, __pyx_L4_error)
14516         __Pyx_GOTREF(__pyx_t_5);
14517         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
14518         __pyx_r = __pyx_t_5;
14519         __pyx_t_5 = 0;
14520         goto __pyx_L8_try_return;
14521
14522         /* "PyClical.pyx":1609
14523  *         return clifford().wrap( glucat.sqrt(toClifford(obj)), toClifford(i)) )
14524  *     else:
14525  *         try:
14526  *             # ««««««««
14527  *             return math.sqrt(obj)
14528  *         except:
14529  */
14530         }
14531         __pyx_L4_error:;
14532         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
14533         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
14534         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
14535
14536         /* "PyClical.pyx":1611
14537  *         try:
14538  *             return math.sqrt(obj)
14539  *         except:
14540  *             # ««««««««
14541  *             return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14542  */
14543         /*except:*/ {
14544             __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14545             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1611, __pyx_L6_except_error)
14546             __Pyx_GOTREF(__pyx_t_5);
14547             __Pyx_GOTREF(__pyx_t_9);
14548             __Pyx_GOTREF(__pyx_t_3);
14549
14550             /* "PyClical.pyx":1612
14551  *         return math.sqrt(obj)
14552  *     except:
14553  *         return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14554  *         # ««««««««
14555  * cpdef inline exp(obj):
14556  */
14557             __Pyx_XDECREF(__pyx_r);
14558             __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1612,
__pyx_L6_except_error)
14559             __Pyx_GOTREF(__pyx_t_10);
14560             __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), sqrt(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1612, __pyx_L6_except_error)
14561             __Pyx_GOTREF(__pyx_t_11);
14562             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
14563             __pyx_r = __pyx_t_11;
14564             __pyx_t_11 = 0;
14565             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14566             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
14567             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
14568             goto __pyx_L7_except_return;
14569         }
14570         __pyx_L6_except_error:;
14571
14572         /* "PyClical.pyx":1609
14573  *         return clifford().wrap( glucat.sqrt(toClifford(obj)), toClifford(i)) )
14574  *     else:
14575  *         try:
14576  *             # ««««««««
14577  *             return math.sqrt(obj)
14578  *         except:
14579  */
14580         __Pyx_XGIVEREF(__pyx_t_6);
14581         __Pyx_XGIVEREF(__pyx_t_7);
14582         __Pyx_XGIVEREF(__pyx_t_8);
14583         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
14584         goto __pyx_L1_error;

```

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14583         __pyx_L8_try_return;;
14584         __Pyx_XGIVEREF(__pyx_t_6);
14585         __Pyx_XGIVEREF(__pyx_t_7);
14586         __Pyx_XGIVEREF(__pyx_t_8);
14587         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
14588         goto __pyx_L0;
14589     __pyx_L7_except_return;;
14590     __Pyx_XGIVEREF(__pyx_t_6);
14591     __Pyx_XGIVEREF(__pyx_t_7);
14592     __Pyx_XGIVEREF(__pyx_t_8);
14593     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
14594     goto __pyx_L0;
14595 }
14596 }
14597
14598     /* "PyCliclcal.pyx":1591
14599     *     return clifford().wrap( glucat.complexifier(toClifford(obj)) )
14600     *
14601     * cpdef inline sqrt(obj, i = None):          # ««««««««
14602     *     """
14603     *     Square root of multivector with optional complexifier.
14604     */
14605
14606     /* function exit code */
14607     __pyx_L1_error;;
14608     __Pyx_XDECREF(__pyx_t_3);
14609     __Pyx_XDECREF(__pyx_t_5);
14610     __Pyx_XDECREF(__pyx_t_9);
14611     __Pyx_XDECREF(__pyx_t_10);
14612     __Pyx_XDECREF(__pyx_t_11);
14613     __Pyx_AddTraceback("PyCliclcal.sqrt", __pyx_clineno, __pyx_lineno, __pyx_filename);
14614     __pyx_r = 0;
14615     __pyx_L0;;
14616     __Pyx_XGIVEREF(__pyx_r);
14617     __Pyx_RefNannyFinishContext();
14618     return __pyx_r;
14619 }
14620
14621     /* Python wrapper */
14622     static PyObject * __pyx_pw_8PyCliclcal_51sqrt(PyObject * __pyx_self, PyObject * __pyx_args,
14623     PyObject * __pyx_kwds); /*proto*/
14624     static char __pyx_doc_8PyCliclcal_50sqrt[] = "\n    Square root of multivector with\n    optional complexifier.\n\n    >> print(sqrt(-1))\n    {-1}\n    >> print(sqrt(clifford(\"2{-1}\")))\n    1+{-1}\n    >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)\n    {1,2,3}\n    -1\n    >> j=sqrt(-1,\"{1,2,3}\"); print(j); print(j*j)\n    {1,2,3}\n    -1\n    ";
14625     static PyObject * __pyx_pw_8PyCliclcal_51sqrt(PyObject * __pyx_self, PyObject * __pyx_args,
14626     PyObject * __pyx_kwds) {
14627         PyObject * __pyx_v_obj = 0;
14628         PyObject * __pyx_v_i = 0;
14629         int __pyx_lineno = 0;
14630         const char * __pyx_filename = NULL;
14631         int __pyx_clineno = 0;
14632         PyObject * __pyx_r = 0;
14633         __Pyx_RefNannyDeclarations
14634         __Pyx_RefNannySetupContext("sqrt (wrapper)", 0);
14635         {
14636             static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
14637             PyObject* values[2] = {0,0};
14638             values[1] = ((PyObject *)Py_None);
14639             if (unlikely(__pyx_kwds)) {
14640                 Py_ssize_t kw_args;
14641                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
14642                 switch (pos_args) {
14643                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14644                     CYTHON_FALLTHROUGH;
14645                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14646                     CYTHON_FALLTHROUGH;
14647                     case 0: break;
14648                     default: goto __pyx_L5_argtuple_error;
14649                 }
14650                 kw_args = PyDict_Size(__pyx_kwds);
14651                 switch (pos_args) {
14652                     case 0:
14653                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
14654                             0)) kw_args--;
14655                         else goto __pyx_L5_argtuple_error;
14656                     CYTHON_FALLTHROUGH;
14657                     case 1:
14658                         if (kw_args > 0) {
14659                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
14660                             if (value) { values[1] = value; kw_args--; }
14661                         }
14662                     if (unlikely(kw_args > 0)) {
14663                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
14664                             values, pos_args, "sqrt") < 0)) __PYX_ERR(0, 1591, __pyx_L3_error)
14665                     }
14666                 }

```

```

14663         } else {
14664             switch (PyTuple_GET_SIZE(__pyx_args)) {
14665                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14666                     CYTHON_FALLTHROUGH;
14667                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14668                     break;
14669                 default: goto __pyx_L5_argtuple_error;
14670             }
14671         }
14672         __pyx_v_obj = values[0];
14673         __pyx_v_i = values[1];
14674     }
14675     goto __pyx_L4_argument_unpacking_done;
14676     __pyx_L5_argtuple_error:;
14677     __Pyx_RaiseArgtupleInvalid("sqrt", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
14678     __PYX_ERR(0, 1591, __pyx_L3_error)
14679     __pyx_L3_error:;
14680     __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno, __pyx_filename);
14681     __Pyx_RefNannyFinishContext();
14682     return NULL;
14683     __pyx_L4_argument_unpacking_done:;
14684     __pyx_r = __pyx_pf_8PyClical_50sqrt(__pyx_self, __pyx_v_obj, __pyx_v_i);
14685
14686     /* function exit code */
14687     __Pyx_RefNannyFinishContext();
14688     return __pyx_r;
14689 }
14690
14691 static PyObject *__pyx_pf_8PyClical_50sqrt(CYTHON_UNUSED PyObject *__pyx_self,
14692 PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
14693     PyObject *__pyx_r = NULL;
14694     __Pyx_RefNannyDeclarations
14695     PyObject *__pyx_t_1 = NULL;
14696     struct __pyx_opt_args_8PyClical_sqrt __pyx_t_2;
14697     int __pyx_lineno = 0;
14698     const char *__pyx_filename = NULL;
14699     int __pyx_clineno = 0;
14700     __Pyx_RefNannySetupContext("sqrt", 0);
14701     __Pyx_XDECREF(__pyx_r);
14702     __pyx_t_2.__pyx_n = 1;
14703     __pyx_t_2.i = __pyx_v_i;
14704     __pyx_t_1 = __pyx_f_8PyClical_sqrt(__pyx_v_obj, 0, &__pyx_t_2); if
14705     (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1591, __pyx_L1_error)
14706     __Pyx_GOTREF(__pyx_t_1);
14707     __pyx_r = __pyx_t_1;
14708     __pyx_t_1 = 0;
14709     goto __pyx_L0;
14710
14711     /* function exit code */
14712     __pyx_L1_error:;
14713     __Pyx_XDECREF(__pyx_t_1);
14714     __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno, __pyx_filename);
14715     __pyx_r = NULL;
14716     __pyx_L0:;
14717     __Pyx_XGIVEREF(__pyx_r);
14718     __Pyx_RefNannyFinishContext();
14719     return __pyx_r;
14720 }
14721
14722 /* "PyClical.pyx":1614
14723 *
14724 * cpdef inline exp(obj): # ««««««
14725 *     """
14726 *     Exponential of multivector.
14727 */
14728
14729 static PyObject *__pyx_pw_8PyClical_53exp(PyObject *__pyx_self, PyObject
14730 *__pyx_v_obj); /*proto*/
14731 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_exp(PyObject *__pyx_v_obj,
14732 CYTHON_UNUSED int __pyx_skip_dispatch) {
14733     PyObject *__pyx_r = NULL;
14734     __Pyx_RefNannyDeclarations
14735     PyObject *__pyx_t_1 = NULL;
14736     PyObject *__pyx_t_2 = NULL;
14737     PyObject *__pyx_t_3 = NULL;
14738     PyObject *__pyx_t_4 = NULL;
14739     PyObject *__pyx_t_5 = NULL;
14740     PyObject *__pyx_t_6 = NULL;
14741     PyObject *__pyx_t_7 = NULL;
14742     PyObject *__pyx_t_8 = NULL;
14743     int __pyx_lineno = 0;
14744     const char *__pyx_filename = NULL;
14745     int __pyx_clineno = 0;
14746     __Pyx_RefNannySetupContext("exp", 0);
14747
14748     /* "PyClical.pyx":1623

```

```

14745 *      {1,2}
14746 *      """
14747 *      try:                  # ««««««««
14748 *          return math.exp(obj)
14749 *      except:
14750 */
14751
14752         {
14753             __Pyx_PyThreadState_declare
14754             __Pyx_PyThreadState_assign
14755             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
14756             __Pyx_XGOTREF(__pyx_t_1);
14757             __Pyx_XGOTREF(__pyx_t_2);
14758             __Pyx_XGOTREF(__pyx_t_3);
14759             /*try:*/ {
14760
14761                 /* "PyClicl.pyx":1624
14762 *
14763 *      try:
14764 *          return math.exp(obj)                  # ««««««««
14765 *      except:
14766 *          return clifford().wrap( glucat.exp(toClifford(obj)) )
14767 */
14768
14769             __Pyx_XDECREF(__pyx_r);
14770             __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
14771 __PYX_ERR(0, 1624, __pyx_L3_error)
14772             __Pyx_GOTREF(__pyx_t_5);
14773             __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_exp); if
14774 (unlikely(!__pyx_t_6)) __PYX_ERR(0, 1624, __pyx_L3_error)
14775             __Pyx_GOTREF(__pyx_t_6);
14776             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
14777             __pyx_t_5 = NULL;
14778             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
14779                 __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
14780                 if (likely(__pyx_t_5)) {
14781                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
14782                     __Pyx_INCREF(__pyx_t_5);
14783                     __Pyx_INCREF(function);
14784                     __Pyx_DECREF_SET(__pyx_t_6, function);
14785                 }
14786             }
14787             __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
14788 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
14789             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
14790             if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1624, __pyx_L3_error)
14791             __Pyx_GOTREF(__pyx_t_4);
14792             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
14793             __pyx_r = __pyx_t_4;
14794             __pyx_t_4 = 0;
14795             goto __pyx_L7_try_return;
14796
14797         /* "PyClicl.pyx":1623
14798 *
14799 *      {1,2}
14800 *      """
14801 *      try:                  # ««««««««
14802 *          return math.exp(obj)
14803 *      except:
14804 */
14805
14806         }
14807         __pyx_L3_error:;
14808         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
14809         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
14810         __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
14811
14812         /* "PyClicl.pyx":1625
14813 *
14814 *      try:
14815 *          return math.exp(obj)
14816 *      except:
14817 *          return clifford().wrap( glucat.exp(toClifford(obj)) )
14818 */
14819
14820         /*except:*/ {
14821             __Pyx_AddTraceback("PyClicl.exp", __pyx_clineno, __pyx_lineno, __pyx_filename);
14822             if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
14823 1625, __pyx_L5_except_error)
14824             __Pyx_GOTREF(__pyx_t_4);
14825             __Pyx_GOTREF(__pyx_t_6);
14826             __Pyx_GOTREF(__pyx_t_5);
14827
14828         /* "PyClicl.pyx":1626
14829 *
14830 *      return math.exp(obj)
14831 *      except:
14832 *          return clifford().wrap( glucat.exp(toClifford(obj)) )
14833 *
14834 * cpdef inline log(obj,i = None):
14835 */
14836
14837             __Pyx_XDECREF(__pyx_r);
14838             __pyx_t_7 = __Pyx_PyObject_CallNoArg((PyObject

```

```

*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1626,
__pyx_L5_except_error)
14828     __Pyx_GOTREF(__pyx_t_7);
14829     __pyx_t_8 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_7), exp(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1626, __pyx_L5_except_error)
14830     __Pyx_GOTREF(__pyx_t_8);
14831     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
14832     __pyx_r = __pyx_t_8;
14833     __pyx_t_8 = 0;
14834     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
14835     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
14836     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
14837     goto __pyx_L6_except_return;
14838 }
14839 __pyx_L5_except_error::
14840
14841     /* "PyClical.pyx":1623
14842  *      {1,2}
14843  *      """
14844  *      try:                # ««««««««
14845  *          return math.exp(obj)
14846  *      except:
14847  */
14848     __Pyx_XGIVEREF(__pyx_t_1);
14849     __Pyx_XGIVEREF(__pyx_t_2);
14850     __Pyx_XGIVEREF(__pyx_t_3);
14851     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14852     goto __pyx_L1_error;
14853     __pyx_L7_try_return:;
14854     __Pyx_XGIVEREF(__pyx_t_1);
14855     __Pyx_XGIVEREF(__pyx_t_2);
14856     __Pyx_XGIVEREF(__pyx_t_3);
14857     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14858     goto __pyx_L0;
14859     __pyx_L6_except_return:;
14860     __Pyx_XGIVEREF(__pyx_t_1);
14861     __Pyx_XGIVEREF(__pyx_t_2);
14862     __Pyx_XGIVEREF(__pyx_t_3);
14863     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14864     goto __pyx_L0;
14865 }
14866
14867     /* "PyClical.pyx":1614
14868  *      return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14869  *
14870  * cpdef inline exp(obj):                # ««««««««
14871  *      """
14872  *      Exponential of multivector.
14873  */
14874
14875     /* function exit code */
14876     __pyx_L1_error::;
14877     __Pyx_XDECREF(__pyx_t_4);
14878     __Pyx_XDECREF(__pyx_t_5);
14879     __Pyx_XDECREF(__pyx_t_6);
14880     __Pyx_XDECREF(__pyx_t_7);
14881     __Pyx_XDECREF(__pyx_t_8);
14882     __Pyx_AddTraceback("PyClical.exp", __pyx_clineno, __pyx_lineno, __pyx_filename);
14883     __pyx_r = 0;
14884     __pyx_L0:;
14885     __Pyx_XGIVEREF(__pyx_r);
14886     __Pyx_RefNannyFinishContext();
14887     return __pyx_r;
14888 }
14889
14890     /* Python wrapper */
14891     static PyObject * __pyx_pw_8PyClical_53exp(PyObject * __pyx_self, PyObject
__pyx_v_obj); /*proto*/
14892     static char __pyx_doc_8PyClical_52exp[] = "\n    Exponential of multivector.\n\n    >>
x=clifford(\"{1,2}\") * pi/4; print(exp(x))\n    0.7071+0.7071{1,2}\n    >> x=clifford(\"{1,2}\") *
pi/2; print(exp(x))\n    {1,2}\n    ";
14893     static PyObject * __pyx_pw_8PyClical_53exp(PyObject * __pyx_self, PyObject * __pyx_v_obj)
{
14894         PyObject * __pyx_r = 0;
14895         __Pyx_RefNannyDeclarations
14896         __Pyx_RefNannySetupContext("exp (wrapper)", 0);
14897         __pyx_r = __pyx_pf_8PyClical_52exp(__pyx_self, ((PyObject *) __pyx_v_obj));
14898
14899         /* function exit code */
14900         __Pyx_RefNannyFinishContext();
14901         return __pyx_r;
14902     }
14903
14904     static PyObject * __pyx_pf_8PyClical_52exp(CYTHON_UNUSED PyObject * __pyx_self, PyObject
__pyx_v_obj) {
14905         PyObject * __pyx_r = NULL;

```

```

14906         __Pyx_RefNannyDeclarations
14907         PyObject *__pyx_t_1 = NULL;
14908         int __pyx_lineno = 0;
14909         const char *__pyx_filename = NULL;
14910         int __pyx_clineno = 0;
14911         __Pyx_RefNannySetupContext("exp", 0);
14912         __Pyx_XDECREF(__pyx_r);
14913         __pyx_t_1 = __pyx_f_8PyClical_exp(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
14914             __PYX_ERR(0, 1614, __pyx_L1_error)
14915         __Pyx_GOTREF(__pyx_t_1);
14916         __pyx_r = __pyx_t_1;
14917         __pyx_t_1 = 0;
14918         goto __pyx_L0;
14919
14920         /* function exit code */
14921         __pyx_L1_error;;
14922         __Pyx_XDECREF(__pyx_t_1);
14923         __Pyx_AddTraceback("PyClical.exp", __pyx_clineno, __pyx_lineno, __pyx_filename);
14924         __pyx_r = NULL;
14925         __pyx_L0;;
14926         __Pyx_XGIVEREF(__pyx_r);
14927         __Pyx_RefNannyFinishContext();
14928         return __pyx_r;
14929     }
14930
14931     /* "PyClical.pyx":1628
14932     *
14933     * cpdef inline log(obj,i = None):
14934     *     """
14935     *     Natural logarithm of multivector with optional complexifier.
14936     */
14937
14938     static PyObject *__pyx_pw_8PyClical_55log(PyObject *__pyx_self, PyObject *__pyx_args,
14939     PyObject *__pyx_kwds); /*proto*/
14940     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_log(PyObject *__pyx_v_obj,
14941     CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_log *__pyx_optional_args) {
14942         PyObject *__pyx_v_i = ((PyObject *)Py_None);
14943         PyObject *__pyx_r = NULL;
14944         __Pyx_RefNannyDeclarations
14945         int __pyx_t_1;
14946         int __pyx_t_2;
14947         PyObject *__pyx_t_3 = NULL;
14948         Clifford __pyx_t_4;
14949         PyObject *__pyx_t_5 = NULL;
14950         PyObject *__pyx_t_6 = NULL;
14951         PyObject *__pyx_t_7 = NULL;
14952         PyObject *__pyx_t_8 = NULL;
14953         PyObject *__pyx_t_9 = NULL;
14954         PyObject *__pyx_t_10 = NULL;
14955         PyObject *__pyx_t_11 = NULL;
14956         int __pyx_lineno = 0;
14957         const char *__pyx_filename = NULL;
14958         int __pyx_clineno = 0;
14959         __Pyx_RefNannySetupContext("log", 0);
14960         if (__pyx_optional_args) {
14961             if (__pyx_optional_args->__pyx_n > 0) {
14962                 __pyx_v_i = __pyx_optional_args->i;
14963             }
14964         }
14965
14966         /* "PyClical.pyx":1643
14967         * RuntimeError: check_complex(val, i): i is not a valid complexifier for val
14968         *
14969         * if not (i is None):
14970         *     return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
14971         * else:
14972         */
14973         __pyx_t_1 = (__pyx_v_i != Py_None);
14974         __pyx_t_2 = (__pyx_t_1 != 0);
14975         if (__pyx_t_2) {
14976             /* "PyClical.pyx":1644
14977             *
14978             * if not (i is None):
14979             *     return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
14980             * else:
14981             *     try:
14982             */
14983             __Pyx_XDECREF(__pyx_r);
14984             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject *)
14985             __pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1644, __pyx_L1_error)
14986             __Pyx_GOTREF(__pyx_t_3);
14987             try {
14988                 __pyx_t_4 = log(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
14989                 __pyx_f_8PyClical_toClifford(__pyx_v_i));
14990             } catch (...) {

```

```

14988         __Pyx_CppExn2PyErr();
14989         __PYX_ERR(0, 1644, __pyx_L1_error)
14990     }
14991     __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1644, __pyx_L1_error)
14992     __Pyx_GOTREF(__pyx_t_5);
14993     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14994     __pyx_r = __pyx_t_5;
14995     __pyx_t_5 = 0;
14996     goto __pyx_L0;
14997
14998     /* "PyClical.pyx":1643
14999     * RuntimeError: check_complex(val, i): i is not a valid complexifier for val
15000     * """
15001     * if not (i is None):                # ««««««««
15002     *     return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15003     * else:
15004     */
15005     }
15006
15007     /* "PyClical.pyx":1646
15008     * return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15009     * else:
15010     *     try:                # ««««««««
15011     *         return math.log(obj)
15012     *     except:
15013     */
15014     /*else*/ {
15015     {
15016         __Pyx_PyThreadState_declare
15017         __Pyx_PyThreadState_assign
15018         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
15019         __Pyx_XGOTREF(__pyx_t_6);
15020         __Pyx_XGOTREF(__pyx_t_7);
15021         __Pyx_XGOTREF(__pyx_t_8);
15022         /*try:*/ {
15023
15024             /* "PyClical.pyx":1647
15025             * else:
15026             *     try:
15027             *         return math.log(obj)                # ««««««««
15028             *     except:
15029             *         return clifford().wrap( glucat.log(toClifford(obj)) )
15030             */
15031             __Pyx_XDECREF(__pyx_r);
15032             __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
15033 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1647, __pyx_L4_error)
15034             __Pyx_GOTREF(__pyx_t_3);
15035             __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_log); if
15036 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1647, __pyx_L4_error)
15037             __Pyx_GOTREF(__pyx_t_9);
15038             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15039             __pyx_t_3 = NULL;
15040             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
15041                 __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
15042                 if (likely(__pyx_t_3)) {
15043                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
15044                     __Pyx_INCREF(__pyx_t_3);
15045                     __Pyx_INCREF(function);
15046                     __Pyx_DECREF_SET(__pyx_t_9, function);
15047                 }
15048             }
15049             __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
15050             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15051             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1647, __pyx_L4_error)
15052             __Pyx_GOTREF(__pyx_t_5);
15053             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15054             __pyx_r = __pyx_t_5;
15055             __pyx_t_5 = 0;
15056             goto __pyx_L8_try_return;
15057
15058             /* "PyClical.pyx":1646
15059             * return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15060             * else:
15061             *     try:                # ««««««««
15062             *         return math.log(obj)
15063             *     except:
15064             */
15065             }
15066             __pyx_L4_error:;
15067             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15068             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15069             __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
15070
15071             /* "PyClical.pyx":1648
15072             * try:

```

```

15071 *         return math.log(obj)
15072 *     except:         # ««««««««
15073 *         return clifford().wrap( glucat.log(toClifford(obj)) )
15074 *
15075 */
15076
15077         /*except:*/ {
15078             __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15079             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1648, __pyx_L6_except_error)
15080             __Pyx_GOTREF(__pyx_t_5);
15081             __Pyx_GOTREF(__pyx_t_9);
15082             __Pyx_GOTREF(__pyx_t_3);
15083
15084             /* "PyClical.pyx":1649
15085 *         return math.log(obj)
15086 *     except:
15087 *         return clifford().wrap( glucat.log(toClifford(obj)) )         # ««««««««
15088 * cpdef inline cos(obj,i = None):
15089 */
15090             __Pyx_XDECREF(__pyx_r);
15091             __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1649,
__pyx_L6_except_error)
15092             __Pyx_GOTREF(__pyx_t_10);
15093             __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), log(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1649, __pyx_L6_except_error)
15094             __Pyx_GOTREF(__pyx_t_11);
15095             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
15096             __pyx_r = __pyx_t_11;
15097             __pyx_t_11 = 0;
15098             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15099             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15100             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15101             goto __pyx_L7_except_return;
15102         }
15103         __pyx_L6_except_error;
15104
15105         /* "PyClical.pyx":1646
15106 *         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15107 *     else:
15108 *         try:         # ««««««««
15109 *             return math.log(obj)
15110 *         except:
15111 */
15112             __Pyx_XGIVEREF(__pyx_t_6);
15113             __Pyx_XGIVEREF(__pyx_t_7);
15114             __Pyx_XGIVEREF(__pyx_t_8);
15115             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15116             goto __pyx_L1_error;
15117             __pyx_L8_try_return;
15118             __Pyx_XGIVEREF(__pyx_t_6);
15119             __Pyx_XGIVEREF(__pyx_t_7);
15120             __Pyx_XGIVEREF(__pyx_t_8);
15121             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15122             goto __pyx_L0;
15123             __pyx_L7_except_return;
15124             __Pyx_XGIVEREF(__pyx_t_6);
15125             __Pyx_XGIVEREF(__pyx_t_7);
15126             __Pyx_XGIVEREF(__pyx_t_8);
15127             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15128             goto __pyx_L0;
15129         }
15130     }
15131
15132     /* "PyClical.pyx":1628
15133 *         return clifford().wrap( glucat.exp(toClifford(obj)) )
15134 *
15135 * cpdef inline log(obj,i = None):         # ««««««««
15136 *     """
15137 *         Natural logarithm of multivector with optional complexifier.
15138 */
15139
15140         /* function exit code */
15141         __pyx_L1_error;
15142         __Pyx_XDECREF(__pyx_t_3);
15143         __Pyx_XDECREF(__pyx_t_5);
15144         __Pyx_XDECREF(__pyx_t_9);
15145         __Pyx_XDECREF(__pyx_t_10);
15146         __Pyx_XDECREF(__pyx_t_11);
15147         __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno, __pyx_filename);
15148         __pyx_r = 0;
15149         __pyx_L0;
15150         __Pyx_XGIVEREF(__pyx_r);
15151         __Pyx_RefNannyFinishContext();

```



```

15152         return __pyx_r;
15153     }
15154
15155     /* Python wrapper */
15156     static PyObject* __pyx_pw_8PyClical_55log(PyObject* __pyx_self, PyObject* __pyx_args,
PyObject* __pyx_kwds); /*proto*/
15157     static char __pyx_doc_8PyClical_54log[] = "\n    Natural logarithm of multivector with
optional complexifier.\n\n    >> x=clifford(\"{-1}\"); print((log(x,\"{-1}\") * 2/pi))\n    {-1}\n
>> x=clifford(\"{1,2}\"); print((log(x,\"{1,2,3}\") * 2/pi))\n    {1,2}\n    >> x=clifford(\"{1,2}\");
print((log(x) * 2/pi))\n    {1,2}\n    >> x=clifford(\"{1,2}\"); print((log(x,\"{1,2}\") * 2/pi))\n
Traceback (most recent call last):\n    ...\n    RuntimeError: check_complex(val, i): i is not a valid
complexifier for val\n    ";
15158     static PyObject* __pyx_pw_8PyClical_55log(PyObject* __pyx_self, PyObject* __pyx_args,
PyObject* __pyx_kwds) {
15159         PyObject* __pyx_v_obj = 0;
15160         PyObject* __pyx_v_i = 0;
15161         int __pyx_lineno = 0;
15162         const char* __pyx_filename = NULL;
15163         int __pyx_clineno = 0;
15164         PyObject* __pyx_r = 0;
15165         __Pyx_RefNannyDeclarations
15166         __Pyx_RefNannySetupContext("log (wrapper)", 0);
15167         {
15168             static PyObject* __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
15169             PyObject* values[2] = {0,0};
15170             values[1] = ((PyObject*)Py_None);
15171             if (unlikely(__pyx_kwds)) {
15172                 Py_ssize_t kw_args;
15173                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
15174                 switch (pos_args) {
15175                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15176                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
15177                     case 0: break;
15178                     default: goto __pyx_L5_argtuple_error;
15179                 }
15180                 kw_args = PyDict_Size(__pyx_kwds);
15181                 switch (pos_args) {
15182                     case 0:
15183                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
15184 0)) kw_args--;
15185
15186                         else goto __pyx_L5_argtuple_error;
15187                         CYTHON_FALLTHROUGH;
15188                     case 1:
15189                         if (kw_args > 0) {
15190                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
15191                             if (value) { values[1] = value; kw_args--; }
15192                         }
15193                     }
15194                     if (unlikely(kw_args > 0)) {
15195                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "log") < 0)) __PYX_ERR(0, 1628, __pyx_L3_error)
15196                     }
15197                     else {
15198                         switch (PyTuple_GET_SIZE(__pyx_args)) {
15199                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15200                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
15201                             case 0: break;
15202                             default: goto __pyx_L5_argtuple_error;
15203                         }
15204                     }
15205                 }
15206                 __pyx_v_obj = values[0];
15207                 __pyx_v_i = values[1];
15208             }
15209             goto __pyx_L4_argument_unpacking_done;
15210             __pyx_L5_argtuple_error:;
15211             __Pyx_RaiseArgtupleInvalid("log", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
__PYX_ERR(0, 1628, __pyx_L3_error)
15212             __pyx_L3_error:;
15213             __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno, __pyx_filename);
15214             __Pyx_RefNannyFinishContext();
15215             return NULL;
15216             __pyx_L4_argument_unpacking_done:;
15217             __pyx_r = __pyx_pf_8PyClical_54log(__pyx_self, __pyx_v_obj, __pyx_v_i);
15218
15219             /* function exit code */
15220             __Pyx_RefNannyFinishContext();
15221             return __pyx_r;
15222         }
15223
15224         static PyObject* __pyx_pf_8PyClical_54log(CYTHON_UNUSED PyObject* __pyx_self, PyObject
*__pyx_v_obj, PyObject* __pyx_v_i) {
15225             PyObject* __pyx_r = NULL;
15226             __Pyx_RefNannyDeclarations
15227             PyObject* __pyx_t_1 = NULL;

```

```

15228         struct __pyx_opt_args_8PyClical_log __pyx_t_2;
15229         int __pyx_lineno = 0;
15230         const char *__pyx_filename = NULL;
15231         int __pyx_clineno = 0;
15232         __Pyx_RefNannySetupContext("log", 0);
15233         __Pyx_XDECREF(__pyx_r);
15234         __pyx_t_2.__pyx_n = 1;
15235         __pyx_t_2.i = __pyx_v_i;
15236         __pyx_t_1 = __pyx_f_8PyClical_log(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1628, __pyx_L1_error)
15237         __Pyx_GOTREF(__pyx_t_1);
15238         __pyx_r = __pyx_t_1;
15239         __pyx_t_1 = 0;
15240         goto __pyx_L0;
15241
15242         /* function exit code */
15243         __pyx_L1_error:;
15244         __Pyx_XDECREF(__pyx_t_1);
15245         __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno, __pyx_filename);
15246         __pyx_r = NULL;
15247         __pyx_L0:;
15248         __Pyx_XGIVEREF(__pyx_r);
15249         __Pyx_RefNannyFinishContext();
15250         return __pyx_r;
15251     }
15252
15253     /* "PyClical.pyx":1651
15254     *
15255     * return clifford().wrap( glucat.log(toClifford(obj)) )
15256     *
15257     * cpdef inline cos(obj,i = None):
15258     *     """
15259     *     Cosine of multivector with optional complexifier.
15260     */
15261
15262     static PyObject *__pyx_pw_8PyClical_57cos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
15263     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cos(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_cos *__pyx_optional_args) {
15264         PyObject *__pyx_v_i = ((PyObject *)Py_None);
15265         PyObject *__pyx_r = NULL;
15266         __Pyx_RefNannyDeclarations
15267         int __pyx_t_1;
15268         int __pyx_t_2;
15269         PyObject *__pyx_t_3 = NULL;
15270         Clifford __pyx_t_4;
15271         PyObject *__pyx_t_5 = NULL;
15272         PyObject *__pyx_t_6 = NULL;
15273         PyObject *__pyx_t_7 = NULL;
15274         PyObject *__pyx_t_8 = NULL;
15275         PyObject *__pyx_t_9 = NULL;
15276         PyObject *__pyx_t_10 = NULL;
15277         PyObject *__pyx_t_11 = NULL;
15278         int __pyx_lineno = 0;
15279         const char *__pyx_filename = NULL;
15280         int __pyx_clineno = 0;
15281         __Pyx_RefNannySetupContext("cos", 0);
15282         if (__pyx_optional_args) {
15283             if (__pyx_optional_args->__pyx_n > 0) {
15284                 __pyx_v_i = __pyx_optional_args->i;
15285             }
15286         }
15287
15288         /* "PyClical.pyx":1660
15289         *
15290         * {1,2}
15291         * """
15292         * if not (i is None):
15293         *     return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15294         * else:
15295         */
15296         __pyx_t_1 = (__pyx_v_i != Py_None);
15297         __pyx_t_2 = (__pyx_t_1 != 0);
15298         if (__pyx_t_2) {
15299
15300             /* "PyClical.pyx":1661
15301             *
15302             * """
15303             * if not (i is None):
15304             *     return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15305             * else:
15306             *     try:
15307             *
15308             *         __Pyx_XDECREF(__pyx_r);
15309             *         __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1661, __pyx_L1_error)
15310             *         __Pyx_GOTREF(__pyx_t_3);
15311             *         try {
15312             *             __pyx_t_4 = cos(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));

```

```

15310         } catch(...) {
15311             __Pyx_CppExn2PyErr();
15312             __PYX_ERR(0, 1661, __pyx_L1_error)
15313         }
15314         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1661, __pyx_L1_error)
15315         __Pyx_GOTREF(__pyx_t_5);
15316         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15317         __pyx_r = __pyx_t_5;
15318         __pyx_t_5 = 0;
15319         goto __pyx_L0;
15320
15321         /* "PyClical.pyx":1660
15322         * {1,2}
15323         * """
15324         * if not (i is None):          # ««««««««
15325         *     return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15326         * else:
15327         */
15328         }
15329
15330         /* "PyClical.pyx":1663
15331         * return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15332         * else:
15333         *     try:          # ««««««««
15334         *         return math.cos(obj)
15335         *     except:
15336         */
15337         /*else*/ {
15338             {
15339                 __Pyx_PyThreadState_declare
15340                 __Pyx_PyThreadState_assign
15341                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
15342                 __Pyx_XGOTREF(__pyx_t_6);
15343                 __Pyx_XGOTREF(__pyx_t_7);
15344                 __Pyx_XGOTREF(__pyx_t_8);
15345                 /*try:*/ {
15346
15347                     /* "PyClical.pyx":1664
15348                     *
15349                     *     try:
15350                     *         return math.cos(obj)          # ««««««««
15351                     *     except:
15352                     *         return clifford().wrap( glucat.cos(toClifford(obj)) )
15353                     */
15354                     __Pyx_XDECREF(__pyx_r);
15355                     __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1664, __pyx_L4_error)
15356                     __Pyx_GOTREF(__pyx_t_3);
15357                     __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_cos); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1664, __pyx_L4_error)
15358                     __Pyx_GOTREF(__pyx_t_9);
15359                     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15360                     __pyx_t_3 = NULL;
15361                     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
15362                         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
15363                         if (likely(__pyx_t_3)) {
15364                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
15365                             __Pyx_INCREF(__pyx_t_3);
15366                             __Pyx_INCREF(function);
15367                             __Pyx_DECREF_SET(__pyx_t_9, function);
15368                         }
15369                     }
15370                     __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
15371                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15372                     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1664, __pyx_L4_error)
15373                     __Pyx_GOTREF(__pyx_t_5);
15374                     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15375                     __pyx_r = __pyx_t_5;
15376                     __pyx_t_5 = 0;
15377                     goto __pyx_L8_try_return;
15378
15379                     /* "PyClical.pyx":1663
15380                     * return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15381                     * else:
15382                     *     try:          # ««««««««
15383                     *         return math.cos(obj)
15384                     *     except:
15385                     */
15386                     }
15387                     __pyx_L4_error:;
15388                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15389                     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15390                     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
15391
15392                     /* "PyClical.pyx":1665

```

```

15393 *         try:
15394 *             return math.cos(obj)
15395 *         except: # ««««««««
15396 *             return clifford().wrap( glucat.cos(toClifford(obj)) )
15397 *
15398 */
15399
15400             /*except:*/ {
15401                 __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15402                 if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1665, __pyx_L6_except_error)
15403                     __Pyx_GOTREF(__pyx_t_5);
15404                     __Pyx_GOTREF(__pyx_t_9);
15405                     __Pyx_GOTREF(__pyx_t_3);
15406
15407                     /* "PyClical.pyx":1666
15408 *             return math.cos(obj)
15409 *         except:
15410 *             return clifford().wrap( glucat.cos(toClifford(obj)) ) # ««««««««
15411 * cpdef inline acos(obj,i = None):
15412 */
15413                 __Pyx_XDECREF(__pyx_r);
15414                 __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1666,
__pyx_L6_except_error)
15415                 __Pyx_GOTREF(__pyx_t_10);
15416                 __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), cos(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1666, __pyx_L6_except_error)
15417                 __Pyx_GOTREF(__pyx_t_11);
15418                 __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
15419                 __pyx_r = __pyx_t_11;
15420                 __pyx_t_11 = 0;
15421                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15422                 __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15423                 __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15424                 goto __pyx_L7_except_return;
15425             }
15426             __pyx_L6_except_error;;
15427
15428             /* "PyClical.pyx":1663
15429 *             return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15430 *         else:
15431 *             try: # ««««««««
15432 *                 return math.cos(obj)
15433 *             except:
15434 */
15435                 __Pyx_XGIVEREF(__pyx_t_6);
15436                 __Pyx_XGIVEREF(__pyx_t_7);
15437                 __Pyx_XGIVEREF(__pyx_t_8);
15438                 __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15439                 goto __pyx_L1_error;
15440                 __pyx_L8_try_return;;
15441                 __Pyx_XGIVEREF(__pyx_t_6);
15442                 __Pyx_XGIVEREF(__pyx_t_7);
15443                 __Pyx_XGIVEREF(__pyx_t_8);
15444                 __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15445                 goto __pyx_L0;
15446                 __pyx_L7_except_return;;
15447                 __Pyx_XGIVEREF(__pyx_t_6);
15448                 __Pyx_XGIVEREF(__pyx_t_7);
15449                 __Pyx_XGIVEREF(__pyx_t_8);
15450                 __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15451                 goto __pyx_L0;
15452             }
15453         }
15454
15455         /* "PyClical.pyx":1651
15456 *             return clifford().wrap( glucat.log(toClifford(obj)) )
15457 *
15458 * cpdef inline cos(obj,i = None): # ««««««««
15459 *     """
15460 *     Cosine of multivector with optional complexifier.
15461 */
15462
15463         /* function exit code */
15464         __pyx_L1_error;;
15465         __Pyx_XDECREF(__pyx_t_3);
15466         __Pyx_XDECREF(__pyx_t_5);
15467         __Pyx_XDECREF(__pyx_t_9);
15468         __Pyx_XDECREF(__pyx_t_10);
15469         __Pyx_XDECREF(__pyx_t_11);
15470         __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15471         __pyx_r = 0;
15472         __pyx_L0;;
15473         __Pyx_XGIVEREF(__pyx_r);

```

```

15474         __Pyx_RefNannyFinishContext();
15475         return __pyx_r;
15476     }
15477
15478     /* Python wrapper */
15479     static PyObject *__pyx_pw_8PyClical_57cos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
15480     static char __pyx_doc_8PyClical_56cos[] = "\n    Cosine of multivector with optional
complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(cos(acos(x),\"{1,2,3}\"))\n    {1,2}\n    >>
x=clifford(\"{1,2}\"); print(cos(acos(x)))\n    {1,2}\n    ";
15481     static PyObject *__pyx_pw_8PyClical_57cos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
15482         PyObject *__pyx_v_obj = 0;
15483         PyObject *__pyx_v_i = 0;
15484         int __pyx_lineno = 0;
15485         const char *__pyx_filename = NULL;
15486         int __pyx_clineno = 0;
15487         PyObject *__pyx_r = 0;
15488         __Pyx_RefNannyDeclarations
15489         __Pyx_RefNannySetupContext("cos (wrapper)", 0);
15490         {
15491             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
15492             PyObject* values[2] = {0,0};
15493             values[1] = (PyObject *)Py_None;
15494             if (unlikely(__pyx_kwds)) {
15495                 Py_ssize_t kw_args;
15496                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
15497                 switch (pos_args) {
15498                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15499                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
15500                     case 0: break;
15501                     default: goto __pyx_L5_argtuple_error;
15502                 }
15503                 kw_args = PyDict_Size(__pyx_kwds);
15504                 switch (pos_args) {
15505                     case 0:
15506                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
15509                         else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
15510                         case 1:
15511                             if (kw_args > 0) {
15512                                 PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
15513                                 if (value) { values[1] = value; kw_args--; }
15514                             }
15515                         }
15516                     if (unlikely(kw_args > 0)) {
15517                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "cos") < 0)) __PYX_ERR(0, 1651, __pyx_L3_error)
15518                     }
15519                     else {
15520                         switch (PyTuple_GET_SIZE(__pyx_args)) {
15521                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15522                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
15523                             case 0: break;
15524                             default: goto __pyx_L5_argtuple_error;
15525                         }
15526                     }
15527                 }
15528                 __pyx_v_obj = values[0];
15529                 __pyx_v_i = values[1];
15530             }
15531             goto __pyx_L4_argument_unpacking_done;
15532             __pyx_L5_argtuple_error:;
15533             __Pyx_RaiseArgtupleInvalid("cos", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
15534             __PYX_ERR(0, 1651, __pyx_L3_error)
15535             __pyx_L3_error:;
15536             __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15537             __Pyx_RefNannyFinishContext();
15538             return NULL;
15539             __pyx_L4_argument_unpacking_done:;
15540             __pyx_r = __pyx_pf_8PyClical_56cos(__pyx_self, __pyx_v_obj, __pyx_v_i);
15541
15542             /* function exit code */
15543             __Pyx_RefNannyFinishContext();
15544             return __pyx_r;
15545         }
15546
15547         static PyObject *__pyx_pf_8PyClical_56cos(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj, PyObject *__pyx_v_i) {
15548             PyObject *__pyx_r = NULL;
15549             __Pyx_RefNannyDeclarations
15550             PyObject *__pyx_t_1 = NULL;
15551             struct __pyx_opt_args_8PyClical_cos __pyx_t_2;
15552             int __pyx_lineno = 0;

```

```

15553         const char *__pyx_filename = NULL;
15554         int __pyx_clineno = 0;
15555         __Pyx_RefNannySetupContext("cos", 0);
15556         __Pyx_XDECREF(__pyx_r);
15557         __pyx_t_2.__pyx_n = 1;
15558         __pyx_t_2.i = __pyx_v_i;
15559         __pyx_t_1 = __pyx_f_8PyClicl_cos(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1651, __pyx_L1_error)
15560         __Pyx_GOTREF(__pyx_t_1);
15561         __pyx_r = __pyx_t_1;
15562         __pyx_t_1 = 0;
15563         goto __pyx_L0;
15564
15565         /* function exit code */
15566         __pyx_L1_error:;
15567         __Pyx_XDECREF(__pyx_t_1);
15568         __Pyx_AddTraceback("PyClicl.cos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15569         __pyx_r = NULL;
15570         __pyx_L0:;
15571         __Pyx_XGIVEREF(__pyx_r);
15572         __Pyx_RefNannyFinishContext();
15573         return __pyx_r;
15574     }
15575
15576     /* "PyClicl.pyx":1668
15577     *
15578     *
15579     * cpdef inline acos(obj,i = None):          # ««««««««
15580     *     """
15581     *     Inverse cosine of multivector with optional complexifier.
15582     */
15583
15584     static PyObject *__pyx_pw_8PyClicl_59acos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
15585     static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_acos(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClicl_acos *__pyx_optional_args) {
15586     PyObject *__pyx_v_i = (PyObject *)Py_None;
15587     PyObject *__pyx_r = NULL;
15588     __Pyx_RefNannyDeclarations
15589     int __pyx_t_1;
15590     int __pyx_t_2;
15591     PyObject *__pyx_t_3 = NULL;
15592     Clifford __pyx_t_4;
15593     PyObject *__pyx_t_5 = NULL;
15594     PyObject *__pyx_t_6 = NULL;
15595     PyObject *__pyx_t_7 = NULL;
15596     PyObject *__pyx_t_8 = NULL;
15597     PyObject *__pyx_t_9 = NULL;
15598     PyObject *__pyx_t_10 = NULL;
15599     PyObject *__pyx_t_11 = NULL;
15600     int __pyx_lineno = 0;
15601     const char *__pyx_filename = NULL;
15602     int __pyx_clineno = 0;
15603     __Pyx_RefNannySetupContext("acos", 0);
15604     if (__pyx_optional_args) {
15605         if (__pyx_optional_args->__pyx_n > 0) {
15606             __pyx_v_i = __pyx_optional_args->i;
15607         }
15608     }
15609
15610     /* "PyClicl.pyx":1681
15611     *     {1,2}
15612     *     """
15613     *     if not (i is None):          # ««««««««
15614     *         return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15615     *     else:
15616     */
15617         __pyx_t_1 = (__pyx_v_i != Py_None);
15618         __pyx_t_2 = (__pyx_t_1 != 0);
15619         if (__pyx_t_2) {
15620
15621             /* "PyClicl.pyx":1682
15622             *
15623             *     if not (i is None):
15624             *         return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15625             *     else:
15626             *         try:
15627             */
15628             __Pyx_XDECREF(__pyx_r);
15629             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1682, __pyx_L1_error)
15630             __Pyx_GOTREF(__pyx_t_3);
15631             try {
15632                 __pyx_t_4 = acos(__pyx_f_8PyClicl_toClifford(__pyx_v_obj),
__pyx_f_8PyClicl_toClifford(__pyx_v_i));
15633             } catch (...) {
15634                 __Pyx_CppExn2PyErr();

```

```

15635         __PYX_ERR(0, 1682, __pyx_L1_error)
15636     }
15637     __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1682, __pyx_L1_error)
15638     __Pyx_GOTREF(__pyx_t_5);
15639     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15640     __pyx_r = __pyx_t_5;
15641     __pyx_t_5 = 0;
15642     goto __pyx_L0;
15643
15644     /* "PyClical.pyx":1681
15645  *      {1,2}
15646  *      """
15647  *      if not (i is None):          # ««««««««
15648  *          return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15649  *      else:
15650  */
15651     }
15652
15653     /* "PyClical.pyx":1684
15654  *      return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15655  *      else:
15656  *          try:          # ««««««««
15657  *              return math.acos(obj)
15658  *          except:
15659  */
15660     /*else*/ {
15661     {
15662         __Pyx_PyThreadState_declare
15663         __Pyx_PyThreadState_assign
15664         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
15665         __Pyx_XGOTREF(__pyx_t_6);
15666         __Pyx_XGOTREF(__pyx_t_7);
15667         __Pyx_XGOTREF(__pyx_t_8);
15668         /*try:*/ {
15669
15670             /* "PyClical.pyx":1685
15671  *          else:
15672  *          try:
15673  *              return math.acos(obj)          # ««««««««
15674  *          except:
15675  *              return clifford().wrap( glucat.acos(toClifford(obj)) )
15676  */
15677         __Pyx_XDECREF(__pyx_r);
15678         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1685, __pyx_L4_error)
15679         __Pyx_GOTREF(__pyx_t_3);
15680         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_acos); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1685, __pyx_L4_error)
15681         __Pyx_GOTREF(__pyx_t_9);
15682         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15683         __pyx_t_3 = NULL;
15684         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
15685             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
15686             if (likely(__pyx_t_3)) {
15687                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
15688                 __Pyx_INCREF(__pyx_t_3);
15689                 __Pyx_INCREF(function);
15690                 __Pyx_DECREF_SET(__pyx_t_9, function);
15691             }
15692         }
15693         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
15694         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15695         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1685, __pyx_L4_error)
15696         __Pyx_GOTREF(__pyx_t_5);
15697         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15698         __pyx_r = __pyx_t_5;
15699         __pyx_t_5 = 0;
15700         goto __pyx_L8_try_return;
15701
15702         /* "PyClical.pyx":1684
15703  *          return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15704  *          else:
15705  *          try:          # ««««««««
15706  *              return math.acos(obj)
15707  *          except:
15708  */
15709     }
15710     __pyx_L4_error:;
15711     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15712     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15713     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
15714
15715     /* "PyClical.pyx":1686
15716  *          try:
15717  *              return math.acos(obj)

```

```

15718 *          except:          # ««««««««
15719 *              return clifford().wrap( glucat.acos(toClifford(obj)) )
15720 *
15721 */
15722
15723          /*except:*/ {
15724              __Pyx_AddTraceback("PyClicl.acos", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15725              if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1686, __pyx_L6_except_error)
15726              __Pyx_GOTREF(__pyx_t_5);
15727              __Pyx_GOTREF(__pyx_t_9);
15728              __Pyx_GOTREF(__pyx_t_3);
15729
15730              /* "PyClicl.pyx":1687
15731 *              return math.acos(obj)
15732 *          except:
15733 *              return clifford().wrap( glucat.acos(toClifford(obj)) )          # ««««««««
15734 * cpdef inline cosh(obj):
15735 */
15736              __Pyx_XDECREF(__pyx_r);
15737              __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1687,
__pyx_L6_except_error)
15738              __Pyx_GOTREF(__pyx_t_10);
15739              __pyx_t_11 = __pyx_f_8PyClicl_8clifford_wrap(((struct
__pyx_obj_8PyClicl_clifford *)__pyx_t_10), acos(__pyx_f_8PyClicl_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1687, __pyx_L6_except_error)
15740              __Pyx_GOTREF(__pyx_t_11);
15741              __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
15742              __pyx_r = __pyx_t_11;
15743              __pyx_t_11 = 0;
15744              __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15745              __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15746              __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15747              goto __pyx_L7_except_return;
15748          }
15749          __pyx_L6_except_error;;
15750
15751          /* "PyClicl.pyx":1684
15752 *          return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15753 *      else:
15754 *          try:          # ««««««««
15755 *              return math.acos(obj)
15756 *          except:
15757 */
15758              __Pyx_XGIVEREF(__pyx_t_6);
15759              __Pyx_XGIVEREF(__pyx_t_7);
15760              __Pyx_XGIVEREF(__pyx_t_8);
15761              __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15762              goto __pyx_L1_error;
15763              __pyx_L8_try_return:;
15764              __Pyx_XGIVEREF(__pyx_t_6);
15765              __Pyx_XGIVEREF(__pyx_t_7);
15766              __Pyx_XGIVEREF(__pyx_t_8);
15767              __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15768              goto __pyx_L0;
15769              __pyx_L7_except_return:;
15770              __Pyx_XGIVEREF(__pyx_t_6);
15771              __Pyx_XGIVEREF(__pyx_t_7);
15772              __Pyx_XGIVEREF(__pyx_t_8);
15773              __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15774              goto __pyx_L0;
15775          }
15776      }
15777
15778      /* "PyClicl.pyx":1668
15779 *      return clifford().wrap( glucat.cos(toClifford(obj)) )
15780 *
15781 * cpdef inline acos(obj, i = None):          # ««««««««
15782 *     """
15783 *     Inverse cosine of multivector with optional complexifier.
15784 */
15785
15786      /* function exit code */
15787      __pyx_L1_error:;
15788      __Pyx_XDECREF(__pyx_t_3);
15789      __Pyx_XDECREF(__pyx_t_5);
15790      __Pyx_XDECREF(__pyx_t_9);
15791      __Pyx_XDECREF(__pyx_t_10);
15792      __Pyx_XDECREF(__pyx_t_11);
15793      __Pyx_AddTraceback("PyClicl.acos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15794      __pyx_r = 0;
15795      __pyx_L0:;
15796      __Pyx_XGIVEREF(__pyx_r);
15797      __Pyx_RefNannyFinishContext();
15798      return __pyx_r;

```



```

15799     }
15800
15801     /* Python wrapper */
15802     static PyObject * __pyx_pw_8PyClical_59acos(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
15803     static char __pyx_doc_8PyClical_58acos[] = "\n    Inverse cosine of multivector with
optional complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(cos(acos(x),\"{1,2,3}\"))\n    {1,2}\n
>> x=clifford(\"{1,2}\"); print(cos(acos(x),\"{-1,1,2,3,4}\"))\n    {1,2}\n    >> print(acos(0) /
pi)\n    0.5\n    >> x=clifford(\"{1,2}\"); print(cos(acos(x))\n    {1,2}\n    ";
15804     static PyObject * __pyx_pw_8PyClical_59acos(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds) {
15805         PyObject * __pyx_v_obj = 0;
15806         PyObject * __pyx_v_i = 0;
15807         int __pyx_lineno = 0;
15808         const char * __pyx_filename = NULL;
15809         int __pyx_clineno = 0;
15810         PyObject * __pyx_r = 0;
15811         __Pyx_RefNannyDeclarations
15812         __Pyx_RefNannySetupContext("acos (wrapper)", 0);
15813         {
15814             static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
15815             PyObject* values[2] = {0,0};
15816             values[1] = ((PyObject *)Py_None);
15817             if (unlikely(__pyx_kwds)) {
15818                 Py_ssize_t kw_args;
15819                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
15820                 switch (pos_args) {
15821                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15822                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
15823                     case 0: break;
15824                     default: goto __pyx_L5_argtuple_error;
15825                 }
15826                 kw_args = PyDict_Size(__pyx_kwds);
15827                 switch (pos_args) {
15828                     case 0:
15829                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
15830
15831                         else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
15832                     case 1:
15833                         if (kw_args > 0) {
15834                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
15835                             if (value) { values[1] = value; kw_args--; }
15836                         }
15837                     if (unlikely(kw_args > 0)) {
15838                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "acos") < 0)) __PYX_ERR(0, 1668, __pyx_L3_error)
15839                     }
15840                     } else {
15841                         switch (PyTuple_GET_SIZE(__pyx_args)) {
15842                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15843                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
break;
15844                             default: goto __pyx_L5_argtuple_error;
15845                         }
15846                     }
15847                 __pyx_v_obj = values[0];
15848                 __pyx_v_i = values[1];
15849             }
15850             goto __pyx_L4_argument_unpacking_done;
15851             __pyx_L5_argtuple_error:;
15852             __Pyx_RaiseArgtupleInvalid("acos", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
15853             __PYX_ERR(0, 1668, __pyx_L3_error)
15854             __pyx_L3_error:;
15855             __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15856             __Pyx_RefNannyFinishContext();
15857             return NULL;
15858             __pyx_L4_argument_unpacking_done:;
15859             __pyx_r = __pyx_pf_8PyClical_58acos(__pyx_self, __pyx_v_obj, __pyx_v_i);
15860
15861             /* function exit code */
15862             __Pyx_RefNannyFinishContext();
15863             return __pyx_r;
15864         }
15865     }
15866
15867     static PyObject * __pyx_pf_8PyClical_58acos(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj, PyObject * __pyx_v_i) {
15868         PyObject * __pyx_r = NULL;
15869         __Pyx_RefNannyDeclarations
15870         PyObject * __pyx_t_1 = NULL;
15871         struct __pyx_opt_args_8PyClical_acos __pyx_t_2;
15872         int __pyx_lineno = 0;
15873         const char * __pyx_filename = NULL;

```

```

15877         int __pyx_clineno = 0;
15878         __Pyx_RefNannySetupContext("acos", 0);
15879         __Pyx_XDECREF(__pyx_r);
15880         __pyx_t_2.__pyx_n = 1;
15881         __pyx_t_2.i = __pyx_v_i;
15882         __pyx_t_1 = __pyx_f_8PyClical_acos(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1668, __pyx_L1_error)
15883         __Pyx_GOTREF(__pyx_t_1);
15884         __pyx_r = __pyx_t_1;
15885         __pyx_t_1 = 0;
15886         goto __pyx_L0;
15887
15888         /* function exit code */
15889         __Pyx_L1_error:;
15890         __Pyx_XDECREF(__pyx_t_1);
15891         __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15892         __pyx_r = NULL;
15893         __pyx_L0:;
15894         __Pyx_XGIVEREF(__pyx_r);
15895         __Pyx_RefNannyFinishContext();
15896         return __pyx_r;
15897     }
15898
15899     /* "PyClical.pyx":1689
15900     *
15901     * return clifford().wrap( glucat.acos(toClifford(obj)) )
15902     * cpdef inline cosh(obj):
15903     *     """
15904     *     Hyperbolic cosine of multivector.
15905     */
15906
15907     static PyObject * __pyx_pw_8PyClical_6lcosh(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
15908     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_cosh(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
15909         PyObject * __pyx_r = NULL;
15910         __Pyx_RefNannyDeclarations
15911         PyObject * __pyx_t_1 = NULL;
15912         PyObject * __pyx_t_2 = NULL;
15913         PyObject * __pyx_t_3 = NULL;
15914         PyObject * __pyx_t_4 = NULL;
15915         PyObject * __pyx_t_5 = NULL;
15916         PyObject * __pyx_t_6 = NULL;
15917         PyObject * __pyx_t_7 = NULL;
15918         PyObject * __pyx_t_8 = NULL;
15919         int __pyx_lineno = 0;
15920         const char * __pyx_filename = NULL;
15921         int __pyx_clineno = 0;
15922         __Pyx_RefNannySetupContext("cosh", 0);
15923
15924         /* "PyClical.pyx":1700
15925     *     {1,2}
15926     *     """
15927     *     try:
15928     *         return math.cosh(obj)
15929     *     except:
15930     */
15931         {
15932             __Pyx_PyThreadState_declare
15933             __Pyx_PyThreadState_assign
15934             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
15935             __Pyx_XGOTREF(__pyx_t_1);
15936             __Pyx_XGOTREF(__pyx_t_2);
15937             __Pyx_XGOTREF(__pyx_t_3);
15938             /*try:*/ {
15939
15940                 /* "PyClical.pyx":1701
15941     *     """
15942     *     try:
15943     *         return math.cosh(obj)
15944     *     except:
15945     *         return clifford().wrap( glucat.cosh(toClifford(obj)) )
15946     */
15947             __Pyx_XDECREF(__pyx_r);
15948             __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 1701, __pyx_L3_error)
15949             __Pyx_GOTREF(__pyx_t_5);
15950             __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_cosh); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 1701, __pyx_L3_error)
15951             __Pyx_GOTREF(__pyx_t_6);
15952             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15953             __pyx_t_5 = NULL;
15954             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
15955                 __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
15956                 if (likely(__pyx_t_5)) {
15957                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
15958                     __Pyx_INCREF(__pyx_t_5);

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15959         __Pyx_INCREF(function);
15960         __Pyx_DECREF_SET(__pyx_t_6, function);
15961     }
15962 }
15963 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
15964     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15965     if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1701, __pyx_L3_error)
15966     __Pyx_GOTREF(__pyx_t_4);
15967     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
15968     __pyx_r = __pyx_t_4;
15969     __pyx_t_4 = 0;
15970     goto __pyx_L7_try_return;
15971
15972     /* "PyClical.pyx":1700
15973  *      {1,2}
15974  *      """
15975  *      try:                # ««««««««
15976  *          return math.cosh(obj)
15977  *      except:
15978  */
15979     }
15980     __pyx_L3_error;;
15981     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
15982     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15983     __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
15984
15985     /* "PyClical.pyx":1702
15986  *      try:
15987  *          return math.cosh(obj)
15988  *      except:                # ««««««««
15989  *          return clifford().wrap( glucat.cosh(toClifford(obj)) )
15990  *
15991  */
15992     /*except:*/ {
15993     __Pyx_AddTraceback("PyClical.cosh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15994     if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
1702, __pyx_L5_except_error)
15995     __Pyx_GOTREF(__pyx_t_4);
15996     __Pyx_GOTREF(__pyx_t_6);
15997     __Pyx_GOTREF(__pyx_t_5);
15998
15999     /* "PyClical.pyx":1703
16000  *          return math.cosh(obj)
16001  *      except:
16002  *          return clifford().wrap( glucat.cosh(toClifford(obj)) )
16003  *
16004  * cpdef inline acosh(obj,i = None):
16005  */
16006     __Pyx_XDECREF(__pyx_r);
16007     __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1703,
__pyx_L5_except_error)
16008     __Pyx_GOTREF(__pyx_t_7);
16009     __pyx_t_8 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_7), cosh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1703, __pyx_L5_except_error)
16010     __Pyx_GOTREF(__pyx_t_8);
16011     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
16012     __pyx_r = __pyx_t_8;
16013     __pyx_t_8 = 0;
16014     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
16015     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16016     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
16017     goto __pyx_L6_except_return;
16018 }
16019 __pyx_L5_except_error;;
16020
16021     /* "PyClical.pyx":1700
16022  *      {1,2}
16023  *      """
16024  *      try:                # ««««««««
16025  *          return math.cosh(obj)
16026  *      except:
16027  */
16028     __Pyx_XGIVEREF(__pyx_t_1);
16029     __Pyx_XGIVEREF(__pyx_t_2);
16030     __Pyx_XGIVEREF(__pyx_t_3);
16031     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
16032     goto __pyx_L1_error;
16033     __pyx_L7_try_return;
16034     __Pyx_XGIVEREF(__pyx_t_1);
16035     __Pyx_XGIVEREF(__pyx_t_2);
16036     __Pyx_XGIVEREF(__pyx_t_3);
16037     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
16038     goto __pyx_L0;

```

```

16039         __pyx_L6_except_return;;
16040         __Pyx_XGIVEREF(__pyx_t_1);
16041         __Pyx_XGIVEREF(__pyx_t_2);
16042         __Pyx_XGIVEREF(__pyx_t_3);
16043         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
16044         goto __pyx_L0;
16045     }
16046
16047     /* "PyClical.pyx":1689
16048     *         return clifford().wrap( glucat.acos(toClifford(obj)) )
16049     *
16050     * cpdef inline cosh(obj):
16051     *     """
16052     *     Hyperbolic cosine of multivector.
16053     */
16054
16055     /* function exit code */
16056     __pyx_L1_error;;
16057     __Pyx_XDECREF(__pyx_t_4);
16058     __Pyx_XDECREF(__pyx_t_5);
16059     __Pyx_XDECREF(__pyx_t_6);
16060     __Pyx_XDECREF(__pyx_t_7);
16061     __Pyx_XDECREF(__pyx_t_8);
16062     __Pyx_AddTraceback("PyClical.cosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16063     __pyx_r = 0;
16064     __pyx_L0;;
16065     __Pyx_XGIVEREF(__pyx_r);
16066     __Pyx_RefNannyFinishContext();
16067     return __pyx_r;
16068 }
16069
16070 /* Python wrapper */
16071 static PyObject *__pyx_pw_8PyClical_61cosh(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
16072 static char __pyx_doc_8PyClical_60cosh[] = "\n    Hyperbolic cosine of
multivector.\n\n    >> x=clifford(\"{1,2}\") * pi; print(cosh(x))\n    -1\n    >>
x=clifford(\"{1,2,3}\"); print(cosh(acosh(x)))\n    {1,2,3}\n    >> x=clifford(\"{1,2}\");
print(cosh(acosh(x)))\n    {1,2}\n    ";
16073 static PyObject *__pyx_pf_8PyClical_61cosh(PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
16074     PyObject *__pyx_r = 0;
16075     __Pyx_RefNannyDeclarations
16076     __Pyx_RefNannySetupContext("cosh (wrapper)", 0);
16077     __pyx_r = __pyx_pf_8PyClical_60cosh(__pyx_self, ((PyObject *)__pyx_v_obj));
16078
16079     /* function exit code */
16080     __Pyx_RefNannyFinishContext();
16081     return __pyx_r;
16082 }
16083
16084 static PyObject *__pyx_pf_8PyClical_60cosh(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
16085     PyObject *__pyx_r = NULL;
16086     __Pyx_RefNannyDeclarations
16087     PyObject *__pyx_t_1 = NULL;
16088     int __pyx_lineno = 0;
16089     const char *__pyx_filename = NULL;
16090     int __pyx_clineno = 0;
16091     __Pyx_RefNannySetupContext("cosh", 0);
16092     __Pyx_XDECREF(__pyx_r);
16093     __pyx_t_1 = __pyx_f_8PyClical_cosh(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1689, __pyx_L1_error)
16094     __Pyx_GOTREF(__pyx_t_1);
16095     __pyx_r = __pyx_t_1;
16096     __pyx_t_1 = 0;
16097     goto __pyx_L0;
16098
16099     /* function exit code */
16100     __pyx_L1_error;;
16101     __Pyx_XDECREF(__pyx_t_1);
16102     __Pyx_AddTraceback("PyClical.cosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16103     __pyx_r = NULL;
16104     __pyx_L0;;
16105     __Pyx_XGIVEREF(__pyx_r);
16106     __Pyx_RefNannyFinishContext();
16107     return __pyx_r;
16108 }
16109
16110 /* "PyClical.pyx":1705
16111 *         return clifford().wrap( glucat.cosh(toClifford(obj)) )
16112 *
16113 * cpdef inline acosh(obj,i = None):
16114 *     """
16115 *     Inverse hyperbolic cosine of multivector with optional complexifier.
16116 */
16117
16118 static PyObject *__pyx_pw_8PyClical_63acosh(PyObject *__pyx_self, PyObject

```

```

    *__pyx_args, PyObject *__pyx_kwds); /*proto*/
16119 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_acosh(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_acosh *__pyx_optional_args) {
16120     PyObject *__pyx_v_i = (PyObject *)Py_None;
16121     PyObject *__pyx_r = NULL;
16122     __Pyx_RefNannyDeclarations
16123     int __pyx_t_1;
16124     int __pyx_t_2;
16125     PyObject *__pyx_t_3 = NULL;
16126     Clifford __pyx_t_4;
16127     PyObject *__pyx_t_5 = NULL;
16128     PyObject *__pyx_t_6 = NULL;
16129     PyObject *__pyx_t_7 = NULL;
16130     PyObject *__pyx_t_8 = NULL;
16131     PyObject *__pyx_t_9 = NULL;
16132     PyObject *__pyx_t_10 = NULL;
16133     PyObject *__pyx_t_11 = NULL;
16134     int __pyx_lineno = 0;
16135     const char *__pyx_filename = NULL;
16136     int __pyx_clineno = 0;
16137     __Pyx_RefNannySetupContext("acosh", 0);
16138     if (__pyx_optional_args) {
16139         if (__pyx_optional_args->__pyx_n > 0) {
16140             __pyx_v_i = __pyx_optional_args->i;
16141         }
16142     }
16143
16144     /* "PyClical.pyx":1720
16145     *
16146     * """
16147     * if not (i is None): # <<<<<<<<
16148     *     return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16149     * else:
16150     */
16151     __pyx_t_1 = (__pyx_v_i != Py_None);
16152     __pyx_t_2 = (__pyx_t_1 != 0);
16153     if (__pyx_t_2) {
16154
16155         /* "PyClical.pyx":1721
16156         * """
16157         * if not (i is None):
16158         *     return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) ) #
16159         * else:
16160         *     try:
16161         */
16162         __Pyx_XDECREF(__pyx_r);
16163         __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
*)__pyx_ptype_8PyClical_clifford); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1721, __pyx_L1_error)
16164         __Pyx_GOTREF(__pyx_t_3);
16165         try {
16166             __pyx_t_4 = acosh(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
16167         } catch (...) {
16168             __Pyx_CppExn2PyErr();
16169             __PYX_ERR(0, 1721, __pyx_L1_error)
16170         }
16171         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1721, __pyx_L1_error)
16172         __Pyx_GOTREF(__pyx_t_5);
16173         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16174         __pyx_r = __pyx_t_5;
16175         __pyx_t_5 = 0;
16176         goto __pyx_L0;
16177
16178         /* "PyClical.pyx":1720
16179         *
16180         * """
16181         * if not (i is None): # <<<<<<<<
16182         *     return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16183         * else:
16184         */
16185         }
16186
16187         /* "PyClical.pyx":1723
16188         *     return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16189         * else:
16190         *     try: # <<<<<<<<
16191         *         return math.acosh(obj)
16192         *     except:
16193         */
16194         /*else*/ {
16195             {
16196                 __Pyx_PyThreadState_declare
16197                 __Pyx_PyThreadState_assign
16198                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
16199                 __Pyx_XGOTREF(__pyx_t_6);

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16200         __Pyx_XGOTREF(__pyx_t_7);
16201         __Pyx_XGOTREF(__pyx_t_8);
16202         /*try:*/ {
16203
16204             /* "PyClical.pyx":1724
16205         *     else:
16206         *         try:
16207         *             return math.acosh(obj)                # ««««««««
16208         *         except:
16209         *             return clifford().wrap( glucat.acosh(toClifford(obj)) )
16210         */
16211
16212         __Pyx_XDECREF(__pyx_r);
16213         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
16214         (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1724, __pyx_L4_error)
16215         __Pyx_GOTREF(__pyx_t_3);
16216         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_acosh); if
16217         (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1724, __pyx_L4_error)
16218         __Pyx_GOTREF(__pyx_t_9);
16219         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16220         __pyx_t_3 = NULL;
16221         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
16222             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
16223             if (likely(__pyx_t_3)) {
16224                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
16225                 __Pyx_INCREF(__pyx_t_3);
16226                 __Pyx_INCREF(function);
16227                 __Pyx_DECREF_SET(__pyx_t_9, function);
16228             }
16229             __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
16230             __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
16231             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16232             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1724, __pyx_L4_error)
16233             __Pyx_GOTREF(__pyx_t_5);
16234             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16235             __pyx_r = __pyx_t_5;
16236             __pyx_t_5 = 0;
16237             goto __pyx_L8_try_return;
16238
16239             /* "PyClical.pyx":1723
16240         *     return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16241         *     else:
16242         *         try:
16243         *             # ««««««««
16244         *             return math.acosh(obj)
16245         *         except:
16246         *             return clifford().wrap( glucat.acosh(toClifford(obj)) )
16247         */
16248
16249         }
16250         __pyx_L4_error:;
16251         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16252         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
16253         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
16254
16255         /* "PyClical.pyx":1725
16256         *     try:
16257         *         return math.acosh(obj)
16258         *     except:
16259         *         # ««««««««
16260         *         return clifford().wrap( glucat.acosh(toClifford(obj)) )
16261         */
16262         /*except:*/ {
16263             __Pyx_AddTraceback("PyClical.acosh", __pyx_clineno, __pyx_lineno,
16264             __pyx_filename);
16265             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
16266             1725, __pyx_L6_except_error)
16267             __Pyx_GOTREF(__pyx_t_5);
16268             __Pyx_GOTREF(__pyx_t_9);
16269             __Pyx_GOTREF(__pyx_t_3);
16270
16271             /* "PyClical.pyx":1726
16272         *     return math.acosh(obj)
16273         *     except:
16274         *         return clifford().wrap( glucat.acosh(toClifford(obj)) )                # ««««««««
16275         *
16276         * cpdef inline sin(obj,i = None):
16277         */
16278
16279         __Pyx_XDECREF(__pyx_r);
16280         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
16281         *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1726,
16282         __pyx_L6_except_error)
16283         __Pyx_GOTREF(__pyx_t_10);
16284         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
16285         __pyx_obj_8PyClical_clifford *)__pyx_t_10), acosh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
16286         (unlikely(!__pyx_t_11)) __PYX_ERR(0, 1726, __pyx_L6_except_error)
16287         __Pyx_GOTREF(__pyx_t_11);
16288         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
16289         __pyx_r = __pyx_t_11;
16290         __pyx_t_11 = 0;

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```

16278         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16279         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16280         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16281         goto __pyx_L7_except_return;
16282     }
16283     __pyx_L6_except_error;;
16284
16285     /* "PyClical.pyx":1723
16286     *         return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16287     *     else:
16288     *         try:
16289     *             # ««««««««
16290     *             return math.acosh(obj)
16291     *         except:
16292     */
16293     __Pyx_XGIVEREF(__pyx_t_6);
16294     __Pyx_XGIVEREF(__pyx_t_7);
16295     __Pyx_XGIVEREF(__pyx_t_8);
16296     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16297     goto __pyx_L1_error;
16298     __pyx_L8_try_return;;
16299     __Pyx_XGIVEREF(__pyx_t_6);
16300     __Pyx_XGIVEREF(__pyx_t_7);
16301     __Pyx_XGIVEREF(__pyx_t_8);
16302     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16303     goto __pyx_L0;
16304     __pyx_L7_except_return;;
16305     __Pyx_XGIVEREF(__pyx_t_6);
16306     __Pyx_XGIVEREF(__pyx_t_7);
16307     __Pyx_XGIVEREF(__pyx_t_8);
16308     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16309     goto __pyx_L0;
16310 }
16311
16312 /* "PyClical.pyx":1705
16313 *         return clifford().wrap( glucat.cosh(toClifford(obj)) )
16314 *
16315 * cpdef inline acosh(obj,i = None):
16316 *     """
16317 *     Inverse hyperbolic cosine of multivector with optional complexifier.
16318 */
16319
16320 /* function exit code */
16321 __pyx_L1_error;;
16322 __Pyx_XDECREF(__pyx_t_3);
16323 __Pyx_XDECREF(__pyx_t_5);
16324 __Pyx_XDECREF(__pyx_t_9);
16325 __Pyx_XDECREF(__pyx_t_10);
16326 __Pyx_XDECREF(__pyx_t_11);
16327 __Pyx_AddTraceback("PyClical.acosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16328 __pyx_r = 0;
16329 __pyx_L0;;
16330 __Pyx_XGIVEREF(__pyx_r);
16331 __Pyx_RefNannyFinishContext();
16332 return __pyx_r;
16333 }
16334
16335 /* Python wrapper */
16336 static PyObject * __pyx_pw_8PyClical_63acosh(PyObject * __pyx_self, PyObject
16337 * __pyx_args, PyObject * __pyx_kwds); /*proto*/
16338 static char __pyx_doc_8PyClical_62acosh[] = "\n    Inverse hyperbolic cosine of
16339 multivector with optional complexifier.\n\n    >> print(acosh(0, \{-2,-1,1\}\n    1.571{-2,-1,1}\n
16340 >> x=clifford(\{1,2,3\}\n    print(cosh(acosh(x, \{-1,1,2,3,4\}\n    {1,2,3}\n    >>
16341 print(acosh(0))\n    1.571{-1}\n    >> x=clifford(\{1,2,3\}\n    print(cosh(acosh(x))\n    {1,2,3}\n
16342 >> x=clifford(\{1,2\}\n    print(cosh(acosh(x))\n    {1,2}\n    "
16343 static PyObject * __pyx_pw_8PyClical_63acosh(PyObject * __pyx_self, PyObject
16344 * __pyx_args, PyObject * __pyx_kwds) {
16345     PyObject * __pyx_v_obj = 0;
16346     PyObject * __pyx_v_i = 0;
16347     int __pyx_lineno = 0;
16348     const char * __pyx_filename = NULL;
16349     int __pyx_clineno = 0;
16350     PyObject * __pyx_r = 0;
16351     __Pyx_RefNannyDeclarations
16352     __Pyx_RefNannySetupContext("acosh (wrapper)", 0);
16353     {
16354         static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
16355         PyObject* values[2] = {0,0};
16356         values[1] = ((PyObject *)Py_None);
16357         if (unlikely(__pyx_kwds)) {
16358             Py_ssize_t kw_args;
16359             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
16360             switch (pos_args) {
16361                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16362                     CYTHON_FALLTHROUGH;
16363                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16364                     CYTHON_FALLTHROUGH;

```

```

16359         case 0: break;
16360         default: goto __pyx_L5_argtuple_error;
16361     }
16362     kw_args = PyDict_Size(__pyx_kwds);
16363     switch (pos_args) {
16364     case 0:
16365         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
16366         else goto __pyx_L5_argtuple_error;
16367         CYTHON_FALLTHROUGH;
16368     case 1:
16369         if (kw_args > 0) {
16370             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
16371             if (value) { values[1] = value; kw_args--; }
16372         }
16373     }
16374     if (unlikely(kw_args > 0)) {
16375         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "acosh") < 0)) __PYX_ERR(0, 1705, __pyx_L3_error)
16376     }
16377     } else {
16378         switch (PyTuple_GET_SIZE(__pyx_args)) {
16379         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16380             CYTHON_FALLTHROUGH;
16381         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16382             break;
16383         default: goto __pyx_L5_argtuple_error;
16384         }
16385     }
16386     __pyx_v_obj = values[0];
16387     __pyx_v_i = values[1];
16388 }
16389 goto __pyx_L4_argument_unpacking_done;
16390 __pyx_L5_argtuple_error:;
16391 __Pyx_RaiseArgtupleInvalid("acosh", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
16392 __PYX_ERR(0, 1705, __pyx_L3_error)
16393 __pyx_L3_error:;
16394 __Pyx_AddTraceback("PyClicl.acosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16395 __Pyx_RefNannyFinishContext();
16396 return NULL;
16397 __pyx_L4_argument_unpacking_done:;
16398 __pyx_r = __pyx_pf_8PyClicl_62acosh(__pyx_self, __pyx_v_obj, __pyx_v_i);
16399
16400 /* function exit code */
16401 __Pyx_RefNannyFinishContext();
16402 return __pyx_r;
16403 }
16404
16405 static PyObject * __pyx_pf_8PyClicl_62acosh(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj, PyObject * __pyx_v_i) {
16406     PyObject * __pyx_r = NULL;
16407     __Pyx_RefNannyDeclarations
16408     PyObject * __pyx_t_1 = NULL;
16409     struct __pyx_opt_args_8PyClicl_acosh __pyx_t_2;
16410     int __pyx_lineno = 0;
16411     const char * __pyx_filename = NULL;
16412     int __pyx_clineno = 0;
16413     __Pyx_RefNannySetupContext("acosh", 0);
16414     __Pyx_XDECREF(__pyx_r);
16415     __pyx_t_2.__pyx_n = 1;
16416     __pyx_t_2.i = __pyx_v_i;
16417     __pyx_t_1 = __pyx_f_8PyClicl_acosh(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1705, __pyx_L1_error)
16418     __Pyx_GOTREF(__pyx_t_1);
16419     __pyx_r = __pyx_t_1;
16420     __pyx_t_1 = 0;
16421     goto __pyx_L0;
16422
16423     /* function exit code */
16424     __pyx_L1_error:;
16425     __Pyx_XDECREF(__pyx_t_1);
16426     __Pyx_AddTraceback("PyClicl.acosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16427     __pyx_r = NULL;
16428     __pyx_L0:;
16429     __Pyx_XGIVEREF(__pyx_r);
16430     __Pyx_RefNannyFinishContext();
16431     return __pyx_r;
16432 }
16433
16434 /* "PyClicl.pyx":1728
16435 *
16436 * cpdef inline sin(obj,i = None): # ««««««
16437 *     """
16438 *     Sine of multivector with optional complexifier.
16439 */
16440

```



```

16441         static PyObject *__pyx_pw_8PyClical_65sin(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
16442         static CYTHON_INLINE PyObject *__pyx_f_8PyClical_sin(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_sin *__pyx_optional_args) {
16443             PyObject *__pyx_v_i = (PyObject *)Py_None;
16444             PyObject *__pyx_r = NULL;
16445             __Pyx_RefNannyDeclarations
16446             int __pyx_t_1;
16447             int __pyx_t_2;
16448             PyObject *__pyx_t_3 = NULL;
16449             Clifford __pyx_t_4;
16450             PyObject *__pyx_t_5 = NULL;
16451             PyObject *__pyx_t_6 = NULL;
16452             PyObject *__pyx_t_7 = NULL;
16453             PyObject *__pyx_t_8 = NULL;
16454             PyObject *__pyx_t_9 = NULL;
16455             PyObject *__pyx_t_10 = NULL;
16456             PyObject *__pyx_t_11 = NULL;
16457             int __pyx_lineno = 0;
16458             const char *__pyx_filename = NULL;
16459             int __pyx_clineno = 0;
16460             __Pyx_RefNannySetupContext("sin", 0);
16461             if (__pyx_optional_args) {
16462                 if (__pyx_optional_args->__pyx_n > 0) {
16463                     __pyx_v_i = __pyx_optional_args->i;
16464                 }
16465             }
16466
16467             /* "PyClical.pyx":1739
16468             *
16469             * {1,2,3}
16470             * if not (i is None): # ««««««««
16471             *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16472             * else:
16473             */
16474             __pyx_t_1 = (__pyx_v_i != Py_None);
16475             __pyx_t_2 = (__pyx_t_1 != 0);
16476             if (__pyx_t_2) {
16477
16478                 /* "PyClical.pyx":1740
16479                 *
16480                 * if not (i is None):
16481                 *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) ) # ««««««««
16482                 * else:
16483                 *     try:
16484                 */
16485                 __Pyx_XDECREF(__pyx_r);
16486                 __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
*)__pyx_ptype_8PyClical_clifford); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1740, __pyx_L1_error)
16487                 __Pyx_GOTREF(__pyx_t_3);
16488                 try {
16489                     __pyx_t_4 = sin(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
16490                 } catch (...) {
16491                     __Pyx_CppExn2PyErr();
16492                     __PYX_ERR(0, 1740, __pyx_L1_error)
16493                 }
16494                 __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1740, __pyx_L1_error)
16495                 __Pyx_GOTREF(__pyx_t_5);
16496                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16497                 __pyx_r = __pyx_t_5;
16498                 __pyx_t_5 = 0;
16499                 goto __pyx_L0;
16500
16501                 /* "PyClical.pyx":1739
16502                 *
16503                 * {1,2,3}
16504                 * if not (i is None): # ««««««««
16505                 *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16506                 * else:
16507                 */
16508                 }
16509
16510                 /* "PyClical.pyx":1742
16511                 *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16512                 * else:
16513                 *     try: # ««««««««
16514                 *         return math.sin(obj)
16515                 *     except:
16516                 */
16517                 /*else*/ {
16518                     {
16519                         __Pyx_PyThreadState_declare
16520                         __Pyx_PyThreadState_assign
16521                         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
16522                         __Pyx_XGOTREF(__pyx_t_6);

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16523         __Pyx_XGOTREF(__pyx_t_7);
16524         __Pyx_XGOTREF(__pyx_t_8);
16525         /*try:*/ {
16526
16527             /* "PyCliclcal.pyx":1743
16528         *
16529         *     try:
16530         *         return math.sin(obj)                # ««««««««
16531         *     except:
16532         *         return clifford().wrap( glucat.sin(toClifford(obj)) )
16533         */
16534
16535         __Pyx_XDECREF(__pyx_r);
16536         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
16537         (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1743, __pyx_L4_error)
16538         __Pyx_GOTREF(__pyx_t_3);
16539         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_sin); if
16540         (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1743, __pyx_L4_error)
16541         __Pyx_GOTREF(__pyx_t_9);
16542         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16543         __pyx_t_3 = NULL;
16544         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
16545             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
16546             if (likely(__pyx_t_3)) {
16547                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
16548                 __Pyx_INCREF(__pyx_t_3);
16549                 __Pyx_INCREF(function);
16550                 __Pyx_DECREF_SET(__pyx_t_9, function);
16551             }
16552             __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
16553             __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
16554             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16555             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1743, __pyx_L4_error)
16556             __Pyx_GOTREF(__pyx_t_5);
16557             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16558             __pyx_r = __pyx_t_5;
16559             __pyx_t_5 = 0;
16560             goto __pyx_L8_try_return;
16561
16562             /* "PyCliclcal.pyx":1742
16563         *
16564         *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16565         *
16566         *     else:
16567         *         try:
16568         *             # ««««««««
16569         *             return math.sin(obj)
16570         *         except:
16571         *             return clifford().wrap( glucat.sin(toClifford(obj)) )
16572         */
16573
16574         }
16575         __pyx_L4_error:;
16576         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16577         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
16578         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
16579
16580         /* "PyCliclcal.pyx":1744
16581         *
16582         *     try:
16583         *         return math.sin(obj)
16584         *     except:
16585         *         # ««««««««
16586         *         return clifford().wrap( glucat.sin(toClifford(obj)) )
16587         */
16588
16589         /*except:*/ {
16590             __Pyx_AddTraceback("PyCliclcal.sin", __pyx_clineno, __pyx_lineno,
16591             __pyx_filename);
16592             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
16593             1744, __pyx_L6_except_error)
16594             __Pyx_GOTREF(__pyx_t_5);
16595             __Pyx_GOTREF(__pyx_t_9);
16596             __Pyx_GOTREF(__pyx_t_3);
16597
16598             /* "PyCliclcal.pyx":1745
16599         *
16600         *     return math.sin(obj)
16601         *     except:
16602         *         return clifford().wrap( glucat.sin(toClifford(obj)) )
16603         *
16604         *     cpdef inline asin(obj,i = None):
16605         */
16606
16607             __Pyx_XDECREF(__pyx_r);
16608             __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
16609             *)__pyx_ptype_8PyCliclcal_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1745,
16610             __pyx_L6_except_error)
16611             __Pyx_GOTREF(__pyx_t_10);
16612             __pyx_t_11 = __pyx_f_8PyCliclcal_8clifford_wrap(((struct
16613             __pyx_obj_8PyCliclcal_clifford *)__pyx_t_10), sin(__pyx_f_8PyCliclcal_toClifford(__pyx_v_obj))); if
16614             (unlikely(!__pyx_t_11)) __PYX_ERR(0, 1745, __pyx_L6_except_error)
16615             __Pyx_GOTREF(__pyx_t_11);
16616             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
16617             __pyx_r = __pyx_t_11;
16618             __pyx_t_11 = 0;

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```

16601         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16602         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16603         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16604         goto __pyx_L7_except_return;
16605     }
16606     __pyx_L6_except_error;;
16607
16608     /* "PyClical.pyx":1742
16609     *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16610     * else:
16611     *     try: # ««««««««
16612     *         return math.sin(obj)
16613     *     except:
16614     */
16615         __Pyx_XGIVEREF(__pyx_t_6);
16616         __Pyx_XGIVEREF(__pyx_t_7);
16617         __Pyx_XGIVEREF(__pyx_t_8);
16618         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16619         goto __pyx_L1_error;
16620     __pyx_L8_try_return;;
16621     __Pyx_XGIVEREF(__pyx_t_6);
16622     __Pyx_XGIVEREF(__pyx_t_7);
16623     __Pyx_XGIVEREF(__pyx_t_8);
16624     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16625     goto __pyx_L0;
16626     __pyx_L7_except_return;;
16627     __Pyx_XGIVEREF(__pyx_t_6);
16628     __Pyx_XGIVEREF(__pyx_t_7);
16629     __Pyx_XGIVEREF(__pyx_t_8);
16630     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16631     goto __pyx_L0;
16632 }
16633 }
16634
16635     /* "PyClical.pyx":1728
16636     *     return clifford().wrap( glucat.acosh(toClifford(obj)) )
16637     *
16638     * cpdef inline sin(obj,i = None): # ««««««««
16639     *     """
16640     *     Sine of multivector with optional complexifier.
16641     */
16642
16643     /* function exit code */
16644     __pyx_L1_error;;
16645     __Pyx_XDECREF(__pyx_t_3);
16646     __Pyx_XDECREF(__pyx_t_5);
16647     __Pyx_XDECREF(__pyx_t_9);
16648     __Pyx_XDECREF(__pyx_t_10);
16649     __Pyx_XDECREF(__pyx_t_11);
16650     __Pyx_AddTraceback("PyClical.sin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16651     __pyx_r = 0;
16652     __pyx_L0;;
16653     __Pyx_XGIVEREF(__pyx_r);
16654     __Pyx_RefNannyFinishContext();
16655     return __pyx_r;
16656 }
16657
16658     /* Python wrapper */
16659     static PyObject *__pyx_pw_8PyClical_65sin(PyObject *__pyx_self, PyObject *__pyx_args,
16660     PyObject *__pyx_kwds); /*proto*/
16661     static char __pyx_doc_8PyClical_64sin[] = "\n    Sine of multivector with optional\n    complexifier.\n\n    >> s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n    {-1}\n    >>\ns=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),\"{-2,-1,1}\"))\n    {-1}\n    >>\nx=clifford(\"{1,2,3}\"); print(asin(sin(x)))\n    {1,2,3}\n    ";
16662     static PyObject *__pyx_pw_8PyClical_65sin(PyObject *__pyx_self, PyObject *__pyx_args,
16663     PyObject *__pyx_kwds) {
16664         PyObject *__pyx_v_obj = 0;
16665         PyObject *__pyx_v_i = 0;
16666         int __pyx_lineno = 0;
16667         const char *__pyx_filename = NULL;
16668         int __pyx_clineno = 0;
16669         PyObject *__pyx_r = 0;
16670         __Pyx_RefNannyDeclarations
16671         __Pyx_RefNannySetupContext("sin (wrapper)", 0);
16672         {
16673             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
16674             PyObject* values[2] = {0,0};
16675             values[1] = ((PyObject *)__Py_None);
16676             if (unlikely(__pyx_kwds)) {
16677                 Py_ssize_t kw_args;
16678                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
16679                 switch (pos_args) {
16680                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16681                     CYTHON_FALLTHROUGH;
16682                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16683                     CYTHON_FALLTHROUGH;
16684                     case 0: break;

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```

16683         default: goto __pyx_L5_argtuple_error;
16684     }
16685     kw_args = PyDict_Size(__pyx_kwds);
16686     switch (pos_args) {
16687     case 0:
16688         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
16689         else goto __pyx_L5_argtuple_error;
16690         CYTHON_FALLTHROUGH;
16691     case 1:
16692         if (kw_args > 0) {
16693             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
16694             if (value) { values[1] = value; kw_args--; }
16695         }
16696     }
16697     if (unlikely(kw_args > 0)) {
16698         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "sin") < 0)) __PYX_ERR(0, 1728, __pyx_L3_error)
16699     } else {
16700         switch (PyTuple_GET_SIZE(__pyx_args)) {
16701         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16702             CYTHON_FALLTHROUGH;
16703         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16704             break;
16705         default: goto __pyx_L5_argtuple_error;
16706         }
16707     }
16708     __pyx_v_obj = values[0];
16709     __pyx_v_i = values[1];
16710 }
16711 goto __pyx_L4_argument_unpacking_done;
16712 __pyx_L5_argtuple_error:;
16713 __Pyx_RaiseArgtupleInvalid("sin", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
16714 __PYX_ERR(0, 1728, __pyx_L3_error)
16715 __pyx_L3_error:;
16716 __Pyx_AddTraceback("PyClical.sin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16717 __Pyx_RefNannyFinishContext();
16718 return NULL;
16719 __pyx_L4_argument_unpacking_done:;
16720 __pyx_r = __pyx_pf_8PyClical_64sin(__pyx_self, __pyx_v_obj, __pyx_v_i);
16721
16722 /* function exit code */
16723 __Pyx_RefNannyFinishContext();
16724 return __pyx_r;
16725 }
16726
16727 static PyObject * __pyx_pf_8PyClical_64sin(CYTHON_UNUSED PyObject * __pyx_self, PyObject
* __pyx_v_obj, PyObject * __pyx_v_i) {
16728     PyObject * __pyx_r = NULL;
16729     __Pyx_RefNannyDeclarations
16730     PyObject * __pyx_t_1 = NULL;
16731     struct __pyx_opt_args_8PyClical_sin __pyx_t_2;
16732     int __pyx_lineno = 0;
16733     const char * __pyx_filename = NULL;
16734     int __pyx_clineno = 0;
16735     __Pyx_RefNannySetupContext("sin", 0);
16736     __Pyx_XDECREf(__pyx_r);
16737     __pyx_t_2.__pyx_n = 1;
16738     __pyx_t_2.i = __pyx_v_i;
16739     __pyx_t_1 = __pyx_pf_8PyClical_sin(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1728, __pyx_L1_error)
16740     __Pyx_GOTREF(__pyx_t_1);
16741     __pyx_r = __pyx_t_1;
16742     __pyx_t_1 = 0;
16743     goto __pyx_L0;
16744
16745     /* function exit code */
16746     __pyx_L1_error:;
16747     __Pyx_XDECREf(__pyx_t_1);
16748     __Pyx_AddTraceback("PyClical.sin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16749     __pyx_r = NULL;
16750     __pyx_L0:;
16751     __Pyx_XGIVEREF(__pyx_r);
16752     __Pyx_RefNannyFinishContext();
16753     return __pyx_r;
16754 }
16755
16756 /* "PyClical.pyx":1747
16757 *
16758 *
16759 * cpdef inline asin(obj, i = None): # ««««««««
16760 *     """
16761 *     Inverse sine of multivector with optional complexifier.
16762 */
16763
16764 static PyObject * __pyx_pw_8PyClical_67asin(PyObject * __pyx_self, PyObject * __pyx_args,

```

```

PyObject *__pyx_kwds); /*proto*/
16765     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_asin(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_asin *__pyx_optional_args) {
16766     PyObject *__pyx_v_i = ((PyObject *)Py_None);
16767     PyObject *__pyx_r = NULL;
16768     __Pyx_RefNannyDeclarations
16769     int __pyx_t_1;
16770     int __pyx_t_2;
16771     PyObject *__pyx_t_3 = NULL;
16772     Clifford __pyx_t_4;
16773     PyObject *__pyx_t_5 = NULL;
16774     PyObject *__pyx_t_6 = NULL;
16775     PyObject *__pyx_t_7 = NULL;
16776     PyObject *__pyx_t_8 = NULL;
16777     PyObject *__pyx_t_9 = NULL;
16778     PyObject *__pyx_t_10 = NULL;
16779     PyObject *__pyx_t_11 = NULL;
16780     int __pyx_lineno = 0;
16781     const char *__pyx_filename = NULL;
16782     int __pyx_clineno = 0;
16783     __Pyx_RefNannySetupContext("asin", 0);
16784     if (__pyx_optional_args) {
16785         if (__pyx_optional_args->__pyx_n > 0) {
16786             __pyx_v_i = __pyx_optional_args->i;
16787         }
16788     }
16789
16790     /* "PyClical.pyx":1760
16791     *     {1,2,3}
16792     *     """
16793     *     if not (i is None):           # ««««««««
16794     *         return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16795     *     else:
16796     */
16797     __pyx_t_1 = (__pyx_v_i != Py_None);
16798     __pyx_t_2 = (__pyx_t_1 != 0);
16799     if (__pyx_t_2) {
16800
16801         /* "PyClical.pyx":1761
16802     *     """
16803     *     if not (i is None):
16804     *         return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )           # ««««««««
16805     *     else:
16806     *         try:
16807     */
16808     __Pyx_XDECREF(__pyx_r);
16809     __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1761, __pyx_L1_error)
16810     __Pyx_GOTREF(__pyx_t_3);
16811     try {
16812         __pyx_t_4 = asin(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
16813     } catch (...) {
16814         __Pyx_CppExn2PyErr();
16815         __PYX_ERR(0, 1761, __pyx_L1_error)
16816     }
16817     __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1761, __pyx_L1_error)
16818     __Pyx_GOTREF(__pyx_t_5);
16819     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16820     __pyx_r = __pyx_t_5;
16821     __pyx_t_5 = 0;
16822     goto __pyx_L0;
16823
16824     /* "PyClical.pyx":1760
16825     *     {1,2,3}
16826     *     """
16827     *     if not (i is None):           # ««««««««
16828     *         return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16829     *     else:
16830     */
16831     }
16832
16833     /* "PyClical.pyx":1763
16834     *     return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16835     *     else:
16836     *         try:           # ««««««««
16837     *             return math.asin(obj)
16838     *         except:
16839     */
16840     /*else*/ {
16841     {
16842         __Pyx_PyThreadState_declare
16843         __Pyx_PyThreadState_assign
16844         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
16845         __Pyx_XGOTREF(__pyx_t_6);
16846         __Pyx_XGOTREF(__pyx_t_7);

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16847         __Pyx_XGOTREF(__pyx_t_8);
16848         /*try:*/ {
16849
16850             /* "PyCliclcal.pyx":1764
16851             *     else:
16852             *         try:
16853             *             return math.asin(obj)                # ««««««««
16854             *         except:
16855             *             return clifford().wrap( glucat.asin(toClifford(obj)) )
16856             */
16857             __Pyx_XDECREF(__pyx_r);
16858             __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1764, __pyx_L4_error)
16859             __Pyx_GOTREF(__pyx_t_3);
16860             __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_asin); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1764, __pyx_L4_error)
16861             __Pyx_GOTREF(__pyx_t_9);
16862             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16863             __pyx_t_3 = NULL;
16864             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
16865                 __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
16866                 if (likely(__pyx_t_3)) {
16867                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
16868                     __Pyx_INCREF(__pyx_t_3);
16869                     __Pyx_INCREF(function);
16870                     __Pyx_DECREF_SET(__pyx_t_9, function);
16871                 }
16872             }
16873             __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
16874             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16875             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1764, __pyx_L4_error)
16876             __Pyx_GOTREF(__pyx_t_5);
16877             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16878             __pyx_r = __pyx_t_5;
16879             __pyx_t_5 = 0;
16880             goto __pyx_L8_try_return;
16881
16882             /* "PyCliclcal.pyx":1763
16883             *     return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16884             *     else:
16885             *         try:
16886             *             return math.asin(obj)
16887             *         except:
16888             */
16889             }
16890             __pyx_L4_error:;
16891             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16892             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
16893             __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
16894
16895             /* "PyCliclcal.pyx":1765
16896             *     try:
16897             *         return math.asin(obj)
16898             *     except:
16899             *         return clifford().wrap( glucat.asin(toClifford(obj)) )
16900             *
16901             */
16902             /*except:*/ {
16903                 __Pyx_AddTraceback("PyCliclcal.asin", __pyx_clineno, __pyx_lineno,
__pyx_filename);
16904                 if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1765, __pyx_L6_except_error)
16905                 __Pyx_GOTREF(__pyx_t_5);
16906                 __Pyx_GOTREF(__pyx_t_9);
16907                 __Pyx_GOTREF(__pyx_t_3);
16908
16909                 /* "PyCliclcal.pyx":1766
16910                 *     return math.asin(obj)
16911                 *     except:
16912                 *         return clifford().wrap( glucat.asin(toClifford(obj)) )
16913                 *
16914                 * cpdef inline sinh(obj):
16915                 */
16916                 __Pyx_XDECREF(__pyx_r);
16917                 __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyCliclcal_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1766,
__pyx_L6_except_error)
16918                 __Pyx_GOTREF(__pyx_t_10);
16919                 __pyx_t_11 = __pyx_f_8PyCliclcal_8clifford_wrap(((struct
__pyx_obj_8PyCliclcal_clifford *)__pyx_t_10), asin(__pyx_f_8PyCliclcal_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1766, __pyx_L6_except_error)
16920                 __Pyx_GOTREF(__pyx_t_11);
16921                 __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
16922                 __pyx_r = __pyx_t_11;
16923                 __pyx_t_11 = 0;
16924                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;

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16925         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16926         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16927         goto __pyx_L7_except_return;
16928     }
16929     __pyx_L6_except_error;;
16930
16931     /* "PyClical.pyx":1763
16932     * return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16933     * else:
16934     *     try:
16935     *         # ««««««««
16936     *         return math.asin(obj)
16937     *     except:
16938     */
16939     __Pyx_XGIVEREF(__pyx_t_6);
16940     __Pyx_XGIVEREF(__pyx_t_7);
16941     __Pyx_XGIVEREF(__pyx_t_8);
16942     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16943     goto __pyx_L1_error;
16944     __pyx_L8_try_return;;
16945     __Pyx_XGIVEREF(__pyx_t_6);
16946     __Pyx_XGIVEREF(__pyx_t_7);
16947     __Pyx_XGIVEREF(__pyx_t_8);
16948     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16949     goto __pyx_L0;
16950     __pyx_L7_except_return;;
16951     __Pyx_XGIVEREF(__pyx_t_6);
16952     __Pyx_XGIVEREF(__pyx_t_7);
16953     __Pyx_XGIVEREF(__pyx_t_8);
16954     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16955     goto __pyx_L0;
16956     }
16957 }
16958
16959 /* "PyClical.pyx":1747
16960 * return clifford().wrap( glucat.sin(toClifford(obj)) )
16961 *
16962 * cpdef inline asin(obj,i = None):
16963 *     # ««««««««
16964 *     """
16965 *     Inverse sine of multivector with optional complexifier.
16966 *
16967     /* function exit code */
16968     __pyx_L1_error;;
16969     __Pyx_XDECREF(__pyx_t_3);
16970     __Pyx_XDECREF(__pyx_t_5);
16971     __Pyx_XDECREF(__pyx_t_9);
16972     __Pyx_XDECREF(__pyx_t_10);
16973     __Pyx_XDECREF(__pyx_t_11);
16974     __Pyx_AddTraceback("PyClical.asin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16975     __pyx_r = 0;
16976     __pyx_L0;;
16977     __Pyx_XGIVEREF(__pyx_r);
16978     __Pyx_RefNannyFinishContext();
16979     return __pyx_r;
16980 }
16981
16982 /* Python wrapper */
16983 static PyObject* __pyx_pw_8PyClical_67asin(PyObject* __pyx_self, PyObject* __pyx_args,
16984 PyObject* __pyx_kwds); /*proto*/
16985 static char __pyx_doc_8PyClical_66asin[] = "\n    Inverse sine of multivector with\n    optional complexifier.\n\n    >> s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n    {-1}\n    >>\n    s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),\"{-2,-1,1}\"))\n    {-1}\n    >> print(asin(1) / pi)\n    0.5\n    >> x=clifford(\"{1,2,3}\"); print(asin(sin(x)))\n    {1,2,3}\n    ";
16986 static PyObject* __pyx_pw_8PyClical_67asin(PyObject* __pyx_self, PyObject* __pyx_args,
16987 PyObject* __pyx_kwds) {
16988     PyObject* __pyx_v_obj = 0;
16989     PyObject* __pyx_v_i = 0;
16990     int __pyx_lineno = 0;
16991     const char* __pyx_filename = NULL;
16992     int __pyx_clineno = 0;
16993     PyObject* __pyx_r = 0;
16994     __Pyx_RefNannyDeclarations
16995     __Pyx_RefNannySetupContext("asin (wrapper)", 0);
16996     {
16997         static PyObject** __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
16998         PyObject* values[2] = {0,0};
16999         values[1] = ((PyObject*)Py_None);
17000         if (unlikely(__pyx_kwds)) {
17001             Py_ssize_t kw_args;
17002             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
17003             switch (pos_args) {
17004                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17005                     CYTHON_FALLTHROUGH;
17006                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17007                     CYTHON_FALLTHROUGH;
17008                 case 0: break;
17009                 default: goto __pyx_L5_argtuple_error;
17010             }
17011         }
17012     }

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```

17007         }
17008         kw_args = PyDict_Size(__pyx_kwds);
17009         switch (pos_args) {
17010             case 0:
17011                 if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
17012                 else goto __pyx_L5_argtuple_error;
17013                 CYTHON_FALLTHROUGH;
17014             case 1:
17015                 if (kw_args > 0) {
17016                     PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
17017                     if (value) { values[1] = value; kw_args--; }
17018                 }
17019             if (unlikely(kw_args > 0)) {
17020                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "asin") < 0)) __PYX_ERR(0, 1747, __pyx_L3_error)
17021             }
17022         } else {
17023             switch (PyTuple_GET_SIZE(__pyx_args)) {
17024                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17025                 CYTHON_FALLTHROUGH;
17026                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17027                 break;
17028                 default: goto __pyx_L5_argtuple_error;
17029             }
17030         }
17031         __pyx_v_obj = values[0];
17032         __pyx_v_i = values[1];
17033     }
17034     goto __pyx_L4_argument_unpacking_done;
17035     __pyx_L5_argtuple_error:;
17036     __Pyx_RaiseArgtupleInvalid("asin", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
17037     __PYX_ERR(0, 1747, __pyx_L3_error)
17038     __pyx_L3_error:;
17039     __Pyx_AddTraceback("PyClical.asin", __pyx_clineno, __pyx_lineno, __pyx_filename);
17040     __Pyx_RefNannyFinishContext();
17041     return NULL;
17042     __pyx_L4_argument_unpacking_done:;
17043     __pyx_r = __pyx_pf_8PyClical_66asin(__pyx_self, __pyx_v_obj, __pyx_v_i);
17044
17045     /* function exit code */
17046     __Pyx_RefNannyFinishContext();
17047     return __pyx_r;
17048 }
17049
17050 static PyObject *__pyx_pf_8PyClical_66asin(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
17051     PyObject *__pyx_r = NULL;
17052     __Pyx_RefNannyDeclarations
17053     PyObject *__pyx_t_1 = NULL;
17054     struct __pyx_opt_args_8PyClical_asin __pyx_t_2;
17055     int __pyx_lineno = 0;
17056     const char *__pyx_filename = NULL;
17057     int __pyx_clineno = 0;
17058     __Pyx_RefNannySetupContext("asin", 0);
17059     __Pyx_XDECREF(__pyx_r);
17060     __pyx_t_2.__pyx_n = 1;
17061     __pyx_t_2.i = __pyx_v_i;
17062     __pyx_t_1 = __pyx_f_8PyClical_asin(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1747, __pyx_L1_error)
17063     __Pyx_GOTREF(__pyx_t_1);
17064     __pyx_r = __pyx_t_1;
17065     __pyx_t_1 = 0;
17066     goto __pyx_L0;
17067
17068     /* function exit code */
17069     __pyx_L1_error:;
17070     __Pyx_XDECREF(__pyx_t_1);
17071     __Pyx_AddTraceback("PyClical.asin", __pyx_clineno, __pyx_lineno, __pyx_filename);
17072     __pyx_r = NULL;
17073     __pyx_L0:;
17074     __Pyx_XGIVEREF(__pyx_r);
17075     __Pyx_RefNannyFinishContext();
17076     return __pyx_r;
17077 }
17078
17079 /* "PyClical.pyx":1768
17080 *         return clifford().wrap( glucat.asin(toClifford(obj)) )
17081 *
17082 * cpdef inline sinh(obj):
17083 *     """
17084 *     Hyperbolic sine of multivector.
17085 */
17086
17087 static PyObject *__pyx_pw_8PyClical_69sinh(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/

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17088         static CYTHON_INLINE PyObject* __pyx_f_8PyClical_sinh(PyObject* __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
17089             PyObject* __pyx_r = NULL;
17090             __Pyx_RefNannyDeclarations
17091             PyObject* __pyx_t_1 = NULL;
17092             PyObject* __pyx_t_2 = NULL;
17093             PyObject* __pyx_t_3 = NULL;
17094             PyObject* __pyx_t_4 = NULL;
17095             PyObject* __pyx_t_5 = NULL;
17096             PyObject* __pyx_t_6 = NULL;
17097             PyObject* __pyx_t_7 = NULL;
17098             PyObject* __pyx_t_8 = NULL;
17099             int __pyx_lineno = 0;
17100             const char* __pyx_filename = NULL;
17101             int __pyx_clineno = 0;
17102             __Pyx_RefNannySetupContext("sinh", 0);
17103
17104             /* "PyClical.pyx":1777
17105             *      0.5{1,2}
17106             *      """
17107             *      try:
17108             *          return math.sinh(obj)
17109             *      except:
17110             */
17111             {
17112                 __Pyx_PyThreadState_declare
17113                 __Pyx_PyThreadState_assign
17114                 __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
17115                 __Pyx_XGOTREF(__pyx_t_1);
17116                 __Pyx_XGOTREF(__pyx_t_2);
17117                 __Pyx_XGOTREF(__pyx_t_3);
17118                 /*try:*/ {
17119
17120                     /* "PyClical.pyx":1778
17121                     *      """
17122                     *      try:
17123                     *          return math.sinh(obj)
17124                     *      except:
17125                     *          return clifford().wrap( glucat.sinh(toClifford(obj)) )
17126                     */
17127                     __Pyx_XDECREF(__pyx_r);
17128                     __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 1778, __pyx_L3_error)
17129                     __Pyx_GOTREF(__pyx_t_5);
17130                     __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_sinh); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 1778, __pyx_L3_error)
17131                     __Pyx_GOTREF(__pyx_t_6);
17132                     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17133                     __pyx_t_5 = NULL;
17134                     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
17135                         __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
17136                         if (likely(__pyx_t_5)) {
17137                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
17138                             __Pyx_INCREF(__pyx_t_5);
17139                             __Pyx_INCREF(function);
17140                             __Pyx_DECREF_SET(__pyx_t_6, function);
17141                         }
17142                     }
17143                     __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
17144                     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17145                     if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1778, __pyx_L3_error)
17146                     __Pyx_GOTREF(__pyx_t_4);
17147                     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
17148                     __pyx_r = __pyx_t_4;
17149                     __pyx_t_4 = 0;
17150                     goto __pyx_L7_try_return;
17151
17152                     /* "PyClical.pyx":1777
17153                     *      0.5{1,2}
17154                     *      """
17155                     *      try:
17156                     *          return math.sinh(obj)
17157                     *      except:
17158                     */
17159                     }
17160                     __pyx_L3_error;;
17161                     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
17162                     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17163                     __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
17164
17165                     /* "PyClical.pyx":1779
17166                     *      try:
17167                     *          return math.sinh(obj)
17168                     *      except:
17169                     *          return clifford().wrap( glucat.sinh(toClifford(obj)) )
17170                     */

```

```

17171 */
17172                                     /*except:*/ {
17173     __Pyx_AddTraceback("PyClical.sinh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
17174     if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
17175 1779, __pyx_L5_except_error)
17176     __Pyx_GOTREF(__pyx_t_4);
17177     __Pyx_GOTREF(__pyx_t_6);
17178     __Pyx_GOTREF(__pyx_t_5);
17179     /* "PyClical.pyx":1780
17180 *         return math.sinh(obj)
17181 *     except:
17182 *         return clifford().wrap( glucat.sinh(toClifford(obj)) )           # ««««««««
17183 *
17184 * cpdef inline asinh(obj,i = None):
17185 */
17186     __Pyx_XDECREF(__pyx_r);
17187     __pyx_t_7 = __Pyx_PyObject_CallNoArg((PyObject
*)__pyx_ptype_8PyClical_clifford); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1780,
__pyx_L5_except_error)
17188     __Pyx_GOTREF(__pyx_t_7);
17189     __pyx_t_8 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_7), sinh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1780, __pyx_L5_except_error)
17190     __Pyx_GOTREF(__pyx_t_8);
17191     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
17192     __pyx_r = __pyx_t_8;
17193     __pyx_t_8 = 0;
17194     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
17195     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17196     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
17197     goto __pyx_L6_except_return;
17198 }
17199 __pyx_L5_except_error:;
17200
17201     /* "PyClical.pyx":1777
17202 *         0.5{1,2}
17203 *         """
17204 *         try:           # ««««««««
17205 *             return math.sinh(obj)
17206 *         except:
17207 */
17208     __Pyx_XGIVEREF(__pyx_t_1);
17209     __Pyx_XGIVEREF(__pyx_t_2);
17210     __Pyx_XGIVEREF(__pyx_t_3);
17211     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
17212     goto __pyx_L1_error;
17213     __pyx_L7_try_return:;
17214     __Pyx_XGIVEREF(__pyx_t_1);
17215     __Pyx_XGIVEREF(__pyx_t_2);
17216     __Pyx_XGIVEREF(__pyx_t_3);
17217     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
17218     goto __pyx_L0;
17219     __pyx_L6_except_return:;
17220     __Pyx_XGIVEREF(__pyx_t_1);
17221     __Pyx_XGIVEREF(__pyx_t_2);
17222     __Pyx_XGIVEREF(__pyx_t_3);
17223     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
17224     goto __pyx_L0;
17225 }
17226
17227     /* "PyClical.pyx":1768
17228 *         return clifford().wrap( glucat.asin(toClifford(obj)) )
17229 *
17230 * cpdef inline sinh(obj):           # ««««««««
17231 *         """
17232 *         Hyperbolic sine of multivector.
17233 */
17234
17235     /* function exit code */
17236     __pyx_L1_error:;
17237     __Pyx_XDECREF(__pyx_t_4);
17238     __Pyx_XDECREF(__pyx_t_5);
17239     __Pyx_XDECREF(__pyx_t_6);
17240     __Pyx_XDECREF(__pyx_t_7);
17241     __Pyx_XDECREF(__pyx_t_8);
17242     __Pyx_AddTraceback("PyClical.sinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17243     __pyx_r = 0;
17244     __pyx_L0:;
17245     __Pyx_XGIVEREF(__pyx_r);
17246     __Pyx_RefNannyFinishContext();
17247     return __pyx_r;
17248 }
17249
17250 /* Python wrapper */
17251 static PyObject *__pyx_pw_8PyClical_69sinh(PyObject *__pyx_self, PyObject

```

```

    *__pyx_v_obj); /*proto*/
17252     static char __pyx_doc_8PyClical_68sinh[] = "\n    Hyperbolic sine of multivector.\n\n
    >> x=clifford('{1,2}') * pi/2; print(sinh(x))\n    {1,2}\n    >> x=clifford('{1,2}') * pi/6;
    print(sinh(x))\n    0.5{1,2}\n    ";
17253     static PyObject *__pyx_pw_8PyClical_69sinh(PyObject *__pyx_self, PyObject
    *__pyx_v_obj) {
17254         PyObject *__pyx_r = 0;
17255         __Pyx_RefNannyDeclarations
17256         __Pyx_RefNannySetupContext("sinh (wrapper)", 0);
17257         __pyx_r = __pyx_pf_8PyClical_68sinh(__pyx_self, ((PyObject *)__pyx_v_obj));
17258
17259         /* function exit code */
17260         __Pyx_RefNannyFinishContext();
17261         return __pyx_r;
17262     }
17263
17264     static PyObject *__pyx_pf_8PyClical_68sinh(CYTHON_UNUSED PyObject *__pyx_self,
    PyObject *__pyx_v_obj) {
17265         PyObject *__pyx_r = NULL;
17266         __Pyx_RefNannyDeclarations
17267         PyObject *__pyx_t_1 = NULL;
17268         int __pyx_lineno = 0;
17269         const char *__pyx_filename = NULL;
17270         int __pyx_clineno = 0;
17271         __Pyx_RefNannySetupContext("sinh", 0);
17272         __Pyx_XDECREF(__pyx_r);
17273         __pyx_t_1 = __pyx_f_8PyClical_sinh(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
    __PYX_ERR(0, 1768, __pyx_L1_error)
17274         __Pyx_GOTREF(__pyx_t_1);
17275         __pyx_r = __pyx_t_1;
17276         __pyx_t_1 = 0;
17277         goto __pyx_L0;
17278
17279         /* function exit code */
17280         __pyx_L1_error:;
17281         __Pyx_XDECREF(__pyx_t_1);
17282         __Pyx_AddTraceback("PyClical.sinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17283         __pyx_r = NULL;
17284         __pyx_L0:;
17285         __Pyx_XGIVEREF(__pyx_r);
17286         __Pyx_RefNannyFinishContext();
17287         return __pyx_r;
17288     }
17289
17290     /* "PyClical.pyx":1782
17291     *         return clifford().wrap( glucat.sinh(toClifford(obj)) )
17292     *
17293     * cpdef inline asinh(obj,i = None):          # <<<<<<<<<
17294     *     """
17295     *     Inverse hyperbolic sine of multivector with optional complexifier.
17296     */
17297
17298     static PyObject *__pyx_pw_8PyClical_71asinh(PyObject *__pyx_self, PyObject
    *__pyx_args, PyObject *__pyx_kwds); /*proto*/
17299     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_asinh(PyObject *__pyx_v_obj,
    CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_asinh *__pyx_optional_args) {
17300         PyObject *__pyx_v_i = ((PyObject *)Py_None);
17301         PyObject *__pyx_r = NULL;
17302         __Pyx_RefNannyDeclarations
17303         int __pyx_t_1;
17304         int __pyx_t_2;
17305         PyObject *__pyx_t_3 = NULL;
17306         Clifford __pyx_t_4;
17307         PyObject *__pyx_t_5 = NULL;
17308         PyObject *__pyx_t_6 = NULL;
17309         PyObject *__pyx_t_7 = NULL;
17310         PyObject *__pyx_t_8 = NULL;
17311         PyObject *__pyx_t_9 = NULL;
17312         PyObject *__pyx_t_10 = NULL;
17313         PyObject *__pyx_t_11 = NULL;
17314         int __pyx_lineno = 0;
17315         const char *__pyx_filename = NULL;
17316         int __pyx_clineno = 0;
17317         __Pyx_RefNannySetupContext("asinh", 0);
17318         if (__pyx_optional_args) {
17319             if (__pyx_optional_args->__pyx_n > 0) {
17320                 __pyx_v_i = __pyx_optional_args->i;
17321             }
17322         }
17323
17324         /* "PyClical.pyx":1793
17325     *     {1,2}
17326     *     """
17327     *     if not (i is None):          # <<<<<<<<<
17328     *         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17329     *     else:
17330     */

```

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17331         __pyx_t_1 = (__pyx_v_i != Py_None);
17332         __pyx_t_2 = (__pyx_t_1 != 0);
17333         if (__pyx_t_2) {
17334
17335             /* "PyCliclal.pyx":1794
17336             *
17337             *     if not (i is None):
17338             *         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17339             *
17340             *     else:
17341             *         try:
17342             *
17343             *             __Pyx_XDECREF(__pyx_r);
17344             *             __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
17345             *)__pyx_ptype_8PyCliclal_clifford); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1794, __pyx_L1_error)
17346             *             __Pyx_GOTREF(__pyx_t_3);
17347             *             try {
17348             *                 __pyx_t_4 = asinh(__pyx_f_8PyCliclal_toClifford(__pyx_v_obj),
17349             *                 __pyx_f_8PyCliclal_toClifford(__pyx_v_i));
17350             *             } catch (...) {
17351             *                 __Pyx_CppExn2PyErr();
17352             *                 __PYX_ERR(0, 1794, __pyx_L1_error)
17353             *             }
17354             *             __pyx_t_5 = __pyx_f_8PyCliclal_8clifford_wrap(((struct __pyx_obj_8PyCliclal_clifford
17355             *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1794, __pyx_L1_error)
17356             *             __Pyx_GOTREF(__pyx_t_5);
17357             *             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17358             *             __pyx_r = __pyx_t_5;
17359             *             __pyx_t_5 = 0;
17360             *             goto __pyx_L0;
17361             *
17362             *         }
17363             *
17364             *     }
17365             *
17366             *         /* "PyCliclal.pyx":1796
17367             *         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17368             *         else:
17369             *             try:
17370             *                 # ««««««««
17371             *                 return math.asinh(obj)
17372             *             except:
17373             *
17374             *                 /*else*/ {
17375             *                     {
17376             *                         __Pyx_PyThreadState_declare
17377             *                         __Pyx_PyThreadState_assign
17378             *                         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
17379             *                         __Pyx_XGOTREF(__pyx_t_6);
17380             *                         __Pyx_XGOTREF(__pyx_t_7);
17381             *                         __Pyx_XGOTREF(__pyx_t_8);
17382             *                     /*try:*/ {
17383             *
17384             *                         /* "PyCliclal.pyx":1797
17385             *                         else:
17386             *                             try:
17387             *                                 return math.asinh(obj)
17388             *                                 # ««««««««
17389             *                             except:
17390             *                                 return clifford().wrap( glucat.asinh(toClifford(obj)) )
17391             *
17392             *                             __Pyx_XDECREF(__pyx_r);
17393             *                             __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
17394             (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1797, __pyx_L4_error)
17395             *                             __Pyx_GOTREF(__pyx_t_3);
17396             *                             __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_asinh); if
17397             (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1797, __pyx_L4_error)
17398             *                             __Pyx_GOTREF(__pyx_t_9);
17399             *                             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17400             *                             __pyx_t_3 = NULL;
17401             *                             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
17402             *                                 __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
17403             *                                 if (likely(__pyx_t_3)) {
17404             *                                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
17405             *                                     __Pyx_INCREF(__pyx_t_3);
17406             *                                     __Pyx_INCREF(function);
17407             *                                     __Pyx_DECREF_SET(__pyx_t_9, function);
17408             *                                 }
17409             *                             }
17410             *                             __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
17411             __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
17412             *                             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17413             *                             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1797, __pyx_L4_error)
17414             *                             __Pyx_GOTREF(__pyx_t_5);

```

```

17411         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17412         __pyx_r = __pyx_t_5;
17413         __pyx_t_5 = 0;
17414         goto __pyx_L8_try_return;
17415
17416         /* "PyClical.pyx":1796
17417  *         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17418  *     else:
17419  *         try:
17420  *             # ««««««««
17421  *             return math.asinh(obj)
17422  *         except:
17423  */
17424         }
17425         __pyx_L4_error:;
17426         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17427         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17428         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
17429         /* "PyClical.pyx":1798
17430  *     try:
17431  *         return math.asinh(obj)
17432  *     except:
17433  *         # ««««««««
17434  *         return clifford().wrap( glucat.asinh(toClifford(obj)) )
17435  */
17436         /*except:*/ {
17437         __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
17438         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1798, __pyx_L6_except_error)
17439         __Pyx_GOTREF(__pyx_t_5);
17440         __Pyx_GOTREF(__pyx_t_9);
17441         __Pyx_GOTREF(__pyx_t_3);
17442
17443         /* "PyClical.pyx":1799
17444  *         return math.asinh(obj)
17445  *     except:
17446  *         return clifford().wrap( glucat.asinh(toClifford(obj)) )
17447  *         # ««««««««
17448  * cpdef inline tan(obj,i = None):
17449  */
17450         __Pyx_XDECREF(__pyx_r);
17451         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1799,
__pyx_L6_except_error)
17452         __Pyx_GOTREF(__pyx_t_10);
17453         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), asinh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1799, __pyx_L6_except_error)
17454         __Pyx_GOTREF(__pyx_t_11);
17455         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
17456         __pyx_r = __pyx_t_11;
17457         __pyx_t_11 = 0;
17458         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17459         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17460         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17461         goto __pyx_L7_except_return;
17462     }
17463     __pyx_L6_except_error:;
17464
17465     /* "PyClical.pyx":1796
17466  *     return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17467  * else:
17468  *     try:
17469  *         # ««««««««
17470  *         return math.asinh(obj)
17471  *     except:
17472  */
17472         __Pyx_XGIVEREF(__pyx_t_6);
17473         __Pyx_XGIVEREF(__pyx_t_7);
17474         __Pyx_XGIVEREF(__pyx_t_8);
17475         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17476         goto __pyx_L1_error;
17477         __pyx_L8_try_return:;
17478         __Pyx_XGIVEREF(__pyx_t_6);
17479         __Pyx_XGIVEREF(__pyx_t_7);
17480         __Pyx_XGIVEREF(__pyx_t_8);
17481         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17482         goto __pyx_L0;
17483         __pyx_L7_except_return:;
17484         __Pyx_XGIVEREF(__pyx_t_6);
17485         __Pyx_XGIVEREF(__pyx_t_7);
17486         __Pyx_XGIVEREF(__pyx_t_8);
17487         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17488         goto __pyx_L0;
17489     }
17490 }
17491

```

```

17492          /* "PyClicl.pyx":1782
17493          *      return clifford().wrap( glucat.sinh(toClifford(obj)) )
17494          *
17495          * cpdef inline asinh(obj,i = None):          # ««««««««
17496          *      """
17497          *      Inverse hyperbolic sine of multivector with optional complexifier.
17498          */
17499
17500          /* function exit code */
17501          __pyx_L1_error++;
17502          __Pyx_XDECREF(__pyx_t_3);
17503          __Pyx_XDECREF(__pyx_t_5);
17504          __Pyx_XDECREF(__pyx_t_9);
17505          __Pyx_XDECREF(__pyx_t_10);
17506          __Pyx_XDECREF(__pyx_t_11);
17507          __Pyx_AddTraceback("PyClicl.asinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17508          __pyx_r = 0;
17509          __pyx_L0:;
17510          __Pyx_XGIVEREF(__pyx_r);
17511          __Pyx_RefNannyFinishContext();
17512          return __pyx_r;
17513      }
17514
17515      /* Python wrapper */
17516      static PyObject *__pyx_pw_8PyClicl_7lasinh(PyObject *__pyx_self, PyObject
17517 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
17518      static char __pyx_doc_8PyClicl_70asinh[] = "\n      Inverse hyperbolic sine of
17519 multivector with optional complexifier.\n\n      >> x=clifford(\"{1,2}\"); print(asinh(x,\"{1,2,3}\") *
2/pi)\n      {1,2}\n      >> x=clifford(\"{1,2}\"); print(asinh(x) * 2/pi)\n      {1,2}\n      >>
x=clifford(\"{1,2}\") / 2; print(asinh(x) * 6/pi)\n      {1,2}\n      ";
17518      static PyObject *__pyx_pw_8PyClicl_7lasinh(PyObject *__pyx_self, PyObject
17519 *__pyx_args, PyObject *__pyx_kwds) {
17520          PyObject *__pyx_v_obj = 0;
17521          PyObject *__pyx_v_i = 0;
17522          int __pyx_lineno = 0;
17523          const char *__pyx_filename = NULL;
17524          int __pyx_clineno = 0;
17525          PyObject *__pyx_r = 0;
17526          __Pyx_RefNannyDeclarations
17527          __Pyx_RefNannySetupContext("asinh (wrapper)", 0);
17528          {
17529              static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
17530              PyObject* values[2] = {0,0};
17531              values[1] = ((PyObject *)Py_None);
17532              if (unlikely(__pyx_kwds)) {
17533                  Py_ssize_t kw_args;
17534                  const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
17535                  switch (pos_args) {
17536                      case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17537                      CYTHON_FALLTHROUGH;
17538                      case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17539                      CYTHON_FALLTHROUGH;
17540                      case 0: break;
17541                      default: goto __pyx_L5_argtuple_error;
17542                  }
17543                  kw_args = PyDict_Size(__pyx_kwds);
17544                  switch (pos_args) {
17545                      case 0:
17546                      if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
17547 0)) kw_args--;
17548                      else goto __pyx_L5_argtuple_error;
17549                      CYTHON_FALLTHROUGH;
17550                      case 1:
17551                      if (kw_args > 0) {
17552                          PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
17553                          if (value) { values[1] = value; kw_args--; }
17554                      }
17555                      if (unlikely(kw_args > 0)) {
17556                          if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
17557 values, pos_args, "asinh") < 0)) __PYX_ERR(0, 1782, __pyx_L3_error)
17558                      }
17559                      } else {
17560                          switch (PyTuple_GET_SIZE(__pyx_args)) {
17561                              case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17562                              CYTHON_FALLTHROUGH;
17563                              case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17564                              break;
17565                              default: goto __pyx_L5_argtuple_error;
17566                          }
17567                      }
17568                  __pyx_v_obj = values[0];
17569                  __pyx_v_i = values[1];
17570              }
17571              goto __pyx_L4_argument_unpacking_done;
17572          __pyx_L5_argtuple_error:;
17573          __Pyx_RaiseArgtupleInvalid("asinh", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));

```

```

17572 __PYX_ERR(0, 1782, __pyx_L3_error)
17573     __pyx_L3_error;
17574     __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17575     __Pyx_RefNannyFinishContext();
17576     return NULL;
17577     __pyx_L4_argument_unpacking_done:
17578     __pyx_r = __pyx_pf_8PyClical_70asinh(__pyx_self, __pyx_v_obj, __pyx_v_i);
17579
17580     /* function exit code */
17581     __Pyx_RefNannyFinishContext();
17582     return __pyx_r;
17583 }
17584
17585 static PyObject *__pyx_pf_8PyClical_70asinh(CYTHON_UNUSED PyObject *__pyx_self,
17586 PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
17587     PyObject *__pyx_r = NULL;
17588     __Pyx_RefNannyDeclarations
17589     PyObject *__pyx_t_1 = NULL;
17590     struct __pyx_opt_args_8PyClical_asinh __pyx_t_2;
17591     int __pyx_lineno = 0;
17592     const char *__pyx_filename = NULL;
17593     int __pyx_clineno = 0;
17594     __Pyx_RefNannySetupContext("asinh", 0);
17595     __Pyx_XDECREF(__pyx_r);
17596     __pyx_t_2.__pyx_n = 1;
17597     __pyx_t_2.i = __pyx_v_i;
17598     __pyx_t_1 = __pyx_f_8PyClical_asinh(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1782, __pyx_L1_error)
17599     __Pyx_GOTREF(__pyx_t_1);
17600     __pyx_r = __pyx_t_1;
17601     __pyx_t_1 = 0;
17602     goto __pyx_L0;
17603
17604     /* function exit code */
17605     __pyx_L1_error;
17606     __Pyx_XDECREF(__pyx_t_1);
17607     __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17608     __pyx_r = NULL;
17609     __pyx_L0;
17610     __Pyx_XGIVEREF(__pyx_r);
17611     __Pyx_RefNannyFinishContext();
17612     return __pyx_r;
17613 }
17614
17615 /* "PyClical.pyx":1801
17616 *
17617 * cpdef inline tan(obj,i = None): # ««««««««
17618 *     """
17619 *     Tangent of multivector with optional complexifier.
17620 *
17621 static PyObject *__pyx_pw_8PyClical_73tan(PyObject *__pyx_self, PyObject *__pyx_args,
17622 PyObject *__pyx_kwds); /*proto*/
17623 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_tan(PyObject *__pyx_v_obj,
17624 CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_tan *__pyx_optional_args) {
17625     PyObject *__pyx_v_i = ((PyObject *)Py_None);
17626     PyObject *__pyx_r = NULL;
17627     __Pyx_RefNannyDeclarations
17628     int __pyx_t_1;
17629     int __pyx_t_2;
17630     PyObject *__pyx_t_3 = NULL;
17631     Clifford __pyx_t_4;
17632     PyObject *__pyx_t_5 = NULL;
17633     PyObject *__pyx_t_6 = NULL;
17634     PyObject *__pyx_t_7 = NULL;
17635     PyObject *__pyx_t_8 = NULL;
17636     PyObject *__pyx_t_9 = NULL;
17637     PyObject *__pyx_t_10 = NULL;
17638     PyObject *__pyx_t_11 = NULL;
17639     int __pyx_lineno = 0;
17640     const char *__pyx_filename = NULL;
17641     int __pyx_clineno = 0;
17642     __Pyx_RefNannySetupContext("tan", 0);
17643     if (__pyx_optional_args) {
17644         if (__pyx_optional_args->__pyx_n > 0) {
17645             __pyx_v_i = __pyx_optional_args->i;
17646         }
17647     }
17648
17649     /* "PyClical.pyx":1810
17650 *
17651 * if not (i is None): # ««««««««
17652 *     return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
17653 * else:
17654 */

```

```

17654         __pyx_t_1 = (__pyx_v_i != Py_None);
17655         __pyx_t_2 = (__pyx_t_1 != 0);
17656         if (__pyx_t_2) {
17657
17658             /* "PyClical.pyx":1811
17659 *      """
17660 *      if not (i is None):
17661 *          return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )      # ««««««««
17662 *      else:
17663 *          try:
17664 */
17665         __Pyx_XDECREF(__pyx_r);
17666         __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
17667 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1811, __pyx_L1_error)
17668         __Pyx_GOTREF(__pyx_t_3);
17669         try {
17670             __pyx_t_4 = tan(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
17671 __pyx_f_8PyClical_toClifford(__pyx_v_i));
17672         } catch (...) {
17673             __Pyx_CppExn2PyErr();
17674             __PYX_ERR(0, 1811, __pyx_L1_error)
17675         }
17676         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
17677 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1811, __pyx_L1_error)
17678         __Pyx_GOTREF(__pyx_t_5);
17679         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17680         __pyx_r = __pyx_t_5;
17681         __pyx_t_5 = 0;
17682         goto __pyx_L0;
17683
17684     /* "PyClical.pyx":1810
17685 *      0.7616{1,2}
17686 *      """
17687 *      if not (i is None):
17688 *          return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )      # ««««««««
17689 *      else:
17690 *          try:
17691 *              return math.tan(obj)
17692 *          except:
17693 */
17694         /*else*/ {
17695             {
17696                 __Pyx_PyThreadState_declare
17697                 __Pyx_PyThreadState_assign
17698                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
17699                 __Pyx_XGOTREF(__pyx_t_6);
17700                 __Pyx_XGOTREF(__pyx_t_7);
17701                 __Pyx_XGOTREF(__pyx_t_8);
17702                 /*try*/ {
17703
17704                     /* "PyClical.pyx":1814
17705 *      else:
17706 *          try:
17707 *              return math.tan(obj)
17708 *          except:
17709 *              return clifford().wrap( glucat.tan(toClifford(obj)) )      # ««««««««
17710 */
17711                 __Pyx_XDECREF(__pyx_r);
17712                 __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
17713 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1814, __pyx_L4_error)
17714                 __Pyx_GOTREF(__pyx_t_3);
17715                 __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_tan); if
17716 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1814, __pyx_L4_error)
17717                 __Pyx_GOTREF(__pyx_t_9);
17718                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17719                 __pyx_t_3 = NULL;
17720                 if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
17721                     __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
17722                     if (likely(__pyx_t_3)) {
17723                         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
17724                         __Pyx_INCREF(__pyx_t_3);
17725                         __Pyx_INCREF(function);
17726                         __Pyx_DECREF_SET(__pyx_t_9, function);
17727                     }
17728                 }
17729                 __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
17730 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
17731                 __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17732                 if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1814, __pyx_L4_error)
17733                 __Pyx_GOTREF(__pyx_t_5);
17734                 __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;

```



```

17735         __pyx_r = __pyx_t_5;
17736         __pyx_t_5 = 0;
17737         goto __pyx_L8_try_return;
17738
17739         /* "PyClical.pyx":1813
17740  *         return clifford().wrap( glucat.tan(toClifford(obj)), toClifford(i)) )
17741  *     else:
17742  *         try:
17743  *             # ««««««««
17744  *             return math.tan(obj)
17745  *         except:
17746  */
17747         }
17748         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17749         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17750         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
17751
17752         /* "PyClical.pyx":1815
17753  *         try:
17754  *             return math.tan(obj)
17755  *         except:
17756  *             # ««««««««
17757  *             return clifford().wrap( glucat.tan(toClifford(obj)) )
17758  */
17759         /*except:*/ {
17760         __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno,
17761         __pyx_filename);
17762         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
17763         1815, __pyx_L6_except_error)
17764         __Pyx_GOTREF(__pyx_t_5);
17765         __Pyx_GOTREF(__pyx_t_9);
17766         __Pyx_GOTREF(__pyx_t_3);
17767
17768         /* "PyClical.pyx":1816
17769  *         return math.tan(obj)
17770  *     except:
17771  *         return clifford().wrap( glucat.tan(toClifford(obj)) )
17772  *             # ««««««««
17773  * cpdef inline atan(obj,i = None):
17774  */
17775         __Pyx_XDECREF(__pyx_r);
17776         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
17777         *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1816,
17778         __pyx_L6_except_error)
17779         __Pyx_GOTREF(__pyx_t_10);
17780         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
17781         __pyx_obj_8PyClical_clifford *)__pyx_t_10), tan(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
17782         (unlikely(!__pyx_t_11)) __PYX_ERR(0, 1816, __pyx_L6_except_error)
17783         __Pyx_GOTREF(__pyx_t_11);
17784         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
17785         __pyx_r = __pyx_t_11;
17786         __pyx_t_11 = 0;
17787         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17788         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17789         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17790         goto __pyx_L7_except_return;
17791     }
17792     __pyx_L6_except_error:;
17793
17794     /* "PyClical.pyx":1813
17795  *         return clifford().wrap( glucat.tan(toClifford(obj)), toClifford(i)) )
17796  *     else:
17797  *         try:
17798  *             # ««««««««
17799  *             return math.tan(obj)
17800  *         except:
17801  */
17802         __Pyx_XGIVEREF(__pyx_t_6);
17803         __Pyx_XGIVEREF(__pyx_t_7);
17804         __Pyx_XGIVEREF(__pyx_t_8);
17805         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17806         goto __pyx_L1_error;
17807         __pyx_L8_try_return:;
17808         __Pyx_XGIVEREF(__pyx_t_6);
17809         __Pyx_XGIVEREF(__pyx_t_7);
17810         __Pyx_XGIVEREF(__pyx_t_8);
17811         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17812         goto __pyx_L0;
17813         __pyx_L7_except_return:;
17814         __Pyx_XGIVEREF(__pyx_t_6);
17815         __Pyx_XGIVEREF(__pyx_t_7);
17816         __Pyx_XGIVEREF(__pyx_t_8);
17817         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17818         goto __pyx_L0;
17819     }
17820 }
17821
17822     /* "PyClical.pyx":1801

```

```

17816 *         return clifford().wrap( glucat.asinh(toClifford(obj)) )
17817 *
17818 * cpdef inline tan(obj,i = None):          # ««««««««
17819 *     """
17820 *     Tangent of multivector with optional complexifier.
17821 */
17822
17823     /* function exit code */
17824     __pyx_L1_error:;
17825     __Pyx_XDECREF(__pyx_t_3);
17826     __Pyx_XDECREF(__pyx_t_5);
17827     __Pyx_XDECREF(__pyx_t_9);
17828     __Pyx_XDECREF(__pyx_t_10);
17829     __Pyx_XDECREF(__pyx_t_11);
17830     __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno, __pyx_filename);
17831     __pyx_r = 0;
17832     __pyx_L0:;
17833     __Pyx_XGIVEREF(__pyx_r);
17834     __Pyx_RefNannyFinishContext();
17835     return __pyx_r;
17836 }
17837
17838 /* Python wrapper */
17839 static PyObject * __pyx_pw_8PyClical_73tan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
17840 static char __pyx_doc_8PyClical_72tan[] = "\n    Tangent of multivector with optional
complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(tan(x,\"{1,2,3}\")\n    0.7616{1,2}\n    >>
x=clifford(\"{1,2}\"); print(tan(x))\n    0.7616{1,2}\n    ";
17841 static PyObject * __pyx_pw_8PyClical_73tan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds) {
17842     PyObject * __pyx_v_obj = 0;
17843     PyObject * __pyx_v_i = 0;
17844     int __pyx_lineno = 0;
17845     const char * __pyx_filename = NULL;
17846     int __pyx_clineno = 0;
17847     PyObject * __pyx_r = 0;
17848     __Pyx_RefNannyDeclarations
17849     __Pyx_RefNannySetupContext("tan (wrapper)", 0);
17850     {
17851         static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
17852         PyObject* values[2] = {0,0};
17853         values[1] = ((PyObject *)Py_None);
17854         if (unlikely(__pyx_kwds)) {
17855             Py_ssize_t kw_args;
17856             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
17857             switch (pos_args) {
17858                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
17859                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
17860                 case 0: break;
17861                 default: goto __pyx_L5_argtuple_error;
17862             }
17863             kw_args = PyDict_Size(__pyx_kwds);
17864             switch (pos_args) {
17865                 case 0:
17866                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
17869                     else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
17870                     case 1:
17871                         if (kw_args > 0) {
17872                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
17873                             if (value) { values[1] = value; kw_args--; }
17874                         }
17875                     }
17876                     if (unlikely(kw_args > 0)) {
17877                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
17878 values, pos_args, "tan") < 0)) __PYX_ERR(0, 1801, __pyx_L3_error)
17879                     }
17880                 } else {
17881                     switch (PyTuple_GET_SIZE(__pyx_args)) {
17882                         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
17883                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17884                         break;
17885                         default: goto __pyx_L5_argtuple_error;
17886                     }
17887                 }
17888                 __pyx_v_obj = values[0];
17889                 __pyx_v_i = values[1];
17890             }
17891             goto __pyx_L4_argument_unpacking_done;
17892             __pyx_L5_argtuple_error:;
17893             __Pyx_RaiseArgtupleInvalid("tan", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
17894             __PYX_ERR(0, 1801, __pyx_L3_error)
17895             __pyx_L3_error:;

```

```

17896         __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno, __pyx_filename);
17897         __Pyx_RefNannyFinishContext();
17898         return NULL;
17899     __pyx_L4_argument_unpacking_done:;
17900     __pyx_r = __pyx_pf_8PyClical_72tan(__pyx_self, __pyx_v_obj, __pyx_v_i);
17901
17902     /* function exit code */
17903     __Pyx_RefNannyFinishContext();
17904     return __pyx_r;
17905 }
17906
17907 static PyObject * __pyx_pf_8PyClical_72tan(CYTHON_UNUSED PyObject * __pyx_self, PyObject
*__pyx_v_obj, PyObject * __pyx_v_i) {
17908     PyObject * __pyx_r = NULL;
17909     __Pyx_RefNannyDeclarations
17910     PyObject * __pyx_t_1 = NULL;
17911     struct __pyx_opt_args_8PyClical_tan __pyx_t_2;
17912     int __pyx_lineno = 0;
17913     const char * __pyx_filename = NULL;
17914     int __pyx_clineno = 0;
17915     __Pyx_RefNannySetupContext("tan", 0);
17916     __Pyx_XDECREF(__pyx_r);
17917     __pyx_t_2.__pyx_n = 1;
17918     __pyx_t_2.i = __pyx_v_i;
17919     __pyx_t_1 = __pyx_f_8PyClical_tan(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1801, __pyx_L1_error)
17920     __Pyx_GOTREF(__pyx_t_1);
17921     __pyx_r = __pyx_t_1;
17922     __pyx_t_1 = 0;
17923     goto __pyx_L0;
17924
17925     /* function exit code */
17926     __pyx_L1_error:;
17927     __Pyx_XDECREF(__pyx_t_1);
17928     __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno, __pyx_filename);
17929     __pyx_r = NULL;
17930     __pyx_L0:;
17931     __Pyx_XGIVEREF(__pyx_r);
17932     __Pyx_RefNannyFinishContext();
17933     return __pyx_r;
17934 }
17935
17936 /* "PyClical.pyx":1818
17937 *
17938 *
17939 * cpdef inline atan(obj,i = None):
17940 *     """
17941 *     Inverse tangent of multivector with optional complexifier.
17942 */
17943
17944 static PyObject * __pyx_pw_8PyClical_75atan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
17945 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_atan(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_atan * __pyx_optional_args) {
17946     PyObject * __pyx_v_i = (PyObject *)Py_None;
17947     PyObject * __pyx_r = NULL;
17948     __Pyx_RefNannyDeclarations
17949     int __pyx_t_1;
17950     int __pyx_t_2;
17951     PyObject * __pyx_t_3 = NULL;
17952     Clifford __pyx_t_4;
17953     PyObject * __pyx_t_5 = NULL;
17954     PyObject * __pyx_t_6 = NULL;
17955     PyObject * __pyx_t_7 = NULL;
17956     PyObject * __pyx_t_8 = NULL;
17957     PyObject * __pyx_t_9 = NULL;
17958     PyObject * __pyx_t_10 = NULL;
17959     PyObject * __pyx_t_11 = NULL;
17960     int __pyx_lineno = 0;
17961     const char * __pyx_filename = NULL;
17962     int __pyx_clineno = 0;
17963     __Pyx_RefNannySetupContext("atan", 0);
17964     if (__pyx_optional_args) {
17965         if (__pyx_optional_args->__pyx_n > 0) {
17966             __pyx_v_i = __pyx_optional_args->1;
17967         }
17968     }
17969
17970     /* "PyClical.pyx":1827
17971 *     {1}
17972 *     """
17973 *     if not (i is None):
17974 *         return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
17975 *     else:
17976 */
17977     __pyx_t_1 = (__pyx_v_i != Py_None);
17978     __pyx_t_2 = (__pyx_t_1 != 0);

```

```

17979         if (__pyx_t_2) {
17980
17981             /* "PyClical.pyx":1828
17982  *      """
17983  *      if not (i is None):
17984  *          return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )      # ««««««««
17985  *      else:
17986  *          try:
17987  */
17988         __Pyx_XDECREF(__pyx_r);
17989         __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
*)__pyx_ptype_8PyClical_clifford); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1828, __pyx_L1_error)
17990         __Pyx_GOTREF(__pyx_t_3);
17991         try {
17992             __pyx_t_4 = atan(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
17993         } catch (...) {
17994             __Pyx_CppExn2PyErr();
17995             __PYX_ERR(0, 1828, __pyx_L1_error)
17996         }
17997         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1828, __pyx_L1_error)
17998         __Pyx_GOTREF(__pyx_t_5);
17999         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18000         __pyx_r = __pyx_t_5;
18001         __pyx_t_5 = 0;
18002         goto __pyx_L0;
18003
18004         /* "PyClical.pyx":1827
18005  *      {1}
18006  *      """
18007  *      if not (i is None):      # ««««««««
18008  *          return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
18009  *      else:
18010  */
18011     }
18012
18013     /* "PyClical.pyx":1830
18014  *      return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
18015  *      else:
18016  *          try:      # ««««««««
18017  *              return math.atan(obj)
18018  *          except:
18019  */
18020     /*else*/ {
18021     {
18022         __Pyx_PyThreadState_declare
18023         __Pyx_PyThreadState_assign
18024         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
18025         __Pyx_XGOTREF(__pyx_t_6);
18026         __Pyx_XGOTREF(__pyx_t_7);
18027         __Pyx_XGOTREF(__pyx_t_8);
18028         /*try:*/ {
18029
18030             /* "PyClical.pyx":1831
18031  *      else:
18032  *          try:
18033  *              return math.atan(obj)      # ««««««««
18034  *          except:
18035  *              return clifford().wrap( glucat.atan(toClifford(obj)) )
18036  */
18037         __Pyx_XDECREF(__pyx_r);
18038         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1831, __pyx_L4_error)
18039         __Pyx_GOTREF(__pyx_t_3);
18040         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_atan); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1831, __pyx_L4_error)
18041         __Pyx_GOTREF(__pyx_t_9);
18042         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18043         __pyx_t_3 = NULL;
18044         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
18045             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
18046             if (likely(__pyx_t_3)) {
18047                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
18048                 __Pyx_INCREF(__pyx_t_3);
18049                 __Pyx_INCREF(function);
18050                 __Pyx_DECREF_SET(__pyx_t_9, function);
18051             }
18052         }
18053         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
18054         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18055         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1831, __pyx_L4_error)
18056         __Pyx_GOTREF(__pyx_t_5);
18057         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18058         __pyx_r = __pyx_t_5;
18059         __pyx_t_5 = 0;

```

```

18060             goto __pyx_L8_try_return;
18061
18062             /* "PyClical.pyx":1830
18063 *         return clifford().wrap( glucat.atan(toClifford(obj)), toClifford(i)) )
18064 *     else:
18065 *         try:             # ««««««««
18066 *             return math.atan(obj)
18067 *         except:
18068 */
18069     }
18070     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18071     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18072     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
18073
18074     /* "PyClical.pyx":1832
18075 *     try:
18076 *         return math.atan(obj)
18077 *     except:             # ««««««««
18078 *         return clifford().wrap( glucat.atan(toClifford(obj)) )
18079 *
18080 */
18081 */
18082     /*except:*/ {
18083         __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18084         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1832, __pyx_L6_except_error)
18085         __Pyx_GOTREF(__pyx_t_5);
18086         __Pyx_GOTREF(__pyx_t_9);
18087         __Pyx_GOTREF(__pyx_t_3);
18088
18089         /* "PyClical.pyx":1833
18090 *         return math.atan(obj)
18091 *     except:
18092 *         return clifford().wrap( glucat.atan(toClifford(obj)) )
18093 *
18094 * cpdef inline tanh(obj):
18095 */
18096         __Pyx_XDECREF(__pyx_r);
18097         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1833,
__pyx_L6_except_error)
18098         __Pyx_GOTREF(__pyx_t_10);
18099         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), atan(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1833, __pyx_L6_except_error)
18100         __Pyx_GOTREF(__pyx_t_11);
18101         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
18102         __pyx_r = __pyx_t_11;
18103         __pyx_t_11 = 0;
18104         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18105         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18106         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18107         goto __pyx_L7_except_return;
18108     }
18109     __pyx_L6_except_error;
18110
18111     /* "PyClical.pyx":1830
18112 *     return clifford().wrap( glucat.atan(toClifford(obj)), toClifford(i)) )
18113 * else:
18114 *     try:             # ««««««««
18115 *         return math.atan(obj)
18116 *     except:
18117 */
18118     __Pyx_XGIVEREF(__pyx_t_6);
18119     __Pyx_XGIVEREF(__pyx_t_7);
18120     __Pyx_XGIVEREF(__pyx_t_8);
18121     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18122     goto __pyx_L1_error;
18123     __pyx_L8_try_return;
18124     __Pyx_XGIVEREF(__pyx_t_6);
18125     __Pyx_XGIVEREF(__pyx_t_7);
18126     __Pyx_XGIVEREF(__pyx_t_8);
18127     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18128     goto __pyx_L0;
18129     __pyx_L7_except_return;
18130     __Pyx_XGIVEREF(__pyx_t_6);
18131     __Pyx_XGIVEREF(__pyx_t_7);
18132     __Pyx_XGIVEREF(__pyx_t_8);
18133     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18134     goto __pyx_L0;
18135 }
18136 }
18137
18138     /* "PyClical.pyx":1818
18139 *     return clifford().wrap( glucat.tan(toClifford(obj)) )
18140 */

```

```

18141 * cpdef inline atan(obj,i = None):          # ««««««««
18142 *      """
18143 *      Inverse tangent of multivector with optional complexifier.
18144 */
18145
18146      /* function exit code */
18147      __pyx_L1_error++;
18148      __Pyx_XDECREF(__pyx_t_3);
18149      __Pyx_XDECREF(__pyx_t_5);
18150      __Pyx_XDECREF(__pyx_t_9);
18151      __Pyx_XDECREF(__pyx_t_10);
18152      __Pyx_XDECREF(__pyx_t_11);
18153      __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno, __pyx_filename);
18154      __pyx_r = 0;
18155      __pyx_L0:;
18156      __Pyx_XGIVEREF(__pyx_r);
18157      __Pyx_RefNannyFinishContext();
18158      return __pyx_r;
18159  }
18160
18161      /* Python wrapper */
18162      static PyObject * __pyx_pw_8PyClical_75atan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
18163      static char __pyx_doc_8PyClical_74atan[] = "\n      Inverse tangent of multivector with
optional complexifier.\n\n      >> s=index_set({1,2,3}); x=clifford(\"{1}\"); print(tan(atan(x,s),s))\n
{1}\n      >> x=clifford(\"{1}\"); print(tan(atan(x)))\n      {1}\n      ";
18164      static PyObject * __pyx_pw_8PyClical_75atan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds) {
18165          PyObject * __pyx_v_obj = 0;
18166          PyObject * __pyx_v_i = 0;
18167          int __pyx_lineno = 0;
18168          const char * __pyx_filename = NULL;
18169          int __pyx_clineno = 0;
18170          PyObject * __pyx_r = 0;
18171          __Pyx_RefNannyDeclarations
18172          __Pyx_RefNannySetupContext("atan (wrapper)", 0);
18173          {
18174              static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
18175              PyObject* values[2] = {0,0};
18176              values[1] = ((PyObject *)Py_None);
18177              if (unlikely(__pyx_kwds)) {
18178                  Py_ssize_t kw_args;
18179                  const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
18180                  switch (pos_args) {
18181                      case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18182                      case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
18183                      case 0: break;
18184                      default: goto __pyx_L5_argtuple_error;
18185                  }
18186                  kw_args = PyDict_Size(__pyx_kwds);
18187                  switch (pos_args) {
18188                      case 0:
18189                      if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
18190
0)) kw_args--;
18192                      else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
18194                      case 1:
18195                      if (kw_args > 0) {
18196                          PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
18197                          if (value) { values[1] = value; kw_args--; }
18198                      }
18199                      if (unlikely(kw_args > 0)) {
18200                          if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
18201
values, pos_args, "atan") < 0)) __PYX_ERR(0, 1818, __pyx_L3_error)
18202                      } else {
18203                          switch (PyTuple_GET_SIZE(__pyx_args)) {
18204                              case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18205                              case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18206                              break;
18207                              default: goto __pyx_L5_argtuple_error;
18208                          }
18209                      }
18210                      __pyx_v_obj = values[0];
18211                      __pyx_v_i = values[1];
18212                  }
18213                  goto __pyx_L4_argument_unpacking_done;
18214                  __pyx_L5_argtuple_error:;
18215                  __Pyx_RaiseArgtupleInvalid("atan", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
18216                  __PYX_ERR(0, 1818, __pyx_L3_error)
18217                  __pyx_L3_error:;
18218                  __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno, __pyx_filename);
18219                  __Pyx_RefNannyFinishContext();
18220

```

```

18221         return NULL;
18222         __pyx_L4_argument_unpacking_done:;
18223         __pyx_r = __pyx_pf_8PyClical_74atan(__pyx_self, __pyx_v_obj, __pyx_v_i);
18224
18225         /* function exit code */
18226         __Pyx_RefNannyFinishContext();
18227         return __pyx_r;
18228     }
18229
18230     static PyObject *__pyx_pf_8PyClical_74atan(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
18231         PyObject *__pyx_r = NULL;
18232         __Pyx_RefNannyDeclarations
18233         PyObject *__pyx_t_1 = NULL;
18234         struct __pyx_opt_args_8PyClical_atan __pyx_t_2;
18235         int __pyx_lineno = 0;
18236         const char *__pyx_filename = NULL;
18237         int __pyx_clineno = 0;
18238         __Pyx_RefNannySetupContext("atan", 0);
18239         __Pyx_XDECREF(__pyx_r);
18240         __pyx_t_2.__pyx_n = 1;
18241         __pyx_t_2.i = __pyx_v_i;
18242         __pyx_t_1 = __pyx_f_8PyClical_atan(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1818, __pyx_L1_error)
18243         __Pyx_GOTREF(__pyx_t_1);
18244         __pyx_r = __pyx_t_1;
18245         __pyx_t_1 = 0;
18246         goto __pyx_L0;
18247
18248         /* function exit code */
18249         __pyx_L1_error:;
18250         __Pyx_XDECREF(__pyx_t_1);
18251         __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno, __pyx_filename);
18252         __pyx_r = NULL;
18253         __pyx_L0:;
18254         __Pyx_XGIVEREF(__pyx_r);
18255         __Pyx_RefNannyFinishContext();
18256         return __pyx_r;
18257     }
18258
18259     /* "PyClical.pyx":1835
18260     *         return clifford().wrap( glucat.atan(toClifford(obj)) )
18261     *
18262     * cpdef inline tanh(obj): # «««««««
18263     *     """
18264     *     Hyperbolic tangent of multivector.
18265     */
18266
18267     static PyObject *__pyx_pw_8PyClical_77tanh(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
18268     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_tanh(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
18269         PyObject *__pyx_r = NULL;
18270         __Pyx_RefNannyDeclarations
18271         PyObject *__pyx_t_1 = NULL;
18272         PyObject *__pyx_t_2 = NULL;
18273         PyObject *__pyx_t_3 = NULL;
18274         PyObject *__pyx_t_4 = NULL;
18275         PyObject *__pyx_t_5 = NULL;
18276         PyObject *__pyx_t_6 = NULL;
18277         PyObject *__pyx_t_7 = NULL;
18278         PyObject *__pyx_t_8 = NULL;
18279         int __pyx_lineno = 0;
18280         const char *__pyx_filename = NULL;
18281         int __pyx_clineno = 0;
18282         __Pyx_RefNannySetupContext("tanh", 0);
18283
18284         /* "PyClical.pyx":1842
18285     *         {1,2}
18286     *         """
18287     *         try: # «««««««
18288     *             return math.tanh(obj)
18289     *         except:
18290     */
18291         {
18292             __Pyx_PyThreadState_declare
18293             __Pyx_PyThreadState_assign
18294             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
18295             __Pyx_XGOTREF(__pyx_t_1);
18296             __Pyx_XGOTREF(__pyx_t_2);
18297             __Pyx_XGOTREF(__pyx_t_3);
18298             /*try:*/ {
18299
18300                 /* "PyClical.pyx":1843
18301     *         """
18302     *         try:
18303     *             return math.tanh(obj) # «««««««

```

```

18304 *      except:
18305 *          return clifford().wrap( glucat.tanh(toClifford(obj)) )
18306 */
18307         __Pyx_XDECREF(__pyx_r);
18308         __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 1843, __pyx_L3_error)
18309         __Pyx_GOTREF(__pyx_t_5);
18310         __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_tanh); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 1843, __pyx_L3_error)
18311         __Pyx_GOTREF(__pyx_t_6);
18312         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18313         __pyx_t_5 = NULL;
18314         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
18315             __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
18316             if (likely(__pyx_t_5)) {
18317                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
18318                 __Pyx_INCREF(__pyx_t_5);
18319                 __Pyx_INCREF(function);
18320                 __Pyx_DECREF_SET(__pyx_t_6, function);
18321             }
18322         }
18323         __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
18324         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18325         if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1843, __pyx_L3_error)
18326         __Pyx_GOTREF(__pyx_t_4);
18327         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
18328         __pyx_r = __pyx_t_4;
18329         __pyx_t_4 = 0;
18330         goto __pyx_L7_try_return;
18331
18332         /* "PyClicl.pyx":1842
18333         *      {1,2}
18334         *      """
18335         *      try:
18336         *          return math.tanh(obj)
18337         *      except:
18338         */
18339         }
18340         __pyx_L3_error;;
18341         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
18342         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18343         __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
18344
18345         /* "PyClicl.pyx":1844
18346         *      try:
18347         *          return math.tanh(obj)
18348         *      except:
18349         *          return clifford().wrap( glucat.tanh(toClifford(obj)) )
18350         *
18351         */
18352         /*except:*/ {
18353             __Pyx_AddTraceback("PyClicl.tanh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18354             if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
1844, __pyx_L5_except_error)
18355             __Pyx_GOTREF(__pyx_t_4);
18356             __Pyx_GOTREF(__pyx_t_6);
18357             __Pyx_GOTREF(__pyx_t_5);
18358
18359             /* "PyClicl.pyx":1845
18360             *          return math.tanh(obj)
18361             *      except:
18362             *          return clifford().wrap( glucat.tanh(toClifford(obj)) )
18363             *
18364             * cpdef inline atanh(obj,i = None):
18365             */
18366             __Pyx_XDECREF(__pyx_r);
18367             __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1845,
__pyx_L5_except_error)
18368             __Pyx_GOTREF(__pyx_t_7);
18369             __pyx_t_8 = __pyx_f_8PyClicl_8clifford_wrap(((struct
__pyx_obj_8PyClicl_clifford *)__pyx_t_7), tanh(__pyx_f_8PyClicl_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1845, __pyx_L5_except_error)
18370             __Pyx_GOTREF(__pyx_t_8);
18371             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
18372             __pyx_r = __pyx_t_8;
18373             __pyx_t_8 = 0;
18374             __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
18375             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18376             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
18377             goto __pyx_L6_except_return;
18378         }
18379         __pyx_L5_except_error;;
18380
18381         /* "PyClicl.pyx":1842

```



```

18382 *      {1,2}
18383 *      """
18384 *      try:          # ««««««««
18385 *          return math.tanh(obj)
18386 *      except:
18387 */
18388         __Pyx_XGIVEREF(__pyx_t_1);
18389         __Pyx_XGIVEREF(__pyx_t_2);
18390         __Pyx_XGIVEREF(__pyx_t_3);
18391         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
18392         goto __pyx_L1_error;
18393         __pyx_L7_try_return:;
18394         __Pyx_XGIVEREF(__pyx_t_1);
18395         __Pyx_XGIVEREF(__pyx_t_2);
18396         __Pyx_XGIVEREF(__pyx_t_3);
18397         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
18398         goto __pyx_L0;
18399         __pyx_L6_except_return:;
18400         __Pyx_XGIVEREF(__pyx_t_1);
18401         __Pyx_XGIVEREF(__pyx_t_2);
18402         __Pyx_XGIVEREF(__pyx_t_3);
18403         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
18404         goto __pyx_L0;
18405     }
18406
18407     /* "PyClical.pyx":1835
18408     *      return clifford().wrap( glucat.atan(toClifford(obj)) )
18409     *
18410     * cpdef inline tanh(obj):          # ««««««««
18411     *      """
18412     *      Hyperbolic tangent of multivector.
18413     */
18414
18415     /* function exit code */
18416     __pyx_L1_error:;
18417     __Pyx_XDECREF(__pyx_t_4);
18418     __Pyx_XDECREF(__pyx_t_5);
18419     __Pyx_XDECREF(__pyx_t_6);
18420     __Pyx_XDECREF(__pyx_t_7);
18421     __Pyx_XDECREF(__pyx_t_8);
18422     __Pyx_AddTraceback("PyClical.tanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18423     __pyx_r = 0;
18424     __pyx_L0:;
18425     __Pyx_XGIVEREF(__pyx_r);
18426     __Pyx_RefNannyFinishContext();
18427     return __pyx_r;
18428 }
18429
18430 /* Python wrapper */
18431 static PyObject * __pyx_pw_8PyClical_77tanh(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
18432 static char __pyx_doc_8PyClical_76tanh[] = "\n      Hyperbolic tangent of
multivector.\n\n      >> x=clifford(\"{1,2}\") * pi/4; print(tanh(x))\n      {1,2}\n";
18433 static PyObject * __pyx_pw_8PyClical_77tanh(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
18434     PyObject * __pyx_r = 0;
18435     __Pyx_RefNannyDeclarations
18436     __Pyx_RefNannySetupContext("tanh (wrapper)", 0);
18437     __pyx_r = __pyx_pf_8PyClical_76tanh(__pyx_self, ((PyObject *) __pyx_v_obj));
18438
18439     /* function exit code */
18440     __Pyx_RefNannyFinishContext();
18441     return __pyx_r;
18442 }
18443
18444 static PyObject * __pyx_pf_8PyClical_76tanh(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
18445     PyObject * __pyx_r = NULL;
18446     __Pyx_RefNannyDeclarations
18447     PyObject * __pyx_t_1 = NULL;
18448     int __pyx_lineno = 0;
18449     const char * __pyx_filename = NULL;
18450     int __pyx_clineno = 0;
18451     __Pyx_RefNannySetupContext("tanh", 0);
18452     __Pyx_XDECREF(__pyx_r);
18453     __pyx_t_1 = __pyx_f_8PyClical_tanh(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1835, __pyx_L1_error)
18454     __Pyx_GOTREF(__pyx_t_1);
18455     __pyx_r = __pyx_t_1;
18456     __pyx_t_1 = 0;
18457     goto __pyx_L0;
18458
18459     /* function exit code */
18460     __pyx_L1_error:;
18461     __Pyx_XDECREF(__pyx_t_1);
18462     __Pyx_AddTraceback("PyClical.tanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18463     __pyx_r = NULL;

```

```

18464         __pyx_L0;;
18465         __Pyx_XGIVEREF(__pyx_r);
18466         __Pyx_RefNannyFinishContext();
18467         return __pyx_r;
18468     }
18469
18470     /* "PyClicl.pyx":1847
18471     *         return clifford().wrap( glucat.tanh(toClifford(obj)) )
18472     *
18473     * cpdef inline atanh(obj,i = None):
18474     *     """
18475     *         Inverse hyperbolic tangent of multivector with optional complexifier.
18476     */
18477
18478     static PyObject * __pyx_pw_8PyClicl_79atanh(PyObject * __pyx_self, PyObject
18479 * __pyx_args, PyObject * __pyx_kwds); /*proto*/
18479     static CYTHON_INLINE PyObject * __pyx_f_8PyClicl_atanh(PyObject * __pyx_v_obj,
18480 CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClicl_atanh * __pyx_optional_args) {
18480     PyObject * __pyx_v_i = (PyObject *)Py_None;
18481     PyObject * __pyx_r = NULL;
18482     __Pyx_RefNannyDeclarations
18483     int __pyx_t_1;
18484     int __pyx_t_2;
18485     PyObject * __pyx_t_3 = NULL;
18486     Clifford __pyx_t_4;
18487     PyObject * __pyx_t_5 = NULL;
18488     PyObject * __pyx_t_6 = NULL;
18489     PyObject * __pyx_t_7 = NULL;
18490     PyObject * __pyx_t_8 = NULL;
18491     PyObject * __pyx_t_9 = NULL;
18492     PyObject * __pyx_t_10 = NULL;
18493     PyObject * __pyx_t_11 = NULL;
18494     int __pyx_lineno = 0;
18495     const char * __pyx_filename = NULL;
18496     int __pyx_clineno = 0;
18497     __Pyx_RefNannySetupContext("atanh", 0);
18498     if (__pyx_optional_args) {
18499         if (__pyx_optional_args->__pyx_n > 0) {
18500             __pyx_v_i = __pyx_optional_args->i;
18501         }
18502     }
18503
18504     /* "PyClicl.pyx":1856
18505     *     {1,2}
18506     *     """
18507     *     if not (i is None):
18508     *         # ««««««««
18509     *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18510     *     else:
18511     */
18511     __pyx_t_1 = (__pyx_v_i != Py_None);
18512     __pyx_t_2 = (__pyx_t_1 != 0);
18513     if (__pyx_t_2) {
18514
18515         /* "PyClicl.pyx":1857
18516         *         """
18517         *         if not (i is None):
18518         *             return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18519         *         #
18520         *         else:
18521         *             try:
18522         */
18522         __Pyx_XDECREF(__pyx_r);
18523         __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
18524 *) __pyx_ptype_8PyClicl_clifford); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1857, __pyx_L1_error)
18524         __Pyx_GOTREF(__pyx_t_3);
18525         try {
18526             __pyx_t_4 = atanh(__pyx_f_8PyClicl_toClifford(__pyx_v_obj),
18527 __pyx_f_8PyClicl_toClifford(__pyx_v_i));
18527         } catch (...) {
18528             __Pyx_CppExn2PyErr();
18529             __PYX_ERR(0, 1857, __pyx_L1_error)
18530         }
18531         __pyx_t_5 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
18532 *) __pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1857, __pyx_L1_error)
18532         __Pyx_GOTREF(__pyx_t_5);
18533         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18534         __pyx_r = __pyx_t_5;
18535         __pyx_t_5 = 0;
18536         goto __pyx_L0;
18537
18538         /* "PyClicl.pyx":1856
18539         *     {1,2}
18540         *     """
18541         *     if not (i is None):
18542         *         # ««««««««
18543         *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18544         *     else:
18545     */

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18627 *
18628 * cpdef inline random_clifford(index_set ixt, fill = 1.0):
18629 */
18630
18631     __Pyx_XDECREF(__pyx_r);
18632     __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
18633 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1862,
__pyx_L6_except_error)
18632     __Pyx_GOTREF(__pyx_t_10);
18633     __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), atanh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1862, __pyx_L6_except_error)
18634     __Pyx_GOTREF(__pyx_t_11);
18635     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
18636     __pyx_r = __pyx_t_11;
18637     __pyx_t_11 = 0;
18638     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18639     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18640     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18641     goto __pyx_L7_except_return;
18642 }
18643 __pyx_L6_except_error;;
18644
18645     /* "PyClical.pyx":1859
18646 *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18647 *     else:
18648 *         try:
18649 *             return math.atanh(obj)
18650 *         except:
18651 */
18652     __Pyx_XGIVEREF(__pyx_t_6);
18653     __Pyx_XGIVEREF(__pyx_t_7);
18654     __Pyx_XGIVEREF(__pyx_t_8);
18655     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18656     goto __pyx_L1_error;
18657     __pyx_L8_try_return;;
18658     __Pyx_XGIVEREF(__pyx_t_6);
18659     __Pyx_XGIVEREF(__pyx_t_7);
18660     __Pyx_XGIVEREF(__pyx_t_8);
18661     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18662     goto __pyx_L0;
18663     __pyx_L7_except_return;;
18664     __Pyx_XGIVEREF(__pyx_t_6);
18665     __Pyx_XGIVEREF(__pyx_t_7);
18666     __Pyx_XGIVEREF(__pyx_t_8);
18667     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18668     goto __pyx_L0;
18669 }
18670 }
18671
18672     /* "PyClical.pyx":1847
18673 *         return clifford().wrap( glucat.tanh(toClifford(obj)) )
18674 *
18675 * cpdef inline atanh(obj,i = None):
18676 *     """
18677 *     Inverse hyperbolic tangent of multivector with optional complexifier.
18678 */
18679
18680     /* function exit code */
18681     __pyx_L1_error;;
18682     __Pyx_XDECREF(__pyx_t_3);
18683     __Pyx_XDECREF(__pyx_t_5);
18684     __Pyx_XDECREF(__pyx_t_9);
18685     __Pyx_XDECREF(__pyx_t_10);
18686     __Pyx_XDECREF(__pyx_t_11);
18687     __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18688     __pyx_r = 0;
18689     __pyx_L0;;
18690     __Pyx_XGIVEREF(__pyx_r);
18691     __Pyx_RefNannyFinishContext();
18692     return __pyx_r;
18693 }
18694
18695     /* Python wrapper */
18696     static PyObject *__pyx_pw_8PyClical_79atanh(PyObject *__pyx_self, PyObject
18697 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
18697     static char __pyx_doc_8PyClical_78atanh[] = "\n    Inverse hyperbolic tangent of
multivector with optional complexifier.\n\n    >> s=index_set({1,2,3}); x=clifford(\"{1,2}\");
print(tanh(atanh(x,s)))\n    {1,2}\n    >> x=clifford(\"{1,2}\"); print(tanh(atanh(x)))\n    {1,2}\n
";
18698     static PyObject *__pyx_pw_8PyClical_79atanh(PyObject *__pyx_self, PyObject
18699 *__pyx_args, PyObject *__pyx_kwds) {
18700     PyObject *__pyx_v_obj = 0;
18701     PyObject *__pyx_v_i = 0;
18702     int __pyx_lineno = 0;
18703     const char *__pyx_filename = NULL;
18704     int __pyx_clineno = 0;
18705     PyObject *__pyx_r = 0;

```

```

18705     __Pyx_RefNannyDeclarations
18706     __Pyx_RefNannySetupContext("atanh (wrapper)", 0);
18707     {
18708         static PyObject* __pyx_pyargnames[] = {&__pyx_n_s_obj, &__pyx_n_s_i, 0};
18709         PyObject* values[2] = {0, 0};
18710         values[1] = (PyObject*)Py_None;
18711         if (unlikely(__pyx_kwds)) {
18712             Py_ssize_t kw_args;
18713             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
18714             switch (pos_args) {
18715                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
18716                     CYTHON_FALLTHROUGH;
18717                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18718                     CYTHON_FALLTHROUGH;
18719                 case 0: break;
18720                 default: goto __pyx_L5_argtuple_error;
18721             }
18722             kw_args = PyDict_Size(__pyx_kwds);
18723             switch (pos_args) {
18724                 case 0:
18725                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
18726                     else goto __pyx_L5_argtuple_error;
18727                     CYTHON_FALLTHROUGH;
18728                 case 1:
18729                     if (kw_args > 0) {
18730                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
18731                         if (value) { values[1] = value; kw_args--; }
18732                     }
18733                     if (unlikely(kw_args > 0)) {
18734                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "atanh") < 0)) __PYX_ERR(0, 1847, __pyx_L3_error)
18735                     }
18736                     } else {
18737                         switch (PyTuple_GET_SIZE(__pyx_args)) {
18738                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
18739                                 CYTHON_FALLTHROUGH;
18740                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18741                                 break;
18742                             default: goto __pyx_L5_argtuple_error;
18743                         }
18744                     }
18745                     __pyx_v_obj = values[0];
18746                     __pyx_v_i = values[1];
18747                 }
18748             goto __pyx_L4_argument_unpacking_done;
18749             __pyx_L5_argtuple_error:;
18750             __Pyx_RaiseArgtupleInvalid("atanh", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
18751             __PYX_ERR(0, 1847, __pyx_L3_error)
18752             __pyx_L3_error:;
18753             __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18754             __Pyx_RefNannyFinishContext();
18755             return NULL;
18756             __pyx_L4_argument_unpacking_done:;
18757             __pyx_r = __pyx_pf_8PyClical_78atanh(__pyx_self, __pyx_v_obj, __pyx_v_i);
18758
18759             /* function exit code */
18760             __Pyx_RefNannyFinishContext();
18761             return __pyx_r;
18762         }
18763
18764         static PyObject* __pyx_pf_8PyClical_78atanh(CYTHON_UNUSED PyObject* __pyx_self,
PyObject* __pyx_v_obj, PyObject* __pyx_v_i) {
18765             PyObject* __pyx_r = NULL;
18766             __Pyx_RefNannyDeclarations
18767             PyObject* __pyx_t_1 = NULL;
18768             struct __pyx_opt_args_8PyClical_atanh __pyx_t_2;
18769             int __pyx_lineno = 0;
18770             const char* __pyx_filename = NULL;
18771             int __pyx_clineno = 0;
18772             __Pyx_RefNannySetupContext("atanh", 0);
18773             __Pyx_XDECREF(__pyx_r);
18774             __pyx_t_2.__pyx_n = 1;
18775             __pyx_t_2.i = __pyx_v_i;
18776             __pyx_t_1 = __pyx_pf_8PyClical_atanh(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1847, __pyx_L1_error)
18777             __Pyx_GOTREF(__pyx_t_1);
18778             __pyx_r = __pyx_t_1;
18779             __pyx_t_1 = 0;
18780             goto __pyx_L0;
18781
18782             /* function exit code */
18783             __pyx_L1_error:;
18784             __Pyx_XDECREF(__pyx_t_1);
18785             __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18786             __pyx_r = NULL;

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18787         __pyx_L0;;
18788         __Pyx_XGIVEREF(__pyx_r);
18789         __Pyx_RefNannyFinishContext();
18790         return __pyx_r;
18791     }
18792
18793     /* "PyClical.pyx":1864
18794     *         return clifford().wrap( glucat.atanh(toClifford(obj)) )
18795     *
18796     * cpdef inline random_clifford(index_set ixt, fill = 1.0):
18797     *         """
18798     *         Random multivector within a frame.
18799     */
18800
18801     static PyObject * __pyx_pw_8PyClical_8lrandom_clifford(PyObject * __pyx_self, PyObject
18802 * __pyx_args, PyObject * __pyx_kwds); /*proto*/
18803     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_random_clifford(struct
18804 __pyx_obj_8PyClical_index_set * __pyx_v_ixt, CYTHON_UNUSED int __pyx_skip_dispatch, struct
18805 __pyx_opt_args_8PyClical_random_clifford * __pyx_optional_args) {
18806     PyObject * __pyx_v_fill = ((PyObject *) __pyx_float_1_0);
18807     PyObject * __pyx_r = NULL;
18808     __Pyx_RefNannyDeclarations
18809     PyObject * __pyx_t_1 = NULL;
18810     PyObject * __pyx_t_2 = NULL;
18811     scalar_t __pyx_t_3;
18812     PyObject * __pyx_t_4 = NULL;
18813     int __pyx_lineno = 0;
18814     const char * __pyx_filename = NULL;
18815     int __pyx_clineno = 0;
18816     __Pyx_RefNannySetupContext("random_clifford", 0);
18817     if (__pyx_optional_args) {
18818         if (__pyx_optional_args->__pyx_n > 0) {
18819             __pyx_v_fill = __pyx_optional_args->fill;
18820         }
18821     }
18822
18823     /* "PyClical.pyx":1871
18824     *         {-3,-1,2}
18825     *         """
18826     *         return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
18827     * #<<<<<<<<<
18828     *
18829     * cpdef inline cga3(obj):
18830     */
18831     __Pyx_XDECREF(__pyx_r);
18832     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford));
18833     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1871, __pyx_L1_error)
18834     __Pyx_GOTREF(__pyx_t_1);
18835     __pyx_t_2 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford));
18836     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1871, __pyx_L1_error)
18837     __Pyx_GOTREF(__pyx_t_2);
18838     __pyx_t_3 = __pyx_PyFloat_AsDouble(__pyx_v_fill); if (unlikely((__pyx_t_3 ==
18839 ((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1871, __pyx_L1_error)
18840     __pyx_t_4 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
18841 *) __pyx_t_1), ((struct __pyx_obj_8PyClical_clifford
18842 *) __pyx_t_2)->instance->random(__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_ixt),
18843 ((scalar_t) __pyx_t_3))); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1871, __pyx_L1_error)
18844     __Pyx_GOTREF(__pyx_t_4);
18845     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
18846     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
18847     __pyx_r = __pyx_t_4;
18848     __pyx_t_4 = 0;
18849     goto __pyx_L0;
18850
18851     /* "PyClical.pyx":1864
18852     *         return clifford().wrap( glucat.atanh(toClifford(obj)) )
18853     *
18854     * cpdef inline random_clifford(index_set ixt, fill = 1.0):
18855     *         """
18856     *         Random multivector within a frame.
18857     */
18858
18859     /* function exit code */
18860     __pyx_L1_error;;
18861     __Pyx_XDECREF(__pyx_t_1);
18862     __Pyx_XDECREF(__pyx_t_2);
18863     __Pyx_XDECREF(__pyx_t_4);
18864     __Pyx_AddTraceback("PyClical.random_clifford", __pyx_clineno, __pyx_lineno,
18865 __pyx_filename);
18866     __pyx_r = 0;
18867     __pyx_L0;;
18868     __Pyx_XGIVEREF(__pyx_r);
18869     __Pyx_RefNannyFinishContext();
18870     return __pyx_r;
18871 }
18872
18873 /* Python wrapper */

```

```

18863         static PyObject *__pyx_pw_8PyClical_81random_clifford(PyObject *__pyx_self, PyObject
18864 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
18864         static char __pyx_doc_8PyClical_80random_clifford[] = "\n    Random multivector within
a frame.\n\n    >> print(random_clifford(index_set({-3,-1,2})).frame())\n    {-3,-1,2}\n    ";
18865         static PyObject *__pyx_pw_8PyClical_81random_clifford(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
18866             struct __pyx_obj_8PyClical_index_set *__pyx_v_ixt = 0;
18867             PyObject *__pyx_v_fill = 0;
18868             int __pyx_lineno = 0;
18869             const char *__pyx_filename = NULL;
18870             int __pyx_clineno = 0;
18871             PyObject *__pyx_r = 0;
18872             __Pyx_RefNannyDeclarations
18873             __Pyx_RefNannySetupContext("random_clifford (wrapper)", 0);
18874             {
18875                 static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_ixt,&__pyx_n_s_fill,0};
18876                 PyObject* values[2] = {0,0};
18877                 values[1] = ((PyObject *)__pyx_float_1_0);
18878                 if (unlikely(__pyx_kwds)) {
18879                     Py_ssize_t kw_args;
18880                     const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
18881                     switch (pos_args) {
18882                         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
18883                             CYTHON_FALLTHROUGH;
18884                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18885                             CYTHON_FALLTHROUGH;
18886                         case 0: break;
18887                         default: goto __pyx_L5_argtuple_error;
18888                     }
18889                     kw_args = PyDict_Size(__pyx_kwds);
18890                     switch (pos_args) {
18891                         case 0:
18892                             if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_ixt)) !=
0)) kw_args--;
18893                             else goto __pyx_L5_argtuple_error;
18894                             CYTHON_FALLTHROUGH;
18895                         case 1:
18896                             if (kw_args > 0) {
18897                                 PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_fill);
18898                                 if (value) { values[1] = value; kw_args--; }
18899                             }
18900                             if (unlikely(kw_args > 0)) {
18901                                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
18902 values, pos_args, "random_clifford") < 0)) __PYX_ERR(0, 1864, __pyx_L3_error)
18903                             }
18904                             else {
18905                                 switch (PyTuple_GET_SIZE(__pyx_args)) {
18906                                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
18907   CYTHON_FALLTHROUGH;
18908                                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18909   break;
18910                                     default: goto __pyx_L5_argtuple_error;
18911                                 }
18912                             }
18913                             __pyx_v_ixt = ((struct __pyx_obj_8PyClical_index_set *)values[0]);
18914                             __pyx_v_fill = values[1];
18915                         }
18916                         goto __pyx_L4_argument_unpacking_done;
18917                         __pyx_L5_argtuple_error:;
18918                         __Pyx_RaiseArgtupleInvalid("random_clifford", 0, 1, 2,
PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0, 1864, __pyx_L3_error)
18919                         __pyx_L3_error:;
18920                         __Pyx_AddTraceback("PyClical.random_clifford", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18921                         __Pyx_RefNannyFinishContext();
18922                         return NULL;
18923                         __pyx_L4_argument_unpacking_done:;
18924                         if (unlikely(!__Pyx_ArgTypeTest(((PyObject *)__pyx_v_ixt),
__pyx_ptype_8PyClical_index_set, 1, "ixt", 0))) __PYX_ERR(0, 1864, __pyx_L1_error)
18925                         __pyx_r = __pyx_pf_8PyClical_80random_clifford(__pyx_self, __pyx_v_ixt,
__pyx_v_fill);
18926
18927                         /* function exit code */
18928                         goto __pyx_L0;
18929                         __pyx_L1_error:;
18930                         __pyx_r = NULL;
18931                         __pyx_L0:;
18932                         __Pyx_RefNannyFinishContext();
18933                         return __pyx_r;
18934                     }
18935
18936                 static PyObject *__pyx_pf_8PyClical_80random_clifford(CYTHON_UNUSED PyObject
*__pyx_self, struct __pyx_obj_8PyClical_index_set *__pyx_v_ixt, PyObject *__pyx_v_fill) {
18937                     PyObject *__pyx_r = NULL;
18938                     __Pyx_RefNannyDeclarations
18939                     PyObject *__pyx_t_1 = NULL;

```

```

18940         struct __pyx_opt_args_8PyClical_random_clifford __pyx_t_2;
18941         int __pyx_lineno = 0;
18942         const char *__pyx_filename = NULL;
18943         int __pyx_clineno = 0;
18944         __Pyx_RefNannySetupContext("random_clifford", 0);
18945         __Pyx_XDECREF(__pyx_r);
18946         __pyx_t_2.__pyx_n = 1;
18947         __pyx_t_2.fill = __pyx_v_fill;
18948         __pyx_t_1 = __pyx_f_8PyClical_random_clifford(__pyx_v_ixt, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1864, __pyx_L1_error)
18949         __Pyx_GOTREF(__pyx_t_1);
18950         __pyx_r = __pyx_t_1;
18951         __pyx_t_1 = 0;
18952         goto __pyx_L0;
18953
18954         /* function exit code */
18955         __pyx_L1_error:;
18956         __Pyx_XDECREF(__pyx_t_1);
18957         __Pyx_AddTraceback("PyClical.random_clifford", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18958         __pyx_r = NULL;
18959         __pyx_L0:;
18960         __Pyx_XGIVEREF(__pyx_r);
18961         __Pyx_RefNannyFinishContext();
18962         return __pyx_r;
18963     }
18964
18965     /* "PyClical.pyx":1873
18966     *     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
18967     *
18968     * cpdef inline cga3(obj):
18969     *     """
18970     *     Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
18971     *     definition.
18972     */
18973     static PyObject *__pyx_pw_8PyClical_83cga3(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
18974     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cga3(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
18975         PyObject *__pyx_r = NULL;
18976         __Pyx_RefNannyDeclarations
18977         PyObject *__pyx_t_1 = NULL;
18978         PyObject *__pyx_t_2 = NULL;
18979         int __pyx_lineno = 0;
18980         const char *__pyx_filename = NULL;
18981         int __pyx_clineno = 0;
18982         __Pyx_RefNannySetupContext("cga3", 0);
18983
18984         /* "PyClical.pyx":1880
18985     *     87{-1}+4{1}+18{2}+2{3}+85{4}
18986     *     """
18987     *     return clifford().wrap( glucat.cga3(toClifford(obj)) )
18988     *
18989     * cpdef inline cga3std(obj):
18990     */
18991         __Pyx_XDECREF(__pyx_r);
18992         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford));
18993         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1880, __pyx_L1_error)
18994         __Pyx_GOTREF(__pyx_t_1);
18995         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), cga3::cga3(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1880, __pyx_L1_error)
18996         __Pyx_GOTREF(__pyx_t_2);
18997         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
18998         __pyx_r = __pyx_t_2;
18999         __pyx_t_2 = 0;
19000         goto __pyx_L0;
19001
19002         /* "PyClical.pyx":1873
19003     *     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
19004     *
19005     * cpdef inline cga3(obj):
19006     *     """
19007     *     Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
19008     *     definition.
19009     */
19010         /* function exit code */
19011         __pyx_L1_error:;
19012         __Pyx_XDECREF(__pyx_t_1);
19013         __Pyx_XDECREF(__pyx_t_2);
19014         __Pyx_AddTraceback("PyClical.cga3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19015         __pyx_r = 0;
19016         __pyx_L0:;
19017         __Pyx_XGIVEREF(__pyx_r);
19018         __Pyx_RefNannyFinishContext();

```



```

19018         return __pyx_r;
19019     }
19020
19021     /* Python wrapper */
19022     static PyObject * __pyx_pw_8PyClical_83cga3(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
19023     static char __pyx_doc_8PyClical_82cga3[] = "\n    Convert Euclidean 3D multivector to
Conformal Geometric Algebra using Doran and Lasenby definition.\n\n    >>
x=clifford(\"2{1}+9{2}+{3}\"); print(cga3(x))\n    87{-1}+4{1}+18{2}+2{3}+85{4}\n    ";
19024     static PyObject * __pyx_pf_8PyClical_83cga3(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
19025         PyObject * __pyx_r = 0;
19026         __Pyx_RefNannyDeclarations
19027         __Pyx_RefNannySetupContext("cga3 (wrapper)", 0);
19028         __pyx_r = __pyx_pf_8PyClical_82cga3(__pyx_self, ((PyObject *) __pyx_v_obj));
19029
19030         /* function exit code */
19031         __Pyx_RefNannyFinishContext();
19032         return __pyx_r;
19033     }
19034
19035     static PyObject * __pyx_pf_8PyClical_82cga3(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
19036         PyObject * __pyx_r = NULL;
19037         __Pyx_RefNannyDeclarations
19038         PyObject * __pyx_t_1 = NULL;
19039         int __pyx_lineno = 0;
19040         const char * __pyx_filename = NULL;
19041         int __pyx_clineno = 0;
19042         __Pyx_RefNannySetupContext("cga3", 0);
19043         __Pyx_XDECREF(__pyx_r);
19044         __pyx_t_1 = __pyx_f_8PyClical_cga3(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1873, __pyx_L1_error)
19045         __Pyx_GOTREF(__pyx_t_1);
19046         __pyx_r = __pyx_t_1;
19047         __pyx_t_1 = 0;
19048         goto __pyx_L0;
19049
19050         /* function exit code */
19051         __pyx_L1_error:;
19052         __Pyx_XDECREF(__pyx_t_1);
19053         __Pyx_AddTraceback("PyClical.cga3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19054         __pyx_r = NULL;
19055         __pyx_L0:;
19056         __Pyx_XGIVEREF(__pyx_r);
19057         __Pyx_RefNannyFinishContext();
19058         return __pyx_r;
19059     }
19060
19061     /* "PyClical.pyx":1882
19062 *     return clifford().wrap( glucat.cga3(toClifford(obj)) )
19063 *
19064 * cpdef inline cga3std(obj): # ««««««««
19065 *     """
19066 *     Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
19067 */
19068
19069     static PyObject * __pyx_pw_8PyClical_85cga3std(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
19070     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_cga3std(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
19071         PyObject * __pyx_r = NULL;
19072         __Pyx_RefNannyDeclarations
19073         PyObject * __pyx_t_1 = NULL;
19074         PyObject * __pyx_t_2 = NULL;
19075         int __pyx_lineno = 0;
19076         const char * __pyx_filename = NULL;
19077         int __pyx_clineno = 0;
19078         __Pyx_RefNannySetupContext("cga3std", 0);
19079
19080         /* "PyClical.pyx":1891
19081 *     0
19082 *     """
19083 *     return clifford().wrap( glucat.cga3std(toClifford(obj)) ) # ««««««««
19084 *
19085 * cpdef inline agc3(obj):
19086 */
19087         __Pyx_XDECREF(__pyx_r);
19088         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford));
19089         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1891, __pyx_L1_error)
19090         __Pyx_GOTREF(__pyx_t_1);
19091         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*) __pyx_t_1), cga3::cga3std(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1891, __pyx_L1_error)
19091         __Pyx_GOTREF(__pyx_t_2);
19092         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
19093         __pyx_r = __pyx_t_2;

```

```

19094         __pyx_t_2 = 0;
19095         goto __pyx_L0;
19096
19097         /* "PyClical.pyx":1882
19098  *      return clifford().wrap( glucat.cga3(toClifford(obj)) )
19099  *
19100  * cpdef inline cga3std(obj):                # ««««««««
19101  *      """
19102  *      Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
19103  */
19104
19105         /* function exit code */
19106         __pyx_L1_error++;
19107         __Pyx_XDECREF(__pyx_t_1);
19108         __Pyx_XDECREF(__pyx_t_2);
19109         __Pyx_AddTraceback("PyClical.cga3std", __pyx_clineno, __pyx_lineno, __pyx_filename);
19110         __pyx_r = 0;
19111         __pyx_L0;
19112         __Pyx_XGIVEREF(__pyx_r);
19113         __Pyx_RefNannyFinishContext();
19114         return __pyx_r;
19115     }
19116
19117     /* Python wrapper */
19118     static PyObject * __pyx_pw_8PyClical_85cga3std(PyObject * __pyx_self, PyObject
19119 * __pyx_v_obj); /*proto*/
19120     static char __pyx_doc_8PyClical_84cga3std[] = "\n      Convert CGA3 null vector to
standard conformal null vector using Doran and Lasenby definition.\n\n      >>
x=clifford(\"2{1}+9{2}+{3}\"); print(cga3std(cga3(x)))\n      87{-1}+4{1}+18{2}+2{3}+85{4}\n      >>
x=clifford(\"2{1}+9{2}+{3}\"); print(cga3std(cga3(x))-cga3(x))\n      0\n      ";
19121     static PyObject * __pyx_pf_8PyClical_85cga3std(PyObject * __pyx_self, PyObject
* __pyx_v_obj) {
19122         PyObject * __pyx_r = 0;
19123         __Pyx_RefNannyDeclarations
19124         __Pyx_RefNannySetupContext("cga3std (wrapper)", 0);
19125         __pyx_r = __pyx_pf_8PyClical_84cga3std(__pyx_self, ((PyObject *) __pyx_v_obj));
19126
19127         /* function exit code */
19128         __Pyx_RefNannyFinishContext();
19129         return __pyx_r;
19130     }
19131
19132     static PyObject * __pyx_pf_8PyClical_84cga3std(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
19133         PyObject * __pyx_r = NULL;
19134         __Pyx_RefNannyDeclarations
19135         PyObject * __pyx_t_1 = NULL;
19136         int __pyx_lineno = 0;
19137         const char * __pyx_filename = NULL;
19138         int __pyx_clineno = 0;
19139         __Pyx_RefNannySetupContext("cga3std", 0);
19140         __Pyx_XDECREF(__pyx_r);
19141         __pyx_t_1 = __pyx_f_8PyClical_cga3std(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1882, __pyx_L1_error)
19142         __Pyx_GOTREF(__pyx_t_1);
19143         __pyx_r = __pyx_t_1;
19144         __pyx_t_1 = 0;
19145         goto __pyx_L0;
19146
19147         /* function exit code */
19148         __pyx_L1_error++;
19149         __Pyx_XDECREF(__pyx_t_1);
19150         __Pyx_AddTraceback("PyClical.cga3std", __pyx_clineno, __pyx_lineno, __pyx_filename);
19151         __pyx_r = NULL;
19152         __pyx_L0;
19153         __Pyx_XGIVEREF(__pyx_r);
19154         __Pyx_RefNannyFinishContext();
19155         return __pyx_r;
19156     }
19157
19158     /* "PyClical.pyx":1893
19159  *      return clifford().wrap( glucat.cga3std(toClifford(obj)) )
19160  *
19161  * cpdef inline agc3(obj):                # ««««««««
19162  *      """
19163  *      Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
19164  */
19165
19166     static PyObject * __pyx_pw_8PyClical_87agc3(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
19167     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_agc3(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
19168         PyObject * __pyx_r = NULL;
19169         __Pyx_RefNannyDeclarations
19170         PyObject * __pyx_t_1 = NULL;
19171         PyObject * __pyx_t_2 = NULL;
19172         int __pyx_lineno = 0;

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```

19172         const char *__pyx_filename = NULL;
19173         int __pyx_clineno = 0;
19174         __Pyx_RefNannySetupContext("agc3", 0);
19175
19176         /* "PyClical.pyx":1902
19177         *      0
19178         *      """
19179         *      return clifford().wrap( glucat.agc3(toClifford(obj)) ) # ««««««««
19180         *
19181         * # Some abbreviations.
19182         */
19183         __Pyx_XDECREF(__pyx_r);
19184         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford));
19185         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1902, __pyx_L1_error)
19186         __Pyx_GOTREF(__pyx_t_1);
19187         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
19188         *) __pyx_t_1), cga3::agc3(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
19189         __PYX_ERR(0, 1902, __pyx_L1_error)
19190         __Pyx_GOTREF(__pyx_t_2);
19191         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
19192         __pyx_r = __pyx_t_2;
19193         __pyx_t_2 = 0;
19194         goto __pyx_L0;
19195
19196         /* "PyClical.pyx":1893
19197         *      return clifford().wrap( glucat.cga3std(toClifford(obj)) )
19198         *
19199         * cpdef inline agc3(obj): # ««««««««
19200         *      """
19201         *      Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
19202         */
19203
19204         /* function exit code */
19205         __pyx_L1_error++;
19206         __Pyx_XDECREF(__pyx_t_1);
19207         __Pyx_XDECREF(__pyx_t_2);
19208         __Pyx_AddTraceback("PyClical.agc3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19209         __pyx_r = 0;
19210         __pyx_L0++;
19211         __Pyx_XGIVEREF(__pyx_r);
19212         __Pyx_RefNannyFinishContext();
19213         return __pyx_r;
19214     }
19215
19216     /* Python wrapper */
19217     static PyObject *__pyx_pw_8PyClical_87agc3(PyObject *__pyx_self, PyObject
19218     *__pyx_v_obj); /*proto*/
19219     static char __pyx_doc_8PyClical_86agc3[] = "\n    Convert CGA3 null vector to
19220     Euclidean 3D vector using Doran and Lasenby definition.\n\n    >> x=clifford(\"2{1}+9{2}+{3}\");
19221     print(agc3(cga3(x)))\n    2{1}+9{2}+{3}\n    >> x=clifford(\"2{1}+9{2}+{3}\");
19222     print(agc3(cga3(x))-x)\n    0\n    ";
19223     static PyObject *__pyx_pw_8PyClical_87agc3(PyObject *__pyx_self, PyObject
19224     *__pyx_v_obj) {
19225         PyObject *__pyx_r = 0;
19226         __Pyx_RefNannyDeclarations
19227         __Pyx_RefNannySetupContext("agc3 (wrapper)", 0);
19228         __pyx_r = __pyx_f_8PyClical_86agc3(__pyx_self, ((PyObject *) __pyx_v_obj));
19229
19230         /* function exit code */
19231         __Pyx_RefNannyFinishContext();
19232         return __pyx_r;
19233     }
19234
19235     static PyObject *__pyx_pf_8PyClical_86agc3(CYTHON_UNUSED PyObject *__pyx_self,
19236     PyObject *__pyx_v_obj) {
19237         PyObject *__pyx_r = NULL;
19238         __Pyx_RefNannyDeclarations
19239         PyObject *__pyx_t_1 = NULL;
19240         int __pyx_lineno = 0;
19241         const char *__pyx_filename = NULL;
19242         int __pyx_clineno = 0;
19243         __Pyx_RefNannySetupContext("agc3", 0);
19244         __Pyx_XDECREF(__pyx_r);
19245         __pyx_t_1 = __pyx_f_8PyClical_agc3(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
19246         __PYX_ERR(0, 1893, __pyx_L1_error)
19247         __Pyx_GOTREF(__pyx_t_1);
19248         __pyx_r = __pyx_t_1;
19249         __pyx_t_1 = 0;
19250         goto __pyx_L0;
19251
19252         /* function exit code */
19253         __pyx_L1_error++;
19254         __Pyx_XDECREF(__pyx_t_1);
19255         __Pyx_AddTraceback("PyClical.agc3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19256         __pyx_r = NULL;
19257         __pyx_L0++;
19258         __Pyx_XGIVEREF(__pyx_r);

```

```

19249         __Pyx_RefNannyFinishContext();
19250         return __pyx_r;
19251     }
19252
19253     /* "PyClical.pyx":1936
19254     * """
19255     *
19256     * def e(obj):          # ««««««««
19257     *     """
19258     *     Abbreviation for clifford(index_set(obj)).
19259     */
19260
19261     /* Python wrapper */
19262     static PyObject *__pyx_pw_8PyClical_89e(PyObject *__pyx_self, PyObject *__pyx_v_obj);
19263
19264     /*proto*/
19265     static char __pyx_doc_8PyClical_88e[] = "\n    Abbreviation for
19266 clifford(index_set(obj)).\n\n    >> print(e(1))\n    {1}\n    >> print(e(-1))\n    {-1}\n    >>
19267 print(e(0))\n    1\n    ";
19268     static PyMethodDef __pyx_mdef_8PyClical_89e = {"e",
19269 (PyCFunction) __pyx_pw_8PyClical_89e, METH_O, __pyx_doc_8PyClical_88e};
19270     static PyObject *__pyx_pw_8PyClical_89e(PyObject *__pyx_self, PyObject *__pyx_v_obj) {
19271     PyObject *__pyx_r = 0;
19272     __Pyx_RefNannyDeclarations
19273     __Pyx_RefNannySetupContext("e (wrapper)", 0);
19274     __pyx_r = __pyx_pf_8PyClical_88e(__pyx_self, ((PyObject *) __pyx_v_obj));
19275
19276     /* function exit code */
19277     __Pyx_RefNannyFinishContext();
19278     return __pyx_r;
19279 }
19280
19281     static PyObject *__pyx_pf_8PyClical_88e(CYTHON_UNUSED PyObject *__pyx_self, PyObject
19282 *__pyx_v_obj) {
19283     PyObject *__pyx_r = NULL;
19284     __Pyx_RefNannyDeclarations
19285     PyObject *__pyx_t_1 = NULL;
19286     PyObject *__pyx_t_2 = NULL;
19287     int __pyx_lineno = 0;
19288     const char *__pyx_filename = NULL;
19289     int __pyx_clineno = 0;
19290     __Pyx_RefNannySetupContext("e", 0);
19291
19292     /* "PyClical.pyx":1947
19293     *     1
19294     *     """
19295     *     return clifford(index_set(obj))          # ««««««««
19296     *
19297     * def istpq(p, q):
19298     */
19299     __Pyx_XDECREF(__pyx_r);
19300     __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *) __pyx_ptype_8PyClical_index_set),
19301 __pyx_v_obj); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1947, __pyx_L1_error)
19302     __Pyx_GOTREF(__pyx_t_1);
19303     __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *) __pyx_ptype_8PyClical_clifford),
19304 __pyx_t_1); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1947, __pyx_L1_error)
19305     __Pyx_GOTREF(__pyx_t_2);
19306     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
19307     __pyx_r = __pyx_t_2;
19308     __pyx_t_2 = 0;
19309     goto __pyx_L0;
19310
19311     /* "PyClical.pyx":1936
19312     * """
19313     *
19314     * def e(obj):          # ««««««««
19315     *     """
19316     *     Abbreviation for clifford(index_set(obj)).
19317     */
19318
19319     /* function exit code */
19320     __pyx_L1_error:;
19321     __Pyx_XDECREF(__pyx_t_1);
19322     __Pyx_XDECREF(__pyx_t_2);
19323     __Pyx_AddTraceback("PyClical.e", __pyx_clineno, __pyx_lineno, __pyx_filename);
19324     __pyx_r = NULL;
19325     __pyx_L0:;
19326     __Pyx_XGIVEREF(__pyx_r);
19327     __Pyx_RefNannyFinishContext();
19328     return __pyx_r;
19329 }
19330
19331     /* "PyClical.pyx":1949
19332     *     return clifford(index_set(obj))
19333     *
19334     * def istpq(p, q):          # ««««««««
19335     *     """
19336     *     Abbreviation for index_set({-q,...p}).

```

```

19329  */
19330
19331      /* Python wrapper */
19332      static PyObject * __pyx_pw_8PyClical_9listpq(PyObject * __pyx_self, PyObject
19333  * __pyx_args, PyObject * __pyx_kwds); /*proto*/
19334      static char __pyx_doc_8PyClical_90listpq[] = "\n    Abbreviation for
index_set({-q,...p}).\n\n    >> print(listpq(2,3))\n    {-3,-2,-1,1,2}\n    ";
19335      static PyMethodDef __pyx_mdef_8PyClical_9listpq = {"listpq",
(PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_9listpq, METH_VARARGS|METH_KEYWORDS,
__pyx_doc_8PyClical_90listpq};
19336      static PyObject * __pyx_pw_8PyClical_9listpq(PyObject * __pyx_self, PyObject
19337  * __pyx_args, PyObject * __pyx_kwds) {
19338          PyObject * __pyx_v_p = 0;
19339          PyObject * __pyx_v_q = 0;
19340          int __pyx_lineno = 0;
19341          const char * __pyx_filename = NULL;
19342          int __pyx_clineno = 0;
19343          PyObject * __pyx_r = 0;
19344          __Pyx_RefNannyDeclarations
19345          __Pyx_RefNannySetupContext("listpq (wrapper)", 0);
19346          {
19347              static PyObject ** __pyx_pyargnames[] = {&__pyx_n_s_p, &__pyx_n_s_q, 0};
19348              PyObject* values[2] = {0, 0};
19349              if (unlikely(__pyx_kwds)) {
19350                  Py_ssize_t kw_args;
19351                  const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
19352                  switch (pos_args) {
19353                      case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
19354                      case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
19355                      case 0: break;
19356                      default: goto __pyx_L5_argtuple_error;
19357                  }
19358                  kw_args = PyDict_Size(__pyx_kwds);
19359                  switch (pos_args) {
19360                      case 0:
19361                          if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_p)) !=
0)) kw_args--;
19362                          else goto __pyx_L5_argtuple_error;
19363                          CYTHON_FALLTHROUGH;
19364                      case 1:
19365                          if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_q)) !=
0)) kw_args--;
19366                          else {
19367                              __Pyx_RaiseArgtupleInvalid("listpq", 1, 2, 2, 1); __PYX_ERR(0, 1949,
__pyx_L3_error)
19368                          }
19369                      if (unlikely(kw_args > 0)) {
19370                          if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "listpq") < 0)) __PYX_ERR(0, 1949, __pyx_L3_error)
19371                      }
19372                      } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
19373                          goto __pyx_L5_argtuple_error;
19374                      } else {
19375                          values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
19376                          values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
19377                      }
19378                      __pyx_v_p = values[0];
19379                      __pyx_v_q = values[1];
19380                  }
19381                  goto __pyx_L4_argument_unpacking_done;
19382                  __pyx_L5_argtuple_error:;
19383                  __Pyx_RaiseArgtupleInvalid("listpq", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args));
19384                  __PYX_ERR(0, 1949, __pyx_L3_error)
19385                  __pyx_L3_error:;
19386                  __Pyx_AddTraceback("PyClical.listpq", __pyx_clineno, __pyx_lineno, __pyx_filename);
19387                  __Pyx_RefNannyFinishContext();
19388                  return NULL;
19389                  __pyx_L4_argument_unpacking_done:;
19390                  __pyx_r = __pyx_pf_8PyClical_90listpq(__pyx_self, __pyx_v_p, __pyx_v_q);
19391
19392                  /* function exit code */
19393                  __Pyx_RefNannyFinishContext();
19394                  return __pyx_r;
19395              }
19396
19397      static PyObject * __pyx_pf_8PyClical_90listpq(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_p, PyObject * __pyx_v_q) {
19398          PyObject * __pyx_r = NULL;
19399          __Pyx_RefNannyDeclarations
19400          PyObject * __pyx_t_1 = NULL;
19401          PyObject * __pyx_t_2 = NULL;
19402          PyObject * __pyx_t_3 = NULL;
19403          int __pyx_lineno = 0;
19404          const char * __pyx_filename = NULL;

```

```

19405         int __pyx_clineno = 0;
19406         __Pyx_RefNannySetupContext("istpq", 0);
19407
19408         /* "PyClical.pyx":1956
19409         *      {-3,-2,-1,1,2}
19410         *      """
19411         *      return index_set(set(range(-q,p+1)))          # ««««««««
19412         *
19413         *      ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
19414         */
19415         __Pyx_XDECREF(__pyx_r);
19416         __pyx_t_1 = PyNumber_Negative(__pyx_v_q); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
19417 1956, __pyx_L1_error)
19418         __Pyx_GOTREF(__pyx_t_1);
19419         __pyx_t_2 = __Pyx_PyInt_AddObjC(__pyx_v_p, __pyx_int_1, 1, 0, 0); if
19420 (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1956, __pyx_L1_error)
19421         __Pyx_GOTREF(__pyx_t_2);
19422         __pyx_t_3 = PyTuple_New(2); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1956,
19423 __pyx_L1_error)
19424         __Pyx_GOTREF(__pyx_t_3);
19425         __Pyx_GIVEREF(__pyx_t_1);
19426         PyTuple_SET_ITEM(__pyx_t_3, 0, __pyx_t_1);
19427         __Pyx_GIVEREF(__pyx_t_2);
19428         PyTuple_SET_ITEM(__pyx_t_3, 1, __pyx_t_2);
19429         __pyx_t_1 = 0;
19430         __pyx_t_2 = 0;
19431         __pyx_t_2 = __Pyx_PyObject_Call(__pyx_builtin_range, __pyx_t_3, NULL); if
19432 (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1956, __pyx_L1_error)
19433         __Pyx_GOTREF(__pyx_t_2);
19434         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
19435         __pyx_t_3 = PySet_New(__pyx_t_2); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1956,
19436 __pyx_L1_error)
19437         __Pyx_GOTREF(__pyx_t_3);
19438         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
19439         __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_index_set),
19440 __pyx_t_3); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1956, __pyx_L1_error)
19441         __Pyx_GOTREF(__pyx_t_2);
19442         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
19443         __pyx_r = __pyx_t_2;
19444         __pyx_t_2 = 0;
19445         goto __pyx_L0;
19446
19447         /* "PyClical.pyx":1949
19448         *      return clifford(index_set(obj))
19449         *
19450         *      def istpq(p, q):          # ««««««««
19451         *      """
19452         *      Abbreviation for index_set({-q,...p}).
19453         */
19454
19455         /* function exit code */
19456         __pyx_L1_error;
19457         __Pyx_XDECREF(__pyx_t_1);
19458         __Pyx_XDECREF(__pyx_t_2);
19459         __Pyx_XDECREF(__pyx_t_3);
19460         __Pyx_AddTraceback("PyClical.istpq", __pyx_clineno, __pyx_lineno, __pyx_filename);
19461         __pyx_r = NULL;
19462         __pyx_L0;
19463         __Pyx_XGIVEREF(__pyx_r);
19464         __Pyx_RefNannyFinishContext();
19465         return __pyx_r;
19466     }
19467
19468     /* "PyClical.pyx":1962
19469     *
19470     * # Doctest interface.
19471     * def _test():          # ««««««««
19472     *     import PyClical, doctest
19473     *     return doctest.testmod(PyClical)
19474     */
19475
19476     /* Python wrapper */
19477     static PyObject * __pyx_pw_8PyClical_93_test(PyObject * __pyx_self, CYTHON_UNUSED
19478 PyObject *unused); /*proto*/
19479     static PyMethodDef __pyx_mdef_8PyClical_93_test = {"_test",
19480 (PyCFunction) __pyx_pw_8PyClical_93_test, METH_NOARGS, 0};
19481     static PyObject * __pyx_pw_8PyClical_93_test(PyObject * __pyx_self, CYTHON_UNUSED
19482 PyObject *unused) {
19483         PyObject * __pyx_r = 0;
19484         __Pyx_RefNannyDeclarations
19485         __Pyx_RefNannySetupContext("_test (wrapper)", 0);
19486         __pyx_r = __pyx_pf_8PyClical_92_test(__pyx_self);
19487
19488         /* function exit code */
19489         __Pyx_RefNannyFinishContext();
19490         return __pyx_r;
19491     }

```

```

19483
19484 static PyObject *__pyx_pf_8PyClical_92_test(CYTHON_UNUSED PyObject *__pyx_self) {
19485     PyObject *__pyx_v_PyClical = NULL;
19486     PyObject *__pyx_v_doctest = NULL;
19487     PyObject *__pyx_r = NULL;
19488     __Pyx_RefNannyDeclarations
19489     PyObject *__pyx_t_1 = NULL;
19490     PyObject *__pyx_t_2 = NULL;
19491     PyObject *__pyx_t_3 = NULL;
19492     int __pyx_lineno = 0;
19493     const char *__pyx_filename = NULL;
19494     int __pyx_clineno = 0;
19495     __Pyx_RefNannySetupContext("_test", 0);
19496
19497     /* "PyClical.pyx":1963
19498  * # Doctest interface.
19499  * def _test():
19500  *     import PyClical, doctest                # ««««««««
19501  *     return doctest.testmod(PyClical)
19502  *
19503  */
19504     __pyx_t_1 = __Pyx_Import(__pyx_n_s_PyClical, 0, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1963, __pyx_L1_error)
19505     __Pyx_GOTREF(__pyx_t_1);
19506     __pyx_v_PyClical = __pyx_t_1;
19507     __pyx_t_1 = 0;
19508     __pyx_t_1 = __Pyx_Import(__pyx_n_s_doctest, 0, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1963, __pyx_L1_error)
19509     __Pyx_GOTREF(__pyx_t_1);
19510     __pyx_v_doctest = __pyx_t_1;
19511     __pyx_t_1 = 0;
19512
19513     /* "PyClical.pyx":1964
19514  * def _test():
19515  *     import PyClical, doctest
19516  *     return doctest.testmod(PyClical)                # ««««««««
19517  *
19518  * if __name__ == "__main__":
19519  */
19520     __Pyx_XDECREF(__pyx_r);
19521     __pyx_t_2 = __Pyx_PyObject_GetAttrStr(__pyx_v_doctest, __pyx_n_s_testmod); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 1964, __pyx_L1_error)
19522     __Pyx_GOTREF(__pyx_t_2);
19523     __pyx_t_3 = NULL;
19524     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_2))) {
19525         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_2);
19526         if (likely(__pyx_t_3)) {
19527             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_2);
19528             __Pyx_INCREF(__pyx_t_3);
19529             __Pyx_INCREF(function);
19530             __Pyx_DECREF_SET(__pyx_t_2, function);
19531         }
19532     }
19533     __pyx_t_1 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_2, __pyx_t_3,
__pyx_v_PyClical) : __Pyx_PyObject_CallOneArg(__pyx_t_2, __pyx_v_PyClical);
19534     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
19535     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1964, __pyx_L1_error)
19536     __Pyx_GOTREF(__pyx_t_1);
19537     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
19538     __pyx_r = __pyx_t_1;
19539     __pyx_t_1 = 0;
19540     goto __pyx_L0;
19541
19542     /* "PyClical.pyx":1962
19543  *
19544  * # Doctest interface.
19545  * def _test():                # ««««««««
19546  *     import PyClical, doctest
19547  *     return doctest.testmod(PyClical)
19548  */
19549
19550     /* function exit code */
19551     __pyx_L1_error:;
19552     __Pyx_XDECREF(__pyx_t_1);
19553     __Pyx_XDECREF(__pyx_t_2);
19554     __Pyx_XDECREF(__pyx_t_3);
19555     __Pyx_AddTraceback("PyClical._test", __pyx_clineno, __pyx_lineno, __pyx_filename);
19556     __pyx_r = NULL;
19557     __pyx_L0:;
19558     __Pyx_XDECREF(__pyx_v_PyClical);
19559     __Pyx_XDECREF(__pyx_v_doctest);
19560     __Pyx_XGIVEREF(__pyx_r);
19561     __Pyx_RefNannyFinishContext();
19562     return __pyx_r;
19563 }
19564
19565     /* "string.to_py":31

```

```

19566 *
19567 * @cname("__pyx_convert_PyObject_string_to_py_std__in_string")
19568 * cdef inline object __pyx_convert_PyObject_string_to_py_std__in_string(const string& s):
19569 # ««««««««
19569 *     return __Pyx_PyObject_FromStringAndSize(s.data(), s.size())
19570 * cdef extern from *:
19571 */
19572
19573     static CYTHON_INLINE PyObject
19574 *__pyx_convert_PyObject_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19574     PyObject *__pyx_r = NULL;
19575     __Pyx_RefNannyDeclarations
19576     PyObject *__pyx_t_1 = NULL;
19577     int __pyx_lineno = 0;
19578     const char *__pyx_filename = NULL;
19579     int __pyx_clineno = 0;
19580     __Pyx_RefNannySetupContext("__pyx_convert_PyObject_string_to_py_std__in_string", 0);
19581
19582     /* "string.to_py":32
19583 * @cname("__pyx_convert_PyObject_string_to_py_std__in_string")
19584 * cdef inline object __pyx_convert_PyObject_string_to_py_std__in_string(const string& s):
19585 *     return __Pyx_PyObject_FromStringAndSize(s.data(), s.size()) # ««««««««
19586 * cdef extern from *:
19587 *     cdef object __Pyx_PyUnicode_FromStringAndSize(const char*, size_t)
19588 */
19589     __Pyx_XDECREF(__pyx_r);
19590     __pyx_t_1 = __Pyx_PyObject_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size()); if
19591 (unlikely(!__pyx_t_1)) __PYX_ERR(1, 32, __pyx_L1_error)
19592     __Pyx_GOTREF(__pyx_t_1);
19593     __pyx_r = __pyx_t_1;
19594     __pyx_t_1 = 0;
19595     goto __pyx_L0;
19596
19597     /* "string.to_py":31
19598 * @cname("__pyx_convert_PyObject_string_to_py_std__in_string")
19599 * cdef inline object __pyx_convert_PyObject_string_to_py_std__in_string(const string& s):
19600 # ««««««««
19600 *     return __Pyx_PyObject_FromStringAndSize(s.data(), s.size())
19601 * cdef extern from *:
19602 */
19603
19604     /* function exit code */
19605     __pyx_L1_error;
19606     __Pyx_XDECREF(__pyx_t_1);
19607
19608     __Pyx_AddTraceback("string.to_py.__pyx_convert_PyObject_string_to_py_std__in_string", __pyx_clineno,
19609 __pyx_lineno, __pyx_filename);
19610     __pyx_r = 0;
19611     __pyx_L0;
19612     __Pyx_XGIVEREF(__pyx_r);
19613     __Pyx_RefNannyFinishContext();
19614     return __pyx_r;
19615 }
19616
19617     /* "string.to_py":37
19618 * @cname("__pyx_convert_PyUnicode_string_to_py_std__in_string")
19619 * cdef inline object __pyx_convert_PyUnicode_string_to_py_std__in_string(const string& s):
19620 # ««««««««
19620 *     return __Pyx_PyUnicode_FromStringAndSize(s.data(), s.size())
19621 * cdef extern from *:
19622 */
19623
19624     static CYTHON_INLINE PyObject
19625 *__pyx_convert_PyUnicode_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19625     PyObject *__pyx_r = NULL;
19626     __Pyx_RefNannyDeclarations
19627     PyObject *__pyx_t_1 = NULL;
19628     int __pyx_lineno = 0;
19629     const char *__pyx_filename = NULL;
19630     int __pyx_clineno = 0;
19631     __Pyx_RefNannySetupContext("__pyx_convert_PyUnicode_string_to_py_std__in_string",
19632 0);
19633
19634     /* "string.to_py":38
19635 * @cname("__pyx_convert_PyUnicode_string_to_py_std__in_string")
19636 * cdef inline object __pyx_convert_PyUnicode_string_to_py_std__in_string(const string& s):
19637 *     return __Pyx_PyUnicode_FromStringAndSize(s.data(), s.size()) # ««««««««
19638 * cdef extern from *:
19639 *     cdef object __Pyx_PyStr_FromStringAndSize(const char*, size_t)
19640 */
19641     __Pyx_XDECREF(__pyx_r);
19642     __pyx_t_1 = __Pyx_PyUnicode_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size());
19643 if (unlikely(!__pyx_t_1)) __PYX_ERR(1, 38, __pyx_L1_error)
19644     __Pyx_GOTREF(__pyx_t_1);
19645     __pyx_r = __pyx_t_1;

```



```

19643         __pyx_t_1 = 0;
19644         goto __pyx_L0;
19645
19646         /* "string.to_py":37
19647  *
19648  * @cname("__pyx_convert_PyUnicode_string_to_py_std__in_string")
19649  * cdef inline object __pyx_convert_PyUnicode_string_to_py_std__in_string(const string& s):
19650  *     return __Pyx_PyUnicode_FromStringAndSize(s.data(), s.size())
19651  * cdef extern from *:
19652  */
19653
19654         /* function exit code */
19655         __pyx_L1_error++;
19656         __Pyx_XDECREF(__pyx_t_1);
19657
19658         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyUnicode_string_to_py_std__in_string", __pyx_clineno,
19659         __pyx_lineno, __pyx_filename);
19660         __pyx_r = 0;
19661         __pyx_L0;
19662         __Pyx_XGIVEREF(__pyx_r);
19663         __Pyx_RefNannyFinishContext();
19664         return __pyx_r;
19665     }
19666
19667     /* "string.to_py":43
19668  *
19669  * @cname("__pyx_convert_PyStr_string_to_py_std__in_string")
19670  * cdef inline object __pyx_convert_PyStr_string_to_py_std__in_string(const string& s):
19671  *     return __Pyx_PyStr_FromStringAndSize(s.data(), s.size())
19672  * cdef extern from *:
19673  */
19674
19675     static CYTHON_INLINE PyObject
19676     *__pyx_convert_PyStr_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19677         PyObject *__pyx_r = NULL;
19678         __Pyx_RefNannyDeclarations
19679         PyObject *__pyx_t_1 = NULL;
19680         int __pyx_lineno = 0;
19681         const char *__pyx_filename = NULL;
19682         int __pyx_clineno = 0;
19683         __Pyx_RefNannySetupContext("__pyx_convert_PyStr_string_to_py_std__in_string", 0);
19684
19685         /* "string.to_py":44
19686  * @cname("__pyx_convert_PyStr_string_to_py_std__in_string")
19687  * cdef inline object __pyx_convert_PyStr_string_to_py_std__in_string(const string& s):
19688  *     return __Pyx_PyStr_FromStringAndSize(s.data(), s.size())
19689  * cdef extern from *:
19690  *     cdef object __Pyx_PyBytes_FromStringAndSize(const char*, size_t)
19691  */
19692         __Pyx_XDECREF(__pyx_r);
19693         __pyx_t_1 = __Pyx_PyStr_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size()); if
19694         (unlikely(!__pyx_t_1)) __PYX_ERR(1, 44, __pyx_L1_error)
19695         __Pyx_GOTREF(__pyx_t_1);
19696         __pyx_r = __pyx_t_1;
19697         __pyx_t_1 = 0;
19698         goto __pyx_L0;
19699
19700         /* "string.to_py":43
19701  *
19702  * @cname("__pyx_convert_PyStr_string_to_py_std__in_string")
19703  * cdef inline object __pyx_convert_PyStr_string_to_py_std__in_string(const string& s):
19704  *     return __Pyx_PyStr_FromStringAndSize(s.data(), s.size())
19705  * cdef extern from *:
19706  */
19707
19708         /* function exit code */
19709         __pyx_L1_error++;
19710         __Pyx_XDECREF(__pyx_t_1);
19711         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyStr_string_to_py_std__in_string",
19712         __pyx_clineno, __pyx_lineno, __pyx_filename);
19713         __pyx_r = 0;
19714         __pyx_L0;
19715         __Pyx_XGIVEREF(__pyx_r);
19716         __Pyx_RefNannyFinishContext();
19717         return __pyx_r;
19718     }
19719
19720     /* "string.to_py":49
19721  *
19722  * @cname("__pyx_convert_PyBytes_string_to_py_std__in_string")
19723  * cdef inline object __pyx_convert_PyBytes_string_to_py_std__in_string(const string& s):
19724  *     return __Pyx_PyBytes_FromStringAndSize(s.data(), s.size())
19725  * cdef extern from *:

```

```

19721 */
19722
19723     static CYTHON_INLINE PyObject
19724 *__pyx_convert_PyBytes_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19725     PyObject *__pyx_r = NULL;
19726     __Pyx_RefNannyDeclarations
19727     PyObject *__pyx_t_1 = NULL;
19728     int __pyx_lineno = 0;
19729     const char *__pyx_filename = NULL;
19730     int __pyx_clineno = 0;
19731     __Pyx_RefNannySetupContext("__pyx_convert_PyBytes_string_to_py_std__in_string", 0);
19732
19733     /* "string.to_py":50
19734 * @cname("__pyx_convert_PyBytes_string_to_py_std__in_string")
19735 * cdef inline object __pyx_convert_PyBytes_string_to_py_std__in_string(const string& s):
19736 *     return __Pyx_PyBytes_FromStringAndSize(s.data(), s.size()) # <<<<<<<<
19737 * cdef extern from *:
19738 *     cdef object __Pyx_PyByteArray_FromStringAndSize(const char*, size_t)
19739 */
19740     __Pyx_XDECREF(__pyx_r);
19741     __pyx_t_1 = __Pyx_PyBytes_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size()); if
19742 (unlikely(!__pyx_t_1)) __PYX_ERR(1, 50, __pyx_L1_error)
19743     __Pyx_GOTREF(__pyx_t_1);
19744     __pyx_r = __pyx_t_1;
19745     __pyx_t_1 = 0;
19746     goto __pyx_L0;
19747
19748     /* "string.to_py":49
19749 * @cname("__pyx_convert_PyBytes_string_to_py_std__in_string")
19750 * cdef inline object __pyx_convert_PyBytes_string_to_py_std__in_string(const string& s):
19751 *     return __Pyx_PyBytes_FromStringAndSize(s.data(), s.size())
19752 * cdef extern from *:
19753 */
19754     /* function exit code */
19755     __pyx_L1_error:;
19756     __Pyx_XDECREF(__pyx_t_1);
19757     __Pyx_AddTraceback("string.to_py.__pyx_convert_PyBytes_string_to_py_std__in_string",
19758 __pyx_clineno, __pyx_lineno, __pyx_filename);
19759     __pyx_r = 0;
19760     __pyx_L0:;
19761     __Pyx_XGIVEREF(__pyx_r);
19762     __Pyx_RefNannyFinishContext();
19763     return __pyx_r;
19764 }
19765
19766     /* "string.to_py":55
19767 * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
19768 * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
19769 *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
19770 *
19771 */
19772
19773     static CYTHON_INLINE PyObject
19774 *__pyx_convert_PyByteArray_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19775     PyObject *__pyx_r = NULL;
19776     __Pyx_RefNannyDeclarations
19777     PyObject *__pyx_t_1 = NULL;
19778     int __pyx_lineno = 0;
19779     const char *__pyx_filename = NULL;
19780     int __pyx_clineno = 0;
19781     __Pyx_RefNannySetupContext("__pyx_convert_PyByteArray_string_to_py_std__in_string",
19782 0);
19783
19784     /* "string.to_py":56
19785 * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
19786 * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
19787 *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size()) # <<<<<<<<
19788 */
19789     __Pyx_XDECREF(__pyx_r);
19790     __pyx_t_1 = __Pyx_PyByteArray_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size());
19791 if (unlikely(!__pyx_t_1)) __PYX_ERR(1, 56, __pyx_L1_error)
19792     __Pyx_GOTREF(__pyx_t_1);
19793     __pyx_r = __pyx_t_1;
19794     __pyx_t_1 = 0;
19795     goto __pyx_L0;
19796
19797     /* "string.to_py":55
19798 * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
19799 * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
19800 *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
19801 *
19802 */

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```

19799 *      return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
19800 *
19801 */
19802
19803         /* function exit code */
19804         __pyx_ll_error;
19805         __Pyx_XDECREF(__pyx_t_1);
19806
19807         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyByteArray_string_to_py_std__in_string",
19808         __pyx_clineno, __pyx_lineno, __pyx_filename);
19809         __pyx_r = 0;
19810         __pyx_L0:;
19811         __Pyx_XGIVEREF(__pyx_r);
19812         __Pyx_RefNannyFinishContext();
19813         return __pyx_r;
19814
19815 static struct __pyx_vtabstruct_8PyClical_index_set __pyx_vtable_8PyClical_index_set;
19816
19817 static PyObject* __pyx_tp_new_8PyClical_index_set(PyTypeObject *t, PyObject *a,
19818 PyObject *k) {
19819     struct __pyx_obj_8PyClical_index_set *p;
19820     PyObject *o;
19821     if (likely((t->tp_flags & Py_TPFLAGS_IS_ABSTRACT) == 0)) {
19822         o = (*t->tp_alloc)(t, 0);
19823     } else {
19824         o = (PyObject *) PyBaseObject_Type.tp_new(t, __pyx_empty_tuple, 0);
19825     }
19826     if (unlikely(!o)) return 0;
19827     p = ((struct __pyx_obj_8PyClical_index_set *)o);
19828     p->__pyx_vtab = __pyx_vtabptr_8PyClical_index_set;
19829     if (unlikely(__pyx_pw_8PyClical_9index_set_3__cinit__(o, a, k) < 0)) goto bad;
19830     return o;
19831 bad:
19832     Py_DECREF(o); o = 0;
19833     return NULL;
19834 }
19835
19836 static void __pyx_tp_dealloc_8PyClical_index_set(PyObject *o) {
19837     #if CYTHON_USE_TP_FINALIZE
19838     if (unlikely(PyType_HasFeature(Py_TYPE(o), Py_TPFLAGS_HAVE_FINALIZE) &&
19839     Py_TYPE(o)->tp_finalize) && (!PyType_IS_GC(Py_TYPE(o)) || !_PyGC_FINALIZED(o))) {
19840         if (PyObject_CallFinalizerFromDealloc(o)) return;
19841     }
19842     #endif
19843     {
19844         PyObject *etype, *eval, *etb;
19845         PyErr_Fetch(&etype, &eval, &etb);
19846         __Pyx_SET_REFCNT(o, Py_REFCNT(o) + 1);
19847         __pyx_pw_8PyClical_9index_set_5__dealloc__(o);
19848         __Pyx_SET_REFCNT(o, Py_REFCNT(o) - 1);
19849         PyErr_Restore(etype, eval, etb);
19850     }
19851     (*Py_TYPE(o)->tp_free)(o);
19852 }
19853
19854 static PyObject* __pyx_sq_item_8PyClical_index_set(PyObject *o, Py_ssize_t i) {
19855     PyObject *r;
19856     PyObject *x = PyInt_FromSsize_t(i); if (!x) return 0;
19857     r = Py_TYPE(o)->tp_as_mapping->mp_subscript(o, x);
19858     Py_DECREF(x);
19859     return r;
19860 }
19861
19862 static int __pyx_mp_ass_subscript_8PyClical_index_set(PyObject *o, PyObject *i,
19863 PyObject *v) {
19864     if (v) {
19865         return __pyx_pw_8PyClical_9index_set_9__setitem__(o, i, v);
19866     }
19867     else {
19868         PyErr_Format(PyExc_NotImplementedError,
19869         "Subscript deletion not supported by %.200s", Py_TYPE(o)->tp_name);
19870         return -1;
19871     }
19872 }
19873
19874 static PyMethodDef __pyx_methods_8PyClical_index_set[] = {
19875     {"copy", (PyCFunction) __pyx_pw_8PyClical_9index_set_1copy, METH_NOARGS,
19876     __pyx_doc_8PyClical_9index_set_copy},
19877     {"count", (PyCFunction) __pyx_pw_8PyClical_9index_set_32count, METH_NOARGS,
19878     __pyx_doc_8PyClical_9index_set_31count},
19879     {"count_neg", (PyCFunction) __pyx_pw_8PyClical_9index_set_34count_neg, METH_NOARGS,
19880     __pyx_doc_8PyClical_9index_set_33count_neg},
19881     {"count_pos", (PyCFunction) __pyx_pw_8PyClical_9index_set_36count_pos, METH_NOARGS,
19882     __pyx_doc_8PyClical_9index_set_35count_pos},
19883     {"min", (PyCFunction) __pyx_pw_8PyClical_9index_set_38min, METH_NOARGS,
19884     __pyx_doc_8PyClical_9index_set_37min},
19885     {"max", (PyCFunction) __pyx_pw_8PyClical_9index_set_40max, METH_NOARGS,
19886     __pyx_doc_8PyClical_9index_set_39max},

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19875         {"hash_fn", (PyCFunction) __pyx_pw_8PyClical_9index_set_42hash_fn, METH_NOARGS,
__pyx_doc_8PyClical_9index_set_41hash_fn},
19876         {"sign_of_mult", (PyCFunction) __pyx_pw_8PyClical_9index_set_44sign_of_mult, METH_O,
__pyx_doc_8PyClical_9index_set_43sign_of_mult},
19877         {"sign_of_square", (PyCFunction) __pyx_pw_8PyClical_9index_set_46sign_of_square,
METH_NOARGS, __pyx_doc_8PyClical_9index_set_45sign_of_square},
19878         {"__reduce_cython__",
(PyCFunction) __pyx_pw_8PyClical_9index_set_52__reduce_cython__, METH_NOARGS, 0},
19879         {"__setstate_cython__",
(PyCFunction) __pyx_pw_8PyClical_9index_set_54__setstate_cython__, METH_O, 0},
19880         {0, 0, 0, 0}
19881     };
19882
19883     static PyNumberMethods __pyx_tp_as_number_index_set = {
19884         0, /*nb_add*/
19885         0, /*nb_subtract*/
19886         0, /*nb_multiply*/
19887         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19888         0, /*nb_divide*/
19889         #endif
19890         0, /*nb_remainder*/
19891         0, /*nb_divmod*/
19892         0, /*nb_power*/
19893         0, /*nb_negative*/
19894         0, /*nb_positive*/
19895         0, /*nb_absolute*/
19896         0, /*nb_nonzero*/
19897         __pyx_pw_8PyClical_9index_set_18__invert__, /*nb_invert*/
19898         0, /*nb_lshift*/
19899         0, /*nb_rshift*/
19900         __pyx_pw_8PyClical_9index_set_24__and__, /*nb_and*/
19901         __pyx_pw_8PyClical_9index_set_20__xor__, /*nb_xor*/
19902         __pyx_pw_8PyClical_9index_set_28__or__, /*nb_or*/
19903         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19904         0, /*nb_coerce*/
19905         #endif
19906         0, /*nb_int*/
19907         #if PY_MAJOR_VERSION < 3
19908         0, /*nb_long*/
19909         #else
19910         0, /*reserved*/
19911         #endif
19912         0, /*nb_float*/
19913         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19914         0, /*nb_oct*/
19915         #endif
19916         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19917         0, /*nb_hex*/
19918         #endif
19919         0, /*nb_inplace_add*/
19920         0, /*nb_inplace_subtract*/
19921         0, /*nb_inplace_multiply*/
19922         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19923         0, /*nb_inplace_divide*/
19924         #endif
19925         0, /*nb_inplace_remainder*/
19926         0, /*nb_inplace_power*/
19927         0, /*nb_inplace_lshift*/
19928         0, /*nb_inplace_rshift*/
19929         __pyx_pw_8PyClical_9index_set_26__iand__, /*nb_inplace_and*/
19930         __pyx_pw_8PyClical_9index_set_22__ixor__, /*nb_inplace_xor*/
19931         __pyx_pw_8PyClical_9index_set_30__ior__, /*nb_inplace_or*/
19932         0, /*nb_floor_divide*/
19933         0, /*nb_true_divide*/
19934         0, /*nb_inplace_floor_divide*/
19935         0, /*nb_inplace_true_divide*/
19936         0, /*nb_index*/
19937         #if PY_VERSION_HEX >= 0x03050000
19938         0, /*nb_matrix_multiply*/
19939         #endif
19940         #if PY_VERSION_HEX >= 0x03050000
19941         0, /*nb_inplace_matrix_multiply*/
19942         #endif
19943     };
19944
19945     static PySequenceMethods __pyx_tp_as_sequence_index_set = {
19946         0, /*sq_length*/
19947         0, /*sq_concat*/
19948         0, /*sq_repeat*/
19949         __pyx_sq_item_8PyClical_index_set, /*sq_item*/
19950         0, /*sq_slice*/
19951         0, /*sq_ass_item*/

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19952         0, /*sq_ass_slice*/
19953         __pyx_pw_8PyClical_9index_set_13__contains__, /*sq_contains*/
19954         0, /*sq_inplace_concat*/
19955         0, /*sq_inplace_repeat*/
19956     };
19957
19958     static PyMappingMethods __pyx_tp_as_mapping_index_set = {
19959         0, /*mp_length*/
19960         __pyx_pw_8PyClical_9index_set_11__getitem__, /*mp_subscript*/
19961         __pyx_mp_ass_subscript_8PyClical_index_set, /*mp_ass_subscript*/
19962     };
19963
19964     static PyTypeObject __pyx_type_8PyClical_index_set = {
19965         PyVarObject_HEAD_INIT(0, 0)
19966         "PyClical.index_set", /*tp_name*/
19967         sizeof(struct __pyx_obj_8PyClical_index_set), /*tp_basicsize*/
19968         0, /*tp_itemsize*/
19969         __pyx_tp_dealloc_8PyClical_index_set, /*tp_dealloc*/
19970         #if PY_VERSION_HEX < 0x030800b4
19971         0, /*tp_print*/
19972         #endif
19973         #if PY_VERSION_HEX >= 0x030800b4
19974         0, /*tp_vectorcall_offset*/
19975         #endif
19976         0, /*tp_getattr*/
19977         0, /*tp_setattr*/
19978         #if PY_MAJOR_VERSION < 3
19979         0, /*tp_compare*/
19980         #endif
19981         #if PY_MAJOR_VERSION >= 3
19982         0, /*tp_as_async*/
19983         #endif
19984         __pyx_pw_8PyClical_9index_set_48__repr__, /*tp_repr*/
19985         &__pyx_tp_as_number_index_set, /*tp_as_number*/
19986         &__pyx_tp_as_sequence_index_set, /*tp_as_sequence*/
19987         &__pyx_tp_as_mapping_index_set, /*tp_as_mapping*/
19988         0, /*tp_hash*/
19989         0, /*tp_call*/
19990         __pyx_pw_8PyClical_9index_set_50__str__, /*tp_str*/
19991         0, /*tp_getattro*/
19992         0, /*tp_setattro*/
19993         0, /*tp_as_buffer*/
19994
19995         Py_TPFLAGS_DEFAULT|Py_TPFLAGS_HAVE_VERSION_TAG|Py_TPFLAGS_CHECKTYPES|Py_TPFLAGS_HAVE_NEWBUFFER|Py_TPFLAGS_BASETYPE,
19996         /*tp_flags*/
19997         "\n    Python class index_set wraps C++ class IndexSet.\n    ", /*tp_doc*/
19998         0, /*tp_traverse*/
19999         0, /*tp_clear*/
20000         __pyx_pw_8PyClical_9index_set_7__richcmp__, /*tp_richcompare*/
20001         0, /*tp_weaklistoffset*/
20002         __pyx_pw_8PyClical_9index_set_15__iter__, /*tp_iter*/
20003         0, /*tp_iternext*/
20004         __pyx_methods_8PyClical_index_set, /*tp_methods*/
20005         0, /*tp_members*/
20006         0, /*tp_getset*/
20007         0, /*tp_base*/
20008         0, /*tp_dict*/
20009         0, /*tp_descr_get*/
20010         0, /*tp_descr_set*/
20011         0, /*tp_dictoffset*/
20012         0, /*tp_init*/
20013         0, /*tp_alloc*/
20014         __pyx_tp_new_8PyClical_index_set, /*tp_new*/
20015         0, /*tp_free*/
20016         0, /*tp_is_gc*/
20017         0, /*tp_bases*/
20018         0, /*tp_mro*/
20019         0, /*tp_cache*/
20020         0, /*tp_subclasses*/
20021         0, /*tp_weaklist*/
20022         0, /*tp_del*/
20023         0, /*tp_version_tag*/
20024         #if PY_VERSION_HEX >= 0x030400a1
20025         0, /*tp_finalize*/
20026         #endif
20027         #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM
20028         >= 0x07030800)
20029         0, /*tp_vectorcall*/
20030         #endif
20031         #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
20032         0, /*tp_print*/
20033         #endif
20034         #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
20035         0, /*tp_pypy_flags*/
20036         #endif
20037     };
20038     static struct __pyx_vtabstruct_8PyClical_clifford __pyx_vtable_8PyClical_clifford;

```

```

20036
20037     static PyObject * __pyx_tp_new_8PyClical_clifford(PyTypeObject *t, PyObject *a,
PyObject *k) {
20038         struct __pyx_obj_8PyClical_clifford *p;
20039         PyObject *o;
20040         if (likely((t->tp_flags & Py_TPFLAGS_IS_ABSTRACT) == 0)) {
20041             o = (*t->tp_alloc)(t, 0);
20042         } else {
20043             o = (PyObject *) PyBaseObject_Type.tp_new(t, __pyx_empty_tuple, 0);
20044         }
20045         if (unlikely(!o)) return 0;
20046         p = ((struct __pyx_obj_8PyClical_clifford *)o);
20047         p->__pyx_vtab = __pyx_vtabptr_8PyClical_clifford;
20048         if (unlikely(__pyx_pw_8PyClical_8clifford_3__cinit__(o, a, k) < 0)) goto bad;
20049         return o;
20050     bad:
20051         Py_DECREF(o); o = 0;
20052         return NULL;
20053     }
20054
20055     static void __pyx_tp_dealloc_8PyClical_clifford(PyObject *o) {
20056         #if CYTHON_USE_TP_FINALIZE
20057         if (unlikely(PyType_HasFeature(Py_TYPE(o), Py_TPFLAGS_HAVE_FINALIZE) &&
Py_TYPE(o)->tp_finalize) && (!PyType_IS_GC(Py_TYPE(o)) || !_PyGC_FINALIZED(o))) {
20058             if (PyObject_CallFinalizerFromDealloc(o)) return;
20059         }
20060         #endif
20061         {
20062             PyObject *etype, *eval, *etb;
20063             PyErr_Fetch(&etype, &eval, &etb);
20064             __Pyx_SET_REFCNT(o, Py_REFCNT(o) + 1);
20065             __pyx_pw_8PyClical_8clifford_5__dealloc__(o);
20066             __Pyx_SET_REFCNT(o, Py_REFCNT(o) - 1);
20067             PyErr_Restore(etype, eval, etb);
20068         }
20069         (*Py_TYPE(o)->tp_free)(o);
20070     }
20071     static PyObject * __pyx_sq_item_8PyClical_clifford(PyObject *o, Py_ssize_t i) {
20072         PyObject *r;
20073         PyObject *x = PyInt_FromSsize_t(i); if(!x) return 0;
20074         r = Py_TYPE(o)->tp_as_mapping->mp_subscript(o, x);
20075         Py_DECREF(x);
20076         return r;
20077     }
20078
20079     static PyMethodDef __pyx_methods_8PyClical_clifford[] = {
20080         {"copy", (PyCFunction) __pyx_pw_8PyClical_8clifford_1copy, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_1copy},
20081         {"reframe", (PyCFunction) __pyx_pw_8PyClical_8clifford_11reframe, METH_O,
__pyx_doc_8PyClical_8clifford_10reframe},
20082         {"inv", (PyCFunction) __pyx_pw_8PyClical_8clifford_49inv, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_48inv},
20083         {"pow", (PyCFunction) __pyx_pw_8PyClical_8clifford_57pow, METH_O,
__pyx_doc_8PyClical_8clifford_56pow},
20084         {"outer_pow", (PyCFunction) __pyx_pw_8PyClical_8clifford_59outer_pow, METH_O,
__pyx_doc_8PyClical_8clifford_58outer_pow},
20085         {"scalar", (PyCFunction) __pyx_pw_8PyClical_8clifford_63scalar, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_62scalar},
20086         {"pure", (PyCFunction) __pyx_pw_8PyClical_8clifford_65pure, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_64pure},
20087         {"even", (PyCFunction) __pyx_pw_8PyClical_8clifford_67even, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_66even},
20088         {"odd", (PyCFunction) __pyx_pw_8PyClical_8clifford_69odd, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_68odd},
20089         {"vector_part",
(PyCFunction)(void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_8clifford_71vector_part,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_8clifford_70vector_part},
20090         {"involute", (PyCFunction) __pyx_pw_8PyClical_8clifford_73involute, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_72involute},
20091         {"reverse", (PyCFunction) __pyx_pw_8PyClical_8clifford_75reverse, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_74reverse},
20092         {"conj", (PyCFunction) __pyx_pw_8PyClical_8clifford_77conj, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_76conj},
20093         {"quad", (PyCFunction) __pyx_pw_8PyClical_8clifford_79quad, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_78quad},
20094         {"norm", (PyCFunction) __pyx_pw_8PyClical_8clifford_81norm, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_80norm},
20095         {"abs", (PyCFunction) __pyx_pw_8PyClical_8clifford_83abs, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_82abs},
20096         {"max_abs", (PyCFunction) __pyx_pw_8PyClical_8clifford_85max_abs, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_84max_abs},
20097         {"truncated", (PyCFunction) __pyx_pw_8PyClical_8clifford_87truncated, METH_O,
__pyx_doc_8PyClical_8clifford_86truncated},
20098         {"isinf", (PyCFunction) __pyx_pw_8PyClical_8clifford_89isinf, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_88isinf},
20099         {"isnan", (PyCFunction) __pyx_pw_8PyClical_8clifford_91isnan, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_90isnan},

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20100         {"frame", (PyCFunction) __pyx_pw_8PyClical_8clifford_93frame, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_92frame},
20101         {"__reduce_cython__", (PyCFunction) __pyx_pw_8PyClical_8clifford_99__reduce_cython__,
METH_NOARGS, 0},
20102         {"__setstate_cython__",
(PyCFunction) __pyx_pw_8PyClical_8clifford_101__setstate_cython__, METH_O, 0},
20103         {0, 0, 0, 0}
20104     };
20105
20106     static PyNumberMethods __pyx_tp_as_number_clifford = {
20107         __pyx_pw_8PyClical_8clifford_21__add__, /*nb_add*/
20108         __pyx_pw_8PyClical_8clifford_25__sub__, /*nb_subtract*/
20109         __pyx_pw_8PyClical_8clifford_29__mul__, /*nb_multiply*/
20110         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20111             0, /*nb_divide*/
20112             #endif
20113         __pyx_pw_8PyClical_8clifford_33__mod__, /*nb_remainder*/
20114             0, /*nb_divmod*/
20115         __pyx_pw_8PyClical_8clifford_55__pow__, /*nb_power*/
20116         __pyx_pw_8PyClical_8clifford_17__neg__, /*nb_negative*/
20117         __pyx_pw_8PyClical_8clifford_19__pos__, /*nb_positive*/
20118             0, /*nb_absolute*/
20119             0, /*nb_nonzero*/
20120             0, /*nb_invert*/
20121             0, /*nb_lshift*/
20122             0, /*nb_rshift*/
20123         __pyx_pw_8PyClical_8clifford_37__and__, /*nb_and*/
20124         __pyx_pw_8PyClical_8clifford_41__xor__, /*nb_xor*/
20125         __pyx_pw_8PyClical_8clifford_51__or__, /*nb_or*/
20126         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20127             0, /*nb_coerce*/
20128             #endif
20129             0, /*nb_int*/
20130             #if PY_MAJOR_VERSION < 3
20131             0, /*nb_long*/
20132             #else
20133             0, /*reserved*/
20134             #endif
20135             0, /*nb_float*/
20136             #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20137             0, /*nb_oct*/
20138             #endif
20139             #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20140             0, /*nb_hex*/
20141             #endif
20142         __pyx_pw_8PyClical_8clifford_23__iadd__, /*nb_inplace_add*/
20143         __pyx_pw_8PyClical_8clifford_27__isub__, /*nb_inplace_subtract*/
20144         __pyx_pw_8PyClical_8clifford_31__imul__, /*nb_inplace_multiply*/
20145         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20146         __pyx_pw_8PyClical_8clifford_47__idiv__, /*nb_inplace_divide*/
20147         #endif
20148         __pyx_pw_8PyClical_8clifford_35__imod__, /*nb_inplace_remainder*/
20149             0, /*nb_inplace_power*/
20150             0, /*nb_inplace_lshift*/
20151             0, /*nb_inplace_rshift*/
20152         __pyx_pw_8PyClical_8clifford_39__iand__, /*nb_inplace_and*/
20153         __pyx_pw_8PyClical_8clifford_43__ixor__, /*nb_inplace_xor*/
20154         __pyx_pw_8PyClical_8clifford_53__ior__, /*nb_inplace_or*/
20155             0, /*nb_floor_divide*/
20156         __pyx_pw_8PyClical_8clifford_45__truediv__, /*nb_true_divide*/
20157             0, /*nb_inplace_floor_divide*/
20158             0, /*nb_inplace_true_divide*/
20159             0, /*nb_index*/
20160             #if PY_VERSION_HEX >= 0x03050000
20161             0, /*nb_matrix_multiply*/
20162             #endif
20163             #if PY_VERSION_HEX >= 0x03050000
20164             0, /*nb_inplace_matrix_multiply*/
20165             #endif
20166     };
20167
20168     static PySequenceMethods __pyx_tp_as_sequence_clifford = {
20169         0, /*sq_length*/
20170         0, /*sq_concat*/
20171         0, /*sq_repeat*/
20172         __pyx_sq_item_8PyClical_clifford, /*sq_item*/
20173         0, /*sq_slice*/
20174         0, /*sq_ass_item*/
20175         0, /*sq_ass_slice*/
20176         __pyx_pw_8PyClical_8clifford_7__contains__, /*sq_contains*/
20177         0, /*sq_inplace_concat*/
20178         0, /*sq_inplace_repeat*/

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```

20179         };
20180
20181     static PyMappingMethods __pyx_tp_as_mapping_clifford = {
20182         0, /*mp_length*/
20183         __pyx_pw_8PyClical_8clifford_15__getitem__, /*mp_subscript*/
20184         0, /*mp_ass_subscript*/
20185     };
20186
20187     static PyTypeObject __pyx_type_8PyClical_clifford = {
20188         PyVarObject_HEAD_INIT(0, 0)
20189         "PyClical.clifford", /*tp_name*/
20190         sizeof(struct __pyx_obj_8PyClical_clifford), /*tp_basicsize*/
20191         0, /*tp_itemsize*/
20192         __pyx_tp_dealloc_8PyClical_clifford, /*tp_dealloc*/
20193         #if PY_VERSION_HEX < 0x030800b4
20194         0, /*tp_print*/
20195         #endif
20196         #if PY_VERSION_HEX >= 0x030800b4
20197         0, /*tp_vectorcall_offset*/
20198         #endif
20199         0, /*tp_getattr*/
20200         0, /*tp_setattr*/
20201         #if PY_MAJOR_VERSION < 3
20202         0, /*tp_compare*/
20203         #endif
20204         #if PY_MAJOR_VERSION >= 3
20205         0, /*tp_as_async*/
20206         #endif
20207         __pyx_pw_8PyClical_8clifford_95__repr__, /*tp_repr*/
20208         &__pyx_tp_as_number_clifford, /*tp_as_number*/
20209         &__pyx_tp_as_sequence_clifford, /*tp_as_sequence*/
20210         &__pyx_tp_as_mapping_clifford, /*tp_as_mapping*/
20211         0, /*tp_hash*/
20212         __pyx_pw_8PyClical_8clifford_61__call__, /*tp_call*/
20213         __pyx_pw_8PyClical_8clifford_97__str__, /*tp_str*/
20214         0, /*tp_getattro*/
20215         0, /*tp_setattro*/
20216         0, /*tp_as_buffer*/
20217
20218     Py_TPFLAGS_DEFAULT|Py_TPFLAGS_HAVE_VERSION_TAG|Py_TPFLAGS_CHECKTYPES|Py_TPFLAGS_HAVE_NEWBUFFER|Py_TPFLAGS_BASETYPE,
20219     /*tp_flags*/
20220     "\n    Python class clifford wraps C++ class Clifford.\n    ", /*tp_doc*/
20221     0, /*tp_traverse*/
20222     0, /*tp_clear*/
20223     __pyx_pw_8PyClical_8clifford_13__richcmp__, /*tp_richcompare*/
20224     0, /*tp_weaklistoffset*/
20225     __pyx_pw_8PyClical_8clifford_9__iter__, /*tp_iter*/
20226     0, /*tp_iternext*/
20227     __pyx_methods_8PyClical_clifford, /*tp_methods*/
20228     0, /*tp_members*/
20229     0, /*tp_getset*/
20230     0, /*tp_base*/
20231     0, /*tp_dict*/
20232     0, /*tp_descr_get*/
20233     0, /*tp_descr_set*/
20234     0, /*tp_dictoffset*/
20235     0, /*tp_init*/
20236     0, /*tp_alloc*/
20237     __pyx_tp_new_8PyClical_clifford, /*tp_new*/
20238     0, /*tp_free*/
20239     0, /*tp_is_gc*/
20240     0, /*tp_bases*/
20241     0, /*tp_mro*/
20242     0, /*tp_cache*/
20243     0, /*tp_subclasses*/
20244     0, /*tp_weaklist*/
20245     0, /*tp_del*/
20246     0, /*tp_version_tag*/
20247     #if PY_VERSION_HEX >= 0x030400a1
20248     0, /*tp_finalize*/
20249     #endif
20250     #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM
20251     >= 0x07030800)
20252     0, /*tp_vectorcall*/
20253     #endif
20254     #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
20255     0, /*tp_print*/
20256     #endif
20257     #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
20258     0, /*tp_pypy_flags*/
20259     #endif
20260     };
20261
20262     static struct __pyx_obj_8PyClical__pyx_scope_struct____iter__
20263     *__pyx_freelist_8PyClical__pyx_scope_struct____iter__[8];
20264     static int __pyx_freecount_8PyClical__pyx_scope_struct____iter__ = 0;

```



```

20262         static PyObject * __pyx_tp_new_8PyClical__pyx_scope_struct____iter__(PyTypeObject *t,
CYTHON_UNUSED PyObject *a, CYTHON_UNUSED PyObject *k) {
20263             PyObject *o;
20264             if (CYTHON_COMPILING_IN_CPYTHON &&
likely((__pyx_freecount_8PyClical__pyx_scope_struct____iter__ > 0) & (t->tp_basicsize ==
sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__))) {
20265                 o =
(PyObject*) __pyx_freelist_8PyClical__pyx_scope_struct____iter__[--__pyx_freecount_8PyClical__pyx_scope_struct____iter.
20266                 memset(o, 0, sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__));
20267                 (void) PyObject_INIT(o, t);
20268                 PyObject_GC_Track(o);
20269             } else {
20270                 o = (*t->tp_alloc)(t, 0);
20271                 if (unlikely(!o)) return 0;
20272             }
20273             return o;
20274         }
20275
20276         static void __pyx_tp_dealloc_8PyClical__pyx_scope_struct____iter__(PyObject *o) {
20277             struct __pyx_obj_8PyClical__pyx_scope_struct____iter__ *p = (struct
__pyx_obj_8PyClical__pyx_scope_struct____iter__ *)o;
20278             PyObject_GC_UnTrack(o);
20279             Py_CLEAR(p->__pyx_v_idx);
20280             Py_CLEAR(p->__pyx_v_self);
20281             Py_CLEAR(p->__pyx_t_0);
20282             if (CYTHON_COMPILING_IN_CPYTHON &&
((__pyx_freecount_8PyClical__pyx_scope_struct____iter__ < 8) & (Py_TYPE(o)->tp_basicsize ==
sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__))) {
20283                 __pyx_freelist_8PyClical__pyx_scope_struct____iter__[__pyx_freecount_8PyClical__pyx_scope_struct____iter__++]
= ((struct __pyx_obj_8PyClical__pyx_scope_struct____iter__ *)o);
20284             } else {
20285                 (*Py_TYPE(o)->tp_free)(o);
20286             }
20287         }
20288
20289         static int __pyx_tp_traverse_8PyClical__pyx_scope_struct____iter__(PyObject *o,
visitproc v, void *a) {
20290             int e;
20291             struct __pyx_obj_8PyClical__pyx_scope_struct____iter__ *p = (struct
__pyx_obj_8PyClical__pyx_scope_struct____iter__ *)o;
20292             if (p->__pyx_v_idx) {
20293                 e = (*v)(p->__pyx_v_idx, a); if (e) return e;
20294             }
20295             if (p->__pyx_v_self) {
20296                 e = (*v)((PyObject *)p->__pyx_v_self, a); if (e) return e;
20297             }
20298             if (p->__pyx_t_0) {
20299                 e = (*v)(p->__pyx_t_0, a); if (e) return e;
20300             }
20301             return 0;
20302         }
20303
20304         static PyTypeObject __pyx_type_8PyClical__pyx_scope_struct____iter__ = {
20305             PyVarObject_HEAD_INIT(0, 0)
20306             "PyClical.__pyx_scope_struct____iter__", /*tp_name*/
20307             sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__), /*tp_basicsize*/
20308             0, /*tp_itemsize*/
20309             __pyx_tp_dealloc_8PyClical__pyx_scope_struct____iter__, /*tp_dealloc*/
20310             #if PY_VERSION_HEX < 0x030800b4
20311             0, /*tp_print*/
20312             #endif
20313             #if PY_VERSION_HEX >= 0x030800b4
20314             0, /*tp_vectorcall_offset*/
20315             #endif
20316             0, /*tp_getattr*/
20317             0, /*tp_setattr*/
20318             #if PY_MAJOR_VERSION < 3
20319             0, /*tp_compare*/
20320             #endif
20321             #if PY_MAJOR_VERSION >= 3
20322             0, /*tp_as_async*/
20323             #endif
20324             0, /*tp_repr*/
20325             0, /*tp_as_number*/
20326             0, /*tp_as_sequence*/
20327             0, /*tp_as_mapping*/
20328             0, /*tp_hash*/
20329             0, /*tp_call*/
20330             0, /*tp_str*/
20331             0, /*tp_getattro*/
20332             0, /*tp_setattro*/
20333             0, /*tp_as_buffer*/
20334
Py_TPFLAGS_DEFAULT|Py_TPFLAGS_HAVE_VERSION_TAG|Py_TPFLAGS_CHECKTYPES|Py_TPFLAGS_HAVE_NEWBUFFER|Py_TPFLAGS_HAVE_GC,
/*tp_flags*/
20335             0, /*tp_doc*/

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20336     __pyx_tp_traverse_8PyClical__pyx_scope_struct____iter__, /*tp_traverse*/
20337     0, /*tp_clear*/
20338     0, /*tp_richcompare*/
20339     0, /*tp_weaklistoffset*/
20340     0, /*tp_iter*/
20341     0, /*tp_iternext*/
20342     0, /*tp_methods*/
20343     0, /*tp_members*/
20344     0, /*tp_getset*/
20345     0, /*tp_base*/
20346     0, /*tp_dict*/
20347     0, /*tp_descr_get*/
20348     0, /*tp_descr_set*/
20349     0, /*tp_dictoffset*/
20350     0, /*tp_init*/
20351     0, /*tp_alloc*/
20352     __pyx_tp_new_8PyClical__pyx_scope_struct____iter__, /*tp_new*/
20353     0, /*tp_free*/
20354     0, /*tp_is_gc*/
20355     0, /*tp_bases*/
20356     0, /*tp_mro*/
20357     0, /*tp_cache*/
20358     0, /*tp_subclasses*/
20359     0, /*tp_weaklist*/
20360     0, /*tp_del*/
20361     0, /*tp_version_tag*/
20362     #if PY_VERSION_HEX >= 0x030400a1
20363     0, /*tp_finalize*/
20364     #endif
20365     #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM
>= 0x07030800)
20366     0, /*tp_vectorcall*/
20367     #endif
20368     #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
20369     0, /*tp_print*/
20370     #endif
20371     #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
20372     0, /*tp_pypy_flags*/
20373     #endif
20374     };
20375
20376     static PyMethodDef __pyx_methods[] = {
20377         {"compare",
20378         (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_3compare, METH_VARARGS|METH_KEYWORDS,
20379         __pyx_doc_8PyClical_2compare},
20380         {"min_neg", (PyCFunction)__pyx_pw_8PyClical_5min_neg, METH_O,
20381         __pyx_doc_8PyClical_4min_neg},
20382         {"max_pos", (PyCFunction)__pyx_pw_8PyClical_7max_pos, METH_O,
20383         __pyx_doc_8PyClical_6max_pos},
20384         {"error_squared_tol", (PyCFunction)__pyx_pw_8PyClical_11error_squared_tol, METH_O,
20385         __pyx_doc_8PyClical_10error_squared_tol},
20386         {"error_squared",
20387         (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_13error_squared,
20388         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_12error_squared},
20389         {"approx_equal",
20390         (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_15approx_equal,
20391         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_14approx_equal},
20392         {"inv", (PyCFunction)__pyx_pw_8PyClical_17inv, METH_O, __pyx_doc_8PyClical_16inv},
20393         {"scalar", (PyCFunction)__pyx_pw_8PyClical_19scalar, METH_O,
20394         __pyx_doc_8PyClical_18scalar},
20395         {"real", (PyCFunction)__pyx_pw_8PyClical_21real, METH_O,
20396         __pyx_doc_8PyClical_20real},
20397         {"imag", (PyCFunction)__pyx_pw_8PyClical_23imag, METH_O,
20398         __pyx_doc_8PyClical_22imag},
20399         {"pure", (PyCFunction)__pyx_pw_8PyClical_25pure, METH_O,
20400         __pyx_doc_8PyClical_24pure},
20401         {"even", (PyCFunction)__pyx_pw_8PyClical_27even, METH_O,
20402         __pyx_doc_8PyClical_26even},
20403         {"odd", (PyCFunction)__pyx_pw_8PyClical_29odd, METH_O, __pyx_doc_8PyClical_28odd},
20404         {"involute", (PyCFunction)__pyx_pw_8PyClical_31involute, METH_O,
20405         __pyx_doc_8PyClical_30involute},
20406         {"reverse", (PyCFunction)__pyx_pw_8PyClical_33reverse, METH_O,
20407         __pyx_doc_8PyClical_32reverse},
20408         {"conj", (PyCFunction)__pyx_pw_8PyClical_35conj, METH_O,
20409         __pyx_doc_8PyClical_34conj},
20410         {"quad", (PyCFunction)__pyx_pw_8PyClical_37quad, METH_O,
20411         __pyx_doc_8PyClical_36quad},
20412         {"norm", (PyCFunction)__pyx_pw_8PyClical_39norm, METH_O,
20413         __pyx_doc_8PyClical_38norm},
20414         {"abs", (PyCFunction)__pyx_pw_8PyClical_41abs, METH_O, __pyx_doc_8PyClical_40abs},
20415         {"max_abs", (PyCFunction)__pyx_pw_8PyClical_43max_abs, METH_O,
20416         __pyx_doc_8PyClical_42max_abs},
20417         {"pow", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_45pow,
20418         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_44pow},
20419         {"outer_pow",
20420         (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_47outer_pow,
20421         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_46outer_pow},

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20399         {"complexifier", (PyCFunction) __pyx_pw_8PyClical_49complexifier, METH_O,
__pyx_doc_8PyClical_48complexifier},
20400         {"sqrt", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_51sqrt,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_50sqrt},
20401         {"exp", (PyCFunction) __pyx_pw_8PyClical_53exp, METH_O, __pyx_doc_8PyClical_52exp},
20402         {"log", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_55log,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_54log},
20403         {"cos", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_57cos,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_56cos},
20404         {"acos", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_59acos,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_58acos},
20405         {"cosh", (PyCFunction) __pyx_pw_8PyClical_61cosh, METH_O,
__pyx_doc_8PyClical_60cosh},
20406         {"acosh", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_63acosh,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_62acosh},
20407         {"sin", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_65sin,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_64sin},
20408         {"asin", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_67asin,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_66asin},
20409         {"sinh", (PyCFunction) __pyx_pw_8PyClical_69sinh, METH_O,
__pyx_doc_8PyClical_68sinh},
20410         {"asinh", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_71asinh,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_70asinh},
20411         {"tan", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_73tan,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_72tan},
20412         {"atan", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_75atan,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_74atan},
20413         {"tanh", (PyCFunction) __pyx_pw_8PyClical_77tanh, METH_O,
__pyx_doc_8PyClical_76tanh},
20414         {"atanh", (PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_79atanh,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_78atanh},
20415         {"random_clifford",
(PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_81random_clifford,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_80random_clifford},
20416         {"cga3", (PyCFunction) __pyx_pw_8PyClical_83cga3, METH_O,
__pyx_doc_8PyClical_82cga3},
20417         {"cga3std", (PyCFunction) __pyx_pw_8PyClical_85cga3std, METH_O,
__pyx_doc_8PyClical_84cga3std},
20418         {"agc3", (PyCFunction) __pyx_pw_8PyClical_87agc3, METH_O,
__pyx_doc_8PyClical_86agc3},
20419         {0, 0, 0, 0}
20420     };
20421
20422     #if PY_MAJOR_VERSION >= 3
20423     #if CYTHON_PEP489_MULTI_PHASE_INIT
20424     static PyObject* __pyx_pymod_create(PyObject *spec, PyModuleDef *def); /*proto*/
20425     static int __pyx_pymod_exec_PyClical(PyObject* module); /*proto*/
20426     static PyModuleDef_Slot __pyx_moduledef_slots[] = {
20427         {Py_mod_create, (void*) __pyx_pymod_create},
20428         {Py_mod_exec, (void*) __pyx_pymod_exec_PyClical},
20429         {0, NULL}
20430     };
20431     #endif
20432
20433     static struct PyModuleDef __pyx_moduledef = {
20434         PyModuleDef_HEAD_INIT,
20435         "PyClical",
20436         0, /* m_doc */
20437         #if CYTHON_PEP489_MULTI_PHASE_INIT
20438         0, /* m_size */
20439         #else
20440         -1, /* m_size */
20441         #endif
20442         __pyx_methods /* m_methods */,
20443         #if CYTHON_PEP489_MULTI_PHASE_INIT
20444         __pyx_moduledef_slots, /* m_slots */
20445         #else
20446         NULL, /* m_reload */
20447         #endif
20448         NULL, /* m_traverse */
20449         NULL, /* m_clear */
20450         NULL /* m_free */
20451     };
20452     #endif
20453     #ifndef CYTHON_SMALL_CODE
20454     #if defined(__clang__)
20455         #define CYTHON_SMALL_CODE
20456     #elif defined(__GNUC__) && (__GNUC__ > 4 || (__GNUC__ == 4 && __GNUC_MINOR__ >= 3))
20457         #define CYTHON_SMALL_CODE __attribute__((cold))
20458     #else
20459         #define CYTHON_SMALL_CODE
20460     #endif
20461     #endif
20462
20463     static __Pyx_StringTabEntry __pyx_string_tab[] = {
20464         {&__pyx_kp_u_ __pyx_k, sizeof(__pyx_k), 0, 1, 0, 0},
20465         {&__pyx_kp_u_Abbreviation_for_clifford_index, __pyx_k_Abbreviation_for_clifford_index,

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    sizeof(__pyx_k_Abbreviation_for_clifford_index), 0, 1, 0, 0),
20466 {&__pyx_kp_u_Abbreviation_for_index_set_q_p, __pyx_k_Abbreviation_for_index_set_q_p,
    sizeof(__pyx_k_Abbreviation_for_index_set_q_p), 0, 1, 0, 0),
20467 {&__pyx_kp_u_Absolute_value_of_multivector_m, __pyx_k_Absolute_value_of_multivector_m,
    sizeof(__pyx_k_Absolute_value_of_multivector_m), 0, 1, 0, 0),
20468 {&__pyx_kp_u_Absolute_value_square_root_of_n, __pyx_k_Absolute_value_square_root_of_n,
    sizeof(__pyx_k_Absolute_value_square_root_of_n), 0, 1, 0, 0),
20469 {&__pyx_kp_u_Cannot_initialize_clifford_objec, __pyx_k_Cannot_initialize_clifford_objec,
    sizeof(__pyx_k_Cannot_initialize_clifford_objec), 0, 1, 0, 0),
20470 {&__pyx_kp_u_Cannot_initialize_index_set_obje, __pyx_k_Cannot_initialize_index_set_obje,
    sizeof(__pyx_k_Cannot_initialize_index_set_obje), 0, 1, 0, 0),
20471 {&__pyx_kp_u_Cannot_reframe, __pyx_k_Cannot_reframe, sizeof(__pyx_k_Cannot_reframe), 0, 1, 0, 0),
20472 {&__pyx_kp_u_Cannot_take_vector_part_of, __pyx_k_Cannot_take_vector_part_of,
    sizeof(__pyx_k_Cannot_take_vector_part_of), 0, 1, 0, 0),
20473 {&__pyx_kp_u_Cardinality_Number_of_indices_i, __pyx_k_Cardinality_Number_of_indices_i,
    sizeof(__pyx_k_Cardinality_Number_of_indices_i), 0, 1, 0, 0),
20474 {&__pyx_kp_u_Check_if_a_multivector_contains, __pyx_k_Check_if_a_multivector_contains,
    sizeof(__pyx_k_Check_if_a_multivector_contains), 0, 1, 0, 0),
20475 {&__pyx_kp_u_Check_if_a_multivector_contains_2, __pyx_k_Check_if_a_multivector_contains_2,
    sizeof(__pyx_k_Check_if_a_multivector_contains_2), 0, 1, 0, 0),
20476 {&__pyx_kp_u_Conjugation_reverse_o_involute, __pyx_k_Conjugation_reverse_o_involute,
    sizeof(__pyx_k_Conjugation_reverse_o_involute), 0, 1, 0, 0),
20477 {&__pyx_kp_u_Conjugation_reverse_o_involute_2, __pyx_k_Conjugation_reverse_o_involute_2,
    sizeof(__pyx_k_Conjugation_reverse_o_involute_2), 0, 1, 0, 0),
20478 {&__pyx_kp_u_Contraction_print_clifford_l_cl, __pyx_k_Contraction_print_clifford_l_cl,
    sizeof(__pyx_k_Contraction_print_clifford_l_cl), 0, 1, 0, 0),
20479 {&__pyx_kp_u_Contraction_x_clifford_l_x_clif, __pyx_k_Contraction_x_clifford_l_x_clif,
    sizeof(__pyx_k_Contraction_x_clifford_l_x_clif), 0, 1, 0, 0),
20480 {&__pyx_kp_u_Convert_CGA3_null_vector_to_Euc, __pyx_k_Convert_CGA3_null_vector_to_Euc,
    sizeof(__pyx_k_Convert_CGA3_null_vector_to_Euc), 0, 1, 0, 0),
20481 {&__pyx_kp_u_Convert_CGA3_null_vector_to_sta, __pyx_k_Convert_CGA3_null_vector_to_sta,
    sizeof(__pyx_k_Convert_CGA3_null_vector_to_sta), 0, 1, 0, 0),
20482 {&__pyx_kp_u_Convert_Euclidean_3D_multivecto, __pyx_k_Convert_Euclidean_3D_multivecto,
    sizeof(__pyx_k_Convert_Euclidean_3D_multivecto), 0, 1, 0, 0),
20483 {&__pyx_kp_u_Copy_this_clifford_object_x_cli, __pyx_k_Copy_this_clifford_object_x_cli,
    sizeof(__pyx_k_Copy_this_clifford_object_x_cli), 0, 1, 0, 0),
20484 {&__pyx_kp_u_Copy_this_index_set_object_s_in, __pyx_k_Copy_this_index_set_object_s_in,
    sizeof(__pyx_k_Copy_this_index_set_object_s_in), 0, 1, 0, 0),
20485 {&__pyx_kp_u_Cosine_of_multivector_with_opti, __pyx_k_Cosine_of_multivector_with_opti,
    sizeof(__pyx_k_Cosine_of_multivector_with_opti), 0, 1, 0, 0),
20486 {&__pyx_kp_u_Even_part_of_multivector_sum_of, __pyx_k_Even_part_of_multivector_sum_of,
    sizeof(__pyx_k_Even_part_of_multivector_sum_of), 0, 1, 0, 0),
20487 {&__pyx_kp_u_Even_part_of_multivector_sum_of_2, __pyx_k_Even_part_of_multivector_sum_of_2,
    sizeof(__pyx_k_Even_part_of_multivector_sum_of_2), 0, 1, 0, 0),
20488 {&__pyx_kp_u_Exponential_of_multivector_x_cl, __pyx_k_Exponential_of_multivector_x_cl,
    sizeof(__pyx_k_Exponential_of_multivector_x_cl), 0, 1, 0, 0),
20489 {&__pyx_kp_u_Geometric_difference_print_clif, __pyx_k_Geometric_difference_print_clif,
    sizeof(__pyx_k_Geometric_difference_print_clif), 0, 1, 0, 0),
20490 {&__pyx_kp_u_Geometric_difference_x_clifford, __pyx_k_Geometric_difference_x_clifford,
    sizeof(__pyx_k_Geometric_difference_x_clifford), 0, 1, 0, 0),
20491 {&__pyx_kp_u_Geometric_multiplicative_invers, __pyx_k_Geometric_multiplicative_invers,
    sizeof(__pyx_k_Geometric_multiplicative_invers), 0, 1, 0, 0),
20492 {&__pyx_kp_u_Geometric_multiplicative_invers_2, __pyx_k_Geometric_multiplicative_invers_2,
    sizeof(__pyx_k_Geometric_multiplicative_invers_2), 0, 1, 0, 0),
20493 {&__pyx_kp_u_Geometric_product_print_cliffor, __pyx_k_Geometric_product_print_cliffor,
    sizeof(__pyx_k_Geometric_product_print_cliffor), 0, 1, 0, 0),
20494 {&__pyx_kp_u_Geometric_product_x_clifford_2, __pyx_k_Geometric_product_x_clifford_2,
    sizeof(__pyx_k_Geometric_product_x_clifford_2), 0, 1, 0, 0),
20495 {&__pyx_kp_u_Geometric_quotient_print_cliffo, __pyx_k_Geometric_quotient_print_cliffo,
    sizeof(__pyx_k_Geometric_quotient_print_cliffo), 0, 1, 0, 0),
20496 {&__pyx_kp_u_Geometric_quotient_x_clifford_l, __pyx_k_Geometric_quotient_x_clifford_l,
    sizeof(__pyx_k_Geometric_quotient_x_clifford_l), 0, 1, 0, 0),
20497 {&__pyx_kp_u_Geometric_sum_print_clifford_l, __pyx_k_Geometric_sum_print_clifford_l,
    sizeof(__pyx_k_Geometric_sum_print_clifford_l), 0, 1, 0, 0),
20498 {&__pyx_kp_u_Geometric_sum_x_clifford_l_x_cl, __pyx_k_Geometric_sum_x_clifford_l_x_cl,
    sizeof(__pyx_kp_u_Geometric_sum_x_clifford_l_x_cl), 0, 1, 0, 0),
20499 {&__pyx_kp_u_Get_the_value_of_an_index_set_o, __pyx_k_Get_the_value_of_an_index_set_o,
    sizeof(__pyx_k_Get_the_value_of_an_index_set_o), 0, 1, 0, 0),
20500 {&__pyx_kp_u_Hyperbolic_cosine_of_multivecto, __pyx_k_Hyperbolic_cosine_of_multivecto,
    sizeof(__pyx_k_Hyperbolic_cosine_of_multivecto), 0, 1, 0, 0),
20501 {&__pyx_kp_u_Hyperbolic_sine_of_multivector, __pyx_k_Hyperbolic_sine_of_multivector,
    sizeof(__pyx_k_Hyperbolic_sine_of_multivector), 0, 1, 0, 0),
20502 {&__pyx_kp_u_Hyperbolic_tangent_of_multivect, __pyx_k_Hyperbolic_tangent_of_multivect,
    sizeof(__pyx_k_Hyperbolic_tangent_of_multivect), 0, 1, 0, 0),
20503 {&__pyx_kp_u_Imaginary_part_deprecated_alway, __pyx_k_Imaginary_part_deprecated_alway,
    sizeof(__pyx_k_Imaginary_part_deprecated_alway), 0, 1, 0, 0),
20504 {&__pyx_n_s_IndexError, __pyx_k_IndexError, sizeof(__pyx_k_IndexError), 0, 0, 1, 1),
20505 {&__pyx_kp_u_Inner_product_print_clifford_l, __pyx_k_Inner_product_print_clifford_l,
    sizeof(__pyx_k_Inner_product_print_clifford_l), 0, 1, 0, 0),
20506 {&__pyx_kp_u_Inner_product_x_clifford_l_x_cl, __pyx_k_Inner_product_x_clifford_l_x_cl,
    sizeof(__pyx_k_Inner_product_x_clifford_l_x_cl), 0, 1, 0, 0),
20507 {&__pyx_kp_u_Integer_power_of_multivector_ob, __pyx_k_Integer_power_of_multivector_ob,
    sizeof(__pyx_k_Integer_power_of_multivector_ob), 0, 1, 0, 0),
20508 {&__pyx_n_s_Integral, __pyx_k_Integral, sizeof(__pyx_k_Integral), 0, 0, 1, 1),
20509 {&__pyx_kp_u_Inverse_cosine_of_multivector_w, __pyx_k_Inverse_cosine_of_multivector_w,
    sizeof(__pyx_k_Inverse_cosine_of_multivector_w), 0, 1, 0, 0),
20510 {&__pyx_kp_u_Inverse_hyperbolic_cosine_of_mu, __pyx_k_Inverse_hyperbolic_cosine_of_mu,

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    sizeof(__pyx_k_Inverse_hyperbolic_cosine_of_mu), 0, 1, 0, 0),
20511 {&__pyx_kp_u_Inverse_hyperbolic_sine_of_mult, __pyx_k_Inverse_hyperbolic_sine_of_mult,
    sizeof(__pyx_k_Inverse_hyperbolic_sine_of_mult), 0, 1, 0, 0),
20512 {&__pyx_kp_u_Inverse_hyperbolic_tangent_of_m, __pyx_k_Inverse_hyperbolic_tangent_of_m,
    sizeof(__pyx_k_Inverse_hyperbolic_tangent_of_m), 0, 1, 0, 0),
20513 {&__pyx_kp_u_Inverse_sine_of_multivector_wit, __pyx_k_Inverse_sine_of_multivector_wit,
    sizeof(__pyx_k_Inverse_sine_of_multivector_wit), 0, 1, 0, 0),
20514 {&__pyx_kp_u_Inverse_tangent_of_multivector, __pyx_k_Inverse_tangent_of_multivector,
    sizeof(__pyx_k_Inverse_tangent_of_multivector), 0, 1, 0, 0),
20515 {&__pyx_kp_u_Iterate_over_the_indices_of_an, __pyx_k_Iterate_over_the_indices_of_an,
    sizeof(__pyx_k_Iterate_over_the_indices_of_an), 0, 1, 0, 0),
20516 {&__pyx_kp_u_Main_involution_each_i_is_repla, __pyx_k_Main_involution_each_i_is_repla,
    sizeof(__pyx_k_Main_involution_each_i_is_repla), 0, 1, 0, 0),
20517 {&__pyx_kp_u_Main_involution_each_i_is_repla_2, __pyx_k_Main_involution_each_i_is_repla_2,
    sizeof(__pyx_k_Main_involution_each_i_is_repla_2), 0, 1, 0, 0),
20518 {&__pyx_kp_u_Maximum_absolute_value_of_coord, __pyx_k_Maximum_absolute_value_of_coord,
    sizeof(__pyx_k_Maximum_absolute_value_of_coord), 0, 1, 0, 0),
20519 {&__pyx_kp_u_Maximum_member_index_set_1_1_2, __pyx_k_Maximum_member_index_set_1_1_2,
    sizeof(__pyx_k_Maximum_member_index_set_1_1_2), 0, 1, 0, 0),
20520 {&__pyx_kp_u_Maximum_of_absolute_values_of_c, __pyx_k_Maximum_of_absolute_values_of_c,
    sizeof(__pyx_k_Maximum_of_absolute_values_of_c), 0, 1, 0, 0),
20521 {&__pyx_kp_u_Maximum_positive_index_or_0_if, __pyx_k_Maximum_positive_index_or_0_if,
    sizeof(__pyx_k_Maximum_positive_index_or_0_if), 0, 1, 0, 0),
20522 {&__pyx_kp_u_Minimum_member_index_set_1_1_2, __pyx_k_Minimum_member_index_set_1_1_2,
    sizeof(__pyx_k_Minimum_member_index_set_1_1_2), 0, 1, 0, 0),
20523 {&__pyx_kp_u_Minimum_negative_index_or_0_if, __pyx_k_Minimum_negative_index_or_0_if,
    sizeof(__pyx_k_Minimum_negative_index_or_0_if), 0, 1, 0, 0),
20524 {&__pyx_kp_u_Natural_logarithm_of_multivecto, __pyx_k_Natural_logarithm_of_multivecto,
    sizeof(__pyx_k_Natural_logarithm_of_multivecto), 0, 1, 0, 0),
20525 {&__pyx_kp_u_Norm_sum_of_squares_of_coordina, __pyx_k_Norm_sum_of_squares_of_coordina,
    sizeof(__pyx_k_Norm_sum_of_squares_of_coordina), 0, 1, 0, 0),
20526 {&__pyx_n_s_NotImplemented, __pyx_k_NotImplemented, sizeof(__pyx_k_NotImplemented), 0, 0, 1, 1},
20527 {&__pyx_kp_u_Not_applicable, __pyx_k_Not_applicable, sizeof(__pyx_k_Not_applicable), 0, 1, 0, 0),
20528 {&__pyx_kp_u_Not_applicable_for_a_in_cliffor, __pyx_k_Not_applicable_for_a_in_cliffor,
    sizeof(__pyx_k_Not_applicable_for_a_in_cliffor), 0, 1, 0, 0),
20529 {&__pyx_kp_u_Number_of_negative_indices_incl, __pyx_k_Number_of_negative_indices_incl,
    sizeof(__pyx_k_Number_of_negative_indices_incl), 0, 1, 0, 0),
20530 {&__pyx_kp_u_Number_of_positive_indices_incl, __pyx_k_Number_of_positive_indices_incl,
    sizeof(__pyx_k_Number_of_positive_indices_incl), 0, 1, 0, 0),
20531 {&__pyx_kp_u_Odd_part_of_multivector_sum_of, __pyx_k_Odd_part_of_multivector_sum_of,
    sizeof(__pyx_k_Odd_part_of_multivector_sum_of), 0, 1, 0, 0),
20532 {&__pyx_kp_u_Odd_part_of_multivector_sum_of_2, __pyx_k_Odd_part_of_multivector_sum_of_2,
    sizeof(__pyx_k_Odd_part_of_multivector_sum_of_2), 0, 1, 0, 0),
20533 {&__pyx_kp_u_Outer_product_power_of_multivec, __pyx_k_Outer_product_power_of_multivec,
    sizeof(__pyx_k_Outer_product_power_of_multivec), 0, 1, 0, 0),
20534 {&__pyx_kp_u_Outer_product_power_x_clifford, __pyx_k_Outer_product_power_x_clifford,
    sizeof(__pyx_k_Outer_product_power_x_clifford), 0, 1, 0, 0),
20535 {&__pyx_kp_u_Outer_product_print_clifford_1, __pyx_k_Outer_product_print_clifford_1,
    sizeof(__pyx_k_Outer_product_print_clifford_1), 0, 1, 0, 0),
20536 {&__pyx_kp_u_Outer_product_x_clifford_1_x_cl, __pyx_k_Outer_product_x_clifford_1_x_cl,
    sizeof(__pyx_k_Outer_product_x_clifford_1_x_cl), 0, 1, 0, 0),
20537 {&__pyx_kp_u_Power_self_to_the_m_x_clifford, __pyx_k_Power_self_to_the_m_x_clifford,
    sizeof(__pyx_k_Power_self_to_the_m_x_clifford), 0, 1, 0, 0),
20538 {&__pyx_kp_u_Power_self_to_the_m_x_clifford_2, __pyx_k_Power_self_to_the_m_x_clifford_2,
    sizeof(__pyx_k_Power_self_to_the_m_x_clifford_2), 0, 1, 0, 0),
20539 {&__pyx_kp_u_Pure_grade_vector_part_print_cl, __pyx_k_Pure_grade_vector_part_print_cl,
    sizeof(__pyx_k_Pure_grade_vector_part_print_cl), 0, 1, 0, 0),
20540 {&__pyx_kp_u_Pure_part_print_clifford_1_1_1, __pyx_k_Pure_part_print_clifford_1_1_1,
    sizeof(__pyx_k_Pure_part_print_clifford_1_1_1), 0, 1, 0, 0),
20541 {&__pyx_kp_u_Pure_part_print_pure_clifford_1, __pyx_k_Pure_part_print_pure_clifford_1,
    sizeof(__pyx_k_Pure_part_print_pure_clifford_1), 0, 1, 0, 0),
20542 {&__pyx_kp_u_Put_self_into_a_larger_frame_co, __pyx_k_Put_self_into_a_larger_frame_co,
    sizeof(__pyx_k_Put_self_into_a_larger_frame_co), 0, 1, 0, 0),
20543 {&__pyx_n_s_PyClical, __pyx_k_PyClical, sizeof(__pyx_k_PyClical), 0, 0, 1, 1},
20544 {&__pyx_kp_s_PyClical_pyx, __pyx_k_PyClical_pyx, sizeof(__pyx_k_PyClical_pyx), 0, 0, 1, 0),
20545 {&__pyx_kp_u_Quadratic_form_rev_x_x_0_print, __pyx_k_Quadratic_form_rev_x_x_0_print,
    sizeof(__pyx_k_Quadratic_form_rev_x_x_0_print), 0, 1, 0, 0),
20546 {&__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2, __pyx_k_Quadratic_form_rev_x_x_0_print_2,
    sizeof(__pyx_k_Quadratic_form_rev_x_x_0_print_2), 0, 1, 0, 0),
20547 {&__pyx_kp_u_Quadratic_norm_error_tolerance, __pyx_k_Quadratic_norm_error_tolerance,
    sizeof(__pyx_k_Quadratic_norm_error_tolerance), 0, 1, 0, 0),
20548 {&__pyx_kp_u_Random_multivector_within_a_fra, __pyx_k_Random_multivector_within_a_fra,
    sizeof(__pyx_k_Random_multivector_within_a_fra), 0, 1, 0, 0),
20549 {&__pyx_n_s_Real, __pyx_k_Real, sizeof(__pyx_k_Real), 0, 0, 1, 1},
20550 {&__pyx_kp_u_Real_part_synonym_for_scalar_pa, __pyx_k_Real_part_synonym_for_scalar_pa,
    sizeof(__pyx_k_Real_part_synonym_for_scalar_pa), 0, 1, 0, 0),
20551 {&__pyx_kp_u_Relative_or_absolute_error_usin, __pyx_k_Relative_or_absolute_error_usin,
    sizeof(__pyx_k_Relative_or_absolute_error_usin), 0, 1, 0, 0),
20552 {&__pyx_kp_u_Remove_all_terms_of_self_with_r, __pyx_k_Remove_all_terms_of_self_with_r,
    sizeof(__pyx_k_Remove_all_terms_of_self_with_r), 0, 1, 0, 0),
20553 {&__pyx_kp_u_Reversion_eg_1_2_2_1_print_reve, __pyx_k_Reversion_eg_1_2_2_1_print_reve,
    sizeof(__pyx_k_Reversion_eg_1_2_2_1_print_reve), 0, 1, 0, 0),
20554 {&__pyx_kp_u_Reversion_eg_clifford_1_cliffor, __pyx_k_Reversion_eg_clifford_1_cliffor,
    sizeof(__pyx_k_Reversion_eg_clifford_1_cliffor), 0, 1, 0, 0),
20555 {&__pyx_n_s_RuntimeError, __pyx_k_RuntimeError, sizeof(__pyx_k_RuntimeError), 0, 0, 1, 1},
20556 {&__pyx_kp_u_Scalar_part_clifford_1_1_2_sc, __pyx_k_Scalar_part_clifford_1_1_2_sc,
    sizeof(__pyx_k_Scalar_part_clifford_1_1_2_sc), 0, 1, 0, 0),

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20557     {&__pyx_kp_u_Scalar_part_scalar_clifford_1_1, __pyx_k_Scalar_part_scalar_clifford_1_1,
sizeof(__pyx_k_Scalar_part_scalar_clifford_1_1), 0, 1, 0, 0},
20558     {&__pyx_n_s_Sequence, __pyx_k_Sequence, sizeof(__pyx_k_Sequence), 0, 0, 1, 1},
20559     {&__pyx_kp_u_Set_complement_not_print_index, __pyx_k_Set_complement_not_print_index,
sizeof(__pyx_k_Set_complement_not_print_index), 0, 1, 0, 0},
20560     {&__pyx_kp_u_Set_intersection_and_print_index, __pyx_k_Set_intersection_and_print_index,
sizeof(__pyx_k_Set_intersection_and_print_index), 0, 1, 0, 0},
20561     {&__pyx_kp_u_Set_intersection_and_x_index_se, __pyx_k_Set_intersection_and_x_index_se,
sizeof(__pyx_k_Set_intersection_and_x_index_se), 0, 1, 0, 0},
20562     {&__pyx_kp_u_Set_the_value_of_an_index_set_o, __pyx_k_Set_the_value_of_an_index_set_o,
sizeof(__pyx_k_Set_the_value_of_an_index_set_o), 0, 1, 0, 0},
20563     {&__pyx_kp_u_Set_union_or_print_index_set_1, __pyx_k_Set_union_or_print_index_set_1,
sizeof(__pyx_k_Set_union_or_print_index_set_1), 0, 1, 0, 0},
20564     {&__pyx_kp_u_Set_union_or_x_index_set_1_x_in, __pyx_k_Set_union_or_x_index_set_1_x_in,
sizeof(__pyx_k_Set_union_or_x_index_set_1_x_in), 0, 1, 0, 0},
20565     {&__pyx_kp_u_Sign_of_geometric_product_of_tw, __pyx_k_Sign_of_geometric_product_of_tw,
sizeof(__pyx_k_Sign_of_geometric_product_of_tw), 0, 1, 0, 0},
20566     {&__pyx_kp_u_Sign_of_geometric_square_of_a_C, __pyx_k_Sign_of_geometric_square_of_a_C,
sizeof(__pyx_k_Sign_of_geometric_square_of_a_C), 0, 1, 0, 0},
20567     {&__pyx_kp_u_Sine_of_multivector_with_option, __pyx_k_Sine_of_multivector_with_option,
sizeof(__pyx_k_Sine_of_multivector_with_option), 0, 1, 0, 0},
20568     {&__pyx_kp_u_Square_root_of_1_which_commutates, __pyx_k_Square_root_of_1_which_commutates,
sizeof(__pyx_k_Square_root_of_1_which_commutates), 0, 1, 0, 0},
20569     {&__pyx_kp_u_Square_root_of_multivector_with, __pyx_k_Square_root_of_multivector_with,
sizeof(__pyx_k_Square_root_of_multivector_with), 0, 1, 0, 0},
20570     {&__pyx_kp_u_Subalgebra_generated_by_all_gen, __pyx_k_Subalgebra_generated_by_all_gen,
sizeof(__pyx_k_Subalgebra_generated_by_all_gen), 0, 1, 0, 0},
20571     {&__pyx_kp_u_Subscripting_map_from_index_set, __pyx_k_Subscripting_map_from_index_set,
sizeof(__pyx_k_Subscripting_map_from_index_set), 0, 1, 0, 0},
20572     {&__pyx_kp_u_Symmetric_set_difference_exclus, __pyx_k_Symmetric_set_difference_exclus,
sizeof(__pyx_k_Symmetric_set_difference_exclus), 0, 1, 0, 0},
20573     {&__pyx_kp_u_Symmetric_set_difference_exclus_2, __pyx_k_Symmetric_set_difference_exclus_2,
sizeof(__pyx_k_Symmetric_set_difference_exclus_2), 0, 1, 0, 0},
20574     {&__pyx_kp_u_Tangent_of_multivector_with_opt, __pyx_k_Tangent_of_multivector_with_opt,
sizeof(__pyx_k_Tangent_of_multivector_with_opt), 0, 1, 0, 0},
20575     {&__pyx_kp_u_Test_for_approximate_equality_o, __pyx_k_Test_for_approximate_equality_o,
sizeof(__pyx_k_Test_for_approximate_equality_o), 0, 1, 0, 0},
20576     {&__pyx_kp_u_Tests_for_functions_that_Doctes, __pyx_k_Tests_for_functions_that_Doctes,
sizeof(__pyx_k_Tests_for_functions_that_Doctes), 0, 1, 0, 0},
20577     {&__pyx_kp_u_Tests_for_functions_that_Doctes_2, __pyx_k_Tests_for_functions_that_Doctes_2,
sizeof(__pyx_k_Tests_for_functions_that_Doctes_2), 0, 1, 0, 0},
20578     {&__pyx_kp_u_The_informal_string_representat, __pyx_k_The_informal_string_representat,
sizeof(__pyx_k_The_informal_string_representat), 0, 1, 0, 0},
20579     {&__pyx_kp_u_The_informal_string_representat_2, __pyx_k_The_informal_string_representat_2,
sizeof(__pyx_k_The_informal_string_representat_2), 0, 1, 0, 0},
20580     {&__pyx_kp_u_The_official_string_representat, __pyx_k_The_official_string_representat,
sizeof(__pyx_k_The_official_string_representat), 0, 1, 0, 0},
20581     {&__pyx_kp_u_The_official_string_representat_2, __pyx_k_The_official_string_representat_2,
sizeof(__pyx_k_The_official_string_representat_2), 0, 1, 0, 0},
20582     {&__pyx_kp_u_This_comparison_operator_is_not, __pyx_k_This_comparison_operator_is_not,
sizeof(__pyx_k_This_comparison_operator_is_not), 0, 1, 0, 0},
20583     {&__pyx_kp_u_Transform_left_hand_side_using, __pyx_k_Transform_left_hand_side_using,
sizeof(__pyx_k_Transform_left_hand_side_using), 0, 1, 0, 0},
20584     {&__pyx_kp_u_Transform_left_hand_side_using_2, __pyx_k_Transform_left_hand_side_using_2,
sizeof(__pyx_k_Transform_left_hand_side_using_2), 0, 1, 0, 0},
20585     {&__pyx_n_s_TypeError, __pyx_k_TypeError, sizeof(__pyx_k_TypeError), 0, 0, 1, 1},
20586     {&__pyx_kp_u_UTF_8, __pyx_k_UTF_8, sizeof(__pyx_k_UTF_8), 0, 1, 0, 0},
20587     {&__pyx_kp_u_Unary_print_clifford_1_1, __pyx_k_Unary_print_clifford_1_1,
sizeof(__pyx_k_Unary_print_clifford_1_1), 0, 1, 0, 0},
20588     {&__pyx_kp_u_Unary_print_clifford_1_1_2, __pyx_k_Unary_print_clifford_1_1_2,
sizeof(__pyx_k_Unary_print_clifford_1_1_2), 0, 1, 0, 0},
20589     {&__pyx_n_s_ValueError, __pyx_k_ValueError, sizeof(__pyx_k_ValueError), 0, 0, 1, 1},
20590     {&__pyx_kp_u_Vector_part_of_multivector_as_a, __pyx_k_Vector_part_of_multivector_as_a,
sizeof(__pyx_k_Vector_part_of_multivector_as_a), 0, 1, 0, 0},
20591     {&__pyx_kp_u_2, __pyx_k_2, sizeof(__pyx_k_2), 0, 1, 0, 0},
20592     {&__pyx_kp_u_5, __pyx_k_5, sizeof(__pyx_k_5), 0, 1, 0, 0},
20593     {&__pyx_kp_u_6, __pyx_k_6, sizeof(__pyx_k_6), 0, 1, 0, 0},
20594     {&__pyx_kp_u_7, __pyx_k_7, sizeof(__pyx_k_7), 0, 1, 0, 0},
20595     {&__pyx_kp_u_8, __pyx_k_8, sizeof(__pyx_k_8), 0, 1, 0, 0},
20596     {&__pyx_kp_u_9, __pyx_k_9, sizeof(__pyx_k_9), 0, 1, 0, 0},
20597     {&__pyx_n_s_abc, __pyx_k_abc, sizeof(__pyx_k_abc), 0, 0, 1, 1},
20598     {&__pyx_kp_u_abs_line_1522, __pyx_k_abs_line_1522, sizeof(__pyx_k_abs_line_1522), 0, 1, 0, 0},
20599     {&__pyx_n_s_acos, __pyx_k_acos, sizeof(__pyx_k_acos), 0, 0, 1, 1},
20600     {&__pyx_kp_u_acos_line_1668, __pyx_k_acos_line_1668, sizeof(__pyx_k_acos_line_1668), 0, 1, 0, 0},
20601     {&__pyx_n_s_acosh, __pyx_k_acosh, sizeof(__pyx_k_acosh), 0, 0, 1, 1},
20602     {&__pyx_kp_u_acosh_line_1705, __pyx_k_acosh_line_1705, sizeof(__pyx_k_acosh_line_1705), 0, 1, 0, 0},
20603     {&__pyx_kp_u_agc3_line_1893, __pyx_k_agc3_line_1893, sizeof(__pyx_k_agc3_line_1893), 0, 1, 0, 0},
20604     {&__pyx_kp_u_approx_equal_line_1359, __pyx_k_approx_equal_line_1359,
sizeof(__pyx_k_approx_equal_line_1359), 0, 1, 0, 0},
20605     {&__pyx_n_s_args, __pyx_k_args, sizeof(__pyx_k_args), 0, 0, 1, 1},
20606     {&__pyx_kp_u_as_frame, __pyx_k_as_frame, sizeof(__pyx_k_as_frame), 0, 1, 0, 0},
20607     {&__pyx_n_s_asin, __pyx_k_asin, sizeof(__pyx_k_asin), 0, 0, 1, 1},
20608     {&__pyx_kp_u_asin_line_1747, __pyx_k_asin_line_1747, sizeof(__pyx_k_asin_line_1747), 0, 1, 0, 0},
20609     {&__pyx_n_s_asinh, __pyx_k_asinh, sizeof(__pyx_k_asinh), 0, 0, 1, 1},
20610     {&__pyx_kp_u_asinh_line_1782, __pyx_k_asinh_line_1782, sizeof(__pyx_k_asinh_line_1782), 0, 1, 0, 0},
20611     {&__pyx_n_s_atan, __pyx_k_atan, sizeof(__pyx_k_atan), 0, 0, 1, 1},
20612     {&__pyx_kp_u_atan_line_1818, __pyx_k_atan_line_1818, sizeof(__pyx_k_atan_line_1818), 0, 1, 0, 0},

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20613     {&_pyx_n_s_atanh, __pyx_k_atanh, sizeof(__pyx_k_atanh), 0, 0, 1, 1},
20614     {&_pyx_kp_u_atanh_line_1847, __pyx_k_atanh_line_1847, sizeof(__pyx_k_atanh_line_1847), 0, 1, 0, 0},
20615     {&_pyx_kp_u_cga3_line_1873, __pyx_k_cga3_line_1873, sizeof(__pyx_k_cga3_line_1873), 0, 1, 0, 0},
20616     {&_pyx_kp_u_cga3std_line_1882, __pyx_k_cga3std_line_1882, sizeof(__pyx_k_cga3std_line_1882), 0, 1,
0, 0},
20617     {&_pyx_n_s_cl, __pyx_k_cl, sizeof(__pyx_k_cl), 0, 0, 1, 1},
20618     {&_pyx_n_s_clifford, __pyx_k_clifford, sizeof(__pyx_k_clifford), 0, 0, 1, 1},
20619     {&_pyx_kp_u_clifford_add_line_740, __pyx_k_clifford_add_line_740,
sizeof(__pyx_k_clifford_add_line_740), 0, 1, 0, 0},
20620     {&_pyx_kp_u_clifford_and_line_836, __pyx_k_clifford_and_line_836,
sizeof(__pyx_k_clifford_and_line_836), 0, 1, 0, 0},
20621     {&_pyx_kp_u_clifford_call_line_1020, __pyx_k_clifford_call_line_1020,
sizeof(__pyx_k_clifford_call_line_1020), 0, 1, 0, 0},
20622     {&_pyx_kp_u_clifford_getitem_line_707, __pyx_k_clifford_getitem_line_707,
sizeof(__pyx_k_clifford_getitem_line_707), 0, 1, 0, 0},
20623     {&_pyx_kp_u_clifford_iadd_line_751, __pyx_k_clifford_iadd_line_751,
sizeof(__pyx_k_clifford_iadd_line_751), 0, 1, 0, 0},
20624     {&_pyx_kp_u_clifford_iand_line_851, __pyx_k_clifford_iand_line_851,
sizeof(__pyx_k_clifford_iand_line_851), 0, 1, 0, 0},
20625     {&_pyx_kp_u_clifford_idiv_line_911, __pyx_k_clifford_idiv_line_911,
sizeof(__pyx_k_clifford_idiv_line_911), 0, 1, 0, 0},
20626     {&_pyx_kp_u_clifford_imod_line_821, __pyx_k_clifford_imod_line_821,
sizeof(__pyx_k_clifford_imod_line_821), 0, 1, 0, 0},
20627     {&_pyx_kp_u_clifford_imul_line_793, __pyx_k_clifford_imul_line_793,
sizeof(__pyx_k_clifford_imul_line_793), 0, 1, 0, 0},
20628     {&_pyx_kp_u_clifford_ior_line_950, __pyx_k_clifford_ior_line_950,
sizeof(__pyx_k_clifford_ior_line_950), 0, 1, 0, 0},
20629     {&_pyx_kp_u_clifford_isub_line_771, __pyx_k_clifford_isub_line_771,
sizeof(__pyx_k_clifford_isub_line_771), 0, 1, 0, 0},
20630     {&_pyx_kp_u_clifford_iter_line_638, __pyx_k_clifford_iter_line_638,
sizeof(__pyx_k_clifford_iter_line_638), 0, 1, 0, 0},
20631     {&_pyx_kp_u_clifford_ior_line_881, __pyx_k_clifford_ior_line_881,
sizeof(__pyx_k_clifford_ior_line_881), 0, 1, 0, 0},
20632     {&_pyx_kp_u_clifford_mod_line_806, __pyx_k_clifford_mod_line_806,
sizeof(__pyx_k_clifford_mod_line_806), 0, 1, 0, 0},
20633     {&_pyx_kp_u_clifford_mul_line_780, __pyx_k_clifford_mul_line_780,
sizeof(__pyx_k_clifford_mul_line_780), 0, 1, 0, 0},
20634     {&_pyx_kp_u_clifford_neg_line_722, __pyx_k_clifford_neg_line_722,
sizeof(__pyx_k_clifford_neg_line_722), 0, 1, 0, 0},
20635     {&_pyx_kp_u_clifford_or_line_939, __pyx_k_clifford_or_line_939,
sizeof(__pyx_k_clifford_or_line_939), 0, 1, 0, 0},
20636     {&_pyx_kp_u_clifford_pos_line_731, __pyx_k_clifford_pos_line_731,
sizeof(__pyx_k_clifford_pos_line_731), 0, 1, 0, 0},
20637     {&_pyx_kp_u_clifford_pow_line_961, __pyx_k_clifford_pow_line_961,
sizeof(__pyx_k_clifford_pow_line_961), 0, 1, 0, 0},
20638     {&_pyx_kp_u_clifford_repr_line_1235, __pyx_k_clifford_repr_line_1235,
sizeof(__pyx_k_clifford_repr_line_1235), 0, 1, 0, 0},
20639     {&_pyx_kp_u_clifford_str_line_1244, __pyx_k_clifford_str_line_1244,
sizeof(__pyx_k_clifford_str_line_1244), 0, 1, 0, 0},
20640     {&_pyx_kp_u_clifford_sub_line_760, __pyx_k_clifford_sub_line_760,
sizeof(__pyx_k_clifford_sub_line_760), 0, 1, 0, 0},
20641     {&_pyx_kp_u_clifford_truediv_line_896, __pyx_k_clifford_truediv_line_896,
sizeof(__pyx_k_clifford_truediv_line_896), 0, 1, 0, 0},
20642     {&_pyx_kp_u_clifford_xor_line_866, __pyx_k_clifford_xor_line_866,
sizeof(__pyx_k_clifford_xor_line_866), 0, 1, 0, 0},
20643     {&_pyx_kp_u_clifford_abs_line_1175, __pyx_k_clifford_abs_line_1175,
sizeof(__pyx_k_clifford_abs_line_1175), 0, 1, 0, 0},
20644     {&_pyx_kp_u_clifford_conj_line_1138, __pyx_k_clifford_conj_line_1138,
sizeof(__pyx_k_clifford_conj_line_1138), 0, 1, 0, 0},
20645     {&_pyx_kp_u_clifford_copy_line_556, __pyx_k_clifford_copy_line_556,
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20646     {&_pyx_kp_u_clifford_even_line_1061, __pyx_k_clifford_even_line_1061,
sizeof(__pyx_k_clifford_even_line_1061), 0, 1, 0, 0},
20647     {&_pyx_kp_u_clifford_frame_line_1224, __pyx_k_clifford_frame_line_1224,
sizeof(__pyx_k_clifford_frame_line_1224), 0, 1, 0, 0},
20648     {&_pyx_n_s_clifford_hidden_doctests, __pyx_k_clifford_hidden_doctests,
sizeof(__pyx_k_clifford_hidden_doctests), 0, 0, 1, 1},
20649     {&_pyx_kp_u_clifford_hidden_doctests_line_12, __pyx_k_clifford_hidden_doctests_line_12,
sizeof(__pyx_k_clifford_hidden_doctests_line_12), 0, 1, 0, 0},
20650     {&_pyx_kp_u_clifford_inv_line_926, __pyx_k_clifford_inv_line_926,
sizeof(__pyx_k_clifford_inv_line_926), 0, 1, 0, 0},
20651     {&_pyx_kp_u_clifford_involute_line_1107, __pyx_k_clifford_involute_line_1107,
sizeof(__pyx_k_clifford_involute_line_1107), 0, 1, 0, 0},
20652     {&_pyx_kp_u_clifford_isinf_line_1206, __pyx_k_clifford_isinf_line_1206,
sizeof(__pyx_k_clifford_isinf_line_1206), 0, 1, 0, 0},
20653     {&_pyx_kp_u_clifford_isnan_line_1215, __pyx_k_clifford_isnan_line_1215,
sizeof(__pyx_k_clifford_isnan_line_1215), 0, 1, 0, 0},
20654     {&_pyx_kp_u_clifford_max_abs_line_1184, __pyx_k_clifford_max_abs_line_1184,
sizeof(__pyx_k_clifford_max_abs_line_1184), 0, 1, 0, 0},
20655     {&_pyx_kp_u_clifford_norm_line_1164, __pyx_k_clifford_norm_line_1164,
sizeof(__pyx_k_clifford_norm_line_1164), 0, 1, 0, 0},
20656     {&_pyx_kp_u_clifford_odd_line_1070, __pyx_k_clifford_odd_line_1070,
sizeof(__pyx_k_clifford_odd_line_1070), 0, 1, 0, 0},
20657     {&_pyx_kp_u_clifford_outer_pow_line_1004, __pyx_k_clifford_outer_pow_line_1004,
sizeof(__pyx_k_clifford_outer_pow_line_1004), 0, 1, 0, 0},
20658     {&_pyx_kp_u_clifford_pow_line_980, __pyx_k_clifford_pow_line_980,
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20659     {&__pyx_kp_u_clifford_pure_line_1050, __pyx_k_clifford_pure_line_1050,
sizeof(__pyx_k_clifford_pure_line_1050), 0, 1, 0, 0},
20660     {&__pyx_kp_u_clifford_quad_line_1153, __pyx_k_clifford_quad_line_1153,
sizeof(__pyx_k_clifford_quad_line_1153), 0, 1, 0, 0},
20661     {&__pyx_kp_u_clifford_reframe_line_649, __pyx_k_clifford_reframe_line_649,
sizeof(__pyx_k_clifford_reframe_line_649), 0, 1, 0, 0},
20662     {&__pyx_kp_u_clifford_reverse_line_1123, __pyx_k_clifford_reverse_line_1123,
sizeof(__pyx_k_clifford_reverse_line_1123), 0, 1, 0, 0},
20663     {&__pyx_kp_u_clifford_scalar_line_1039, __pyx_k_clifford_scalar_line_1039,
sizeof(__pyx_k_clifford_scalar_line_1039), 0, 1, 0, 0},
20664     {&__pyx_kp_u_clifford_truncated_line_1195, __pyx_k_clifford_truncated_line_1195,
sizeof(__pyx_k_clifford_truncated_line_1195), 0, 1, 0, 0},
20665     {&__pyx_kp_u_clifford_vector_part_line_1079, __pyx_k_clifford_vector_part_line_1079,
sizeof(__pyx_k_clifford_vector_part_line_1079), 0, 1, 0, 0},
20666     {&__pyx_n_s_cline_in_traceback, __pyx_k_cline_in_traceback, sizeof(__pyx_k_cline_in_traceback), 0,
0, 1, 1},
20667     {&__pyx_n_s_close, __pyx_k_close, sizeof(__pyx_k_close), 0, 0, 1, 1},
20668     {&__pyx_n_s_collections, __pyx_k_collections, sizeof(__pyx_k_collections), 0, 0, 1, 1},
20669     {&__pyx_kp_u_compare_line_492, __pyx_k_compare_line_492, sizeof(__pyx_k_compare_line_492), 0, 1, 0,
0},
20670     {&__pyx_kp_u_complexifier_line_1576, __pyx_k_complexifier_line_1576,
sizeof(__pyx_k_complexifier_line_1576), 0, 1, 0, 0},
20671     {&__pyx_n_s_conj, __pyx_k_conj, sizeof(__pyx_k_conj), 0, 0, 1, 1},
20672     {&__pyx_kp_u_conj_line_1485, __pyx_k_conj_line_1485, sizeof(__pyx_k_conj_line_1485), 0, 1, 0, 0},
20673     {&__pyx_n_s_copy, __pyx_k_copy, sizeof(__pyx_k_copy), 0, 0, 1, 1},
20674     {&__pyx_n_s_cos, __pyx_k_cos, sizeof(__pyx_k_cos), 0, 0, 1, 1},
20675     {&__pyx_kp_u_cos_line_1651, __pyx_k_cos_line_1651, sizeof(__pyx_k_cos_line_1651), 0, 1, 0, 0},
20676     {&__pyx_n_s_cosh, __pyx_k_cosh, sizeof(__pyx_k_cosh), 0, 0, 1, 1},
20677     {&__pyx_kp_u_cosh_line_1689, __pyx_k_cosh_line_1689, sizeof(__pyx_k_cosh_line_1689), 0, 1, 0, 0},
20678     {&__pyx_n_s_doctest, __pyx_k_doctest, sizeof(__pyx_k_doctest), 0, 0, 1, 1},
20679     {&__pyx_n_s_e, __pyx_k_e, sizeof(__pyx_k_e), 0, 0, 1, 1},
20680     {&__pyx_kp_u_e_line_1936, __pyx_k_e_line_1936, sizeof(__pyx_k_e_line_1936), 0, 1, 0, 0},
20681     {&__pyx_n_s_encode, __pyx_k_encode, sizeof(__pyx_k_encode), 0, 0, 1, 1},
20682     {&__pyx_kp_u_error_squared_line_1346, __pyx_k_error_squared_line_1346,
sizeof(__pyx_k_error_squared_line_1346), 0, 1, 0, 0},
20683     {&__pyx_kp_u_error_squared_tol_line_1337, __pyx_k_error_squared_tol_line_1337,
sizeof(__pyx_k_error_squared_tol_line_1337), 0, 1, 0, 0},
20684     {&__pyx_n_s_even, __pyx_k_even, sizeof(__pyx_k_even), 0, 0, 1, 1},
20685     {&__pyx_kp_u_even_line_1437, __pyx_k_even_line_1437, sizeof(__pyx_k_even_line_1437), 0, 1, 0, 0},
20686     {&__pyx_n_s_exp, __pyx_k_exp, sizeof(__pyx_k_exp), 0, 0, 1, 1},
20687     {&__pyx_kp_u_exp_line_1614, __pyx_k_exp_line_1614, sizeof(__pyx_k_exp_line_1614), 0, 1, 0, 0},
20688     {&__pyx_n_s_fill, __pyx_k_fill, sizeof(__pyx_k_fill), 0, 0, 1, 1},
20689     {&__pyx_n_s_frm, __pyx_k_frm, sizeof(__pyx_k_frm), 0, 0, 1, 1},
20690     {&__pyx_kp_u_from, __pyx_k_from, sizeof(__pyx_k_from), 0, 1, 0, 0},
20691     {&__pyx_n_s_getstate, __pyx_k_getstate, sizeof(__pyx_k_getstate), 0, 0, 1, 1},
20692     {&__pyx_n_s_grade, __pyx_k_grade, sizeof(__pyx_k_grade), 0, 0, 1, 1},
20693     {&__pyx_n_s_i, __pyx_k_i, sizeof(__pyx_k_i), 0, 0, 1, 1},
20694     {&__pyx_kp_u_imag_line_1415, __pyx_k_imag_line_1415, sizeof(__pyx_k_imag_line_1415), 0, 1, 0, 0},
20695     {&__pyx_n_s_import, __pyx_k_import, sizeof(__pyx_k_import), 0, 0, 1, 1},
20696     {&__pyx_n_s_index_set, __pyx_k_index_set, sizeof(__pyx_k_index_set), 0, 0, 1, 1},
20697     {&__pyx_kp_u_index_set_and_line_271, __pyx_k_index_set_and_line_271,
sizeof(__pyx_k_index_set_and_line_271), 0, 1, 0, 0},
20698     {&__pyx_kp_u_index_set_getitem_line_191, __pyx_k_index_set_getitem_line_191,
sizeof(__pyx_k_index_set_getitem_line_191), 0, 1, 0, 0},
20699     {&__pyx_kp_u_index_set_iand_line_282, __pyx_k_index_set_iand_line_282,
sizeof(__pyx_k_index_set_iand_line_282), 0, 1, 0, 0},
20700     {&__pyx_kp_u_index_set_invert_line_240, __pyx_k_index_set_invert_line_240,
sizeof(__pyx_k_index_set_invert_line_240), 0, 1, 0, 0},
20701     {&__pyx_kp_u_index_set_ior_line_304, __pyx_k_index_set_ior_line_304,
sizeof(__pyx_k_index_set_ior_line_304), 0, 1, 0, 0},
20702     {&__pyx_n_s_index_set_iter, __pyx_k_index_set_iter, sizeof(__pyx_k_index_set_iter), 0, 0, 1,
1},
20703     {&__pyx_kp_u_index_set_iter_line_229, __pyx_k_index_set_iter_line_229,
sizeof(__pyx_k_index_set_iter_line_229), 0, 1, 0, 0},
20704     {&__pyx_kp_u_index_set_ixor_line_260, __pyx_k_index_set_ixor_line_260,
sizeof(__pyx_k_index_set_ixor_line_260), 0, 1, 0, 0},
20705     {&__pyx_kp_u_index_set_or_line_293, __pyx_k_index_set_or_line_293,
sizeof(__pyx_k_index_set_or_line_293), 0, 1, 0, 0},
20706     {&__pyx_kp_u_index_set_repr_line_384, __pyx_k_index_set_repr_line_384,
sizeof(__pyx_k_index_set_repr_line_384), 0, 1, 0, 0},
20707     {&__pyx_kp_u_index_set_setitem_line_179, __pyx_k_index_set_setitem_line_179,
sizeof(__pyx_k_index_set_setitem_line_179), 0, 1, 0, 0},
20708     {&__pyx_kp_u_index_set_str_line_395, __pyx_k_index_set_str_line_395,
sizeof(__pyx_k_index_set_str_line_395), 0, 1, 0, 0},
20709     {&__pyx_kp_u_index_set_xor_line_249, __pyx_k_index_set_xor_line_249,
sizeof(__pyx_k_index_set_xor_line_249), 0, 1, 0, 0},
20710     {&__pyx_kp_u_index_set_copy_line_65, __pyx_k_index_set_copy_line_65,
sizeof(__pyx_k_index_set_copy_line_65), 0, 1, 0, 0},
20711     {&__pyx_kp_u_index_set_count_line_315, __pyx_k_index_set_count_line_315,
sizeof(__pyx_k_index_set_count_line_315), 0, 1, 0, 0},
20712     {&__pyx_kp_u_index_set_count_neg_line_324, __pyx_k_index_set_count_neg_line_324,
sizeof(__pyx_k_index_set_count_neg_line_324), 0, 1, 0, 0},
20713     {&__pyx_kp_u_index_set_count_pos_line_333, __pyx_k_index_set_count_pos_line_333,
sizeof(__pyx_k_index_set_count_pos_line_333), 0, 1, 0, 0},
20714     {&__pyx_n_s_index_set_hidden_doctests, __pyx_k_index_set_hidden_doctests,
sizeof(__pyx_k_index_set_hidden_doctests), 0, 0, 1, 1},
20715     {&__pyx_kp_u_index_set_hidden_doctests_line_4, __pyx_k_index_set_hidden_doctests_line_4,

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    sizeof(__pyx_k_index_set_hidden_doctests_line_4), 0, 1, 0, 0),
20716 {&__pyx_kp_u_index_set_max_line_351, __pyx_k_index_set_max_line_351,
    sizeof(__pyx_k_index_set_max_line_351), 0, 1, 0, 0),
20717 {&__pyx_kp_u_index_set_min_line_342, __pyx_k_index_set_min_line_342,
    sizeof(__pyx_k_index_set_min_line_342), 0, 1, 0, 0),
20718 {&__pyx_kp_u_index_set_sign_of_mult_line_366, __pyx_k_index_set_sign_of_mult_line_366,
    sizeof(__pyx_k_index_set_sign_of_mult_line_366), 0, 1, 0, 0),
20719 {&__pyx_kp_u_index_set_sign_of_square_line_37, __pyx_k_index_set_sign_of_square_line_37,
    sizeof(__pyx_k_index_set_sign_of_square_line_37), 0, 1, 0, 0),
20720 {&__pyx_n_s_inv, __pyx_k_inv, sizeof(__pyx_k_inv), 0, 0, 1, 1},
20721 {&__pyx_kp_u_inv_line_1378, __pyx_k_inv_line_1378, sizeof(__pyx_k_inv_line_1378), 0, 1, 0, 0),
20722 {&__pyx_kp_u_invalid, __pyx_k_invalid, sizeof(__pyx_k_invalid), 0, 1, 0, 0),
20723 {&__pyx_kp_u_invalid_string, __pyx_k_invalid_string, sizeof(__pyx_k_invalid_string), 0, 1, 0, 0),
20724 {&__pyx_n_s_involute, __pyx_k_involute, sizeof(__pyx_k_involute), 0, 0, 1, 1},
20725 {&__pyx_kp_u_involute_line_1455, __pyx_k_involute_line_1455, sizeof(__pyx_k_involute_line_1455), 0,
    1, 0, 0),
20726 {&__pyx_n_s_ist, __pyx_k_ist, sizeof(__pyx_k_ist), 0, 0, 1, 1},
20727 {&__pyx_n_s_istpq, __pyx_k_istpq, sizeof(__pyx_k_istpq), 0, 0, 1, 1},
20728 {&__pyx_kp_u_istpq_line_1949, __pyx_k_istpq_line_1949, sizeof(__pyx_k_istpq_line_1949), 0, 1, 0, 0),
20729 {&__pyx_n_s_iter, __pyx_k_iter, sizeof(__pyx_k_iter), 0, 0, 1, 1},
20730 {&__pyx_n_s_ixt, __pyx_k_ixt, sizeof(__pyx_k_ixt), 0, 0, 1, 1},
20731 {&__pyx_kp_u_lexicographic_compare_eg_3_4_5, __pyx_k_lexicographic_compare_eg_3_4_5,
    sizeof(__pyx_k_lexicographic_compare_eg_3_4_5), 0, 1, 0, 0),
20732 {&__pyx_n_s_lhs, __pyx_k_lhs, sizeof(__pyx_k_lhs), 0, 0, 1, 1},
20733 {&__pyx_n_s_log, __pyx_k_log, sizeof(__pyx_k_log), 0, 0, 1, 1},
20734 {&__pyx_kp_u_log_line_1628, __pyx_k_log_line_1628, sizeof(__pyx_k_log_line_1628), 0, 1, 0, 0),
20735 {&__pyx_n_s_m, __pyx_k_m, sizeof(__pyx_k_m), 0, 0, 1, 1},
20736 {&__pyx_n_s_main, __pyx_k_main, sizeof(__pyx_k_main), 0, 0, 1, 1},
20737 {&__pyx_n_u_main, __pyx_k_main, sizeof(__pyx_k_main), 0, 1, 0, 1},
20738 {&__pyx_n_s_math, __pyx_k_math, sizeof(__pyx_k_math), 0, 0, 1, 1},
20739 {&__pyx_n_s_max, __pyx_k_max, sizeof(__pyx_k_max), 0, 0, 1, 1},
20740 {&__pyx_kp_u_max_abs_line_1531, __pyx_k_max_abs_line_1531, sizeof(__pyx_k_max_abs_line_1531), 0, 1,
    0, 0),
20741 {&__pyx_kp_u_max_pos_line_513, __pyx_k_max_pos_line_513, sizeof(__pyx_k_max_pos_line_513), 0, 1, 0,
    0),
20742 {&__pyx_n_s_min, __pyx_k_min, sizeof(__pyx_k_min), 0, 0, 1, 1},
20743 {&__pyx_kp_u_min_neg_line_504, __pyx_k_min_neg_line_504, sizeof(__pyx_k_min_neg_line_504), 0, 1, 0,
    0),
20744 {&__pyx_n_s_name, __pyx_k_name, sizeof(__pyx_k_name), 0, 0, 1, 1},
20745 {&__pyx_n_s_nbar3, __pyx_k_nbar3, sizeof(__pyx_k_nbar3), 0, 0, 1, 1},
20746 {&__pyx_n_s_ninf3, __pyx_k_ninf3, sizeof(__pyx_k_ninf3), 0, 0, 1, 1},
20747 {&__pyx_kp_s_no_default_reduce_due_to_non, __pyx_k_no_default_reduce_due_to_non,
    sizeof(__pyx_k_no_default_reduce_due_to_non), 0, 0, 1, 0),
20748 {&__pyx_n_s_norm, __pyx_k_norm, sizeof(__pyx_k_norm), 0, 0, 1, 1},
20749 {&__pyx_kp_u_norm_line_1511, __pyx_k_norm_line_1511, sizeof(__pyx_k_norm_line_1511), 0, 1, 0, 0),
20750 {&__pyx_kp_u_norm_sum_of_squares_of_coordina, __pyx_k_norm_sum_of_squares_of_coordina,
    sizeof(__pyx_k_norm_sum_of_squares_of_coordina), 0, 1, 0, 0),
20751 {&__pyx_n_s_numbers, __pyx_k_numbers, sizeof(__pyx_k_numbers), 0, 0, 1, 1},
20752 {&__pyx_n_s_obj, __pyx_k_obj, sizeof(__pyx_k_obj), 0, 0, 1, 1},
20753 {&__pyx_n_s_odd, __pyx_k_odd, sizeof(__pyx_k_odd), 0, 0, 1, 1},
20754 {&__pyx_kp_u_odd_line_1446, __pyx_k_odd_line_1446, sizeof(__pyx_k_odd_line_1446), 0, 1, 0, 0),
20755 {&__pyx_n_s_other, __pyx_k_other, sizeof(__pyx_k_other), 0, 0, 1, 1},
20756 {&__pyx_n_s_outer_pow, __pyx_k_outer_pow, sizeof(__pyx_k_outer_pow), 0, 0, 1, 1},
20757 {&__pyx_kp_u_outer_pow_line_1567, __pyx_k_outer_pow_line_1567, sizeof(__pyx_k_outer_pow_line_1567),
    0, 1, 0, 0),
20758 {&__pyx_n_s_p, __pyx_k_p, sizeof(__pyx_k_p), 0, 0, 1, 1},
20759 {&__pyx_n_s_pi, __pyx_k_pi, sizeof(__pyx_k_pi), 0, 0, 1, 1},
20760 {&__pyx_n_s_pow, __pyx_k_pow, sizeof(__pyx_k_pow), 0, 0, 1, 1},
20761 {&__pyx_kp_u_pow_line_1543, __pyx_k_pow_line_1543, sizeof(__pyx_k_pow_line_1543), 0, 1, 0, 0),
20762 {&__pyx_n_s_pure, __pyx_k_pure, sizeof(__pyx_k_pure), 0, 0, 1, 1},
20763 {&__pyx_kp_u_pure_line_1426, __pyx_k_pure_line_1426, sizeof(__pyx_k_pure_line_1426), 0, 1, 0, 0),
20764 {&__pyx_n_s_pyx_vtable, __pyx_k_pyx_vtable, sizeof(__pyx_k_pyx_vtable), 0, 0, 1, 1},
20765 {&__pyx_n_s_q, __pyx_k_q, sizeof(__pyx_k_q), 0, 0, 1, 1},
20766 {&__pyx_n_s_quad, __pyx_k_quad, sizeof(__pyx_k_quad), 0, 0, 1, 1},
20767 {&__pyx_kp_u_quad_line_1500, __pyx_k_quad_line_1500, sizeof(__pyx_k_quad_line_1500), 0, 1, 0, 0),
20768 {&__pyx_kp_u_random_clifford_line_1864, __pyx_k_random_clifford_line_1864,
    sizeof(__pyx_k_random_clifford_line_1864), 0, 1, 0, 0),
20769 {&__pyx_n_s_range, __pyx_k_range, sizeof(__pyx_k_range), 0, 0, 1, 1},
20770 {&__pyx_kp_u_real_line_1404, __pyx_k_real_line_1404, sizeof(__pyx_k_real_line_1404), 0, 1, 0, 0),
20771 {&__pyx_n_s_reduce, __pyx_k_reduce, sizeof(__pyx_k_reduce), 0, 0, 1, 1},
20772 {&__pyx_n_s_reduce_cython, __pyx_k_reduce_cython, sizeof(__pyx_k_reduce_cython), 0, 0, 1, 1},
20773 {&__pyx_n_s_reduce_ex, __pyx_k_reduce_ex, sizeof(__pyx_k_reduce_ex), 0, 0, 1, 1},
20774 {&__pyx_n_s_reverse, __pyx_k_reverse, sizeof(__pyx_k_reverse), 0, 0, 1, 1},
20775 {&__pyx_kp_u_reverse_line_1470, __pyx_k_reverse_line_1470, sizeof(__pyx_k_reverse_line_1470), 0, 1,
    0, 0),
20776 {&__pyx_n_s_rhs, __pyx_k_rhs, sizeof(__pyx_k_rhs), 0, 0, 1, 1},
20777 {&__pyx_n_s_scalar, __pyx_k_scalar, sizeof(__pyx_k_scalar), 0, 0, 1, 1},
20778 {&__pyx_n_s_scalar_epsilon, __pyx_k_scalar_epsilon, sizeof(__pyx_k_scalar_epsilon), 0, 0, 1, 1},
20779 {&__pyx_kp_u_scalar_line_1393, __pyx_k_scalar_line_1393, sizeof(__pyx_k_scalar_line_1393), 0, 1, 0,
    0),
20780 {&__pyx_n_s_send, __pyx_k_send, sizeof(__pyx_k_send), 0, 0, 1, 1},
20781 {&__pyx_n_s_setstate, __pyx_k_setstate, sizeof(__pyx_k_setstate), 0, 0, 1, 1},
20782 {&__pyx_n_s_setstate_cython, __pyx_k_setstate_cython, sizeof(__pyx_k_setstate_cython), 0, 0, 1, 1},
20783 {&__pyx_n_s_sin, __pyx_k_sin, sizeof(__pyx_k_sin), 0, 0, 1, 1},
20784 {&__pyx_kp_u_sin_line_1728, __pyx_k_sin_line_1728, sizeof(__pyx_k_sin_line_1728), 0, 1, 0, 0),
20785 {&__pyx_n_s_sinh, __pyx_k_sinh, sizeof(__pyx_k_sinh), 0, 0, 1, 1},
20786 {&__pyx_kp_u_sinh_line_1768, __pyx_k_sinh_line_1768, sizeof(__pyx_k_sinh_line_1768), 0, 1, 0, 0),

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20787     {&__pyx_n_s_sqrt, __pyx_k_sqrt, sizeof(__pyx_k_sqrt), 0, 0, 1, 1},
20788     {&__pyx_kp_u_sqrt_line_1591, __pyx_k_sqrt_line_1591, sizeof(__pyx_k_sqrt_line_1591), 0, 1, 0, 0},
20789     {&__pyx_n_s_tan, __pyx_k_tan, sizeof(__pyx_k_tan), 0, 0, 1, 1},
20790     {&__pyx_kp_u_tan_line_1801, __pyx_k_tan_line_1801, sizeof(__pyx_k_tan_line_1801), 0, 1, 0, 0},
20791     {&__pyx_n_s_tanh, __pyx_k_tanh, sizeof(__pyx_k_tanh), 0, 0, 1, 1},
20792     {&__pyx_kp_u_tanh_line_1835, __pyx_k_tanh_line_1835, sizeof(__pyx_k_tanh_line_1835), 0, 1, 0, 0},
20793     {&__pyx_n_s_tau, __pyx_k_tau, sizeof(__pyx_k_tau), 0, 0, 1, 1},
20794     {&__pyx_n_s_test, __pyx_k_test, sizeof(__pyx_k_test), 0, 0, 1, 1},
20795     {&__pyx_n_s_test_2, __pyx_k_test_2, sizeof(__pyx_k_test_2), 0, 0, 1, 1},
20796     {&__pyx_n_s_testmod, __pyx_k_testmod, sizeof(__pyx_k_testmod), 0, 0, 1, 1},
20797     {&__pyx_n_s_threshold, __pyx_k_threshold, sizeof(__pyx_k_threshold), 0, 0, 1, 1},
20798     {&__pyx_n_s_throw, __pyx_k_throw, sizeof(__pyx_k_throw), 0, 0, 1, 1},
20799     {&__pyx_kp_u_to_frame, __pyx_k_to_frame, sizeof(__pyx_k_to_frame), 0, 1, 0, 0},
20800     {&__pyx_n_s_tol, __pyx_k_tol, sizeof(__pyx_k_tol), 0, 0, 1, 1},
20801     {&__pyx_kp_u_using, __pyx_k_using, sizeof(__pyx_k_using), 0, 1, 0, 0},
20802     {&__pyx_kp_u_using_invalid, __pyx_k_using_invalid, sizeof(__pyx_k_using_invalid), 0, 1, 0, 0},
20803     {&__pyx_kp_u_utf_8, __pyx_k_utf_8, sizeof(__pyx_k_utf_8), 0, 1, 0, 0},
20804     {&__pyx_kp_u_value, __pyx_k_value, sizeof(__pyx_k_value), 0, 1, 0, 0},
20805     {&__pyx_n_s_version, __pyx_k_version, sizeof(__pyx_k_version), 0, 0, 1, 1},
20806     {&__pyx_n_s_xrange, __pyx_k_xrange, sizeof(__pyx_k_xrange), 0, 0, 1, 1},
20807     {0, 0, 0, 0, 0, 0, 0}
20808 };
20809 static CYTHON_SMALL_CODE int __Pyx_InitCachedBuiltins(void) {
20810     __pyx_builtin_IndexError = __Pyx_GetBuiltinName(__pyx_n_s_IndexError); if
20811     (!__pyx_builtin_IndexError) __PYX_ERR(0, 103, __pyx_L1_error)
20812     __pyx_builtin_RuntimeError = __Pyx_GetBuiltinName(__pyx_n_s_RuntimeError); if
20813     (!__pyx_builtin_RuntimeError) __PYX_ERR(0, 105, __pyx_L1_error)
20814     __pyx_builtin_TypeError = __Pyx_GetBuiltinName(__pyx_n_s_TypeError); if (!__pyx_builtin_TypeError)
20815     __PYX_ERR(0, 105, __pyx_L1_error)
20816     __pyx_builtin_ValueError = __Pyx_GetBuiltinName(__pyx_n_s_ValueError); if
20817     (!__pyx_builtin_ValueError) __PYX_ERR(0, 106, __pyx_L1_error)
20818     __pyx_builtin_NotImplemented = __Pyx_GetBuiltinName(__pyx_n_s_NotImplemented); if
20819     (!__pyx_builtin_NotImplemented) __PYX_ERR(0, 159, __pyx_L1_error)
20820     __pyx_builtin_range = __Pyx_GetBuiltinName(__pyx_n_s_range); if (!__pyx_builtin_range) __PYX_ERR(0,
20821     236, __pyx_L1_error)
20822     #if PY_MAJOR_VERSION >= 3
20823     __pyx_builtin_xrange = __Pyx_GetBuiltinName(__pyx_n_s_range); if (!__pyx_builtin_xrange)
20824     __PYX_ERR(0, 1099, __pyx_L1_error)
20825     #else
20826     __pyx_builtin_xrange = __Pyx_GetBuiltinName(__pyx_n_s_xrange); if (!__pyx_builtin_xrange)
20827     __PYX_ERR(0, 1099, __pyx_L1_error)
20828     #endif
20829     return 0;
20830     __pyx_L1_error:;
20831     return -1;
20832 }
20833
20834 static CYTHON_SMALL_CODE int __Pyx_InitCachedConstants(void) {
20835     __Pyx_RefNannyDeclarations
20836     __Pyx_RefNannySetupContext("__Pyx_InitCachedConstants", 0);
20837
20838     /* "(tree fragment)":2
20839     * def __reduce_cython__(self):
20840     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
20841     * def __setstate_cython__(self, __pyx_state):
20842     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20843     */
20844     __pyx_tuple__3 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
20845     (unlikely(!__pyx_tuple__3)) __PYX_ERR(1, 2, __pyx_L1_error)
20846     __Pyx_GOTREF(__pyx_tuple__3);
20847     __Pyx_GIVEREF(__pyx_tuple__3);
20848
20849     /* "(tree fragment)":4
20850     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20851     * def __setstate_cython__(self, __pyx_state):
20852     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
20853     */
20854     __pyx_tuple__4 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
20855     (unlikely(!__pyx_tuple__4)) __PYX_ERR(1, 4, __pyx_L1_error)
20856     __Pyx_GOTREF(__pyx_tuple__4);
20857     __Pyx_GIVEREF(__pyx_tuple__4);
20858
20859     /* "PyClical.pyx":636
20860     *     raise TypeError: Not applicable.
20861     *     """
20862     *     raise TypeError("Not applicable.") # ««««««««
20863     *
20864     * def __iter__(self):
20865     */
20866     __pyx_tuple__10 = PyTuple_Pack(1, __pyx_kp_u_Not_applicable); if (unlikely(!__pyx_tuple__10))
20867     __PYX_ERR(0, 636, __pyx_L1_error)
20868     __Pyx_GOTREF(__pyx_tuple__10);
20869     __Pyx_GIVEREF(__pyx_tuple__10);
20870
20871     /* "(tree fragment)":2
20872     * def __reduce_cython__(self):
20873     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««

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20863 * def __setstate_cython__(self, __pyx_state):
20864 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20865 */
20866 __pyx_tuple__11 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
(unlikely(!__pyx_tuple__11)) __PYX_ERR(1, 2, __pyx_L1_error)
20867 __Pyx_GOTREF(__pyx_tuple__11);
20868 __Pyx_GIVEREF(__pyx_tuple__11);
20869
20870 /* "tree fragment":4
20871 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20872 * def __setstate_cython__(self, __pyx_state):
20873 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")          # ««««««««
20874 */
20875 __pyx_tuple__12 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
(unlikely(!__pyx_tuple__12)) __PYX_ERR(1, 4, __pyx_L1_error)
20876 __Pyx_GOTREF(__pyx_tuple__12);
20877 __Pyx_GIVEREF(__pyx_tuple__12);
20878
20879 /* "PyClical.pyx":406
20880 *     return index_set_to_str( self.unwrap() ).decode()
20881 *
20882 * def index_set_hidden_doctests():          # ««««««««
20883 *     """
20884 *     Tests for functions that Doctest cannot see.
20885 */
20886 __pyx_codeobj__13 = (PyObject*)__Pyx_PyCode_New(0, 0, 0, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_index_set_hidden_doctests, 406,
__pyx_empty_bytes); if (unlikely(!__pyx_codeobj__13)) __PYX_ERR(0, 406, __pyx_L1_error)
20887
20888 /* "PyClical.pyx":1253
20889 *     return clifford_to_str( self.unwrap() ).decode()
20890 *
20891 * def clifford_hidden_doctests():          # ««««««««
20892 *     """
20893 *     Tests for functions that Doctest cannot see.
20894 */
20895 __pyx_codeobj__14 = (PyObject*)__Pyx_PyCode_New(0, 0, 0, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_clifford_hidden_doctests, 1253,
__pyx_empty_bytes); if (unlikely(!__pyx_codeobj__14)) __PYX_ERR(0, 1253, __pyx_L1_error)
20896
20897 /* "PyClical.pyx":1907
20898 * scalar_epsilon = epsilon
20899 *
20900 * pi = atan(clifford(1.0)) * 4.0          # ««««««««
20901 * tau = atan(clifford(1.0)) * 8.0
20902 *
20903 */
20904 __pyx_tuple__15 = PyTuple_Pack(1, __pyx_float_1_0); if (unlikely(!__pyx_tuple__15)) __PYX_ERR(0,
1907, __pyx_L1_error)
20905 __Pyx_GOTREF(__pyx_tuple__15);
20906 __Pyx_GIVEREF(__pyx_tuple__15);
20907
20908 /* "PyClical.pyx":1936
20909 * """
20910 *
20911 * def e(obj):          # ««««««««
20912 *     """
20913 *     Abbreviation for clifford(index_set(obj)).
20914 */
20915 __pyx_tuple__16 = PyTuple_Pack(1, __pyx_n_s_obj); if (unlikely(!__pyx_tuple__16)) __PYX_ERR(0, 1936,
__pyx_L1_error)
20916 __Pyx_GOTREF(__pyx_tuple__16);
20917 __Pyx_GIVEREF(__pyx_tuple__16);
20918 __pyx_codeobj__17 = (PyObject*)__Pyx_PyCode_New(1, 0, 1, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_tuple__16, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_e, 1936, __pyx_empty_bytes); if
(unlikely(!__pyx_codeobj__17)) __PYX_ERR(0, 1936, __pyx_L1_error)
20919
20920 /* "PyClical.pyx":1949
20921 *     return clifford(index_set(obj))
20922 *
20923 * def istpq(p, q):          # ««««««««
20924 *     """
20925 *     Abbreviation for index_set({-q,...p}).
20926 */
20927 __pyx_tuple__18 = PyTuple_Pack(2, __pyx_n_s_p, __pyx_n_s_q); if (unlikely(!__pyx_tuple__18))
__PYX_ERR(0, 1949, __pyx_L1_error)
20928 __Pyx_GOTREF(__pyx_tuple__18);
20929 __Pyx_GIVEREF(__pyx_tuple__18);
20930 __pyx_codeobj__19 = (PyObject*)__Pyx_PyCode_New(2, 0, 2, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_tuple__18, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_istpq, 1949, __pyx_empty_bytes); if
(unlikely(!__pyx_codeobj__19)) __PYX_ERR(0, 1949, __pyx_L1_error)
20931
20932 /* "PyClical.pyx":1958

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20933 *      return index_set(set(range(-q,p+1)))
20934 *
20935 * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].      #
20936 * nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
20937 *
20938 */
20939 __pyx_tuple__20 = PyTuple_Pack(1, __pyx_int_4); if (unlikely(!__pyx_tuple__20)) __PYX_ERR(0, 1958,
__pyx_L1_error)
20940 __Pyx_GOTREF(__pyx_tuple__20);
20941 __Pyx_GIVEREF(__pyx_tuple__20);
20942 __pyx_tuple__21 = PyTuple_Pack(1, __pyx_int_neg_1); if (unlikely(!__pyx_tuple__21)) __PYX_ERR(0,
1958, __pyx_L1_error)
20943 __Pyx_GOTREF(__pyx_tuple__21);
20944 __Pyx_GIVEREF(__pyx_tuple__21);
20945
20946 /* "PyClicl.pyx":1962
20947 *
20948 * # Doctest interface.
20949 * def _test():      # ««««««
20950 *     import PyClicl, doctest
20951 *     return doctest.testmod(PyClicl)
20952 */
20953 __pyx_tuple__22 = PyTuple_Pack(2, __pyx_n_s_PyClicl, __pyx_n_s_doctest); if
(unlikely(!__pyx_tuple__22)) __PYX_ERR(0, 1962, __pyx_L1_error)
20954 __Pyx_GOTREF(__pyx_tuple__22);
20955 __Pyx_GIVEREF(__pyx_tuple__22);
20956 __pyx_codeobj__23 = (PyObject *)__Pyx_PyCode_New(0, 0, 2, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_tuple__22, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClicl.pyx, __pyx_n_s_test, 1962, __pyx_empty_bytes); if
(unlikely(!__pyx_codeobj__23)) __PYX_ERR(0, 1962, __pyx_L1_error)
20957 __Pyx_RefNannyFinishContext();
20958 return 0;
20959 __pyx_L1_error:;
20960 __Pyx_RefNannyFinishContext();
20961 return -1;
20962 }
20963
20964 static CYTHON_SMALL_CODE int __Pyx_InitGlobals(void) {
20965     if (__Pyx_InitStrings(__pyx_string_tab) < 0) __PYX_ERR(0, 1, __pyx_L1_error);
20966     __pyx_float_0_0 = PyFloat_FromDouble(0.0); if (unlikely(!__pyx_float_0_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20967     __pyx_float_1_0 = PyFloat_FromDouble(1.0); if (unlikely(!__pyx_float_1_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20968     __pyx_float_4_0 = PyFloat_FromDouble(4.0); if (unlikely(!__pyx_float_4_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20969     __pyx_float_8_0 = PyFloat_FromDouble(8.0); if (unlikely(!__pyx_float_8_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20970     __pyx_int_0 = PyInt_FromLong(0); if (unlikely(!__pyx_int_0)) __PYX_ERR(0, 1, __pyx_L1_error)
20971     __pyx_int_1 = PyInt_FromLong(1); if (unlikely(!__pyx_int_1)) __PYX_ERR(0, 1, __pyx_L1_error)
20972     __pyx_int_4 = PyInt_FromLong(4); if (unlikely(!__pyx_int_4)) __PYX_ERR(0, 1, __pyx_L1_error)
20973     __pyx_int_neg_1 = PyInt_FromLong(-1); if (unlikely(!__pyx_int_neg_1)) __PYX_ERR(0, 1,
__pyx_L1_error)
20974     return 0;
20975     __pyx_L1_error:;
20976     return -1;
20977 }
20978
20979 static CYTHON_SMALL_CODE int __Pyx_modinit_global_init_code(void); /*proto*/
20980 static CYTHON_SMALL_CODE int __Pyx_modinit_variable_export_code(void); /*proto*/
20981 static CYTHON_SMALL_CODE int __Pyx_modinit_function_export_code(void); /*proto*/
20982 static CYTHON_SMALL_CODE int __Pyx_modinit_type_init_code(void); /*proto*/
20983 static CYTHON_SMALL_CODE int __Pyx_modinit_type_import_code(void); /*proto*/
20984 static CYTHON_SMALL_CODE int __Pyx_modinit_variable_import_code(void); /*proto*/
20985 static CYTHON_SMALL_CODE int __Pyx_modinit_function_import_code(void); /*proto*/
20986
20987 static int __Pyx_modinit_global_init_code(void) {
20988     __Pyx_RefNannyDeclarations
20989     __Pyx_RefNannySetupContext("__Pyx_modinit_global_init_code", 0);
20990     /*--- Global init code ---*/
20991     __Pyx_RefNannyFinishContext();
20992     return 0;
20993 }
20994
20995 static int __Pyx_modinit_variable_export_code(void) {
20996     __Pyx_RefNannyDeclarations
20997     __Pyx_RefNannySetupContext("__Pyx_modinit_variable_export_code", 0);
20998     /*--- Variable export code ---*/
20999     __Pyx_RefNannyFinishContext();
21000     return 0;
21001 }
21002
21003 static int __Pyx_modinit_function_export_code(void) {
21004     __Pyx_RefNannyDeclarations
21005     __Pyx_RefNannySetupContext("__Pyx_modinit_function_export_code", 0);
21006     /*--- Function export code ---*/
21007     __Pyx_RefNannyFinishContext();

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21008     return 0;
21009 }
21010
21011 static int __Pyx_modinit_type_init_code(void) {
21012     __Pyx_RefNannyDeclarations
21013     int __pyx_lineno = 0;
21014     const char *__pyx_filename = NULL;
21015     int __pyx_clineno = 0;
21016     __Pyx_RefNannySetupContext("__Pyx_modinit_type_init_code", 0);
21017     /*--- Type init code ---*/
21018     __pyx_vtabptr_8PyClical_index_set = &__pyx_vtable_8PyClical_index_set;
21019     __pyx_vtable_8PyClical_index_set.wrap = (PyObject *) (struct __pyx_obj_8PyClical_index_set *,
IndexSet)) __pyx_f_8PyClical_9index_set_wrap;
21020     __pyx_vtable_8PyClical_index_set.unwrap = (IndexSet *) (struct __pyx_obj_8PyClical_index_set
*) __pyx_f_8PyClical_9index_set_unwrap;
21021     __pyx_vtable_8PyClical_index_set.copy = (PyObject *) (struct __pyx_obj_8PyClical_index_set *, int
__pyx_skip_dispatch)) __pyx_f_8PyClical_9index_set_copy;
21022     if (PyType_Ready(&__pyx_type_8PyClical_index_set) < 0) __PYX_ERR(0, 46, __pyx_L1_error)
21023     #if PY_VERSION_HEX < 0x030800B1
21024     __pyx_type_8PyClical_index_set.tp_print = 0;
21025     #endif
21026     if ((CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP) &&
likely(!__pyx_type_8PyClical_index_set.tp_dictoffset && __pyx_type_8PyClical_index_set.tp_getattro ==
PyObject_GenericGetAttr)) {
21027         __pyx_type_8PyClical_index_set.tp_getattro = __Pyx_PyObject_GenericGetAttr;
21028     }
21029     #if CYTHON_COMPILING_IN_CPYTHON
21030     {
21031         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
"__setitem__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21032         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21033             __pyx_wrapperbase_8PyClical_9index_set_8__setitem__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21034             __pyx_wrapperbase_8PyClical_9index_set_8__setitem__.doc =
__pyx_doc_8PyClical_9index_set_8__setitem__;
21035             ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_8__setitem__;
21036         }
21037     }
21038     #endif
21039     #if CYTHON_COMPILING_IN_CPYTHON
21040     {
21041         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
"__getitem__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21042         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21043             __pyx_wrapperbase_8PyClical_9index_set_10__getitem__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21044             __pyx_wrapperbase_8PyClical_9index_set_10__getitem__.doc =
__pyx_doc_8PyClical_9index_set_10__getitem__;
21045             ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_10__getitem__;
21046         }
21047     }
21048     #endif
21049     #if CYTHON_COMPILING_IN_CPYTHON
21050     {
21051         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
"__contains__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21052         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21053             __pyx_wrapperbase_8PyClical_9index_set_12__contains__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21054             __pyx_wrapperbase_8PyClical_9index_set_12__contains__.doc =
__pyx_doc_8PyClical_9index_set_12__contains__;
21055             ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_12__contains__;
21056         }
21057     }
21058     #endif
21059     #if CYTHON_COMPILING_IN_CPYTHON
21060     {
21061         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
"__iter__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21062         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21063             __pyx_wrapperbase_8PyClical_9index_set_14__iter__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21064             __pyx_wrapperbase_8PyClical_9index_set_14__iter__.doc =
__pyx_doc_8PyClical_9index_set_14__iter__;
21065             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_14__iter__;
21066         }
21067     }
21068     #endif
21069     #if CYTHON_COMPILING_IN_CPYTHON
21070     {
21071         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
"__invert__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21072         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21073             __pyx_wrapperbase_8PyClical_9index_set_17__invert__ = *((PyWrapperDescrObject
*)wrapper)->d_base;

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21074     __pyx_wrapperbase_8PyClical_9index_set_17__invert___.doc =
__pyx_doc_8PyClical_9index_set_17__invert__;
21075     ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_17__invert__;
21076     }
21077     }
21078     #endif
21079     #if CYTHON_COMPILING_IN_CPYTHON
21080     {
21081         PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
"__xor__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21082         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21083             __pyx_wrapperbase_8PyClical_9index_set_19__xor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21084             __pyx_wrapperbase_8PyClical_9index_set_19__xor___.doc = __pyx_doc_8PyClical_9index_set_19__xor__;
21085             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_19__xor__;
21086         }
21087     }
21088     #endif
21089     #if CYTHON_COMPILING_IN_CPYTHON
21090     {
21091         PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
"__ixor__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21092         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21093             __pyx_wrapperbase_8PyClical_9index_set_21__ixor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21094             __pyx_wrapperbase_8PyClical_9index_set_21__ixor___.doc =
__pyx_doc_8PyClical_9index_set_21__ixor__;
21095             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_21__ixor__;
21096         }
21097     }
21098     #endif
21099     #if CYTHON_COMPILING_IN_CPYTHON
21100     {
21101         PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
"__and__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21102         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21103             __pyx_wrapperbase_8PyClical_9index_set_23__and__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21104             __pyx_wrapperbase_8PyClical_9index_set_23__and___.doc = __pyx_doc_8PyClical_9index_set_23__and__;
21105             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_23__and__;
21106         }
21107     }
21108     #endif
21109     #if CYTHON_COMPILING_IN_CPYTHON
21110     {
21111         PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
"__iand__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21112         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21113             __pyx_wrapperbase_8PyClical_9index_set_25__iand__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21114             __pyx_wrapperbase_8PyClical_9index_set_25__iand___.doc =
__pyx_doc_8PyClical_9index_set_25__iand__;
21115             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_25__iand__;
21116         }
21117     }
21118     #endif
21119     #if CYTHON_COMPILING_IN_CPYTHON
21120     {
21121         PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set, "__or__");
if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21122         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21123             __pyx_wrapperbase_8PyClical_9index_set_27__or__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21124             __pyx_wrapperbase_8PyClical_9index_set_27__or___.doc = __pyx_doc_8PyClical_9index_set_27__or__;
21125             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_27__or__;
21126         }
21127     }
21128     #endif
21129     #if CYTHON_COMPILING_IN_CPYTHON
21130     {
21131         PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
"__ior__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21132         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21133             __pyx_wrapperbase_8PyClical_9index_set_29__ior__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21134             __pyx_wrapperbase_8PyClical_9index_set_29__ior___.doc = __pyx_doc_8PyClical_9index_set_29__ior__;
21135             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_29__ior__;
21136         }
21137     }
21138     #endif
21139     #if CYTHON_COMPILING_IN_CPYTHON
21140     {
21141         PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
"__repr__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21142         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21143             __pyx_wrapperbase_8PyClical_9index_set_47__repr__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21144             __pyx_wrapperbase_8PyClical_9index_set_47__repr___.doc =
__pyx_doc_8PyClical_9index_set_47__repr__;
21145             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_47__repr__;
21146         }
21147     }
21148     #endif

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21149     #if CYTHON_COMPILING_IN_CPYTHON
21150     {
21151         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21152         "__str__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21153         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21154             __pyx_wrapperbase_8PyClical_9index_set_49_str__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21155             __pyx_wrapperbase_8PyClical_9index_set_49_str__.doc = __pyx_doc_8PyClical_9index_set_49_str__;
21156             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_49_str__;
21157         }
21158     }
21159     #endif
21160     if (__Pyx_SetVtable(__pyx_type_8PyClical_index_set.tp_dict, __pyx_vtabptr_8PyClical_index_set) < 0)
21161     __PYX_ERR(0, 46, __pyx_L1_error)
21162     if (PyObject_SetAttr(__pyx_m, __pyx_n_s_index_set, (PyObject *) &__pyx_type_8PyClical_index_set) < 0)
21163     __PYX_ERR(0, 46, __pyx_L1_error)
21164     if (__Pyx_setup_reduce((PyObject *) &__pyx_type_8PyClical_index_set) < 0) __PYX_ERR(0, 46,
21165     __pyx_L1_error)
21166     __pyx_ptype_8PyClical_index_set = &__pyx_type_8PyClical_index_set;
21167     __pyx_vtabptr_8PyClical_clifford = &__pyx_vtable_8PyClical_clifford;
21168     __pyx_vtable_8PyClical_clifford.wrap = (PyObject *) (struct __pyx_obj_8PyClical_clifford *,
21169     Clifford) __pyx_f_8PyClical_8clifford_wrap;
21170     __pyx_vtable_8PyClical_clifford.unwrap = (Clifford *) (struct __pyx_obj_8PyClical_clifford
21171     *) __pyx_f_8PyClical_8clifford_unwrap;
21172     __pyx_vtable_8PyClical_clifford.copy = (PyObject *) (struct __pyx_obj_8PyClical_clifford *, int
21173     __pyx_skip_dispatch) __pyx_f_8PyClical_8clifford_copy;
21174     if (PyType_Ready(&__pyx_type_8PyClical_clifford) < 0) __PYX_ERR(0, 537, __pyx_L1_error)
21175     #if PY_VERSION_HEX < 0x030800B1
21176     __pyx_type_8PyClical_clifford.tp_print = 0;
21177     #endif
21178     if ((CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP) &&
21179     likely(!__pyx_type_8PyClical_clifford.tp_dictoffset && __pyx_type_8PyClical_clifford.tp_getattro ==
21180     PyObject_GenericGetAttr)) {
21181         __pyx_type_8PyClical_clifford.tp_getattro = __Pyx_PyObject_GenericGetAttr;
21182     }
21183     #if CYTHON_COMPILING_IN_CPYTHON
21184     {
21185         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21186         "__contains__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21187         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21188             __pyx_wrapperbase_8PyClical_8clifford_6__contains__ = *((PyWrapperDescrObject
21189             *)wrapper)->d_base;
21190             __pyx_wrapperbase_8PyClical_8clifford_6__contains__.doc =
21191             __pyx_doc_8PyClical_8clifford_6__contains__;
21192             ((PyWrapperDescrObject *)wrapper)->d_base =
21193             &__pyx_wrapperbase_8PyClical_8clifford_6__contains__;
21194         }
21195     }
21196     #endif
21197     #if CYTHON_COMPILING_IN_CPYTHON
21198     {
21199         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21200         "__iter__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21201         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21202             __pyx_wrapperbase_8PyClical_8clifford_8_iter__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21203             __pyx_wrapperbase_8PyClical_8clifford_8_iter__.doc = __pyx_doc_8PyClical_8clifford_8_iter__;
21204             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_8_iter__;
21205         }
21206     }
21207     #endif
21208     #if CYTHON_COMPILING_IN_CPYTHON
21209     {
21210         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21211         "__getitem__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21212         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21213             __pyx_wrapperbase_8PyClical_8clifford_14__getitem__ = *((PyWrapperDescrObject
21214             *)wrapper)->d_base;
21215             __pyx_wrapperbase_8PyClical_8clifford_14__getitem__.doc =
21216             __pyx_doc_8PyClical_8clifford_14__getitem__;
21217             ((PyWrapperDescrObject *)wrapper)->d_base =
21218             &__pyx_wrapperbase_8PyClical_8clifford_14__getitem__;
21219         }
21220     }
21221     #endif
21222     #if CYTHON_COMPILING_IN_CPYTHON
21223     {
21224         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__neg__");
21225         if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21226         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21227             __pyx_wrapperbase_8PyClical_8clifford_16_neg__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21228             __pyx_wrapperbase_8PyClical_8clifford_16_neg__.doc = __pyx_doc_8PyClical_8clifford_16_neg__;
21229             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_16_neg__;
21230         }
21231     }
21232     #endif
21233     #if CYTHON_COMPILING_IN_CPYTHON
21234     {
21235         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__pos__");

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    if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21217     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21218         __pyx_wrapperbase_8PyClical_8clifford_18__pos__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21219         __pyx_wrapperbase_8PyClical_8clifford_18__pos__.doc = __pyx_doc_8PyClical_8clifford_18__pos__;
21220         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_18__pos__;
21221     }
21222 }
21223 #endif
21224 #if CYTHON_COMPILING_IN_CPYTHON
21225 {
21226     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__add__");
21227     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21228     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21229         __pyx_wrapperbase_8PyClical_8clifford_20__add__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21230         __pyx_wrapperbase_8PyClical_8clifford_20__add__.doc = __pyx_doc_8PyClical_8clifford_20__add__;
21231         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_20__add__;
21232     }
21233 }
21234 #endif
21235 #if CYTHON_COMPILING_IN_CPYTHON
21236 {
21237     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
21238         "__iadd__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21239     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21240         __pyx_wrapperbase_8PyClical_8clifford_22__iadd__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21241         __pyx_wrapperbase_8PyClical_8clifford_22__iadd__.doc = __pyx_doc_8PyClical_8clifford_22__iadd__;
21242         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_22__iadd__;
21243     }
21244 }
21245 #endif
21246 #if CYTHON_COMPILING_IN_CPYTHON
21247 {
21248     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__sub__");
21249     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21250     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21251         __pyx_wrapperbase_8PyClical_8clifford_24__sub__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21252         __pyx_wrapperbase_8PyClical_8clifford_24__sub__.doc = __pyx_doc_8PyClical_8clifford_24__sub__;
21253         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_24__sub__;
21254     }
21255 }
21256 #endif
21257 #if CYTHON_COMPILING_IN_CPYTHON
21258 {
21259     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
21260         "__isub__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21261     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21262         __pyx_wrapperbase_8PyClical_8clifford_26__isub__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21263         __pyx_wrapperbase_8PyClical_8clifford_26__isub__.doc = __pyx_doc_8PyClical_8clifford_26__isub__;
21264         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_26__isub__;
21265     }
21266 }
21267 #endif
21268 #if CYTHON_COMPILING_IN_CPYTHON
21269 {
21270     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__mul__");
21271     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21272     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21273         __pyx_wrapperbase_8PyClical_8clifford_28__mul__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21274         __pyx_wrapperbase_8PyClical_8clifford_28__mul__.doc = __pyx_doc_8PyClical_8clifford_28__mul__;
21275         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_28__mul__;
21276     }
21277 }
21278 #endif
21279 #if CYTHON_COMPILING_IN_CPYTHON
21280 {
21281     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
21282         "__imul__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21283     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21284         __pyx_wrapperbase_8PyClical_8clifford_30__imul__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21285         __pyx_wrapperbase_8PyClical_8clifford_30__imul__.doc = __pyx_doc_8PyClical_8clifford_30__imul__;
21286         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_30__imul__;
21287     }
21288 }
21289 #endif
21290 #if CYTHON_COMPILING_IN_CPYTHON
21291 {
21292     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__mod__");
21293     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21294     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21295         __pyx_wrapperbase_8PyClical_8clifford_32__mod__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21296         __pyx_wrapperbase_8PyClical_8clifford_32__mod__.doc = __pyx_doc_8PyClical_8clifford_32__mod__;
21297         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_32__mod__;
21298     }
21299 }
21300 #endif
21301 #if CYTHON_COMPILING_IN_CPYTHON
21302 {

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21296     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21297         "__imod__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21297     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21298         __pyx_wrapperbase_8PyClical_8clifford_34__imod__ = *((PyWrapperDescrObject *) wrapper)->d_base;
21299         __pyx_wrapperbase_8PyClical_8clifford_34__imod__.doc = __pyx_doc_8PyClical_8clifford_34__imod__;
21300         ((PyWrapperDescrObject *) wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_34__imod__;
21301     }
21302 }
21303 #endif
21304 #if CYTHON_COMPILING_IN_CPYTHON
21305 {
21306     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__and__");
21307     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21307     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21308         __pyx_wrapperbase_8PyClical_8clifford_36__and__ = *((PyWrapperDescrObject *) wrapper)->d_base;
21309         __pyx_wrapperbase_8PyClical_8clifford_36__and__.doc = __pyx_doc_8PyClical_8clifford_36__and__;
21310         ((PyWrapperDescrObject *) wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_36__and__;
21311     }
21312 }
21313 #endif
21314 #if CYTHON_COMPILING_IN_CPYTHON
21315 {
21316     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21317         "__iand__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21317     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21318         __pyx_wrapperbase_8PyClical_8clifford_38__iand__ = *((PyWrapperDescrObject *) wrapper)->d_base;
21319         __pyx_wrapperbase_8PyClical_8clifford_38__iand__.doc = __pyx_doc_8PyClical_8clifford_38__iand__;
21320         ((PyWrapperDescrObject *) wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_38__iand__;
21321     }
21322 }
21323 #endif
21324 #if CYTHON_COMPILING_IN_CPYTHON
21325 {
21326     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__xor__");
21327     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21327     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21328         __pyx_wrapperbase_8PyClical_8clifford_40__xor__ = *((PyWrapperDescrObject *) wrapper)->d_base;
21329         __pyx_wrapperbase_8PyClical_8clifford_40__xor__.doc = __pyx_doc_8PyClical_8clifford_40__xor__;
21330         ((PyWrapperDescrObject *) wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_40__xor__;
21331     }
21332 }
21333 #endif
21334 #if CYTHON_COMPILING_IN_CPYTHON
21335 {
21336     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21337         "__ixor__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21337     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21338         __pyx_wrapperbase_8PyClical_8clifford_42__ixor__ = *((PyWrapperDescrObject *) wrapper)->d_base;
21339         __pyx_wrapperbase_8PyClical_8clifford_42__ixor__.doc = __pyx_doc_8PyClical_8clifford_42__ixor__;
21340         ((PyWrapperDescrObject *) wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_42__ixor__;
21341     }
21342 }
21343 #endif
21344 #if CYTHON_COMPILING_IN_CPYTHON
21345 {
21346     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21347         "__truediv__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21347     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21348         __pyx_wrapperbase_8PyClical_8clifford_44__truediv__ = *((PyWrapperDescrObject *) wrapper)->d_base;
21349         __pyx_wrapperbase_8PyClical_8clifford_44__truediv__.doc =
21350         __pyx_doc_8PyClical_8clifford_44__truediv__;
21350         ((PyWrapperDescrObject *) wrapper)->d_base =
21351         &__pyx_wrapperbase_8PyClical_8clifford_44__truediv__;
21351     }
21352 }
21353 #endif
21354 #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
21355 #if CYTHON_COMPILING_IN_CPYTHON
21356 {
21357     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21358         "__idiv__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21357     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21358         __pyx_wrapperbase_8PyClical_8clifford_46__idiv__ = *((PyWrapperDescrObject *) wrapper)->d_base;
21359         __pyx_wrapperbase_8PyClical_8clifford_46__idiv__.doc = __pyx_doc_8PyClical_8clifford_46__idiv__;
21360         ((PyWrapperDescrObject *) wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_46__idiv__;
21361     }
21362 }
21363 #endif
21364 #endif
21365 #if CYTHON_COMPILING_IN_CPYTHON
21366 {
21367     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__or__");
21368     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21367     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21368         __pyx_wrapperbase_8PyClical_8clifford_50__or__ = *((PyWrapperDescrObject *) wrapper)->d_base;
21369         __pyx_wrapperbase_8PyClical_8clifford_50__or__.doc = __pyx_doc_8PyClical_8clifford_50__or__;
21370     }
21371 }

```

```

21372     ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_50__or__;
21373 }
21374 }
21375 #endif
21376 #if CYTHON_COMPILING_IN_CPYTHON
21377 {
21378     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__ior__");
21379     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21380     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21381         __pyx_wrapperbase_8PyClical_8clifford_52__ior__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21382         __pyx_wrapperbase_8PyClical_8clifford_52__ior__.doc = __pyx_doc_8PyClical_8clifford_52__ior__;
21383         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_52__ior__;
21384     }
21385 #endif
21386 #if CYTHON_COMPILING_IN_CPYTHON
21387 {
21388     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__pow__");
21389     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21390     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21391         __pyx_wrapperbase_8PyClical_8clifford_54__pow__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21392         __pyx_wrapperbase_8PyClical_8clifford_54__pow__.doc = __pyx_doc_8PyClical_8clifford_54__pow__;
21393         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_54__pow__;
21394     }
21395 #endif
21396 #if CYTHON_COMPILING_IN_CPYTHON
21397 {
21398     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
21399     "__call__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21400     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21401         __pyx_wrapperbase_8PyClical_8clifford_60__call__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21402         __pyx_wrapperbase_8PyClical_8clifford_60__call__.doc = __pyx_doc_8PyClical_8clifford_60__call__;
21403         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_60__call__;
21404     }
21405 #endif
21406 #if CYTHON_COMPILING_IN_CPYTHON
21407 {
21408     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
21409     "__repr__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21410     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21411         __pyx_wrapperbase_8PyClical_8clifford_94__repr__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21412         __pyx_wrapperbase_8PyClical_8clifford_94__repr__.doc = __pyx_doc_8PyClical_8clifford_94__repr__;
21413         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_94__repr__;
21414     }
21415 #endif
21416 #if CYTHON_COMPILING_IN_CPYTHON
21417 {
21418     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__str__");
21419     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21420     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21421         __pyx_wrapperbase_8PyClical_8clifford_96__str__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21422         __pyx_wrapperbase_8PyClical_8clifford_96__str__.doc = __pyx_doc_8PyClical_8clifford_96__str__;
21423         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_96__str__;
21424     }
21425 #endif
21426 if (__Pyx_SetVtable(__pyx_type_8PyClical_clifford.tp_dict, __pyx_vtabptr_8PyClical_clifford) < 0)
21427     __PYX_ERR(0, 537, __pyx_L1_error)
21428 if (PyObject_SetAttr(__pyx_m, __pyx_n_s_clifford, (PyObject *)&__pyx_type_8PyClical_clifford) < 0)
21429     __PYX_ERR(0, 537, __pyx_L1_error)
21430 if (__Pyx_setup_reduce((PyObject *)&__pyx_type_8PyClical_clifford) < 0) __PYX_ERR(0, 537,
21431     __pyx_L1_error)
21432 __pyx_type_8PyClical_clifford = &__pyx_type_8PyClical_clifford;
21433 if (PyType_Ready(&__pyx_type_8PyClical__pyx_scope_struct__iter__) < 0) __PYX_ERR(0, 229,
21434     __pyx_L1_error)
21435 #if PY_VERSION_HEX < 0x030800B1
21436     __pyx_type_8PyClical__pyx_scope_struct__iter__.tp_print = 0;
21437 #endif
21438 if ((CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP) &&
21439     likely(!__pyx_type_8PyClical__pyx_scope_struct__iter__.tp_dictoffset &&
21440     __pyx_type_8PyClical__pyx_scope_struct__iter__.tp_getattro == PyObject_GenericGetAttr)) {
21441     __pyx_type_8PyClical__pyx_scope_struct__iter__.tp_getattro =
21442     __Pyx_PyObject_GenericGetAttrNoDict;
21443 }
21444 __pyx_ptype_8PyClical__pyx_scope_struct__iter__ =
21445 &__pyx_type_8PyClical__pyx_scope_struct__iter__;
21446 __Pyx_RefNannyFinishContext();
21447 return 0;
21448 __pyx_L1_error:;
21449 __Pyx_RefNannyFinishContext();
21450 return -1;
21451 }
21452 }
21453 static int __Pyx_modinit_type_import_code(void) {

```

```

21446 __Pyx_RefNannyDeclarations
21447 __Pyx_RefNannySetupContext("__Pyx_modinit_type_import_code", 0);
21448 /*--- Type import code ---*/
21449 __Pyx_RefNannyFinishContext();
21450 return 0;
21451 }
21452
21453 static int __Pyx_modinit_variable_import_code(void) {
21454     __Pyx_RefNannyDeclarations
21455     __Pyx_RefNannySetupContext("__Pyx_modinit_variable_import_code", 0);
21456     /*--- Variable import code ---*/
21457     __Pyx_RefNannyFinishContext();
21458     return 0;
21459 }
21460
21461 static int __Pyx_modinit_function_import_code(void) {
21462     __Pyx_RefNannyDeclarations
21463     __Pyx_RefNannySetupContext("__Pyx_modinit_function_import_code", 0);
21464     /*--- Function import code ---*/
21465     __Pyx_RefNannyFinishContext();
21466     return 0;
21467 }
21468
21469
21470 #ifndef CYTHON_NO_PYINIT_EXPORT
21471 #define __Pyx_PyMODINIT_FUNC PyMODINIT_FUNC
21472 #elif PY_MAJOR_VERSION < 3
21473 #define __cplusplus
21474 #define __Pyx_PyMODINIT_FUNC extern "C" void
21475 #else
21476 #define __Pyx_PyMODINIT_FUNC void
21477 #endif
21478 #else
21479 #define __cplusplus
21480 #define __Pyx_PyMODINIT_FUNC extern "C" PyObject *
21481 #else
21482 #define __Pyx_PyMODINIT_FUNC PyObject *
21483 #endif
21484 #endif
21485
21486
21487 #if PY_MAJOR_VERSION < 3
21488 __Pyx_PyMODINIT_FUNC initPyClical(void) CYTHON_SMALL_CODE; /*proto*/
21489 __Pyx_PyMODINIT_FUNC initPyClical(void)
21490 #else
21491 __Pyx_PyMODINIT_FUNC PyInit_PyClical(void) CYTHON_SMALL_CODE; /*proto*/
21492 __Pyx_PyMODINIT_FUNC PyInit_PyClical(void)
21493 #if CYTHON_PEP489_MULTI_PHASE_INIT
21494 {
21495     return PyModuleDef_Init(&__pyx_moduledef);
21496 }
21497 static CYTHON_SMALL_CODE int __Pyx_check_single_interpreter(void) {
21498     #if PY_VERSION_HEX >= 0x030700A1
21499         static PY_INT64_T main_interpreter_id = -1;
21500         PY_INT64_T current_id = PyInterpreterState_GetID(PyThreadState_Get()->interp);
21501         if (main_interpreter_id == -1) {
21502             main_interpreter_id = current_id;
21503             return (unlikely(current_id == -1)) ? -1 : 0;
21504         } else if (unlikely(main_interpreter_id != current_id))
21505             #else
21506             static PyInterpreterState *main_interpreter = NULL;
21507             PyInterpreterState *current_interpreter = PyThreadState_Get()->interp;
21508             if (!main_interpreter) {
21509                 main_interpreter = current_interpreter;
21510             } else if (unlikely(main_interpreter != current_interpreter))
21511                 #endif
21512             {
21513                 PyErr_SetString(
21514                     PyExc_ImportError,
21515                     "Interpreter change detected - this module can only be loaded into one interpreter per
21516 process.");
21517                 return -1;
21518             }
21519         }
21520         return 0;
21521     }
21522 static CYTHON_SMALL_CODE int __Pyx_copy_spec_to_module(PyObject *spec, PyObject *moddict, const char*
21523 from_name, const char* to_name, int allow_none) {
21524     PyObject *value = PyObject_GetAttrString(spec, from_name);
21525     int result = 0;
21526     if (likely(value)) {
21527         if (allow_none || value != Py_None) {
21528             result = PyDict_SetItemString(moddict, to_name, value);
21529         }
21530         Py_DECREF(value);
21531     } else if (PyErr_ExceptionMatches(PyExc_AttributeError)) {
21532         PyErr_Clear();
21533     } else {

```

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21531         result = -1;
21532     }
21533     return result;
21534 }
21535 static CYTHON_SMALL_CODE PyObject* __pyx_pymod_create(PyObject *spec, CYTHON_UNUSED PyModuleDef *def)
21536 {
21537     PyObject *module = NULL, *moddict, *modname;
21538     if (__Pyx_check_single_interpreter())
21539         return NULL;
21540     if (__pyx_m)
21541         return __Pyx_NewRef(__pyx_m);
21542     modname = PyObject_GetAttrString(spec, "name");
21543     if (unlikely(!modname)) goto bad;
21544     module = PyModule_NewObject(modname);
21545     Py_DECREF(modname);
21546     if (unlikely(!module)) goto bad;
21547     moddict = PyModule_GetDict(module);
21548     if (unlikely(!moddict)) goto bad;
21549     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "loader", "__loader__", 1) < 0)) goto bad;
21550     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "origin", "__file__", 1) < 0)) goto bad;
21551     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "parent", "__package__", 1) < 0)) goto bad;
21552     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "submodule_search_locations", "__path__", 0)
21553 < 0)) goto bad;
21554     return module;
21555 bad:
21556     Py_XDECREF(module);
21557     return NULL;
21558 }
21559 static CYTHON_SMALL_CODE int __pyx_pymod_exec_PyClicl(PyObject *__pyx_pyinit_module)
21560 #endif
21561 #endif
21562 {
21563     PyObject *__pyx_t_1 = NULL;
21564     PyObject *__pyx_t_2 = NULL;
21565     PyObject *__pyx_t_3 = NULL;
21566     int __pyx_t_4;
21567     int __pyx_lineno = 0;
21568     const char *__pyx_filename = NULL;
21569     int __pyx_clineno = 0;
21570     __Pyx_RefNannyDeclarations
21571     #if CYTHON_PEP489_MULTI_PHASE_INIT
21572     if (__pyx_m) {
21573         if (__pyx_m == __pyx_pyinit_module) return 0;
21574         PyErr_SetString(PyExc_RuntimeError, "Module 'PyClicl' has already been imported.
21575 Re-initialisation is not supported.");
21576         return -1;
21577     }
21578     #elif PY_MAJOR_VERSION >= 3
21579     if (__pyx_m) return __Pyx_NewRef(__pyx_m);
21580     #endif
21581     #if CYTHON_REFNANNY
21582     __Pyx_RefNanny = __Pyx_RefNannyImportAPI("refnanny");
21583     if (!__Pyx_RefNanny) {
21584         PyErr_Clear();
21585         __Pyx_RefNanny = __Pyx_RefNannyImportAPI("Cython.Runtime.refnanny");
21586         if (!__Pyx_RefNanny)
21587             Py_FatalError("failed to import 'refnanny' module");
21588     }
21589     #endif
21590     __Pyx_RefNannySetupContext("__Pyx_PyMODINIT_FUNC PyInit_PyClicl(void)", 0);
21591     if (__Pyx_check_binary_version() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21592     #ifdef __Pxy_PyFrame_Initialize_Offsets
21593     __Pxy_PyFrame_Initialize_Offsets();
21594     #endif
21595     __pyx_empty_tuple = PyTuple_New(0); if (unlikely(!__pyx_empty_tuple)) __PYX_ERR(0, 1,
21596 __pyx_L1_error)
21597     __pyx_empty_bytes = PyBytes_FromStringAndSize("", 0); if (unlikely(!__pyx_empty_bytes)) __PYX_ERR(0,
21598 1, __pyx_L1_error)
21599     __pyx_empty_unicode = PyUnicode_FromStringAndSize("", 0); if (unlikely(!__pyx_empty_unicode))
21600 __PYX_ERR(0, 1, __pyx_L1_error)
21601     #ifdef __Pyx_CyFunction_USED
21602     if (__pyx_CyFunction_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21603     #endif
21604     #ifdef __Pyx_FusedFunction_USED
21605     if (__pyx_FusedFunction_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21606     #endif
21607     #ifdef __Pyx_Coroutine_USED
21608     if (__pyx_Coroutine_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21609     #endif
21610     #ifdef __Pyx_Generator_USED
21611     if (__pyx_Generator_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21612     #endif
21613     #ifdef __Pyx_AsyncGen_USED
21614     if (__pyx_AsyncGen_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21615     #endif

```

```

21612 #ifdef __Pyx_StopAsyncIteration_USED
21613 if (__pyx_StopAsyncIteration_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21614 #endif
21615 /*---- Library function declarations ----*/
21616 /*---- Threads initialization code ----*/
21617 #if defined(WITH_THREAD) && PY_VERSION_HEX < 0x030700F0 && defined(__PYX_FORCE_INIT_THREADS) &&
__PYX_FORCE_INIT_THREADS
21618 PyEval_InitThreads();
21619 #endif
21620 /*---- Module creation code ----*/
21621 #if CYTHON_PEP489_MULTI_PHASE_INIT
21622 __pyx_m = __pyx_pyinit_module;
21623 Py_INCREF(__pyx_m);
21624 #else
21625 #if PY_MAJOR_VERSION < 3
21626 __pyx_m = Py_InitModule4("PyClical", __pyx_methods, 0, 0, PYTHON_API_VERSION); Py_XINCREF(__pyx_m);
21627 #else
21628 __pyx_m = PyModule_Create(&__pyx_moduledef);
21629 #endif
21630 if (unlikely(!__pyx_m)) __PYX_ERR(0, 1, __pyx_L1_error)
21631 #endif
21632 __pyx_d = PyModule_GetDict(__pyx_m); if (unlikely(!__pyx_d)) __PYX_ERR(0, 1, __pyx_L1_error)
21633 Py_INCREF(__pyx_d);
21634 __pyx_b = PyImport_AddModule(__Pyx_BUILTIN_MODULE_NAME); if (unlikely(!__pyx_b)) __PYX_ERR(0, 1,
__pyx_L1_error)
21635 Py_INCREF(__pyx_b);
21636 __pyx_cython_runtime = PyImport_AddModule((char *) "cython_runtime"); if
(unlikely(!__pyx_cython_runtime)) __PYX_ERR(0, 1, __pyx_L1_error)
21637 Py_INCREF(__pyx_cython_runtime);
21638 if (PyObject_SetAttrString(__pyx_m, "__builtins__", __pyx_b) < 0) __PYX_ERR(0, 1, __pyx_L1_error);
21639 /*---- Initialize various global constants etc. ----*/
21640 if (__Pyx_InitGlobals() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21641 #if PY_MAJOR_VERSION < 3 && (__PYX_DEFAULT_STRING_ENCODING_IS_ASCII ||
__PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT)
21642 if (__Pyx_init_sys_getdefaultencoding_params() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21643 #endif
21644 if (__pyx_module_is_main_PyClical) {
21645     if (PyObject_SetAttr(__pyx_m, __pyx_n_s_name, __pyx_n_s_main) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21646 }
21647 #if PY_MAJOR_VERSION >= 3
21648 {
21649     PyObject *modules = PyImport_GetModuleDict(); if (unlikely(!modules)) __PYX_ERR(0, 1,
__pyx_L1_error)
21650     if (!PyDict_GetItemString(modules, "PyClical")) {
21651         if (unlikely(PyDict_SetItemString(modules, "PyClical", __pyx_m) < 0)) __PYX_ERR(0, 1,
__pyx_L1_error)
21652     }
21653 }
21654 #endif
21655 /*---- Builtin init code ----*/
21656 if (__Pyx_InitCachedBuiltins() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21657 /*---- Constants init code ----*/
21658 if (__Pyx_InitCachedConstants() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21659 /*---- Global type/function init code ----*/
21660 (void)__Pyx_modinit_global_init_code();
21661 (void)__Pyx_modinit_variable_export_code();
21662 (void)__Pyx_modinit_function_export_code();
21663 if (unlikely(__Pyx_modinit_type_init_code() < 0)) __PYX_ERR(0, 1, __pyx_L1_error)
21664 (void)__Pyx_modinit_type_import_code();
21665 (void)__Pyx_modinit_variable_import_code();
21666 (void)__Pyx_modinit_function_import_code();
21667 /*---- Execution code ----*/
21668 #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
21669 if (__Pyx_patch_abc() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21670 #endif
21671
21672 /* "PyClical.pyx":29
21673 * # C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
21674 *
21675 * import math                # ««««««««
21676 * import numbers
21677 * import collections
21678 */
21679 __pyx_t_1 = __Pyx_Import(__pyx_n_s_math, 0, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 29,
__pyx_L1_error)
21680 __Pyx_GOTREF(__pyx_t_1);
21681 if (PyDict_SetItem(__pyx_d, __pyx_n_s_math, __pyx_t_1) < 0) __PYX_ERR(0, 29, __pyx_L1_error)
21682 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21683
21684 /* "PyClical.pyx":30
21685 *
21686 * import math
21687 * import numbers                # ««««««««
21688 * import collections
21689 *
21690 */
21691 __pyx_t_1 = __Pyx_Import(__pyx_n_s_numbers, 0, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 30,

```

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__pyx_L1_error)
21692 __Pyx_GOTREF(__pyx_t_1);
21693 if (PyDict_SetItem(__pyx_d, __pyx_n_s_numbers, __pyx_t_1) < 0) __PYX_ERR(0, 30, __pyx_L1_error)
21694 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21695
21696 /* "PyClicl.pyx":31
21697 * import math
21698 * import numbers
21699 * import collections          # ««««««««
21700 *
21701 * from PyClicl cimport *
21702 */
21703 __pyx_t_1 = __Pyx_Import(__pyx_n_s_collections, 0, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 31,
__pyx_L1_error)
21704 __Pyx_GOTREF(__pyx_t_1);
21705 if (PyDict_SetItem(__pyx_d, __pyx_n_s_collections, __pyx_t_1) < 0) __PYX_ERR(0, 31, __pyx_L1_error)
21706 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21707
21708 /* "PyClicl.pyx":35
21709 * from PyClicl cimport *
21710 *
21711 * __version__ = str(glucat_package_version,'utf-8')          # ««««««««
21712 *
21713 * # Forward reference
21714 */
21715 __pyx_t_1 = __pyx_convert_PyBytes_string_to_py_std_in_string(glucat_package_version); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 35, __pyx_L1_error)
21716 __Pyx_GOTREF(__pyx_t_1);
21717 __pyx_t_2 = PyTuple_New(2); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 35, __pyx_L1_error)
21718 __Pyx_GOTREF(__pyx_t_2);
21719 __Pyx_GIVEREF(__pyx_t_1);
21720 PyTuple_SET_ITEM(__pyx_t_2, 0, __pyx_t_1);
21721 __Pyx_INCREF(__pyx_kp_u_utf_8);
21722 __Pyx_GIVEREF(__pyx_kp_u_utf_8);
21723 PyTuple_SET_ITEM(__pyx_t_2, 1, __pyx_kp_u_utf_8);
21724 __pyx_t_1 = 0;
21725 __pyx_t_1 = __Pyx_PyObject_Call(((PyObject *)(&PyUnicode_Type)), __pyx_t_2, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 35, __pyx_L1_error)
21726 __Pyx_GOTREF(__pyx_t_1);
21727 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21728 if (PyDict_SetItem(__pyx_d, __pyx_n_s_version, __pyx_t_1) < 0) __PYX_ERR(0, 35, __pyx_L1_error)
21729 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21730
21731 /* "PyClicl.pyx":406
21732 *         return index_set_to_str( self.unwrap() ).decode()
21733 *
21734 * def index_set_hidden_doctests():          # ««««««««
21735 *     """
21736 *     Tests for functions that Doctest cannot see.
21737 */
21738 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClicl_1index_set_hidden_doctests, NULL,
__pyx_n_s_PyClicl); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 406, __pyx_L1_error)
21739 __Pyx_GOTREF(__pyx_t_1);
21740 if (PyDict_SetItem(__pyx_d, __pyx_n_s_index_set_hidden_doctests, __pyx_t_1) < 0) __PYX_ERR(0, 406,
__pyx_L1_error)
21741 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21742
21743 /* "PyClicl.pyx":1253
21744 *         return clifford_to_str( self.unwrap() ).decode()
21745 *
21746 * def clifford_hidden_doctests():          # ««««««««
21747 *     """
21748 *     Tests for functions that Doctest cannot see.
21749 */
21750 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClicl_9clifford_hidden_doctests, NULL,
__pyx_n_s_PyClicl); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1253, __pyx_L1_error)
21751 __Pyx_GOTREF(__pyx_t_1);
21752 if (PyDict_SetItem(__pyx_d, __pyx_n_s_clifford_hidden_doctests, __pyx_t_1) < 0) __PYX_ERR(0, 1253,
__pyx_L1_error)
21753 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21754
21755 /* "PyClicl.pyx":1905
21756 *
21757 * # Some abbreviations.
21758 * scalar_epsilon = epsilon          # ««««««««
21759 *
21760 * pi = atan(clifford(1.0)) * 4.0
21761 */
21762 __pyx_t_1 = PyFloat_FromDouble(epsilon); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1905,
__pyx_L1_error)
21763 __Pyx_GOTREF(__pyx_t_1);
21764 if (PyDict_SetItem(__pyx_d, __pyx_n_s_scalar_epsilon, __pyx_t_1) < 0) __PYX_ERR(0, 1905,
__pyx_L1_error)
21765 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21766
21767 /* "PyClicl.pyx":1907
21768 * scalar_epsilon = epsilon

```

```

21769 *
21770 * pi = atan(clifford(1.0)) * 4.0          # ««««««««
21771 * tau = atan(clifford(1.0)) * 8.0
21772 *
21773 */
21774 __pyx_t_1 = __Pyx_PyObject_Call(((PyObject *)__pyx_ptype_8PyClical_clifford), __pyx_tuple__15,
NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1907, __pyx_L1_error)
21775 __Pyx_GOTREF(__pyx_t_1);
21776 __pyx_t_2 = __pyx_f_8PyClical_atan(__pyx_t_1, 0, NULL); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1907,
__pyx_L1_error)
21777 __Pyx_GOTREF(__pyx_t_2);
21778 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21779 __pyx_t_1 = PyNumber_Multiply(__pyx_t_2, __pyx_float_4_0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1907, __pyx_L1_error)
21780 __Pyx_GOTREF(__pyx_t_1);
21781 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21782 if (PyDict_SetItem(__pyx_d, __pyx_n_s_pi, __pyx_t_1) < 0) __PYX_ERR(0, 1907, __pyx_L1_error)
21783 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21784
21785 /* "PyClical.pyx":1908
21786 *
21787 * pi = atan(clifford(1.0)) * 4.0
21788 * tau = atan(clifford(1.0)) * 8.0          # ««««««««
21789 *
21790 * cl = clifford
21791 */
21792 __pyx_t_1 = __Pyx_PyObject_Call(((PyObject *)__pyx_ptype_8PyClical_clifford), __pyx_tuple__15,
NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1908, __pyx_L1_error)
21793 __Pyx_GOTREF(__pyx_t_1);
21794 __pyx_t_2 = __pyx_f_8PyClical_atan(__pyx_t_1, 0, NULL); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1908,
__pyx_L1_error)
21795 __Pyx_GOTREF(__pyx_t_2);
21796 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21797 __pyx_t_1 = PyNumber_Multiply(__pyx_t_2, __pyx_float_8_0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1908, __pyx_L1_error)
21798 __Pyx_GOTREF(__pyx_t_1);
21799 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21800 if (PyDict_SetItem(__pyx_d, __pyx_n_s_tau, __pyx_t_1) < 0) __PYX_ERR(0, 1908, __pyx_L1_error)
21801 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21802
21803 /* "PyClical.pyx":1910
21804 * tau = atan(clifford(1.0)) * 8.0
21805 *
21806 * cl = clifford          # ««««««««
21807 * """
21808 * Abbreviation for clifford.
21809 */
21810 if (PyDict_SetItem(__pyx_d, __pyx_n_s_cl, ((PyObject *)__pyx_ptype_8PyClical_clifford)) < 0)
__PYX_ERR(0, 1910, __pyx_L1_error)
21811
21812 /* "PyClical.pyx":1928
21813 * """
21814 *
21815 * ist = index_set          # ««««««««
21816 * """
21817 * Abbreviation for index_set.
21818 */
21819 if (PyDict_SetItem(__pyx_d, __pyx_n_s_ist, ((PyObject *)__pyx_ptype_8PyClical_index_set)) < 0)
__PYX_ERR(0, 1928, __pyx_L1_error)
21820
21821 /* "PyClical.pyx":1936
21822 * """
21823 *
21824 * def e(obj):          # ««««««««
21825 *     """
21826 *     Abbreviation for clifford(index_set(obj)).
21827 */
21828 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_89e, NULL, __pyx_n_s_PyClical); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1936, __pyx_L1_error)
21829 __Pyx_GOTREF(__pyx_t_1);
21830 if (PyDict_SetItem(__pyx_d, __pyx_n_s_e, __pyx_t_1) < 0) __PYX_ERR(0, 1936, __pyx_L1_error)
21831 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21832
21833 /* "PyClical.pyx":1949
21834 *     return clifford(index_set(obj))
21835 *
21836 * def istpq(p, q):          # ««««««««
21837 *     """
21838 *     Abbreviation for index_set({-q,...p}).
21839 */
21840 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_91istpq, NULL, __pyx_n_s_PyClical); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1949, __pyx_L1_error)
21841 __Pyx_GOTREF(__pyx_t_1);
21842 if (PyDict_SetItem(__pyx_d, __pyx_n_s_istpq, __pyx_t_1) < 0) __PYX_ERR(0, 1949, __pyx_L1_error)
21843 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21844
21845 /* "PyClical.pyx":1958

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21846 *         return index_set(set(range(-q,p+1)))
21847 *
21848 * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].          #
21849 * nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
21850 *
21851 */
21852 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1958,
__pyx_L1_error)
21853 __Pyx_GOTREF(__pyx_t_1);
21854 __pyx_t_2 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__20, NULL); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1958, __pyx_L1_error)
21855 __Pyx_GOTREF(__pyx_t_2);
21856 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21857 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1958,
__pyx_L1_error)
21858 __Pyx_GOTREF(__pyx_t_1);
21859 __pyx_t_3 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__21, NULL); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 1958, __pyx_L1_error)
21860 __Pyx_GOTREF(__pyx_t_3);
21861 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21862 __pyx_t_1 = PyNumber_Add(__pyx_t_2, __pyx_t_3); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1958,
__pyx_L1_error)
21863 __Pyx_GOTREF(__pyx_t_1);
21864 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21865 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
21866 if (PyDict_SetItem(__pyx_d, __pyx_n_s_ninf3, __pyx_t_1) < 0) __PYX_ERR(0, 1958, __pyx_L1_error)
21867 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21868
21869 /* "PyClicl.pyx":1959
21870 *
21871 * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
21872 * nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].          # ««««««««
21873 *
21874 * # Doctest interface.
21875 */
21876 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1959,
__pyx_L1_error)
21877 __Pyx_GOTREF(__pyx_t_1);
21878 __pyx_t_3 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__20, NULL); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 1959, __pyx_L1_error)
21879 __Pyx_GOTREF(__pyx_t_3);
21880 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21881 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1959,
__pyx_L1_error)
21882 __Pyx_GOTREF(__pyx_t_1);
21883 __pyx_t_2 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__21, NULL); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1959, __pyx_L1_error)
21884 __Pyx_GOTREF(__pyx_t_2);
21885 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21886 __pyx_t_1 = PyNumber_Subtract(__pyx_t_3, __pyx_t_2); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1959,
__pyx_L1_error)
21887 __Pyx_GOTREF(__pyx_t_1);
21888 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
21889 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21890 if (PyDict_SetItem(__pyx_d, __pyx_n_s_nbar3, __pyx_t_1) < 0) __PYX_ERR(0, 1959, __pyx_L1_error)
21891 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21892
21893 /* "PyClicl.pyx":1962
21894 *
21895 * # Doctest interface.
21896 * def _test():          # ««««««««
21897 *     import PyClicl, doctest
21898 *     return doctest.testmod(PyClicl)
21899 */
21900 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClicl_93_test, NULL, __pyx_n_s_PyClicl); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1962, __pyx_L1_error)
21901 __Pyx_GOTREF(__pyx_t_1);
21902 if (PyDict_SetItem(__pyx_d, __pyx_n_s_test, __pyx_t_1) < 0) __PYX_ERR(0, 1962, __pyx_L1_error)
21903 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21904
21905 /* "PyClicl.pyx":1966
21906 *         return doctest.testmod(PyClicl)
21907 *
21908 * if __name__ == "__main__":          # ««««««««
21909 *     _test()
21910 */
21911 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_name); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1966,
__pyx_L1_error)
21912 __Pyx_GOTREF(__pyx_t_1);
21913 __pyx_t_4 = (__Pyx_PyUnicode_Equals(__pyx_t_1, __pyx_n_u_main, Py_EQ)); if (unlikely(__pyx_t_4 < 0))
__PYX_ERR(0, 1966, __pyx_L1_error)
21914 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21915 if (__pyx_t_4) {
21916
21917     /* "PyClicl.pyx":1967
21918     *

```



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21919 * if __name__ == "__main__":
21920 *     _test() # ««««««««
21921 */
21922 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_test); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1967,
__pyx_L1_error)
21923 __Pyx_GOTREF(__pyx_t_1);
21924 __pyx_t_2 = __Pyx_PyObject_CallNoArg(__pyx_t_1); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1967,
__pyx_L1_error)
21925 __Pyx_GOTREF(__pyx_t_2);
21926 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21927 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21928
21929 /* "PyClical.pyx":1966
21930 *     return doctest.testmod(PyClical)
21931 *
21932 * if __name__ == "__main__": # ««««««««
21933 *     _test()
21934 */
21935 }
21936
21937 /* "PyClical.pyx":1
21938 * # -*- coding: utf-8 -*- # ««««««««
21939 * # cython: language_level=3
21940 * # distutils: language = c++
21941 */
21942 __pyx_t_2 = __Pyx_PyDict_NewPresized(111); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1, __pyx_L1_error)
21943 __Pyx_GOTREF(__pyx_t_2);
21944 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_copy_line_65,
__pyx_kp_u_Copy_this_index_set_object_s_in) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21945 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_setitem_line_179,
__pyx_kp_u_Set_the_value_of_an_index_set_o) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21946 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_getitem_line_191,
__pyx_kp_u_Get_the_value_of_an_index_set_o) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21947 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_iter_line_229,
__pyx_kp_u_Iterate_over_the_indices_of_an) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21948 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_invert_line_240,
__pyx_kp_u_Set_complement_not_print_index) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21949 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_xor_line_249,
__pyx_kp_u_Symmetric_set_difference_exclus) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21950 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_ixor_line_260,
__pyx_kp_u_Symmetric_set_difference_exclus_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21951 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_and_line_271,
__pyx_kp_u_Set_intersection_and_print_inde) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21952 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_iand_line_282,
__pyx_kp_u_Set_intersection_and_x_index_se) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21953 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_or_line_293,
__pyx_kp_u_Set_union_or_print_index_set_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21954 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_ior_line_304,
__pyx_kp_u_Set_union_or_x_index_set_1_x_in) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21955 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_count_line_315,
__pyx_kp_u_Cardinality_Number_of_indices_i) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21956 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_count_neg_line_324,
__pyx_kp_u_Number_of_negative_indices_incl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21957 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_count_pos_line_333,
__pyx_kp_u_Number_of_positive_indices_incl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21958 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_min_line_342,
__pyx_kp_u_Minimum_member_index_set_1_1_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21959 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_max_line_351,
__pyx_kp_u_Maximum_member_index_set_1_1_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21960 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_sign_of_mult_line_366,
__pyx_kp_u_Sign_of_geometric_product_of_tw) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21961 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_sign_of_square_line_37,
__pyx_kp_u_Sign_of_geometric_square_of_a_C) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21962 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_repr_line_384,
__pyx_kp_u_The_official_string_representat) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21963 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_str_line_395,
__pyx_kp_u_The_informal_string_representat) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21964 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_hidden_doctests_line_4,
__pyx_kp_u_Tests_for_functions_that_Doctes) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21965 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_compare_line_492,
__pyx_kp_u_lexicographic_compare_eg_3_4_5) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21966 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_min_neg_line_504,
__pyx_kp_u_Minimum_negative_index_or_0_if) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21967 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_max_pos_line_513,
__pyx_kp_u_Maximum_positive_index_or_0_if) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21968 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_copy_line_556,
__pyx_kp_u_Copy_this_clifford_object_x_cli) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21969 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_iter_line_638,
__pyx_kp_u_Not_applicable_for_a_in_cliffor) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21970 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_reframe_line_649,
__pyx_kp_u_Put_self_into_a_larger_frame_co) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21971 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_getitem_line_707,
__pyx_kp_u_Subscripting_map_from_index_set) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21972 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_neg_line_722,
__pyx_kp_u_Unary_print_clifford_1_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21973 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_pos_line_731,
__pyx_kp_u_Unary_print_clifford_1_1_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)

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21974     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__add__line_740,
21975         _pyx_kp_u_Geometric_sum_print_clifford_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21976     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__iadd__line_751,
21977         _pyx_kp_u_Geometric_sum_x_clifford_1_x_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21978     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__sub__line_760,
21979         _pyx_kp_u_Geometric_difference_print_clif) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21980     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__isub__line_771,
21981         _pyx_kp_u_Geometric_difference_x_clifford) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21982     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__mul__line_780,
21983         _pyx_kp_u_Geometric_product_print_cliffor) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21984     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__imul__line_793,
21985         _pyx_kp_u_Geometric_product_x_clifford_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21986     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__mod__line_806,
21987         _pyx_kp_u_Contraction_print_clifford_1_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21988     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__imod__line_821,
21989         _pyx_kp_u_Contraction_x_clifford_1_x_clif) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21990     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__and__line_836,
21991         _pyx_kp_u_Inner_product_print_clifford_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21992     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__iand__line_851,
21993         _pyx_kp_u_Inner_product_x_clifford_1_x_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21994     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__xor__line_866,
21995         _pyx_kp_u_Outer_product_print_clifford_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21996     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__ixor__line_881,
21997         _pyx_kp_u_Outer_product_x_clifford_1_x_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21998     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__truediv__line_896,
21999         _pyx_kp_u_Geometric_quotient_print_cliffo) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22000     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__idiv__line_911,
22001         _pyx_kp_u_Geometric_quotient_x_clifford_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22002     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__inv_line_926,
22003         _pyx_kp_u_Geometric_multiplicative_invers) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22004     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__or__line_939,
22005         _pyx_kp_u_Transform_left_hand_side_using) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22006     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__ior__line_950,
22007         _pyx_kp_u_Transform_left_hand_side_using_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22008     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__pow__line_961,
22009         _pyx_kp_u_Power_self_to_the_m_x_clifford) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22010     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__pow_line_980,
22011         _pyx_kp_u_Power_self_to_the_m_x_clifford_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22012     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__outer_pow_line_1004,
22013         _pyx_kp_u_Outer_product_power_x_clifford) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22014     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__call__line_1020,
22015         _pyx_kp_u_Pure_grade_vector_part_print_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22016     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__scalar_line_1039,
22017         _pyx_kp_u_Scalar_part_clifford_1_l_1_2_sc) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22018     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__pure_line_1050,
22019         _pyx_kp_u_Pure_part_print_clifford_1_l_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22020     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__even_line_1061,
22021         _pyx_kp_u_Even_part_of_multivector_sum_of) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22022     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__odd_line_1070,
22023         _pyx_kp_u_Odd_part_of_multivector_sum_of) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22024     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__vector_part_line_1079,
22025         _pyx_kp_u_Vector_part_of_multivector_as_a) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22026     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__involute_line_1107,
22027         _pyx_kp_u_Main_involution_each_i_is_repla) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22028     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__reverse_line_1123,
22029         _pyx_kp_u_Reversion_eg_clifford_1_cliffor) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22030     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__conj_line_1138,
22031         _pyx_kp_u_Conjugation_reverse_o_involute) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22032     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__quad_line_1153,
22033         _pyx_kp_u_Quadratic_form_rev_x_x_0_print) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22034     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__norm_line_1164,
22035         _pyx_kp_u_Norm_sum_of_squares_of_coordina) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22036     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__abs_line_1175,
22037         _pyx_kp_u_Absolute_value_square_root_of_n) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22038     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__max_abs_line_1184,
22039         _pyx_kp_u_Maximum_of_absolute_values_of_c) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22040     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__truncated_line_1195,
22041         _pyx_kp_u_Remove_all_terms_of_self_with_r) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22042     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__isinf_line_1206,
22043         _pyx_kp_u_Check_if_a_multivector_contains) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22044     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__isnan_line_1215,
22045         _pyx_kp_u_Check_if_a_multivector_contains_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22046     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__frame_line_1224,
22047         _pyx_kp_u_Subalgebra_generated_by_all_gen) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22048     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__repr__line_1235,
22049         _pyx_kp_u_The_official_string_representat_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22050     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__str__line_1244,
22051         _pyx_kp_u_The_informal_string_representat_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22052     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_clifford__hidden_doctests_line_12,
22053         _pyx_kp_u_Tests_for_functions_that_Doctes_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22054     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_error_squared_tol_line_1337,
22055         _pyx_kp_u_Quadratic_norm_error_tolerance) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22056     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_error_squared_line_1346,
22057         _pyx_kp_u_Relative_or_absolute_error_usin) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22058     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_approx_equal_line_1359,
22059         _pyx_kp_u_Test_for_approximate_equality_o) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22060     if (PyDict_SetItem(_pyx_t_2, _pyx_kp_u_inv_line_1378,

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__pyx_kp_u_Geometric_multiplicative_invers_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22018 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_scalar_line_1393,
__pyx_kp_u_Scalar_part_scalar_clifford_1_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22019 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_real_line_1404, __pyx_kp_u_Real_part_synonym_for_scalar_pa
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22020 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_imag_line_1415, __pyx_kp_u_Imaginary_part_deprecated_alway
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22021 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_pure_line_1426, __pyx_kp_u_Pure_part_print_pure_clifford_1)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22022 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_even_line_1437,
__pyx_kp_u_Even_part_of_multivector_sum_of_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22023 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_odd_line_1446, __pyx_kp_u_Odd_part_of_multivector_sum_of_2)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22024 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_involute_line_1455,
__pyx_kp_u_Main_involution_each_i_is_repla_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22025 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_reverse_line_1470,
__pyx_kp_u_Reversion_eg_1_2_2_1_print_reve) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22026 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_conj_line_1485,
__pyx_kp_u_Conjugation_reverse_o_involute_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22027 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_quad_line_1500,
__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22028 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_norm_line_1511, __pyx_kp_u_norm_sum_of_squares_of_coordina
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22029 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_abs_line_1522, __pyx_kp_u_Absolute_value_of_multivector_m)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22030 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_max_abs_line_1531,
__pyx_kp_u_Maximum_absolute_value_of_coord) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22031 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_pow_line_1543, __pyx_kp_u_Integer_power_of_multivector_ob)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22032 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_outer_pow_line_1567,
__pyx_kp_u_Outer_product_power_of_multivec) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22033 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_complexifier_line_1576,
__pyx_kp_u_Square_root_of_1_which_commutates) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22034 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_sqrt_line_1591, __pyx_kp_u_Square_root_of_multivector_with)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22035 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_exp_line_1614, __pyx_kp_u_Exponential_of_multivector_x_cl)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22036 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_log_line_1628, __pyx_kp_u_Natural_logarithm_of_multivecto)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22037 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cos_line_1651, __pyx_kp_u_Cosine_of_multivector_with_opti)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22038 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_acos_line_1668, __pyx_kp_u_Inverse_cosine_of_multivector_w)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22039 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cosh_line_1689, __pyx_kp_u_Hyperbolic_cosine_of_multivecto)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22040 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_acosh_line_1705,
__pyx_kp_u_Inverse_hyperbolic_cosine_of_mu) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22041 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_sin_line_1728, __pyx_kp_u_Sine_of_multivector_with_option)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22042 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_asin_line_1747, __pyx_kp_u_Inverse_sine_of_multivector_wit)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22043 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_sinh_line_1768, __pyx_kp_u_Hyperbolic_sine_of_multivector)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22044 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_asinh_line_1782,
__pyx_kp_u_Inverse_hyperbolic_sine_of_mult) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22045 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_tan_line_1801, __pyx_kp_u_Tangent_of_multivector_with_opt)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22046 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_atan_line_1818, __pyx_kp_u_Inverse_tangent_of_multivector)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22047 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_tanh_line_1835, __pyx_kp_u_Hyperbolic_tangent_of_multivect)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22048 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_atanh_line_1847,
__pyx_kp_u_Inverse_hyperbolic_tangent_of_m) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22049 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_random_clifford_line_1864,
__pyx_kp_u_Random_multivector_within_a_fra) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22050 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cga3_line_1873, __pyx_kp_u_Convert_Euclidean_3D_multivecto)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22051 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cga3std_line_1882,
__pyx_kp_u_Convert_CGA3_null_vector_to_sta) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22052 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_agc3_line_1893, __pyx_kp_u_Convert_CGA3_null_vector_to_Euc)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22053 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_e_line_1936, __pyx_kp_u_Abbreviation_for_clifford_index) <
0) __PYX_ERR(0, 1, __pyx_L1_error)
22054 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_istpq_line_1949, __pyx_kp_u_Abbreviation_for_index_set_q_p)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22055 if (PyDict_SetItem(__pyx_d, __pyx_n_s_test_2, __pyx_t_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22056 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
22057
22058 /* "string.to_py":55
22059 *
22060 * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
22061 * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
22062 # ««««««««
22062 *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
22063 *
22064 */
22065

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22066  /*--- Wrapped vars code ---*/
22067
22068  goto __pyx_L0;
22069  __pyx_L1_error:;
22070  __Pyx_XDECREF(__pyx_t_1);
22071  __Pyx_XDECREF(__pyx_t_2);
22072  __Pyx_XDECREF(__pyx_t_3);
22073  if (__pyx_m) {
22074      if (__pyx_d) {
22075          __Pyx_AddTraceback("init PyClical", __pyx_clineno, __pyx_lineno, __pyx_filename);
22076      }
22077      Py_CLEAR(__pyx_m);
22078  } else if (!PyErr_Occurred()) {
22079      PyErr_SetString(PyExc_ImportError, "init PyClical");
22080  }
22081  __pyx_L0:;
22082  __Pyx_RefNannyFinishContext();
22083  #if CYTHON_PEP489_MULTI_PHASE_INIT
22084  return (__pyx_m != NULL) ? 0 : -1;
22085  #elif PY_MAJOR_VERSION >= 3
22086  return __pyx_m;
22087  #else
22088  return;
22089  #endif
22090 }
22091
22092 /* --- Runtime support code --- */
22093 /* Refnanny */
22094 #if CYTHON_REFNANNY
22095 static __Pyx_RefNannyAPIStruct *__Pyx_RefNannyImportAPI(const char *modname) {
22096     PyObject *m = NULL, *p = NULL;
22097     void *r = NULL;
22098     m = PyImport_ImportModule(modname);
22099     if (!m) goto end;
22100     p = PyObject_GetAttrString(m, "RefNannyAPI");
22101     if (!p) goto end;
22102     r = PyLong_AsVoidPtr(p);
22103 end:
22104     Py_XDECREF(p);
22105     Py_XDECREF(m);
22106     return ((__Pyx_RefNannyAPIStruct *)r);
22107 }
22108 #endif
22109
22110 /* PyObjectGetAttrStr */
22111 #if CYTHON_USE_TYPE_SLOTS
22112 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStr(PyObject* obj, PyObject* attr_name) {
22113     PyTypeObject* tp = Py_TYPE(obj);
22114     if (likely(tp->tp_getattro))
22115         return tp->tp_getattro(obj, attr_name);
22116 #if PY_MAJOR_VERSION < 3
22117     if (likely(tp->tp_getattr))
22118         return tp->tp_getattr(obj, PyString_AS_STRING(attr_name));
22119 #endif
22120     return PyObject_GetAttr(obj, attr_name);
22121 }
22122 #endif
22123
22124 /* GetBuiltinName */
22125 static PyObject* __Pyx_GetBuiltinName(PyObject *name) {
22126     PyObject* result = __Pyx_PyObject_GetAttrStr(__pyx_b, name);
22127     if (unlikely(!result)) {
22128         PyErr_Format(PyExc_NameError,
22129             "name '%U' is not defined", name);
22130     }
22131 #else
22132     "name '%.200s' is not defined", PyString_AS_STRING(name));
22133 #endif
22134 }
22135 return result;
22136 }
22137
22138 /* PyCFunctionFastCall */
22139 #if CYTHON_FAST_PYCALL
22140 static CYTHON_INLINE PyObject* __Pyx_PyCFunction_FastCall(PyObject *func_obj, PyObject **args,
22141     Py_ssize_t nargs) {
22142     PyCFunctionObject *func = (PyCFunctionObject*)func_obj;
22143     PyCFunction meth = PyCFunction_GET_FUNCTION(func);
22144     PyObject *self = PyCFunction_GET_SELF(func);
22145     int flags = PyCFunction_GET_FLAGS(func);
22146     assert(PyCFunction_Check(func));
22147     assert(METH_FASTCALL == (flags & ~(METH_CLASS | METH_STATIC | METH_COEXIST | METH_KEYWORDS |
22148         METH_STACKLESS)));
22149     assert(nargs >= 0);
22150     assert(nargs == 0 || args != NULL);
22151     /* __Pyx_PyCFunction_FastCallDict() must not be called with an exception set,
22152        because it may clear it (directly or indirectly) and so the

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22151         caller loses its exception */
22152         assert(!PyErr_Occurred());
22153         if ((PY_VERSION_HEX < 0x030700A0) || unlikely(flags & METH_KEYWORDS)) {
22154             return ((*((__Pyx_PyCFunctionFastWithKeywords)(void*)meth)) (self, args, nargs, NULL));
22155         } else {
22156             return ((*((__Pyx_PyCFunctionFast)(void*)meth)) (self, args, nargs));
22157         }
22158     }
22159 #endif
22160
22161 /* PyFunctionFastCall */
22162 #if CYTHON_FAST_PYCALL
22163 static PyObject* __Pyx_PyFunction_FastCallNoKw(PyCodeObject *co, PyObject **args, Py_ssize_t na,
22164   PyObject *globals) {
22165     PyFrameObject *f;
22166     PyThreadState *tstate = __Pyx_PyThreadState_Current;
22167     PyObject **fastlocals;
22168     Py_ssize_t i;
22169     PyObject *result;
22170     assert(globals != NULL);
22171     /* XXX Perhaps we should create a specialized
22172        PyFrame_New() that doesn't take locals, but does
22173        take builtins without sanity checking them.
22174        */
22175     assert(tstate != NULL);
22176     f = PyFrame_New(tstate, co, globals, NULL);
22177     if (f == NULL) {
22178         return NULL;
22179     }
22180     fastlocals = __Pyx_PyFrame_GetLocalsplus(f);
22181     for (i = 0; i < na; i++) {
22182         Py_INCREF(*args);
22183         fastlocals[i] = *args++;
22184     }
22185     result = PyEval_EvalFrameEx(f, 0);
22186     ++tstate->recursion_depth;
22187     Py_DECREF(f);
22188     --tstate->recursion_depth;
22189     return result;
22190 }
22191 #if 1 || PY_VERSION_HEX < 0x030600B1
22192 static PyObject* __Pyx_PyFunction_FastCallDict(PyObject *func, PyObject **args, Py_ssize_t nargs,
22193   PyObject *kwargs) {
22194     PyCodeObject *co = (PyCodeObject *)PyFunction_GET_CODE(func);
22195     PyObject *globals = PyFunction_GET_GLOBALS(func);
22196     PyObject *argdefs = PyFunction_GET_DEFAULTS(func);
22197     PyObject *closure;
22198     #if PY_MAJOR_VERSION >= 3
22199     PyObject *kwdefs;
22200     #endif
22201     PyObject *kwtuple, **k;
22202     PyObject **d;
22203     Py_ssize_t nd;
22204     Py_ssize_t nk;
22205     PyObject *result;
22206     assert(kwargs == NULL || PyDict_Check(kwargs));
22207     nk = kwargs ? PyDict_Size(kwargs) : 0;
22208     if (Py_EnterRecursiveCall((char*)" while calling a Python object")) {
22209         return NULL;
22210     }
22211     if (
22212     #if PY_MAJOR_VERSION >= 3
22213         co->co_kwonlyargcount == 0 &&
22214     #endif
22215         likely(kwargs == NULL || nk == 0) &&
22216         co->co_flags == (CO_OPTIMIZED | CO_NEWLOCALS | CO_NOFREE)) {
22217         if (argdefs == NULL && co->co_argcount == nargs) {
22218             result = __Pyx_PyFunction_FastCallNoKw(co, args, nargs, globals);
22219             goto done;
22220         }
22221         else if (nargs == 0 && argdefs != NULL
22222                 && co->co_argcount == Py_SIZE(argdefs)) {
22223             /* function called with no arguments, but all parameters have
22224                a default value: use default values as arguments .*/
22225             args = &PyTuple_GET_ITEM(argdefs, 0);
22226             result = __Pyx_PyFunction_FastCallNoKw(co, args, Py_SIZE(argdefs), globals);
22227             goto done;
22228         }
22229     }
22230     if (kwargs != NULL) {
22231         Py_ssize_t pos, i;
22232         kwtuple = PyTuple_New(2 * nk);
22233         if (kwtuple == NULL) {
22234             result = NULL;
22235             goto done;
22236         }
22237         k = &PyTuple_GET_ITEM(kwtuple, 0);

```

```

22237         pos = i = 0;
22238         while (PyDict_Next(kwargs, &pos, &k[i], &k[i+1])) {
22239             Py_INCREF(k[i]);
22240             Py_INCREF(k[i+1]);
22241             i += 2;
22242         }
22243         nk = i / 2;
22244     }
22245     else {
22246         kwtuple = NULL;
22247         k = NULL;
22248     }
22249     closure = PyFunction_GET_CLOSURE(func);
22250 #if PY_MAJOR_VERSION >= 3
22251     kwdefs = PyFunction_GET_KW_DEFAULTS(func);
22252 #endif
22253     if (argdefs != NULL) {
22254         d = &PyTuple_GET_ITEM(argdefs, 0);
22255         nd = Py_SIZE(argdefs);
22256     }
22257     else {
22258         d = NULL;
22259         nd = 0;
22260     }
22261 #if PY_MAJOR_VERSION >= 3
22262     result = PyEval_EvalCodeEx((PyObject*)co, globals, (PyObject *)NULL,
22263                                args, (int)nargs,
22264                                k, (int)nk,
22265                                d, (int)nd, kwdefs, closure);
22266 #else
22267     result = PyEval_EvalCodeEx(co, globals, (PyObject *)NULL,
22268                                args, (int)nargs,
22269                                k, (int)nk,
22270                                d, (int)nd, closure);
22271 #endif
22272     Py_XDECREF(kwtuple);
22273 done:
22274     Py_LeaveRecursiveCall();
22275     return result;
22276 }
22277 #endif
22278 #endif
22279
22280 /* PyObjectCall */
22281 #if CYTHON_COMPILING_IN_CPYTHON
22282 static CYTHON_INLINE PyObject* __Pyx_PyObject_Call(PyObject *func, PyObject *arg, PyObject *kw) {
22283     PyObject *result;
22284     ternaryfunc call = Py_TYPE(func)->tp_call;
22285     if (unlikely(!call))
22286         return PyObject_Call(func, arg, kw);
22287     if (unlikely(Py_EnterRecursiveCall((char*)" while calling a Python object")))
22288         return NULL;
22289     result = (*call)(func, arg, kw);
22290     Py_LeaveRecursiveCall();
22291     if (unlikely(!result) && unlikely(!PyErr_Occurred())) {
22292         PyErr_SetString(
22293             PyExc_SystemError,
22294             "NULL result without error in PyObject_Call");
22295     }
22296     return result;
22297 }
22298 #endif
22299
22300 /* PyObjectCallMethO */
22301 #if CYTHON_COMPILING_IN_CPYTHON
22302 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallMethO(PyObject *func, PyObject *arg) {
22303     PyObject *self, *result;
22304     PyCFunction cfunc;
22305     cfunc = PyCFunction_GET_FUNCTION(func);
22306     self = PyCFunction_GET_SELF(func);
22307     if (unlikely(Py_EnterRecursiveCall((char*)" while calling a Python object")))
22308         return NULL;
22309     result = cfunc(self, arg);
22310     Py_LeaveRecursiveCall();
22311     if (unlikely(!result) && unlikely(!PyErr_Occurred())) {
22312         PyErr_SetString(
22313             PyExc_SystemError,
22314             "NULL result without error in PyObject_Call");
22315     }
22316     return result;
22317 }
22318 #endif
22319
22320 /* PyObjectCallOneArg */
22321 #if CYTHON_COMPILING_IN_CPYTHON
22322 static PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg) {
22323     PyObject *result;

```

```

22324     PyObject *args = PyTuple_New(1);
22325     if (unlikely(!args)) return NULL;
22326     Py_INCREF(arg);
22327     PyTuple_SET_ITEM(args, 0, arg);
22328     result = __Pyx_PyObject_Call(func, args, NULL);
22329     Py_DECREF(args);
22330     return result;
22331 }
22332 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg) {
22333 #if CYTHON_FAST_PYCALL
22334     if (PyFunction_Check(func)) {
22335         return __Pyx_PyFunction_FastCall(func, &arg, 1);
22336     }
22337 #endif
22338     if (likely(PyCFunction_Check(func))) {
22339         if (likely(PyCFunction_GET_FLAGS(func) & METH_O)) {
22340             return __Pyx_PyObject_CallMethO(func, arg);
22341 #if CYTHON_FAST_PYCALL
22342         } else if (__Pyx_PyFastCFunction_Check(func)) {
22343             return __Pyx_PyCFunction_FastCall(func, &arg, 1);
22344 #endif
22345         }
22346     }
22347     return __Pyx_PyObject_CallOneArg(func, arg);
22348 }
22349 #else
22350 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg) {
22351     PyObject *result;
22352     PyObject *args = PyTuple_Pack(1, arg);
22353     if (unlikely(!args)) return NULL;
22354     result = __Pyx_PyObject_Call(func, args, NULL);
22355     Py_DECREF(args);
22356     return result;
22357 }
22358 #endif
22359
22360 /* PyErrFetchRestore */
22361 #if CYTHON_FAST_THREAD_STATE
22362 static CYTHON_INLINE void __Pyx_ErrRestoreInState(PyThreadState *tstate, PyObject *type, PyObject
22363 *value, PyObject *tb) {
22364     PyObject *tmp_type, *tmp_value, *tmp_tb;
22365     tmp_type = tstate->curexc_type;
22366     tmp_value = tstate->curexc_value;
22367     tmp_tb = tstate->curexc_traceback;
22368     tstate->curexc_type = type;
22369     tstate->curexc_value = value;
22370     tstate->curexc_traceback = tb;
22371     Py_XDECREF(tmp_type);
22372     Py_XDECREF(tmp_value);
22373     Py_XDECREF(tmp_tb);
22374 }
22375 static CYTHON_INLINE void __Pyx_ErrFetchInState(PyThreadState *tstate, PyObject **type, PyObject
22376 **value, PyObject **tb) {
22377     *type = tstate->curexc_type;
22378     *value = tstate->curexc_value;
22379     *tb = tstate->curexc_traceback;
22380     tstate->curexc_type = 0;
22381     tstate->curexc_value = 0;
22382     tstate->curexc_traceback = 0;
22383 }
22384 #endif
22385
22386 /* WriteUnraisableException */
22387 static void __Pyx_WriteUnraisable(const char *name, CYTHON_UNUSED int clineno,
22388 CYTHON_UNUSED int lineno, CYTHON_UNUSED const char *filename,
22389 int full_traceback, CYTHON_UNUSED int nogil) {
22390     PyObject *old_exc, *old_val, *old_tb;
22391     PyObject *ctx;
22392     __Pyx_PyThreadState_declare
22393 #ifdef WITH_THREAD
22394     PyGILState_STATE state;
22395     if (nogil)
22396         state = PyGILState_Ensure();
22397 #endif
22398 #ifdef _MSC_VER
22399     else state = (PyGILState_STATE)-1;
22400 #endif
22401 #endif
22402     __Pyx_PyThreadState_assign
22403     __Pyx_ErrFetch(&old_exc, &old_val, &old_tb);
22404     if (full_traceback) {
22405         Py_XINCRREF(old_exc);
22406         Py_XINCRREF(old_val);
22407         Py_XINCRREF(old_tb);
22408         __Pyx_ErrRestore(old_exc, old_val, old_tb);
22409         PyErr_PrintEx(1);
22410     }
22411 #if PY_MAJOR_VERSION < 3

```



```

22409     ctx = PyString_FromString(name);
22410     #else
22411     ctx = PyUnicode_FromString(name);
22412     #endif
22413     __Pyx_ErrRestore(old_exc, old_val, old_tb);
22414     if (!ctx) {
22415         PyErr_WriteUnraisable(Py_None);
22416     } else {
22417         PyErr_WriteUnraisable(ctx);
22418         Py_DECREF(ctx);
22419     }
22420 #ifdef WITH_THREAD
22421     if (nogil)
22422         PyGILState_Release(state);
22423 #endif
22424 }
22425
22426 /* PyDictVersioning */
22427 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_TYPE_SLOTS
22428 static CYTHON_INLINE PY_UINT64_T __Pyx_get_tp_dict_version(PyObject *obj) {
22429     PyObject *dict = Py_TYPE(obj)->tp_dict;
22430     return likely(dict) ? __PYX_GET_DICT_VERSION(dict) : 0;
22431 }
22432 static CYTHON_INLINE PY_UINT64_T __Pyx_get_object_dict_version(PyObject *obj) {
22433     PyObject **dictptr = NULL;
22434     Py_ssize_t offset = Py_TYPE(obj)->tp_dictoffset;
22435     if (offset) {
22436 #if CYTHON_COMPILING_IN_CPYTHON
22437         dictptr = (likely(offset > 0)) ? (PyObject **) ((char *)obj + offset) :
                _PyObject_GetDictPtr(obj);
22438     #else
22439         dictptr = _PyObject_GetDictPtr(obj);
22440     #endif
22441     }
22442     return (dictptr && *dictptr) ? __PYX_GET_DICT_VERSION(*dictptr) : 0;
22443 }
22444 static CYTHON_INLINE int __Pyx_object_dict_version_matches(PyObject* obj, PY_UINT64_T tp_dict_version,
PY_UINT64_T obj_dict_version) {
22445     PyObject *dict = Py_TYPE(obj)->tp_dict;
22446     if (unlikely(!dict) || unlikely(tp_dict_version != __PYX_GET_DICT_VERSION(dict)))
22447         return 0;
22448     return obj_dict_version == __Pyx_get_object_dict_version(obj);
22449 }
22450 #endif
22451
22452 /* PyObjectCallNoArg */
22453 #if CYTHON_COMPILING_IN_CPYTHON
22454 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallNoArg(PyObject *func) {
22455     #if CYTHON_FAST_PYCALL
22456         if (PyFunction_Check(func)) {
22457             return __Pyx_PyFunction_FastCall(func, NULL, 0);
22458         }
22459     #endif
22460     #ifdef __Pyx_CyFunction_USED
22461     if (likely(PyCFunction_Check(func) || __Pyx_CyFunction_Check(func)))
22462     #else
22463     if (likely(PyCFunction_Check(func)))
22464     #endif
22465     {
22466         if (likely(PyCFunction_GET_FLAGS(func) & METH_NOARGS)) {
22467             return __Pyx_PyObject_CallMethO(func, NULL);
22468         }
22469     }
22470     return __Pyx_PyObject_Call(func, __pyx_empty_tuple, NULL);
22471 }
22472 #endif
22473
22474 /* RaiseDoubleKeywords */
22475 static void __Pyx_RaiseDoubleKeywordsError(
22476     const char* func_name,
22477     PyObject* kw_name)
22478 {
22479     PyErr_Format(PyExc_TypeError,
22480         #if PY_MAJOR_VERSION >= 3
22481         "%s() got multiple values for keyword argument '%U'", func_name, kw_name);
22482         #else
22483         "%s() got multiple values for keyword argument '%s'", func_name,
22484         PyString_AsString(kw_name));
22485         #endif
22486 }
22487
22488 /* ParseKeywords */
22489 static int __Pyx_ParseOptionalKeywords(
22490     PyObject *kwds,
22491     PyObject **argnames[],
22492     PyObject *kwds2,
22493     PyObject *values[],

```



```

22494     Py_ssize_t num_pos_args,
22495     const char* function_name)
22496 {
22497     PyObject *key = 0, *value = 0;
22498     Py_ssize_t pos = 0;
22499     PyObject*** name;
22500     PyObject*** first_kw_arg = argnames + num_pos_args;
22501     while (PyDict_Next(kwds, &pos, &key, &value)) {
22502         name = first_kw_arg;
22503         while (*name && (**name != key)) name++;
22504         if (*name) {
22505             values[name-argnames] = value;
22506             continue;
22507         }
22508         name = first_kw_arg;
22509         #if PY_MAJOR_VERSION < 3
22510         if (likely(PyString_Check(key))) {
22511             while (*name) {
22512                 if ((CYTHON_COMPILING_IN_PYPY || PyString_GET_SIZE(**name) == PyString_GET_SIZE(key))
22513                     && _PyString_Eq(**name, key)) {
22514                     values[name-argnames] = value;
22515                     break;
22516                 }
22517                 name++;
22518             }
22519             if (*name) continue;
22520             else {
22521                 PyObject*** argname = argnames;
22522                 while (argname != first_kw_arg) {
22523                     if (**argname == key) || (
22524                         CYTHON_COMPILING_IN_PYPY || PyString_GET_SIZE(**argname) ==
22525                         PyString_GET_SIZE(key))
22526                         && _PyString_Eq(**argname, key)) {
22527                         goto arg_passed_twice;
22528                     }
22529                     argname++;
22530                 }
22531             } else
22532             #endif
22533             if (likely(PyUnicode_Check(key))) {
22534                 while (*name) {
22535                     int cmp = (**name == key) ? 0 :
22536                         #if !CYTHON_COMPILING_IN_PYPY && PY_MAJOR_VERSION >= 3
22537                         (__Pyx_PyUnicode_GET_LENGTH(**name) != __Pyx_PyUnicode_GET_LENGTH(key)) ? 1 :
22538                         #endif
22539                         PyUnicode_Compare(**name, key);
22540                     if (cmp < 0 && unlikely(PyErr_Occurred())) goto bad;
22541                     if (cmp == 0) {
22542                         values[name-argnames] = value;
22543                         break;
22544                     }
22545                     name++;
22546                 }
22547                 if (*name) continue;
22548                 else {
22549                     PyObject*** argname = argnames;
22550                     while (argname != first_kw_arg) {
22551                         int cmp = (**argname == key) ? 0 :
22552                             #if !CYTHON_COMPILING_IN_PYPY && PY_MAJOR_VERSION >= 3
22553                             (__Pyx_PyUnicode_GET_LENGTH(**argname) != __Pyx_PyUnicode_GET_LENGTH(key)) ? 1 :
22554                             #endif
22555                             PyUnicode_Compare(**argname, key);
22556                         if (cmp < 0 && unlikely(PyErr_Occurred())) goto bad;
22557                         if (cmp == 0) goto arg_passed_twice;
22558                         argname++;
22559                     }
22560                 }
22561             } else
22562             goto invalid_keyword_type;
22563         if (kwds2) {
22564             if (unlikely(PyDict_SetItem(kwds2, key, value))) goto bad;
22565         } else {
22566             goto invalid_keyword;
22567         }
22568     }
22569     return 0;
22570 arg_passed_twice:
22571     __Pyx_RaiseDoubleKeywordsError(function_name, key);
22572     goto bad;
22573 invalid_keyword_type:
22574     PyErr_Format(PyExc_TypeError,
22575         "%.200s() keywords must be strings", function_name);
22576     goto bad;
22577 invalid_keyword:
22578     PyErr_Format(PyExc_TypeError,

```

```

22579     #if PY_MAJOR_VERSION < 3
22580         "%.200s() got an unexpected keyword argument '%.200s'",
22581         function_name, PyString_AsString(key));
22582     #else
22583         "%s() got an unexpected keyword argument '%U'",
22584         function_name, key);
22585     #endif
22586 bad:
22587     return -1;
22588 }
22589
22590 /* RaiseArgTupleInvalid */
22591 static void __Pyx_RaiseArgtupleInvalid(
22592     const char* func_name,
22593     int exact,
22594     Py_ssize_t num_min,
22595     Py_ssize_t num_max,
22596     Py_ssize_t num_found)
22597 {
22598     Py_ssize_t num_expected;
22599     const char *more_or_less;
22600     if (num_found < num_min) {
22601         num_expected = num_min;
22602         more_or_less = "at least";
22603     } else {
22604         num_expected = num_max;
22605         more_or_less = "at most";
22606     }
22607     if (exact) {
22608         more_or_less = "exactly";
22609     }
22610     PyErr_Format(PyExc_TypeError,
22611         "%.200s() takes %.8s %" CYTHON_FORMAT_SSIZE_T "d positional argument%.1s (%"
CYTHON_FORMAT_SSIZE_T "d given)",
22612         func_name, more_or_less, num_expected,
22613         (num_expected == 1) ? "" : "s", num_found);
22614 }
22615
22616 /* GetModuleGlobalName */
22617 #if CYTHON_USE_DICT_VERSIONS
22618 static PyObject * __Pyx_GetModuleGlobalName(PyObject *name, PY_UINT64_T *dict_version, PyObject
**dict_cached_value)
22619 #else
22620 static CYTHON_INLINE PyObject * __Pyx_GetModuleGlobalName(PyObject *name)
22621 #endif
22622 {
22623     PyObject *result;
22624     #if !CYTHON_AVOID_BORROWED_REFS
22625     #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x030500A1
22626         result = _PyDict_GetItem_KnownHash(__pyx_d, name, ((PyASCIIObject *) name)->hash);
22627         __PYX_UPDATE_DICT_CACHE(__pyx_d, result, *dict_cached_value, *dict_version)
22628         if (likely(result)) {
22629             return __Pyx_NewRef(result);
22630         } else if (unlikely(PyErr_Occurred())) {
22631             return NULL;
22632         }
22633     #else
22634         result = PyDict_GetItem(__pyx_d, name);
22635         __PYX_UPDATE_DICT_CACHE(__pyx_d, result, *dict_cached_value, *dict_version)
22636         if (likely(result)) {
22637             return __Pyx_NewRef(result);
22638         }
22639     #endif
22640     #else
22641         result = PyObject_GetItem(__pyx_d, name);
22642         __PYX_UPDATE_DICT_CACHE(__pyx_d, result, *dict_cached_value, *dict_version)
22643         if (likely(result)) {
22644             return __Pyx_NewRef(result);
22645         }
22646         PyErr_Clear();
22647     #endif
22648     return __Pyx_GetBuiltinName(name);
22649 }
22650
22651 /* GetTopmostException */
22652 #if CYTHON_USE_EXC_INFO_STACK
22653 static _PyErr_StackItem *
22654 __Pyx_PyErr_GetTopmostException(PyThreadState *tstate)
22655 {
22656     _PyErr_StackItem *exc_info = tstate->exc_info;
22657     while ((exc_info->exc_type == NULL || exc_info->exc_type == Py_None) &&
22658         exc_info->previous_item != NULL)
22659     {
22660         exc_info = exc_info->previous_item;
22661     }
22662     return exc_info;
22663 }

```

```

22664 #endif
22665
22666 /* SaveResetException */
22667 #if CYTHON_FAST_THREAD_STATE
22668 static CYTHON_INLINE void __Pyx_ExceptionSave(PyThreadState *tstate, PyObject **type, PyObject
**value, PyObject **tb) {
22669     #if CYTHON_USE_EXC_INFO_STACK
22670         _PyErr_StackItem *exc_info = __Pyx_PyErr_GetTopmostException(tstate);
22671         *type = exc_info->exc_type;
22672         *value = exc_info->exc_value;
22673         *tb = exc_info->exc_traceback;
22674     #else
22675         *type = tstate->exc_type;
22676         *value = tstate->exc_value;
22677         *tb = tstate->exc_traceback;
22678     #endif
22679     Py_XINCREF(*type);
22680     Py_XINCREF(*value);
22681     Py_XINCREF(*tb);
22682 }
22683 static CYTHON_INLINE void __Pyx_ExceptionReset(PyThreadState *tstate, PyObject *type, PyObject
*value, PyObject *tb) {
22684     PyObject *tmp_type, *tmp_value, *tmp_tb;
22685     #if CYTHON_USE_EXC_INFO_STACK
22686         _PyErr_StackItem *exc_info = tstate->exc_info;
22687         tmp_type = exc_info->exc_type;
22688         tmp_value = exc_info->exc_value;
22689         tmp_tb = exc_info->exc_traceback;
22690         exc_info->exc_type = type;
22691         exc_info->exc_value = value;
22692         exc_info->exc_traceback = tb;
22693     #else
22694         tmp_type = tstate->exc_type;
22695         tmp_value = tstate->exc_value;
22696         tmp_tb = tstate->exc_traceback;
22697         tstate->exc_type = type;
22698         tstate->exc_value = value;
22699         tstate->exc_traceback = tb;
22700     #endif
22701     Py_XDECREF(tmp_type);
22702     Py_XDECREF(tmp_value);
22703     Py_XDECREF(tmp_tb);
22704 }
22705 #endif
22706
22707 /* PyErrExceptionMatches */
22708 #if CYTHON_FAST_THREAD_STATE
22709 static int __Pyx_PyErr_ExceptionMatchesTuple(PyObject *exc_type, PyObject *tuple) {
22710     Py_ssize_t i, n;
22711     n = PyTuple_GET_SIZE(tuple);
22712     #if PY_MAJOR_VERSION >= 3
22713     for (i=0; i<n; i++) {
22714         if (exc_type == PyTuple_GET_ITEM(tuple, i)) return 1;
22715     }
22716 #endif
22717     for (i=0; i<n; i++) {
22718         if (__Pyx_PyErr_GivenExceptionMatches(exc_type, PyTuple_GET_ITEM(tuple, i))) return 1;
22719     }
22720     return 0;
22721 }
22722 static CYTHON_INLINE int __Pyx_PyErr_ExceptionMatchesInState(PyThreadState* tstate, PyObject* err) {
22723     PyObject *exc_type = tstate->curexc_type;
22724     if (exc_type == err) return 1;
22725     if (unlikely(!exc_type)) return 0;
22726     if (unlikely(PyTuple_Check(err)))
22727         return __Pyx_PyErr_ExceptionMatchesTuple(exc_type, err);
22728     return __Pyx_PyErr_GivenExceptionMatches(exc_type, err);
22729 }
22730 #endif
22731
22732 /* GetException */
22733 #if CYTHON_FAST_THREAD_STATE
22734 static int __Pyx_GetException(PyThreadState *tstate, PyObject **type, PyObject **value, PyObject
**tb)
22735 #else
22736 static int __Pyx_GetException(PyObject **type, PyObject **value, PyObject **tb)
22737 #endif
22738 {
22739     PyObject *local_type, *local_value, *local_tb;
22740     #if CYTHON_FAST_THREAD_STATE
22741     PyObject *tmp_type, *tmp_value, *tmp_tb;
22742     local_type = tstate->curexc_type;
22743     local_value = tstate->curexc_value;
22744     local_tb = tstate->curexc_traceback;
22745     tstate->curexc_type = 0;
22746     tstate->curexc_value = 0;
22747     tstate->curexc_traceback = 0;

```

```

22748 #else
22749     PyErr_Fetch(&local_type, &local_value, &local_tb);
22750 #endif
22751     PyErr_NormalizeException(&local_type, &local_value, &local_tb);
22752 #if CYTHON_FAST_THREAD_STATE
22753     if (unlikely(tstate->curexc_type))
22754 #else
22755     if (unlikely(PyErr_Occurred()))
22756 #endif
22757     goto bad;
22758     #if PY_MAJOR_VERSION >= 3
22759     if (local_tb) {
22760         if (unlikely(PyException_SetTraceback(local_value, local_tb) < 0))
22761             goto bad;
22762     }
22763     #endif
22764     Py_XINCREF(local_tb);
22765     Py_XINCREF(local_type);
22766     Py_XINCREF(local_value);
22767     *type = local_type;
22768     *value = local_value;
22769     *tb = local_tb;
22770 #if CYTHON_FAST_THREAD_STATE
22771 #if CYTHON_USE_EXC_INFO_STACK
22772     {
22773         _PyErr_StackItem *exc_info = tstate->exc_info;
22774         tmp_type = exc_info->exc_type;
22775         tmp_value = exc_info->exc_value;
22776         tmp_tb = exc_info->exc_traceback;
22777         exc_info->exc_type = local_type;
22778         exc_info->exc_value = local_value;
22779         exc_info->exc_traceback = local_tb;
22780     }
22781     #else
22782     tmp_type = tstate->exc_type;
22783     tmp_value = tstate->exc_value;
22784     tmp_tb = tstate->exc_traceback;
22785     tstate->exc_type = local_type;
22786     tstate->exc_value = local_value;
22787     tstate->exc_traceback = local_tb;
22788     #endif
22789     Py_XDECREF(tmp_type);
22790     Py_XDECREF(tmp_value);
22791     Py_XDECREF(tmp_tb);
22792 #else
22793     PyErr_SetExcInfo(local_type, local_value, local_tb);
22794 #endif
22795     return 0;
22796 bad:
22797     *type = 0;
22798     *value = 0;
22799     *tb = 0;
22800     Py_XDECREF(local_type);
22801     Py_XDECREF(local_value);
22802     Py_XDECREF(local_tb);
22803     return -1;
22804 }
22805
22806 /* RaiseException */
22807 #if PY_MAJOR_VERSION < 3
22808 static void __Pyx_Raise(PyObject *type, PyObject *value, PyObject *tb,
22809                        CYTHON_UNUSED PyObject *cause) {
22810     __Pyx_PyThreadState_declare
22811     Py_XINCREF(type);
22812     if (!value || value == Py_None)
22813         value = NULL;
22814     else
22815         Py_INCREF(value);
22816     if (!tb || tb == Py_None)
22817         tb = NULL;
22818     else {
22819         Py_INCREF(tb);
22820         if (!PyTraceBack_Check(tb)) {
22821             PyErr_SetString(PyExc_TypeError,
22822                             "raise: arg 3 must be a traceback or None");
22823             goto raise_error;
22824         }
22825     }
22826     if (PyType_Check(type)) {
22827 #if CYTHON_COMPILING_IN_PYPY
22828         if (!value) {
22829             Py_INCREF(Py_None);
22830             value = Py_None;
22831         }
22832 #endif
22833     }
22834     PyErr_NormalizeException(&type, &value, &tb);
22835 } else {

```

```

22835         if (value) {
22836             PyErr_SetString(PyExc_TypeError,
22837                 "instance exception may not have a separate value");
22838             goto raise_error;
22839         }
22840         value = type;
22841         type = (PyObject*) Py_TYPE(type);
22842         Py_INCREF(type);
22843         if (!PyType_IsSubtype((PyTypeObject *)type, (PyTypeObject *)PyExc_BaseException)) {
22844             PyErr_SetString(PyExc_TypeError,
22845                 "raise: exception class must be a subclass of BaseException");
22846             goto raise_error;
22847         }
22848     }
22849     __Pyx_PyThreadState_assign
22850     __Pyx_ErrRestore(type, value, tb);
22851     return;
22852 raise_error:
22853     Py_XDECREF(value);
22854     Py_XDECREF(type);
22855     Py_XDECREF(tb);
22856     return;
22857 }
22858 #else
22859 static void __Pyx_Raise(PyObject *type, PyObject *value, PyObject *tb, PyObject *cause) {
22860     PyObject* owned_instance = NULL;
22861     if (tb == Py_None) {
22862         tb = 0;
22863     } else if (tb && !PyTraceBack_Check(tb)) {
22864         PyErr_SetString(PyExc_TypeError,
22865             "raise: arg 3 must be a traceback or None");
22866         goto bad;
22867     }
22868     if (value == Py_None)
22869         value = 0;
22870     if (PyExceptionInstance_Check(type)) {
22871         if (value) {
22872             PyErr_SetString(PyExc_TypeError,
22873                 "instance exception may not have a separate value");
22874             goto bad;
22875         }
22876         value = type;
22877         type = (PyObject*) Py_TYPE(value);
22878     } else if (PyExceptionClass_Check(type)) {
22879         PyObject *instance_class = NULL;
22880         if (value && PyExceptionInstance_Check(value)) {
22881             instance_class = (PyObject*) Py_TYPE(value);
22882             if (instance_class != type) {
22883                 int is_subclass = PyObject_IsSubclass(instance_class, type);
22884                 if (!is_subclass) {
22885                     instance_class = NULL;
22886                 } else if (unlikely(is_subclass == -1)) {
22887                     goto bad;
22888                 } else {
22889                     type = instance_class;
22890                 }
22891             }
22892         }
22893         if (!instance_class) {
22894             PyObject *args;
22895             if (!value)
22896                 args = PyTuple_New(0);
22897             else if (PyTuple_Check(value)) {
22898                 Py_INCREF(value);
22899                 args = value;
22900             } else
22901                 args = PyTuple_Pack(1, value);
22902             if (!args)
22903                 goto bad;
22904             owned_instance = PyObject_Call(type, args, NULL);
22905             Py_DECREF(args);
22906             if (!owned_instance)
22907                 goto bad;
22908             value = owned_instance;
22909             if (!PyExceptionInstance_Check(value)) {
22910                 PyErr_Format(PyExc_TypeError,
22911                     "calling %R should have returned an instance of "
22912                     "BaseException, not %R",
22913                     type, Py_TYPE(value));
22914                 goto bad;
22915             }
22916         }
22917     } else {
22918         PyErr_SetString(PyExc_TypeError,
22919             "raise: exception class must be a subclass of BaseException");
22920         goto bad;
22921     }

```

```

22922     if (cause) {
22923         PyObject *fixed_cause;
22924         if (cause == Py_None) {
22925             fixed_cause = NULL;
22926         } else if (PyExceptionClass_Check(cause)) {
22927             fixed_cause = PyObject_CallObject(cause, NULL);
22928             if (fixed_cause == NULL)
22929                 goto bad;
22930         } else if (PyExceptionInstance_Check(cause)) {
22931             fixed_cause = cause;
22932             Py_INCREF(fixed_cause);
22933         } else {
22934             PyErr_SetString(PyExc_TypeError,
22935                             "exception causes must derive from "
22936                             "BaseException");
22937             goto bad;
22938         }
22939         PyException_SetCause(value, fixed_cause);
22940     }
22941     PyErr_SetObject(type, value);
22942     if (tb) {
22943 #if CYTHON_COMPILING_IN_PYPY
22944         PyObject *tmp_type, *tmp_value, *tmp_tb;
22945         PyErr_Fetch(&tmp_type, &tmp_value, &tmp_tb);
22946         Py_INCREF(tb);
22947         PyErr_Restore(tmp_type, tmp_value, tb);
22948         Py_XDECREF(tmp_tb);
22949 #else
22950         PyThreadState *tstate = __Pyx_PyThreadState_Current;
22951         PyObject* tmp_tb = tstate->curexc_traceback;
22952         if (tb != tmp_tb) {
22953             Py_INCREF(tb);
22954             tstate->curexc_traceback = tb;
22955             Py_XDECREF(tmp_tb);
22956         }
22957 #endif
22958     }
22959 bad:
22960     Py_XDECREF(owned_instance);
22961     return;
22962 }
22963 #endif
22964
22965 /* PyObjectCall2Args */
22966 static CYTHON_UNUSED PyObject* __Pyx_PyObject_Call2Args(PyObject* function, PyObject* arg1, PyObject*
arg2) {
22967     PyObject *args, *result = NULL;
22968     #if CYTHON_FAST_PYCALL
22969     if (PyFunction_Check(function)) {
22970         PyObject *args[2] = {arg1, arg2};
22971         return __Pyx_PyFunction_FastCall(function, args, 2);
22972     }
22973     #endif
22974     #if CYTHON_FAST_PYCCALL
22975     if (__Pyx_PyFastCFunction_Check(function)) {
22976         PyObject *args[2] = {arg1, arg2};
22977         return __Pyx_PyCFunction_FastCall(function, args, 2);
22978     }
22979     #endif
22980     args = PyTuple_New(2);
22981     if (unlikely(!args)) goto done;
22982     Py_INCREF(arg1);
22983     PyTuple_SET_ITEM(args, 0, arg1);
22984     Py_INCREF(arg2);
22985     PyTuple_SET_ITEM(args, 1, arg2);
22986     Py_INCREF(function);
22987     result = __Pyx_PyObject_Call(function, args, NULL);
22988     Py_DECREF(args);
22989     Py_DECREF(function);
22990 done:
22991     return result;
22992 }
22993
22994 /* PyIntBinop */
22995 #if !CYTHON_COMPILING_IN_PYPY
22996 static PyObject* __Pyx_PyInt_AddObjC(PyObject *op1, PyObject *op2, CYTHON_UNUSED long intval, int
inplace, int zerodivision_check) {
22997     (void)inplace;
22998     (void)zerodivision_check;
22999     #if PY_MAJOR_VERSION < 3
23000     if (likely(PyInt_CheckExact(op1))) {
23001         const long b = intval;
23002         long x;
23003         long a = PyInt_AS_LONG(op1);
23004         x = (long)((unsigned long)a + b);
23005         if (likely((x^a) >= 0 || (x^b) >= 0))
23006             return PyInt_FromLong(x);

```

```

23007         return PyLong_Type.tp_as_number->nb_add(op1, op2);
23008     }
23009     #endif
23010     #if CYTHON_USE_PYLONG_INTERNALS
23011     if (likely(PyLong_CheckExact(op1))) {
23012         const long b = intval;
23013         long a, x;
23014     #ifdef HAVE_LONG_LONG
23015         const PY_LONG_LONG llb = intval;
23016         PY_LONG_LONG lla, llx;
23017     #endif
23018         const digit* digits = ((PyLongObject*)op1)->ob_digit;
23019         const Py_ssize_t size = Py_SIZE(op1);
23020         if (likely(__Pyx_sst_abs(size) <= 1)) {
23021             a = likely(size) ? digits[0] : 0;
23022             if (size == -1) a = -a;
23023         } else {
23024             switch (size) {
23025                 case -2:
23026                     if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
23027                         a = -(long) (((unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0]);
23028                         break;
23029                     } else if (8 * sizeof(PY_LONG_LONG) - 1 > 2 * PyLong_SHIFT) {
23030                         lla = -(PY_LONG_LONG) (((unsigned PY_LONG_LONG)digits[1]) « PyLong_SHIFT) |
23031 (unsigned PY_LONG_LONG)digits[0]);
23032                         goto long_long;
23033                     }
23034                     CYTHON_FALLTHROUGH;
23035                 case 2:
23036                     if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
23037                         a = (long) (((unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0]);
23038                         break;
23039                     } else if (8 * sizeof(PY_LONG_LONG) - 1 > 2 * PyLong_SHIFT) {
23040                         lla = (PY_LONG_LONG) (((unsigned PY_LONG_LONG)digits[1]) « PyLong_SHIFT) |
23041 (unsigned PY_LONG_LONG)digits[0]);
23042                         goto long_long;
23043                     }
23044                     CYTHON_FALLTHROUGH;
23045                 case -3:
23046                     if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
23047                         a = -(long) (((((unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]);
23048                         break;
23049                     } else if (8 * sizeof(PY_LONG_LONG) - 1 > 3 * PyLong_SHIFT) {
23050                         lla = -(PY_LONG_LONG) (((((unsigned PY_LONG_LONG)digits[2]) « PyLong_SHIFT)
| (unsigned PY_LONG_LONG)digits[1]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[0]);
23051                         goto long_long;
23052                     }
23053                     CYTHON_FALLTHROUGH;
23054                 case 3:
23055                     if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
23056                         a = (long) (((((unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]);
23057                         break;
23058                     } else if (8 * sizeof(PY_LONG_LONG) - 1 > 3 * PyLong_SHIFT) {
23059                         lla = (PY_LONG_LONG) (((((unsigned PY_LONG_LONG)digits[2]) « PyLong_SHIFT)
| (unsigned PY_LONG_LONG)digits[1]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[0]);
23060                         goto long_long;
23061                     }
23062                     CYTHON_FALLTHROUGH;
23063                 case -4:
23064                     if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
23065                         a = -(long) (((((((unsigned long)digits[3]) « PyLong_SHIFT) | (unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0]);
23066                         break;
23067                     } else if (8 * sizeof(PY_LONG_LONG) - 1 > 4 * PyLong_SHIFT) {
23068                         lla = -(PY_LONG_LONG) (((((((unsigned PY_LONG_LONG)digits[3]) «
PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[2]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[1])
« PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[0]);
23069                         goto long_long;
23070                     }
23071                     CYTHON_FALLTHROUGH;
23072                 case 4:
23073                     if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {

```

```

23082         a = (long) (((((((((unsigned long)digits[3]) « PyLong_SHIFT) | (unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0]));
23083         break;
23084 #ifdef HAVE_LONG_LONG
23085     } else if (8 * sizeof(PY_LONG_LONG) - 1 > 4 * PyLong_SHIFT) {
23086         lla = (PY_LONG_LONG) (((((((((unsigned PY_LONG_LONG)digits[3]) « PyLong_SHIFT)
| (unsigned PY_LONG_LONG)digits[2]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[1]) «
PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[0]));
23087         goto long_long;
23088 #endif
23089     }
23090     CYTHON_FALLTHROUGH;
23091     default: return PyLong_Type.tp_as_number->nb_add(op1, op2);
23092 }
23093 }
23094     x = a + b;
23095     return PyLong_FromLong(x);
23096 #ifdef HAVE_LONG_LONG
23097     long_long:
23098         llx = lla + llb;
23099         return PyLong_FromLongLong(llx);
23100 #endif
23101 }
23102 }
23103 #endif
23104 if (PyFloat_CheckExact(op1)) {
23105     const long b = intval;
23106     double a = PyFloat_AS_DOUBLE(op1);
23107     double result;
23108     PyFPE_START_PROTECT("add", return NULL)
23109     result = ((double)a) + (double)b;
23110     PyFPE_END_PROTECT(result)
23111     return PyFloat_FromDouble(result);
23112 }
23113 return (inplace ? PyNumber_InPlaceAdd : PyNumber_Add)(op1, op2);
23114 }
23115 #endif
23116
23117 /* decode_c_bytes */
23118 static CYTHON_INLINE PyObject* __Pyx_decode_c_bytes(
23119     const char* cstring, Py_ssize_t length, Py_ssize_t start, Py_ssize_t stop,
23120     const char* encoding, const char* errors,
23121     PyObject* (*decode_func)(const char *s, Py_ssize_t size, const char *errors)) {
23122     if (unlikely((start < 0) | (stop < 0))) {
23123         if (start < 0) {
23124             start += length;
23125             if (start < 0)
23126                 start = 0;
23127         }
23128         if (stop < 0)
23129             stop += length;
23130     }
23131     if (stop > length)
23132         stop = length;
23133     if (unlikely(stop <= start))
23134         return __Pyx_NewRef(__pyx_empty_unicode);
23135     length = stop - start;
23136     cstring += start;
23137     if (decode_func) {
23138         return decode_func(cstring, length, errors);
23139     } else {
23140         return PyUnicode_Decode(cstring, length, encoding, errors);
23141     }
23142 }
23143 }
23144
23145 /* SwapException */
23146 #if CYTHON_FAST_THREAD_STATE
23147 static CYTHON_INLINE void __Pyx_ExceptionSwap(PyThreadState *tstate, PyObject **type, PyObject
**value, PyObject **tb) {
23148     PyObject *tmp_type, *tmp_value, *tmp_tb;
23149     #if CYTHON_USE_EXC_INFO_STACK
23150     _PyErr_StackItem *exc_info = tstate->exc_info;
23151     tmp_type = exc_info->exc_type;
23152     tmp_value = exc_info->exc_value;
23153     tmp_tb = exc_info->exc_traceback;
23154     exc_info->exc_type = *type;
23155     exc_info->exc_value = *value;
23156     exc_info->exc_traceback = *tb;
23157     #else
23158     tmp_type = tstate->exc_type;
23159     tmp_value = tstate->exc_value;
23160     tmp_tb = tstate->exc_traceback;
23161     tstate->exc_type = *type;
23162     tstate->exc_value = *value;
23163     tstate->exc_traceback = *tb;

```



```

23164     #endif
23165     *type = tmp_type;
23166     *value = tmp_value;
23167     *tb = tmp_tb;
23168 }
23169 #else
23170 static CYTHON_INLINE void __Pyx_ExceptionSwap(PyObject **type, PyObject **value, PyObject **tb) {
23171     PyObject *tmp_type, *tmp_value, *tmp_tb;
23172     PyErr_GetExcInfo(&tmp_type, &tmp_value, &tmp_tb);
23173     PyErr_SetExcInfo(*type, *value, *tb);
23174     *type = tmp_type;
23175     *value = tmp_value;
23176     *tb = tmp_tb;
23177 }
23178 #endif
23179
23180 /* SetItemInt */
23181 static int __Pyx_SetItemInt_Generic(PyObject *o, PyObject *j, PyObject *v) {
23182     int r;
23183     if (!j) return -1;
23184     r = PyObject_SetItem(o, j, v);
23185     Py_DECREF(j);
23186     return r;
23187 }
23188 static CYTHON_INLINE int __Pyx_SetItemInt_Fast(PyObject *o, Py_ssize_t i, PyObject *v, int is_list,
23189     CYTHON_NCP_UNUSED int wraparound, CYTHON_NCP_UNUSED int
23190     boundscheck) {
23191     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS && CYTHON_USE_TYPE_SLOTS
23192     if (is_list || PyList_CheckExact(o)) {
23193         Py_ssize_t n = (!wraparound) ? i : ((likely(i >= 0)) ? i : i + PyList_GET_SIZE(o));
23194         if ((!boundscheck) || likely(__Pyx_is_valid_index(n, PyList_GET_SIZE(o)))) {
23195             PyObject* old = PyList_GET_ITEM(o, n);
23196             Py_INCREF(v);
23197             PyList_SET_ITEM(o, n, v);
23198             Py_DECREF(old);
23199             return 1;
23200         } else {
23201             PySequenceMethods *m = Py_TYPE(o)->tp_as_sequence;
23202             if (likely(m && m->sq_ass_item)) {
23203                 if (wraparound && unlikely(i < 0) && likely(m->sq_length)) {
23204                     Py_ssize_t l = m->sq_length(o);
23205                     if (likely(l >= 0)) {
23206                         i += l;
23207                     } else {
23208                         if (!PyErr_ExceptionMatches(PyExc_OverflowError))
23209                             return -1;
23210                         PyErr_Clear();
23211                     }
23212                 }
23213                 return m->sq_ass_item(o, i, v);
23214             }
23215         }
23216     #else
23217     #if CYTHON_COMPILING_IN_PYPY
23218     if (is_list || (PySequence_Check(o) && !PyDict_Check(o)))
23219     #else
23220     if (is_list || PySequence_Check(o))
23221     #endif
23222     {
23223         return PySequence_SetItem(o, i, v);
23224     }
23225     #endif
23226     return __Pyx_SetItemInt_Generic(o, PyInt_FromSsize_t(i), v);
23227 }
23228
23229 /* ArgTypeTest */
23230 static int __Pyx_ArgTypeTest(PyObject *obj, PyTypeObject *type, const char *name, int exact)
23231 {
23232     if (unlikely(!type)) {
23233         PyErr_SetString(PyExc_SystemError, "Missing type object");
23234         return 0;
23235     }
23236     else if (exact) {
23237         #if PY_MAJOR_VERSION == 2
23238         if ((type == &PyBaseString_Type) && likely(__Pyx_PyBaseString_CheckExact(obj))) return 1;
23239         #endif
23240     }
23241     else {
23242         if (likely(__Pyx_TypeCheck(obj, type))) return 1;
23243     }
23244     PyErr_Format(PyExc_TypeError,
23245         "Argument '%.200s' has incorrect type (expected %.200s, got %.200s)",
23246         name, type->tp_name, Py_TYPE(obj)->tp_name);
23247     return 0;
23248 }
23249

```

```

23250 /* Import */
23251 static PyObject *__Pyx_Import(PyObject *name, PyObject *from_list, int level) {
23252     PyObject *empty_list = 0;
23253     PyObject *module = 0;
23254     PyObject *global_dict = 0;
23255     PyObject *empty_dict = 0;
23256     PyObject *list;
23257     #if PY_MAJOR_VERSION < 3
23258     PyObject *py_import;
23259     py_import = __Pyx_PyObject_GetAttrStr(__pyx_b, __pyx_n_s_import);
23260     if (!py_import)
23261         goto bad;
23262     #endif
23263     if (from_list)
23264         list = from_list;
23265     else {
23266         empty_list = PyList_New(0);
23267         if (!empty_list)
23268             goto bad;
23269         list = empty_list;
23270     }
23271     global_dict = PyModule_GetDict(__pyx_m);
23272     if (!global_dict)
23273         goto bad;
23274     empty_dict = PyDict_New();
23275     if (!empty_dict)
23276         goto bad;
23277     {
23278         #if PY_MAJOR_VERSION >= 3
23279         if (level == -1) {
23280             if ((1) && (strchr(__Pyx_MODULE_NAME, '.'))) {
23281                 module = PyImport_ImportModuleLevelObject(
23282                     name, global_dict, empty_dict, list, 1);
23283                 if (!module) {
23284                     if (!PyErr_ExceptionMatches(PyExc_ImportError))
23285                         goto bad;
23286                     PyErr_Clear();
23287                 }
23288             }
23289             level = 0;
23290         }
23291         #endif
23292         if (!module) {
23293             #if PY_MAJOR_VERSION < 3
23294             PyObject *py_level = PyInt_FromLong(level);
23295             if (!py_level)
23296                 goto bad;
23297             module = PyObject_CallFunctionObjArgs(py_import,
23298                 name, global_dict, empty_dict, list, py_level, (PyObject *)NULL);
23299             Py_DECREF(py_level);
23300             #else
23301             module = PyImport_ImportModuleLevelObject(
23302                 name, global_dict, empty_dict, list, level);
23303             #endif
23304         }
23305     }
23306 bad:
23307     #if PY_MAJOR_VERSION < 3
23308     Py_XDECREF(py_import);
23309     #endif
23310     Py_XDECREF(empty_list);
23311     Py_XDECREF(empty_dict);
23312     return module;
23313 }
23314
23315 /* PyObject_GenericGetAttrNoDict */
23316 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
23317 static PyObject *__Pyx_RaiseGenericGetAttributeError(PyTypeObject *tp, PyObject *attr_name) {
23318     PyErr_Format(PyExc_AttributeError,
23319         #if PY_MAJOR_VERSION >= 3
23320         "'%.50s' object has no attribute '%U'",
23321         tp->tp_name, attr_name);
23322     #else
23323         "'%.50s' object has no attribute '%.400s'",
23324         tp->tp_name, PyString_AS_STRING(attr_name));
23325     #endif
23326     return NULL;
23327 }
23328 static CYTHON_INLINE PyObject* __Pyx_PyObject_GenericGetAttrNoDict(PyObject* obj, PyObject* attr_name)
23329 {
23330     PyObject *descr;
23331     PyTypeObject *tp = Py_TYPE(obj);
23332     if (unlikely(!PyString_Check(attr_name))) {
23333         return PyObject_GenericGetAttr(obj, attr_name);
23334     }
23335     assert(!tp->tp_dictoffset);
23336     descr = _PyType_Lookup(tp, attr_name);

```

```

23336     if (unlikely(!descr)) {
23337         return __Pyx_RaiseGenericGetAttributeError(tp, attr_name);
23338     }
23339     Py_INCREF(descr);
23340     #if PY_MAJOR_VERSION < 3
23341     if (likely(PyType_HasFeature(Py_TYPE(descr), Py_TPFLAGS_HAVE_CLASS)))
23342     #endif
23343     {
23344         descrgetfunc f = Py_TYPE(descr)->tp_descr_get;
23345         if (unlikely(!f)) {
23346             PyObject *res = f(descr, obj, (PyObject *)tp);
23347             Py_DECREF(descr);
23348             return res;
23349         }
23350     }
23351     return descr;
23352 }
23353 #endif
23354
23355 /* PyObject_GenericGetAttr */
23356 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
23357 static PyObject* __Pyx_PyObject_GenericGetAttr(PyObject* obj, PyObject* attr_name) {
23358     if (unlikely(Py_TYPE(obj)->tp_dictoffset)) {
23359         return PyObject_GenericGetAttr(obj, attr_name);
23360     }
23361     return __Pyx_PyObject_GenericGetAttrNoDict(obj, attr_name);
23362 }
23363 #endif
23364
23365 /* SetVTable */
23366 static int __Pyx_SetVtable(PyObject *dict, void *vtable) {
23367     #if PY_VERSION_HEX >= 0x02070000
23368     PyObject *ob = PyCapsule_New(vtable, 0, 0);
23369     #else
23370     PyObject *ob = PyCObject_FromVoidPtr(vtable, 0);
23371     #endif
23372     if (!ob)
23373         goto bad;
23374     if (PyDict_SetItem(dict, __pyx_n_s_pyx_vtable, ob) < 0)
23375         goto bad;
23376     Py_DECREF(ob);
23377     return 0;
23378 bad:
23379     Py_XDECREF(ob);
23380     return -1;
23381 }
23382
23383 /* PyObject_GetAttrStrNoError */
23384 static void __Pyx_PyObject_GetAttrStr_ClearAttributeError(void) {
23385     __Pyx_PyThreadState_declare
23386     __Pyx_PyThreadState_assign
23387     if (likely(__Pyx_PyErr_ExceptionMatches(PyExc_AttributeError)))
23388         __Pyx_PyErr_Clear();
23389 }
23390 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStrNoError(PyObject* obj, PyObject* attr_name) {
23391     PyObject *result;
23392     #if CYTHON_COMPILING_IN_CPYTHON && CYTHON_USE_TYPE_SLOTS && PY_VERSION_HEX >= 0x030700B1
23393     PyTypeObject* tp = Py_TYPE(obj);
23394     if (likely(tp->tp_getattro == PyObject_GenericGetAttr)) {
23395         return PyObject_GenericGetAttrWithDict(obj, attr_name, NULL, 1);
23396     }
23397     #endif
23398     result = PyObject_GetAttrStr(obj, attr_name);
23399     if (unlikely(!result)) {
23400         __Pyx_PyObject_GetAttrStr_ClearAttributeError();
23401     }
23402     return result;
23403 }
23404
23405 /* SetupReduce */
23406 static int __Pyx_setup_reduce_is_named(PyObject* meth, PyObject* name) {
23407     int ret;
23408     PyObject *name_attr;
23409     name_attr = __Pyx_PyObject_GetAttrStr(meth, __pyx_n_s_name);
23410     if (likely(name_attr)) {
23411         ret = PyObject_RichCompareBool(name_attr, name, Py_EQ);
23412     } else {
23413         ret = -1;
23414     }
23415     if (unlikely(ret < 0)) {
23416         PyErr_Clear();
23417         ret = 0;
23418     }
23419     Py_XDECREF(name_attr);
23420     return ret;
23421 }
23422 static int __Pyx_setup_reduce(PyObject* type_obj) {

```

```

23423     int ret = 0;
23424     PyObject *object_reduce = NULL;
23425     PyObject *object_reduce_ex = NULL;
23426     PyObject *reduce = NULL;
23427     PyObject *reduce_ex = NULL;
23428     PyObject *reduce_cython = NULL;
23429     PyObject *setstate = NULL;
23430     PyObject *setstate_cython = NULL;
23431     #if CYTHON_USE_PYTYPE_LOOKUP
23432     if (_PyType_Lookup((PyObject*)type_obj, __pyx_n_s_getstate)) goto __PYX_GOOD;
23433     #else
23434     if (PyObject_HasAttr(type_obj, __pyx_n_s_getstate)) goto __PYX_GOOD;
23435     #endif
23436     #if CYTHON_USE_PYTYPE_LOOKUP
23437     object_reduce_ex = _PyType_Lookup(&PyBaseObject_Type, __pyx_n_s_reduce_ex); if (!object_reduce_ex)
23438     goto __PYX_BAD;
23439     #else
23440     object_reduce_ex = __Pyx_PyObject_GetAttrStr((PyObject*)&PyBaseObject_Type, __pyx_n_s_reduce_ex);
23441     if (!object_reduce_ex) goto __PYX_BAD;
23442     #endif
23443     reduce_ex = __Pyx_PyObject_GetAttrStr(type_obj, __pyx_n_s_reduce_ex); if (unlikely(!reduce_ex))
23444     goto __PYX_BAD;
23445     if (reduce_ex == object_reduce_ex) {
23446     #if CYTHON_USE_PYTYPE_LOOKUP
23447     object_reduce = _PyType_Lookup(&PyBaseObject_Type, __pyx_n_s_reduce); if (!object_reduce) goto
23448     __PYX_BAD;
23449     #else
23450     object_reduce = __Pyx_PyObject_GetAttrStr((PyObject*)&PyBaseObject_Type, __pyx_n_s_reduce); if
23451     (!object_reduce) goto __PYX_BAD;
23452     #endif
23453     reduce = __Pyx_PyObject_GetAttrStr(type_obj, __pyx_n_s_reduce); if (unlikely(!reduce)) goto
23454     __PYX_BAD;
23455     if (reduce == object_reduce || __Pyx_setup_reduce_is_named(reduce, __pyx_n_s_reduce_cython)) {
23456     reduce_cython = __Pyx_PyObject_GetAttrStrNoError(type_obj, __pyx_n_s_reduce_cython);
23457     if (likely(reduce_cython)) {
23458     ret = PyDict_SetItem(((PyObject*)type_obj)->tp_dict, __pyx_n_s_reduce,
23459     reduce_cython); if (unlikely(ret < 0)) goto __PYX_BAD;
23460     ret = PyDict_DelItem(((PyObject*)type_obj)->tp_dict, __pyx_n_s_reduce_cython); if
23461     (unlikely(ret < 0)) goto __PYX_BAD;
23462     } else if (reduce == object_reduce || PyErr_Occurred()) {
23463     goto __PYX_BAD;
23464     }
23465     setstate = __Pyx_PyObject_GetAttrStr(type_obj, __pyx_n_s_setstate);
23466     if (!setstate) PyErr_Clear();
23467     if (!setstate || __Pyx_setup_reduce_is_named(setstate, __pyx_n_s_setstate_cython)) {
23468     setstate_cython = __Pyx_PyObject_GetAttrStrNoError(type_obj,
23469     __pyx_n_s_setstate_cython);
23470     if (likely(setstate_cython)) {
23471     ret = PyDict_SetItem(((PyObject*)type_obj)->tp_dict, __pyx_n_s_setstate,
23472     setstate_cython); if (unlikely(ret < 0)) goto __PYX_BAD;
23473     ret = PyDict_DelItem(((PyObject*)type_obj)->tp_dict,
23474     __pyx_n_s_setstate_cython); if (unlikely(ret < 0)) goto __PYX_BAD;
23475     } else if (!setstate || PyErr_Occurred()) {
23476     goto __PYX_BAD;
23477     }
23478     }
23479     PyType_Modified((PyObject*)type_obj);
23480     }
23481     goto __PYX_GOOD;
23482     __PYX_BAD:
23483     if (!PyErr_Occurred())
23484     PyErr_Format(PyExc_RuntimeError, "Unable to initialize pickling for %s",
23485     ((PyObject*)type_obj)->tp_name);
23486     ret = -1;
23487     __PYX_GOOD:
23488     #if !CYTHON_USE_PYTYPE_LOOKUP
23489     Py_XDECREF(object_reduce);
23490     Py_XDECREF(object_reduce_ex);
23491     #endif
23492     Py_XDECREF(reduce);
23493     Py_XDECREF(reduce_ex);
23494     Py_XDECREF(reduce_cython);
23495     Py_XDECREF(setstate);
23496     Py_XDECREF(setstate_cython);
23497     return ret;
23498 }
23499
23500 /* BytesEquals */
23501 static CYTHON_INLINE int __Pyx_PyBytes_Equals(PyObject* s1, PyObject* s2, int equals) {
23502     #if CYTHON_COMPILING_IN_PYPY
23503     return PyObject_RichCompareBool(s1, s2, equals);
23504     #else
23505     if (s1 == s2) {
23506     return (equals == Py_EQ);
23507     } else if (PyBytes_CheckExact(s1) & PyBytes_CheckExact(s2)) {
23508     const char *ps1, *ps2;

```

```

23498     Py_ssize_t length = PyBytes_GET_SIZE(s1);
23499     if (length != PyBytes_GET_SIZE(s2))
23500         return (equals == Py_NE);
23501     ps1 = PyBytes_AS_STRING(s1);
23502     ps2 = PyBytes_AS_STRING(s2);
23503     if (ps1[0] != ps2[0]) {
23504         return (equals == Py_NE);
23505     } else if (length == 1) {
23506         return (equals == Py_EQ);
23507     } else {
23508         int result;
23509 #if CYTHON_USE_UNICODE_INTERNALS
23510         Py_hash_t hash1, hash2;
23511         hash1 = ((PyBytesObject*)s1)->ob_shash;
23512         hash2 = ((PyBytesObject*)s2)->ob_shash;
23513         if (hash1 != hash2 && hash1 != -1 && hash2 != -1) {
23514             return (equals == Py_NE);
23515         }
23516 #endif
23517         result = memcmp(ps1, ps2, (size_t)length);
23518         return (equals == Py_EQ) ? (result == 0) : (result != 0);
23519     }
23520 } else if ((s1 == Py_None) & PyBytes_CheckExact(s2)) {
23521     return (equals == Py_NE);
23522 } else if ((s2 == Py_None) & PyBytes_CheckExact(s1)) {
23523     return (equals == Py_NE);
23524 } else {
23525     int result;
23526     PyObject* py_result = PyObject_RichCompare(s1, s2, equals);
23527     if (!py_result)
23528         return -1;
23529     result = __Pyx_PyObject_IsTrue(py_result);
23530     Py_DECREF(py_result);
23531     return result;
23532 }
23533 #endif
23534 }
23535
23536 /* UnicodeEquals */
23537 static CYTHON_INLINE int __Pyx_PyUnicode_Equals(PyObject* s1, PyObject* s2, int equals) {
23538 #if CYTHON_COMPILING_IN_PYPY
23539     return PyObject_RichCompareBool(s1, s2, equals);
23540 #else
23541 #if PY_MAJOR_VERSION < 3
23542     PyObject* owned_ref = NULL;
23543 #endif
23544     int s1_is_unicode, s2_is_unicode;
23545     if (s1 == s2) {
23546         goto return_eq;
23547     }
23548     s1_is_unicode = PyUnicode_CheckExact(s1);
23549     s2_is_unicode = PyUnicode_CheckExact(s2);
23550 #if PY_MAJOR_VERSION < 3
23551     if ((s1_is_unicode & (!s2_is_unicode)) && PyString_CheckExact(s2)) {
23552         owned_ref = PyUnicode_FromObject(s2);
23553         if (unlikely(!owned_ref))
23554             return -1;
23555         s2 = owned_ref;
23556         s2_is_unicode = 1;
23557     } else if ((s2_is_unicode & (!s1_is_unicode)) && PyString_CheckExact(s1)) {
23558         owned_ref = PyUnicode_FromObject(s1);
23559         if (unlikely(!owned_ref))
23560             return -1;
23561         s1 = owned_ref;
23562         s1_is_unicode = 1;
23563     } else if (((!s2_is_unicode) & (!s1_is_unicode))) {
23564         return __Pyx_PyBytes_Equals(s1, s2, equals);
23565     }
23566 #endif
23567     if (s1_is_unicode & s2_is_unicode) {
23568         Py_ssize_t length;
23569         int kind;
23570         void *data1, *data2;
23571         if (unlikely(__Pyx_PyUnicode_READY(s1) < 0) || unlikely(__Pyx_PyUnicode_READY(s2) < 0))
23572             return -1;
23573         length = __Pyx_PyUnicode_GET_LENGTH(s1);
23574         if (length != __Pyx_PyUnicode_GET_LENGTH(s2)) {
23575             goto return_ne;
23576         }
23577 #if CYTHON_USE_UNICODE_INTERNALS
23578         {
23579             Py_hash_t hash1, hash2;
23580 #if CYTHON_PEP393_ENABLED
23581             hash1 = ((PyASCIIObject*)s1)->hash;
23582             hash2 = ((PyASCIIObject*)s2)->hash;
23583 #else
23584             hash1 = ((PyUnicodeObject*)s1)->hash;

```

```

23585         hash2 = ((PyUnicodeObject*)s2)->hash;
23586     #endif
23587     if (hash1 != hash2 && hash1 != -1 && hash2 != -1) {
23588         goto return_ne;
23589     }
23590 }
23591 #endif
23592 kind = __Pyx_PyUnicode_KIND(s1);
23593 if (kind != __Pyx_PyUnicode_KIND(s2)) {
23594     goto return_ne;
23595 }
23596 data1 = __Pyx_PyUnicode_DATA(s1);
23597 data2 = __Pyx_PyUnicode_DATA(s2);
23598 if (__Pyx_PyUnicode_READ(kind, data1, 0) != __Pyx_PyUnicode_READ(kind, data2, 0)) {
23599     goto return_ne;
23600 } else if (length == 1) {
23601     goto return_eq;
23602 } else {
23603     int result = memcmp(data1, data2, (size_t)(length * kind));
23604     #if PY_MAJOR_VERSION < 3
23605     Py_XDECREF(owned_ref);
23606     #endif
23607     return (equals == Py_EQ) ? (result == 0) : (result != 0);
23608 }
23609 } else if ((s1 == Py_None) & s2_is_unicode) {
23610     goto return_ne;
23611 } else if ((s2 == Py_None) & s1_is_unicode) {
23612     goto return_ne;
23613 } else {
23614     int result;
23615     PyObject* py_result = PyObject_RichCompare(s1, s2, equals);
23616     #if PY_MAJOR_VERSION < 3
23617     Py_XDECREF(owned_ref);
23618     #endif
23619     if (!py_result)
23620         return -1;
23621     result = __Pyx_PyObject_IsTrue(py_result);
23622     Py_DECREF(py_result);
23623     return result;
23624 }
23625 return_eq:
23626     #if PY_MAJOR_VERSION < 3
23627     Py_XDECREF(owned_ref);
23628     #endif
23629     return (equals == Py_EQ);
23630 return_ne:
23631     #if PY_MAJOR_VERSION < 3
23632     Py_XDECREF(owned_ref);
23633     #endif
23634     return (equals == Py_NE);
23635 #endif
23636 }
23637
23638 /* CLineInTraceback */
23639 #ifndef CYTHON_CLINE_IN_TRACEBACK
23640 static int __Pyx_CLineForTraceback(CYTHON_NCP_UNUSED PyThreadState *tstate, int c_line) {
23641     PyObject *use_cline;
23642     PyObject *ptype, *pvalue, *ptraceback;
23643     #if CYTHON_COMPILING_IN_CPYTHON
23644     PyObject **cython_runtime_dict;
23645     #endif
23646     if (unlikely(!__pyx_cython_runtime)) {
23647         return c_line;
23648     }
23649     __Pyx_ErrFetchInState(tstate, &ptype, &pvalue, &ptraceback);
23650     #if CYTHON_COMPILING_IN_CPYTHON
23651     cython_runtime_dict = _PyObject_GetDictPtr(__pyx_cython_runtime);
23652     if (likely(cython_runtime_dict)) {
23653         __PYX_PY_DICT_LOOKUP_IF_MODIFIED(
23654             use_cline, *cython_runtime_dict,
23655             __Pyx_PyDict_GetItemStr(*cython_runtime_dict, __pyx_n_s_cline_in_traceback))
23656     } else
23657     #endif
23658     {
23659         PyObject *use_cline_obj = __Pyx_PyObject_GetAttrStr(__pyx_cython_runtime,
23660             __pyx_n_s_cline_in_traceback);
23661         if (use_cline_obj) {
23662             use_cline = PyObject_Not(use_cline_obj) ? Py_False : Py_True;
23663             Py_DECREF(use_cline_obj);
23664         } else {
23665             PyErr_Clear();
23666             use_cline = NULL;
23667         }
23668     }
23669     if (!use_cline) {
23670         c_line = 0;
23671         (void) PyObject_SetAttr(__pyx_cython_runtime, __pyx_n_s_cline_in_traceback, Py_False);

```

```

23671     }
23672     else if (use_cline == Py_False || (use_cline != Py_True && PyObject_Not(use_cline) != 0)) {
23673         c_line = 0;
23674     }
23675     __Pyx_ErrRestoreInState(tstate, ptype, pvalue, ptraceback);
23676     return c_line;
23677 }
23678 #endif
23679
23680 /* CodeObjectCache */
23681 static int __pyx_bisect_code_objects(__Pyx_CodeObjectCacheEntry* entries, int count, int code_line) {
23682     int start = 0, mid = 0, end = count - 1;
23683     if (end >= 0 && code_line > entries[end].code_line) {
23684         return count;
23685     }
23686     while (start < end) {
23687         mid = start + (end - start) / 2;
23688         if (code_line < entries[mid].code_line) {
23689             end = mid;
23690         } else if (code_line > entries[mid].code_line) {
23691             start = mid + 1;
23692         } else {
23693             return mid;
23694         }
23695     }
23696     if (code_line <= entries[mid].code_line) {
23697         return mid;
23698     } else {
23699         return mid + 1;
23700     }
23701 }
23702 static PyCodeObject* __pyx_find_code_object(int code_line) {
23703     PyCodeObject* code_object;
23704     int pos;
23705     if (unlikely(!code_line) || unlikely(!__pyx_code_cache.entries)) {
23706         return NULL;
23707     }
23708     pos = __pyx_bisect_code_objects(__pyx_code_cache.entries, __pyx_code_cache.count, code_line);
23709     if (unlikely(pos >= __pyx_code_cache.count) || unlikely(__pyx_code_cache.entries[pos].code_line !=
code_line)) {
23710         return NULL;
23711     }
23712     code_object = __pyx_code_cache.entries[pos].code_object;
23713     Py_INCREF(code_object);
23714     return code_object;
23715 }
23716 static void __pyx_insert_code_object(int code_line, PyCodeObject* code_object) {
23717     int pos, i;
23718     __Pyx_CodeObjectCacheEntry* entries = __pyx_code_cache.entries;
23719     if (unlikely(!code_line)) {
23720         return;
23721     }
23722     if (unlikely(!entries)) {
23723         entries = (__Pyx_CodeObjectCacheEntry*)PyMem_Malloc(64*sizeof(__Pyx_CodeObjectCacheEntry));
23724         if (likely(entries)) {
23725             __pyx_code_cache.entries = entries;
23726             __pyx_code_cache.max_count = 64;
23727             __pyx_code_cache.count = 1;
23728             entries[0].code_line = code_line;
23729             entries[0].code_object = code_object;
23730             Py_INCREF(code_object);
23731         }
23732         return;
23733     }
23734     pos = __pyx_bisect_code_objects(__pyx_code_cache.entries, __pyx_code_cache.count, code_line);
23735     if ((pos < __pyx_code_cache.count) && unlikely(__pyx_code_cache.entries[pos].code_line ==
code_line)) {
23736         PyCodeObject* tmp = entries[pos].code_object;
23737         entries[pos].code_object = code_object;
23738         Py_DECREF(tmp);
23739         return;
23740     }
23741     if (__pyx_code_cache.count == __pyx_code_cache.max_count) {
23742         int new_max = __pyx_code_cache.max_count + 64;
23743         entries = (__Pyx_CodeObjectCacheEntry*)PyMem_Realloc(
23744             __pyx_code_cache.entries, ((size_t)new_max) * sizeof(__Pyx_CodeObjectCacheEntry));
23745         if (unlikely(!entries)) {
23746             return;
23747         }
23748         __pyx_code_cache.entries = entries;
23749         __pyx_code_cache.max_count = new_max;
23750     }
23751     for (i=__pyx_code_cache.count; i>pos; i--) {
23752         entries[i] = entries[i-1];
23753     }
23754     entries[pos].code_line = code_line;
23755     entries[pos].code_object = code_object;

```

```

23756     __pyx_code_cache.count++;
23757     Py_INCREF(code_object);
23758 }
23759
23760 /* AddTraceback */
23761 #include "compile.h"
23762 #include "frameobject.h"
23763 #include "traceback.h"
23764 static PyCodeObject* __Pyx_CreateCodeObjectForTraceback(
23765     const char *funcname, int c_line,
23766     int py_line, const char *filename) {
23767     PyCodeObject *py_code = NULL;
23768     PyObject *py_funcname = NULL;
23769     #if PY_MAJOR_VERSION < 3
23770     PyObject *py_srcfile = NULL;
23771     py_srcfile = PyString_FromString(filename);
23772     if (!py_srcfile) goto bad;
23773     #endif
23774     if (c_line) {
23775         #if PY_MAJOR_VERSION < 3
23776         py_funcname = PyString_FromFormat( "%s (%s:%d)", funcname, __pyx_cfilenm, c_line);
23777         if (!py_funcname) goto bad;
23778         #else
23779         py_funcname = PyUnicode_FromFormat( "%s (%s:%d)", funcname, __pyx_cfilenm, c_line);
23780         if (!py_funcname) goto bad;
23781         funcname = PyUnicode_AsUTF8(py_funcname);
23782         if (!funcname) goto bad;
23783         #endif
23784     }
23785     else {
23786         #if PY_MAJOR_VERSION < 3
23787         py_funcname = PyString_FromString(funcname);
23788         if (!py_funcname) goto bad;
23789         #endif
23790     }
23791     #if PY_MAJOR_VERSION < 3
23792     py_code = __Pyx_PyCode_New(
23793         0,
23794         0,
23795         0,
23796         0,
23797         0,
23798         __pyx_empty_bytes, /*PyObject *code,*/
23799         __pyx_empty_tuple, /*PyObject *consts,*/
23800         __pyx_empty_tuple, /*PyObject *names,*/
23801         __pyx_empty_tuple, /*PyObject *varnames,*/
23802         __pyx_empty_tuple, /*PyObject *freevars,*/
23803         __pyx_empty_tuple, /*PyObject *cellvars,*/
23804         py_srcfile, /*PyObject *filename,*/
23805         py_funcname, /*PyObject *name,*/
23806         py_line,
23807         __pyx_empty_bytes /*PyObject *notab*/
23808     );
23809     Py_DECREF(py_srcfile);
23810     #else
23811     py_code = PyCode_NewEmpty(filename, funcname, py_line);
23812     #endif
23813     Py_XDECREF(py_funcname); // XDECREF since it's only set on Py3 if cline
23814     return py_code;
23815 bad:
23816     Py_XDECREF(py_funcname);
23817     #if PY_MAJOR_VERSION < 3
23818     Py_XDECREF(py_srcfile);
23819     #endif
23820     return NULL;
23821 }
23822 static void __Pyx_AddTraceback(const char *funcname, int c_line,
23823     int py_line, const char *filename) {
23824     PyCodeObject *py_code = 0;
23825     PyFrameObject *py_frame = 0;
23826     PyThreadState *tstate = __Pyx_PyThreadState_Current;
23827     if (c_line) {
23828         c_line = __Pyx_CLineForTraceback(tstate, c_line);
23829     }
23830     py_code = __pyx_find_code_object(c_line ? -c_line : py_line);
23831     if (!py_code) {
23832         py_code = __Pyx_CreateCodeObjectForTraceback(
23833             funcname, c_line, py_line, filename);
23834         if (!py_code) goto bad;
23835         __pyx_insert_code_object(c_line ? -c_line : py_line, py_code);
23836     }
23837     py_frame = PyFrame_New(
23838         tstate, /*PyThreadState *tstate,*/
23839         py_code, /*PyObject *code,*/
23840         __pyx_d, /*PyObject *globals,*/
23841         0 /*PyObject *locals*/
23842     );

```



```

23843     if (!py_frame) goto bad;
23844     __Pyx_PyFrame_SetLineNumber(py_frame, py_line);
23845     PyTraceBack_Here(py_frame);
23846 bad:
23847     Py_XDECREF(py_code);
23848     Py_XDECREF(py_frame);
23849 }
23850
23851 /* CIntFromPyVerify */
23852 #define __PYX_VERIFY_RETURN_INT(target_type, func_type, func_value)\
23853     __PYX_VERIFY_RETURN_INT(target_type, func_type, func_value, 0)
23854 #define __PYX_VERIFY_RETURN_INT_EXC(target_type, func_type, func_value)\
23855     __PYX_VERIFY_RETURN_INT(target_type, func_type, func_value, 1)
23856 #define __PYX_VERIFY_RETURN_INT(target_type, func_type, func_value, exc)\
23857     {\
23858         func_type value = func_value;\
23859         if (sizeof(target_type) < sizeof(func_type)) {\
23860             if (unlikely(value != (func_type) (target_type) value)) {\
23861                 func_type zero = 0;\
23862                 if (exc && unlikely(value == (func_type)-1 && PyErr_Occurred()))\
23863                     return (target_type) -1;\
23864                 if (is_unsigned && unlikely(value < zero))\
23865                     goto raise_neg_overflow;\
23866                 else\
23867                     goto raise_overflow;\
23868             }\
23869         }\
23870         return (target_type) value;\
23871     }
23872
23873 /* CIntFromPy */
23874 static CYTHON_INLINE int __Pyx_PyInt_As_int(PyObject *x) {
23875     #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
23876     #pragma GCC diagnostic push
23877     #pragma GCC diagnostic ignored "-Wconversion"
23878     #endif
23879     const int neg_one = (int) -1, const_zero = (int) 0;
23880     #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
23881     #pragma GCC diagnostic pop
23882     #endif
23883     const int is_unsigned = neg_one > const_zero;
23884     #if PY_MAJOR_VERSION < 3
23885     if (likely(PyInt_Check(x))) {
23886         if (sizeof(int) < sizeof(long)) {
23887             __PYX_VERIFY_RETURN_INT(int, long, PyInt_AS_LONG(x))
23888         } else {
23889             long val = PyInt_AS_LONG(x);
23890             if (is_unsigned && unlikely(val < 0)) {
23891                 goto raise_neg_overflow;
23892             }
23893             return (int) val;
23894         }
23895     } else
23896     #endif
23897     if (likely(PyLong_Check(x))) {
23898         if (is_unsigned) {
23899             #if CYTHON_USE_PYLONG_INTERNALS
23900             const digit* digits = ((PyLongObject*)x)->ob_digit;
23901             switch (Py_SIZE(x)) {
23902                 case 0: return (int) 0;
23903                 case 1: __PYX_VERIFY_RETURN_INT(int, digit, digits[0])
23904                 case 2:
23905                     if (8 * sizeof(int) > 1 * PyLong_SHIFT) {
23906                         if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
23907                             __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((unsigned long)digits[1])
23908 « PyLong_SHIFT) | (unsigned long)digits[0])))
23909                         } else if (8 * sizeof(int) >= 2 * PyLong_SHIFT) {
23910                             return (int) (((((int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
23911                         }
23912                     }
23913                     break;
23914                 case 3:
23915                     if (8 * sizeof(int) > 2 * PyLong_SHIFT) {
23916                         if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
23917                             __PYX_VERIFY_RETURN_INT(int, unsigned long, ((((((unsigned
23918 long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
23919 long)digits[0])))
23920                         } else if (8 * sizeof(int) >= 3 * PyLong_SHIFT) {
23921                             return (int) ((((((int)digits[2]) « PyLong_SHIFT) | (int)digits[1]) «
23922 PyLong_SHIFT) | (int)digits[0]));
23923                         }
23924                     }
23925                     break;
23926                 case 4:
23927                     if (8 * sizeof(int) > 3 * PyLong_SHIFT) {
23928                         if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
23929                             __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((((((unsigned

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long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]))
23926     } else if (8 * sizeof(int) >= 4 * PyLong_SHIFT) {
23927         return (int) (((((((int)digits[3]) « PyLong_SHIFT) | (int)digits[2]) «
PyLong_SHIFT) | (int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
23928     }
23929     }
23930     break;
23931 }
23932 #endif
23933 #if CYTHON_COMPILING_IN_CPYTHON
23934     if (unlikely(Py_SIZE(x) < 0)) {
23935         goto raise_neg_overflow;
23936     }
23937 #else
23938     {
23939         int result = PyObject_RichCompareBool(x, Py_False, Py_LT);
23940         if (unlikely(result < 0))
23941             return (int) -1;
23942         if (unlikely(result == 1))
23943             goto raise_neg_overflow;
23944     }
23945 #endif
23946     if (sizeof(int) <= sizeof(unsigned long)) {
23947         __PYX_VERIFY_RETURN_INT_EXC(int, unsigned long, PyLong_AsUnsignedLong(x))
23948 #ifdef HAVE_LONG_LONG
23949     } else if (sizeof(int) <= sizeof(unsigned PY_LONG_LONG)) {
23950         __PYX_VERIFY_RETURN_INT_EXC(int, unsigned PY_LONG_LONG, PyLong_AsUnsignedLongLong(x))
23951 #endif
23952     }
23953     } else {
23954 #if CYTHON_USE_PYLONG_INTERNALS
23955         const digit* digits = ((PyLongObject*)x)->ob_digit;
23956         switch (Py_SIZE(x)) {
23957             case 0: return (int) 0;
23958             case -1: __PYX_VERIFY_RETURN_INT(int, sdigit, (sdigit) -(sdigit)digits[0]))
23959             case 1: __PYX_VERIFY_RETURN_INT(int, digit, +digits[0])
23960             case -2:
23961                 if (8 * sizeof(int) - 1 > 1 * PyLong_SHIFT) {
23962                     if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
23963                         __PYX_VERIFY_RETURN_INT(int, long, -(long) (((((unsigned long)digits[1]) «
PyLong_SHIFT) | (unsigned long)digits[0])))
23964                     } else if (8 * sizeof(int) - 1 > 2 * PyLong_SHIFT) {
23965                         return (int) (((int)-1)*((((int)digits[1]) « PyLong_SHIFT) |
(int)digits[0]));
23966                     }
23967                     break;
23968                 }
23969             case 2:
23970                 if (8 * sizeof(int) > 1 * PyLong_SHIFT) {
23971                     if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
23972                         __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((unsigned long)digits[1])
« PyLong_SHIFT) | (unsigned long)digits[0])))
23973                     } else if (8 * sizeof(int) - 1 > 2 * PyLong_SHIFT) {
23974                         return (int) (((((int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
23975                     }
23976                     break;
23977                 }
23978             case -3:
23979                 if (8 * sizeof(int) - 1 > 2 * PyLong_SHIFT) {
23980                     if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
23981                         __PYX_VERIFY_RETURN_INT(int, long, -(long) (((((((unsigned long)digits[2])
« PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
23982                     } else if (8 * sizeof(int) - 1 > 3 * PyLong_SHIFT) {
23983                         return (int) (((int)-1)*((((((int)digits[2]) « PyLong_SHIFT) |
(int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
23984                     }
23985                     break;
23986                 }
23987             case 3:
23988                 if (8 * sizeof(int) > 2 * PyLong_SHIFT) {
23989                     if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
23990                         __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((((unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0])))
23991                     } else if (8 * sizeof(int) - 1 > 3 * PyLong_SHIFT) {
23992                         return (int) (((((((int)digits[2]) « PyLong_SHIFT) | (int)digits[1]) «
PyLong_SHIFT) | (int)digits[0]));
23993                     }
23994                     break;
23995                 }
23996             case -4:
23997                 if (8 * sizeof(int) - 1 > 3 * PyLong_SHIFT) {
23998                     if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
23999                         __PYX_VERIFY_RETURN_INT(int, long, -(long) (((((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))

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24000         } else if (8 * sizeof(int) - 1 > 4 * PyLong_SHIFT) {
24001             return (int) (((int)-1)*(((((((int)digits[3]) < PyLong_SHIFT) |
(int)digits[2]) < PyLong_SHIFT) | (int)digits[1]) < PyLong_SHIFT) | (int)digits[0]]));
24002         }
24003     }
24004     break;
24005     case 4:
24006         if (8 * sizeof(int) > 3 * PyLong_SHIFT) {
24007             if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24008                 __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((((((unsigned
long)digits[3]) < PyLong_SHIFT) | (unsigned long)digits[2]) < PyLong_SHIFT) | (unsigned
long)digits[1]) < PyLong_SHIFT) | (unsigned long)digits[0]]))
24009             } else if (8 * sizeof(int) - 1 > 4 * PyLong_SHIFT) {
24010                 return (int) ((((((((((int)digits[3]) < PyLong_SHIFT) | (int)digits[2]) <
PyLong_SHIFT) | (int)digits[1]) < PyLong_SHIFT) | (int)digits[0]]));
24011             }
24012         }
24013     }
24014     break;
24015 #endif
24016     if (sizeof(int) <= sizeof(long)) {
24017         __PYX_VERIFY_RETURN_INT_EXC(int, long, PyLong_AsLong(x))
24018 #ifdef HAVE_LONG_LONG
24019     } else if (sizeof(int) <= sizeof(PY_LONG_LONG)) {
24020         __PYX_VERIFY_RETURN_INT_EXC(int, PY_LONG_LONG, PyLong_AsLongLong(x))
24021 #endif
24022     }
24023 }
24024 {
24025 #if CYTHON_COMPILING_IN_PYPY && !defined(_PyLong_AsByteArray)
24026     PyErr_SetString(PyExc_RuntimeError,
24027         "_PyLong_AsByteArray() not available in PyPy, cannot convert large
numbers");
24028 #else
24029     int val;
24030     PyObject *v = __Pyx_PyNumber_IntOrLong(x);
24031     #if PY_MAJOR_VERSION < 3
24032         if (likely(v) && !PyLong_Check(v)) {
24033             PyObject *tmp = v;
24034             v = PyNumber_Long(tmp);
24035             Py_DECREF(tmp);
24036         }
24037     #endif
24038     if (likely(v)) {
24039         int one = 1; int is_little = (int)*(unsigned char *)&one;
24040         unsigned char *bytes = (unsigned char *)&val;
24041         int ret = _PyLong_AsByteArray((PyLongObject *)v,
24042             bytes, sizeof(val),
24043             is_little, !is_unsigned);
24044         Py_DECREF(v);
24045         if (likely(!ret))
24046             return val;
24047     }
24048 #endif
24049     return (int) -1;
24050 }
24051 } else {
24052     int val;
24053     PyObject *tmp = __Pyx_PyNumber_IntOrLong(x);
24054     if (!tmp) return (int) -1;
24055     val = __Pyx_PyInt_As_int(tmp);
24056     Py_DECREF(tmp);
24057     return val;
24058 }
24059 raise_overflow:
24060     PyErr_SetString(PyExc_OverflowError,
24061         "value too large to convert to int");
24062     return (int) -1;
24063 raise_neg_overflow:
24064     PyErr_SetString(PyExc_OverflowError,
24065         "can't convert negative value to int");
24066     return (int) -1;
24067 }
24068
24069 /* CIntToPy */
24070 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_int(int value) {
24071 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24072 #pragma GCC diagnostic push
24073 #pragma GCC diagnostic ignored "-Wconversion"
24074 #endif
24075     const int neg_one = (int) -1, const_zero = (int) 0;
24076 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24077 #pragma GCC diagnostic pop
24078 #endif
24079     const int is_unsigned = neg_one > const_zero;
24080     if (is_unsigned) {
24081         if (sizeof(int) < sizeof(long)) {

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```

24082         return PyInt_FromLong((long) value);
24083     } else if (sizeof(int) <= sizeof(unsigned long)) {
24084         return PyLong_FromUnsignedLong((unsigned long) value);
24085 #ifdef HAVE_LONG_LONG
24086     } else if (sizeof(int) <= sizeof(unsigned PY_LONG_LONG)) {
24087         return PyLong_FromUnsignedLongLong((unsigned PY_LONG_LONG) value);
24088 #endif
24089     }
24090 } else {
24091     if (sizeof(int) <= sizeof(long)) {
24092         return PyInt_FromLong((long) value);
24093 #ifdef HAVE_LONG_LONG
24094     } else if (sizeof(int) <= sizeof(PY_LONG_LONG)) {
24095         return PyLong_FromLongLong((PY_LONG_LONG) value);
24096 #endif
24097     }
24098 }
24099 {
24100     int one = 1; int little = (int)*(unsigned char *)&one;
24101     unsigned char *bytes = (unsigned char *)&value;
24102     return _PyLong_FromByteArray(bytes, sizeof(int),
24103                                 little, !is_unsigned);
24104 }
24105 }
24106
24107 /* CIntToPy */
24108 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_long(long value) {
24109 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24110 #pragma GCC diagnostic push
24111 #pragma GCC diagnostic ignored "-Wconversion"
24112 #endif
24113     const long neg_one = (long) -1, const_zero = (long) 0;
24114 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24115 #pragma GCC diagnostic pop
24116 #endif
24117     const int is_unsigned = neg_one > const_zero;
24118     if (is_unsigned) {
24119         if (sizeof(long) < sizeof(unsigned long)) {
24120             return PyInt_FromLong((long) value);
24121         } else if (sizeof(long) <= sizeof(unsigned long)) {
24122             return PyLong_FromUnsignedLong((unsigned long) value);
24123 #ifdef HAVE_LONG_LONG
24124         } else if (sizeof(long) <= sizeof(unsigned PY_LONG_LONG)) {
24125             return PyLong_FromUnsignedLongLong((unsigned PY_LONG_LONG) value);
24126 #endif
24127         }
24128     } else {
24129         if (sizeof(long) <= sizeof(long)) {
24130             return PyInt_FromLong((long) value);
24131 #ifdef HAVE_LONG_LONG
24132         } else if (sizeof(long) <= sizeof(PY_LONG_LONG)) {
24133             return PyLong_FromLongLong((PY_LONG_LONG) value);
24134 #endif
24135         }
24136     }
24137     {
24138         int one = 1; int little = (int)*(unsigned char *)&one;
24139         unsigned char *bytes = (unsigned char *)&value;
24140         return _PyLong_FromByteArray(bytes, sizeof(long),
24141                                     little, !is_unsigned);
24142     }
24143 }
24144
24145 /* CIntFromPy */
24146 static CYTHON_INLINE long __Pyx_PyInt_As_long(PyObject *x) {
24147 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24148 #pragma GCC diagnostic push
24149 #pragma GCC diagnostic ignored "-Wconversion"
24150 #endif
24151     const long neg_one = (long) -1, const_zero = (long) 0;
24152 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24153 #pragma GCC diagnostic pop
24154 #endif
24155     const int is_unsigned = neg_one > const_zero;
24156 #if PY_MAJOR_VERSION < 3
24157     if (likely(PyInt_Check(x))) {
24158         if (sizeof(long) < sizeof(unsigned long)) {
24159             __PYX_VERIFY_RETURN_INT(long, long, PyInt_AS_LONG(x))
24160         } else {
24161             long val = PyInt_AS_LONG(x);
24162             if (is_unsigned && unlikely(val < 0)) {
24163                 goto raise_neg_overflow;
24164             }
24165             return (long) val;
24166         }
24167     } else
24168 #endif

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24169     if (likely(PyLong_Check(x))) {
24170         if (is_unsigned) {
24171             #if CYTHON_USE_PYLONG_INTERNALS
24172                 const digit* digits = ((PyLongObject*)x)->ob_digit;
24173                 switch (Py_SIZE(x)) {
24174                     case 0: return (long) 0;
24175                     case 1: __PYX_VERIFY_RETURN_INT(long, digit, digits[0])
24176                     case 2:
24177                         if (8 * sizeof(long) > 1 * PyLong_SHIFT) {
24178                             if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
24179                                 __PYX_VERIFY_RETURN_INT(long, unsigned long, (((unsigned long)digits[1])
24180 « PyLong_SHIFT) | (unsigned long)digits[0]))
24181                             } else if (8 * sizeof(long) >= 2 * PyLong_SHIFT) {
24182                                 return (long) (((long)digits[1]) « PyLong_SHIFT) | (long)digits[0]);
24183                             }
24184                             break;
24185                         case 3:
24186                             if (8 * sizeof(long) > 2 * PyLong_SHIFT) {
24187                                 if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
24188                                     __PYX_VERIFY_RETURN_INT(long, unsigned long, (((((unsigned
24189 long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
24190 long)digits[0]))
24191                                 } else if (8 * sizeof(long) >= 3 * PyLong_SHIFT) {
24192                                     return (long) (((((long)digits[2]) « PyLong_SHIFT) | (long)digits[1]) «
24193 PyLong_SHIFT) | (long)digits[0]));
24194                                 }
24195                             }
24196                             break;
24197                         case 4:
24198                             if (8 * sizeof(long) > 3 * PyLong_SHIFT) {
24199                                 if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24200                                     __PYX_VERIFY_RETURN_INT(long, unsigned long, (((((((unsigned
24201 long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
24202 long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]))
24203                                 } else if (8 * sizeof(long) >= 4 * PyLong_SHIFT) {
24204                                     return (long) (((((((long)digits[3]) « PyLong_SHIFT) | (long)digits[2])
24205 « PyLong_SHIFT) | (long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24206                                 }
24207                             }
24208                             break;
24209                         }
24210                     }
24211             #endif
24212             #if CYTHON_COMPILING_IN_CPYTHON
24213                 if (unlikely(Py_SIZE(x) < 0)) {
24214                     goto raise_neg_overflow;
24215                 }
24216             #else
24217                 if (unlikely(Py_SIZE(x) < 0)) {
24218                     int result = PyObject_RichCompareBool(x, Py_False, Py_LT);
24219                     if (unlikely(result < 0))
24220                         return (long) -1;
24221                     if (unlikely(result == 1))
24222                         goto raise_neg_overflow;
24223                 }
24224             #endif
24225             if (sizeof(long) <= sizeof(unsigned long)) {
24226                 __PYX_VERIFY_RETURN_INT_EXC(long, unsigned long, PyLong_AsUnsignedLong(x))
24227             } else if (sizeof(long) <= sizeof(unsigned PY_LONG_LONG)) {
24228                 __PYX_VERIFY_RETURN_INT_EXC(long, unsigned PY_LONG_LONG, PyLong_AsUnsignedLongLong(x))
24229             }
24230             #endif
24231             #if CYTHON_USE_PYLONG_INTERNALS
24232                 const digit* digits = ((PyLongObject*)x)->ob_digit;
24233                 switch (Py_SIZE(x)) {
24234                     case 0: return (long) 0;
24235                     case -1: __PYX_VERIFY_RETURN_INT(long, sdigit, (sdigit) -(sdigit)digits[0])
24236                     case 1: __PYX_VERIFY_RETURN_INT(long, digit, +digits[0])
24237                     case -2:
24238                         if (8 * sizeof(long) - 1 > 1 * PyLong_SHIFT) {
24239                             if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
24240                                 __PYX_VERIFY_RETURN_INT(long, long, -(long) (((((unsigned long)digits[1])
24241 « PyLong_SHIFT) | (unsigned long)digits[0]))
24242                             } else if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
24243                                 return (long) (((long)-1)*(((long)digits[1]) « PyLong_SHIFT) |
24244 (long)digits[0]));
24245                             }
24246                             }
24247                             }
24248                             break;
24249                         case 2:
24250                             if (8 * sizeof(long) > 1 * PyLong_SHIFT) {
24251                                 if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
24252                                     __PYX_VERIFY_RETURN_INT(long, unsigned long, (((((unsigned long)digits[1])
24253 « PyLong_SHIFT) | (unsigned long)digits[0]))
24254                                 } else if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {

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```

24246         return (long) (((((long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24247     }
24248 }
24249 break;
24250 case -3:
24251     if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
24252         if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
24253             __PYX_VERIFY_RETURN_INT(long, long, -(long) ((((((unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0])))
24254         } else if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
24255             return (long) (((long)-1)*((((((long)digits[2]) « PyLong_SHIFT) |
(long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24256         }
24257     }
24258 break;
24259 case 3:
24260     if (8 * sizeof(long) > 2 * PyLong_SHIFT) {
24261         if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
24262             __PYX_VERIFY_RETURN_INT(long, unsigned long, ((((((unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0])))
24263         } else if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
24264             return (long) ((((((long)digits[2]) « PyLong_SHIFT) | (long)digits[1]) «
PyLong_SHIFT) | (long)digits[0]));
24265         }
24266     }
24267 break;
24268 case -4:
24269     if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
24270         if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24271             __PYX_VERIFY_RETURN_INT(long, long, -(long) (((((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
24272         } else if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
24273             return (long) ((((((long)-1)*(((((((long)digits[3]) « PyLong_SHIFT) |
(long)digits[2]) « PyLong_SHIFT) | (long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24274         }
24275     }
24276 break;
24277 case 4:
24278     if (8 * sizeof(long) > 3 * PyLong_SHIFT) {
24279         if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24280             __PYX_VERIFY_RETURN_INT(long, unsigned long, (((((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
24281         } else if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
24282             return (long) (((((((((long)digits[3]) « PyLong_SHIFT) | (long)digits[2])
« PyLong_SHIFT) | (long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24283         }
24284     }
24285 break;
24286 }
24287 #endif
24288 if (sizeof(long) <= sizeof(long)) {
24289     __PYX_VERIFY_RETURN_INT_EXC(long, long, PyLong_AsLong(x))
24290 #ifdef HAVE_LONG_LONG
24291 } else if (sizeof(long) <= sizeof(PY_LONG_LONG)) {
24292     __PYX_VERIFY_RETURN_INT_EXC(long, PY_LONG_LONG, PyLong_AsLongLong(x))
24293 #endif
24294 }
24295 }
24296 {
24297 #if CYTHON_COMPILING_IN_PYPY && !defined(PyLong_AsByteArray)
24298     PyErr_SetString(PyExc_RuntimeError,
24299         "_PyLong_AsByteArray() not available in PyPy, cannot convert large
numbers");
24300 #else
24301     long val;
24302     PyObject *v = __Pyx_PyNumber_IntOrLong(x);
24303     #if PY_MAJOR_VERSION < 3
24304         if (likely(v) && !PyLong_Check(v)) {
24305             PyObject *tmp = v;
24306             v = PyNumber_Long(tmp);
24307             Py_DECREF(tmp);
24308         }
24309     #endif
24310     if (likely(v)) {
24311         int one = 1; int is_little = (int)*(unsigned char *)&one;
24312         unsigned char *bytes = (unsigned char *)&val;
24313         int ret = _PyLong_AsByteArray((PyLongObject *)v,
24314             bytes, sizeof(val),
24315             is_little, !is_unsigned);
24316         Py_DECREF(v);
24317         if (likely(!ret))
24318             return val;
24319     }

```

```

24320 #endif
24321         return (long) -1;
24322     }
24323 } else {
24324     long val;
24325     PyObject *tmp = __Pyx_PyNumber_IntOrLong(x);
24326     if (!tmp) return (long) -1;
24327     val = __Pyx_PyInt_As_long(tmp);
24328     Py_DECREF(tmp);
24329     return val;
24330 }
24331 raise_overflow:
24332     PyErr_SetString(PyExc_OverflowError,
24333         "value too large to convert to long");
24334     return (long) -1;
24335 raise_neg_overflow:
24336     PyErr_SetString(PyExc_OverflowError,
24337         "can't convert negative value to long");
24338     return (long) -1;
24339 }
24340
24341 /* FastTypeChecks */
24342 #if CYTHON_COMPILING_IN_CPYTHON
24343 static int __Pyx_InBases(PyTypeObject *a, PyTypeObject *b) {
24344     while (a) {
24345         a = a->tp_base;
24346         if (a == b)
24347             return 1;
24348     }
24349     return b == &PyBaseObject_Type;
24350 }
24351 static CYTHON_INLINE int __Pyx_IsSubtype(PyTypeObject *a, PyTypeObject *b) {
24352     PyObject *mro;
24353     if (a == b) return 1;
24354     mro = a->tp_mro;
24355     if (likely(mro)) {
24356         Py_ssize_t i, n;
24357         n = PyTuple_GET_SIZE(mro);
24358         for (i = 0; i < n; i++) {
24359             if (PyTuple_GET_ITEM(mro, i) == (PyObject *)b)
24360                 return 1;
24361         }
24362         return 0;
24363     }
24364     return __Pyx_InBases(a, b);
24365 }
24366 #if PY_MAJOR_VERSION == 2
24367 static int __Pyx_inner_PyErr_GivenExceptionMatches2(PyObject *err, PyObject* exc_type1, PyObject*
exc_type2) {
24368     PyObject *exception, *value, *tb;
24369     int res;
24370     __Pyx_PyThreadState_declare
24371     __Pyx_PyThreadState_assign
24372     __Pyx_ErrFetch(&exception, &value, &tb);
24373     res = exc_type1 ? PyObject_IsSubclass(err, exc_type1) : 0;
24374     if (unlikely(res == -1)) {
24375         PyErr_WriteUnraisable(err);
24376         res = 0;
24377     }
24378     if (!res) {
24379         res = PyObject_IsSubclass(err, exc_type2);
24380         if (unlikely(res == -1)) {
24381             PyErr_WriteUnraisable(err);
24382             res = 0;
24383         }
24384     }
24385     __Pyx_ErrRestore(exception, value, tb);
24386     return res;
24387 }
24388 #else
24389 static CYTHON_INLINE int __Pyx_inner_PyErr_GivenExceptionMatches2(PyObject *err, PyObject* exc_type1,
PyObject *exc_type2) {
24390     int res = exc_type1 ? __Pyx_IsSubtype((PyTypeObject*)err, (PyTypeObject*)exc_type1) : 0;
24391     if (!res) {
24392         res = __Pyx_IsSubtype((PyTypeObject*)err, (PyTypeObject*)exc_type2);
24393     }
24394     return res;
24395 }
24396 #endif
24397 static int __Pyx_PyErr_GivenExceptionMatchesTuple(PyObject *exc_type, PyObject *tuple) {
24398     Py_ssize_t i, n;
24399     assert(PyExceptionClass_Check(exc_type));
24400     n = PyTuple_GET_SIZE(tuple);
24401     #if PY_MAJOR_VERSION >= 3
24402     for (i=0; i<n; i++) {
24403         if (exc_type == PyTuple_GET_ITEM(tuple, i)) return 1;
24404     }

```

```

24405 #endif
24406     for (i=0; i<n; i++) {
24407         PyObject *t = PyTuple_GET_ITEM(tuple, i);
24408         #if PY_MAJOR_VERSION < 3
24409         if (likely(exc_type == t)) return 1;
24410         #endif
24411         if (likely(PyExceptionClass_Check(t))) {
24412             if (__Pyx_inner_PyErr_GivenExceptionMatches2(exc_type, NULL, t)) return 1;
24413         } else {
24414         }
24415     }
24416     return 0;
24417 }
24418 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches(PyObject *err, PyObject* exc_type) {
24419     if (likely(err == exc_type)) return 1;
24420     if (likely(PyExceptionClass_Check(err))) {
24421         if (likely(PyExceptionClass_Check(exc_type))) {
24422             return __Pyx_inner_PyErr_GivenExceptionMatches2(err, NULL, exc_type);
24423         } else if (likely(PyTuple_Check(exc_type))) {
24424             return __Pyx_PyErr_GivenExceptionMatchesTuple(err, exc_type);
24425         } else {
24426         }
24427     }
24428     return PyErr_GivenExceptionMatches(err, exc_type);
24429 }
24430 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches2(PyObject *err, PyObject *exc_type1,
24431     PyObject *exc_type2) {
24432     assert(PyExceptionClass_Check(exc_type1));
24433     assert(PyExceptionClass_Check(exc_type2));
24434     if (likely(err == exc_type1 || err == exc_type2)) return 1;
24435     if (likely(PyExceptionClass_Check(err))) {
24436         return __Pyx_inner_PyErr_GivenExceptionMatches2(err, exc_type1, exc_type2);
24437     }
24438     return (PyErr_GivenExceptionMatches(err, exc_type1) || PyErr_GivenExceptionMatches(err,
24439         exc_type2));
24440 }
24441 #endif
24442 /* FetchCommonType */
24443 static PyTypeObject* __Pyx_FetchCommonType(PyTypeObject* type) {
24444     PyObject* fake_module;
24445     PyTypeObject* cached_type = NULL;
24446     fake_module = PyImport_AddModule((char*) "_cython_" CYTHON_ABI);
24447     if (!fake_module) return NULL;
24448     Py_INCREF(fake_module);
24449     cached_type = (PyTypeObject*) PyObject_GetAttrString(fake_module, type->tp_name);
24450     if (cached_type) {
24451         if (!PyType_Check((PyObject*)cached_type)) {
24452             PyErr_Format(PyExc_TypeError,
24453                 "Shared Cython type %.200s is not a type object",
24454                 type->tp_name);
24455             goto bad;
24456         }
24457         if (cached_type->tp_basicsize != type->tp_basicsize) {
24458             PyErr_Format(PyExc_TypeError,
24459                 "Shared Cython type %.200s has the wrong size, try recompiling",
24460                 type->tp_name);
24461             goto bad;
24462         }
24463     } else {
24464         if (!PyErr_ExceptionMatches(PyExc_AttributeError)) goto bad;
24465         PyErr_Clear();
24466         if (PyType_Ready(type) < 0) goto bad;
24467         if (PyObject_SetAttrString(fake_module, type->tp_name, (PyObject*) type) < 0)
24468             goto bad;
24469         Py_INCREF(type);
24470         cached_type = type;
24471     }
24472 done:
24473     Py_DECREF(fake_module);
24474     return cached_type;
24475 bad:
24476     Py_XDECREF(cached_type);
24477     cached_type = NULL;
24478     goto done;
24479 }
24480 /* PyObjectGetMethod */
24481 static int __Pyx_PyObject_GetMethod(PyObject *obj, PyObject *name, PyObject **method) {
24482     PyObject *attr;
24483     #if CYTHON_UNPACK_METHODS && CYTHON_COMPILING_IN_CPYTHON && CYTHON_USE_PYTYPE_LOOKUP
24484     PyTypeObject *tp = Py_TYPE(obj);
24485     PyObject *descr;
24486     descrgetfunc f = NULL;
24487     PyObject **dictptr, *dict;
24488     int meth_found = 0;
24489     assert (*method == NULL);

```



```

24490     if (unlikely(tp->tp_getattro != PyObject_GenericGetAttr)) {
24491         attr = __Pyx_PyObject_GetAttrStr(obj, name);
24492         goto try_unpack;
24493     }
24494     if (unlikely(tp->tp_dict == NULL) && unlikely(PyType_Ready(tp) < 0)) {
24495         return 0;
24496     }
24497     descr = _PyType_Lookup(tp, name);
24498     if (likely(descr != NULL)) {
24499         Py_INCREF(descr);
24500 #if PY_MAJOR_VERSION >= 3
24501         #ifdef __Pyx_CyFunction_USED
24502         if (likely(PyFunction_Check(descr) || (Py_TYPE(descr) == &PyMethodDescr_Type) ||
24503             __Pyx_CyFunction_Check(descr)))
24504             #else
24505             if (likely(PyFunction_Check(descr) || (Py_TYPE(descr) == &PyMethodDescr_Type)))
24506             #endif
24507         #else
24508         #ifdef __Pyx_CyFunction_USED
24509         if (likely(PyFunction_Check(descr) || __Pyx_CyFunction_Check(descr)))
24510         #else
24511         if (likely(PyFunction_Check(descr)))
24512         #endif
24513         {
24514             meth_found = 1;
24515         } else {
24516             f = Py_TYPE(descr)->tp_descr_get;
24517             if (f != NULL && PyDescr_IsData(descr)) {
24518                 attr = f(descr, obj, (PyObject *)Py_TYPE(obj));
24519                 Py_DECREF(descr);
24520                 goto try_unpack;
24521             }
24522         }
24523     }
24524     dictptr = _PyObject_GetDictPtr(obj);
24525     if (dictptr != NULL && (dict = *dictptr) != NULL) {
24526         Py_INCREF(dict);
24527         attr = __Pyx_PyDict_GetItemStr(dict, name);
24528         if (attr != NULL) {
24529             Py_INCREF(attr);
24530             Py_DECREF(dict);
24531             Py_XDECREF(descr);
24532             goto try_unpack;
24533         }
24534         Py_DECREF(dict);
24535     }
24536     if (meth_found) {
24537         *method = descr;
24538         return 1;
24539     }
24540     if (f != NULL) {
24541         attr = f(descr, obj, (PyObject *)Py_TYPE(obj));
24542         Py_DECREF(descr);
24543         goto try_unpack;
24544     }
24545     if (descr != NULL) {
24546         *method = descr;
24547         return 0;
24548     }
24549     PyErr_Format(PyExc_AttributeError,
24550 #if PY_MAJOR_VERSION >= 3
24551         "'%.50s' object has no attribute '%U'",
24552         tp->tp_name, name);
24553 #else
24554         "'%.50s' object has no attribute '%.400s'",
24555         tp->tp_name, PyString_AS_STRING(name));
24556 #endif
24557     return 0;
24558 #else
24559     attr = __Pyx_PyObject_GetAttrStr(obj, name);
24560     goto try_unpack;
24561 #endif
24562 try_unpack:
24563 #if CYTHON_UNPACK_METHODS
24564     if (likely(attr) && PyMethod_Check(attr) && likely(PyMethod_GET_SELF(attr) == obj)) {
24565         PyObject *function = PyMethod_GET_FUNCTION(attr);
24566         Py_INCREF(function);
24567         Py_DECREF(attr);
24568         *method = function;
24569         return 1;
24570     }
24571 #endif
24572     *method = attr;
24573     return 0;
24574 }
24575

```

```

24576 /* PyObjectCallMethod1 */
24577 static PyObject* __Pyx_PyObject_CallMethod1(PyObject* method, PyObject* arg) {
24578     PyObject *result = __Pyx_PyObject_CallOneArg(method, arg);
24579     Py_DECREF(method);
24580     return result;
24581 }
24582 static PyObject* __Pyx_PyObject_CallMethod1(PyObject* obj, PyObject* method_name, PyObject* arg) {
24583     PyObject *method = NULL, *result;
24584     int is_method = __Pyx_PyObject_GetMethod(obj, method_name, &method);
24585     if (likely(is_method)) {
24586         result = __Pyx_PyObject_Call2Args(method, obj, arg);
24587         Py_DECREF(method);
24588         return result;
24589     }
24590     if (unlikely(!method)) return NULL;
24591     return __Pyx_PyObject_CallMethod1(method, arg);
24592 }
24593
24594 /* CoroutineBase */
24595 #include <structmember.h>
24596 #include <frameobject.h>
24597 #define __Pyx_Coroutine_Undelegate(gen) Py_CLEAR((gen)->yieldfrom)
24598 static int __Pyx_PyGen_FetchStopIterationValue(CYTHON_UNUSED PyThreadState *__pyx_tstate, PyObject
24599     **pvalue) {
24600     PyObject *et, *ev, *tb;
24601     PyObject *value = NULL;
24602     __Pyx_ErrFetch(&et, &ev, &tb);
24603     if (!et) {
24604         Py_XDECREF(tb);
24605         Py_XDECREF(ev);
24606         Py_INCREF(Py_None);
24607         *pvalue = Py_None;
24608         return 0;
24609     }
24610     if (likely(et == PyExc_StopIteration)) {
24611         if (!ev) {
24612             Py_INCREF(Py_None);
24613             value = Py_None;
24614         }
24615         #if PY_VERSION_HEX >= 0x030300A0
24616         else if (Py_TYPE(ev) == (PyTypeObject*)PyExc_StopIteration) {
24617             value = ((PyStopIterationObject *)ev)->value;
24618             Py_INCREF(value);
24619             Py_DECREF(ev);
24620         }
24621         #endif
24622         else if (unlikely(PyTuple_Check(ev))) {
24623             if (PyTuple_GET_SIZE(ev) >= 1) {
24624                 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
24625                 value = PyTuple_GET_ITEM(ev, 0);
24626                 Py_INCREF(value);
24627             }
24628             #endif
24629             } else {
24630                 Py_INCREF(Py_None);
24631                 value = Py_None;
24632             }
24633         Py_DECREF(ev);
24634     }
24635     else if (!__Pyx_TypeCheck(ev, (PyTypeObject*)PyExc_StopIteration)) {
24636         value = ev;
24637     }
24638     if (likely(value)) {
24639         Py_XDECREF(tb);
24640         Py_DECREF(et);
24641         *pvalue = value;
24642         return 0;
24643     }
24644     } else if (!__Pyx_PyErr_GivenExceptionMatches(et, PyExc_StopIteration)) {
24645         __Pyx_ErrRestore(et, ev, tb);
24646         return -1;
24647     }
24648     PyErr_NormalizeException(&et, &ev, &tb);
24649     if (unlikely(!PyObject_TypeCheck(ev, (PyTypeObject*)PyExc_StopIteration))) {
24650         __Pyx_ErrRestore(et, ev, tb);
24651         return -1;
24652     }
24653     Py_XDECREF(tb);
24654     Py_DECREF(et);
24655     #if PY_VERSION_HEX >= 0x030300A0
24656     value = ((PyStopIterationObject *)ev)->value;
24657     Py_INCREF(value);
24658     Py_DECREF(ev);
24659     #else
24660     {
24661         PyObject* args = __Pyx_PyObject_GetAttrStr(ev, __pyx_n_s_args);

```

```

24662         Py_DECREF(ev);
24663         if (likely(args)) {
24664             value = PySequence_GetItem(args, 0);
24665             Py_DECREF(args);
24666         }
24667         if (unlikely(!value)) {
24668             __Pyx_ErrRestore(NULL, NULL, NULL);
24669             Py_INCREF(Py_None);
24670             value = Py_None;
24671         }
24672     }
24673 #endif
24674     *pvalue = value;
24675     return 0;
24676 }
24677 static CYTHON_INLINE
24678 void __Pyx_Coroutine_ExceptionClear(__Pyx_ExcInfoStruct *exc_state) {
24679     PyObject *t, *v, *tb;
24680     t = exc_state->exc_type;
24681     v = exc_state->exc_value;
24682     tb = exc_state->exc_traceback;
24683     exc_state->exc_type = NULL;
24684     exc_state->exc_value = NULL;
24685     exc_state->exc_traceback = NULL;
24686     Py_XDECREF(t);
24687     Py_XDECREF(v);
24688     Py_XDECREF(tb);
24689 }
24690 #define __Pyx_Coroutine_AlreadyRunningError(gen) (__Pyx__Coroutine_AlreadyRunningError(gen),
(PyObject*)NULL)
24691 static void __Pyx__Coroutine_AlreadyRunningError(CYTHON_UNUSED __pyx_CoroutineObject *gen) {
24692     const char *msg;
24693     if ((0)) {
24694         #ifdef __Pyx_Coroutine_USED
24695     } else if (__Pyx_Coroutine_Check((PyObject*)gen)) {
24696         msg = "coroutine already executing";
24697     #endif
24698     #ifdef __Pyx_AsyncGen_USED
24699     } else if (__Pyx_AsyncGen_CheckExact((PyObject*)gen)) {
24700         msg = "async generator already executing";
24701     #endif
24702     } else {
24703         msg = "generator already executing";
24704     }
24705     PyErr_SetString(PyExc_ValueError, msg);
24706 }
24707 #define __Pyx_Coroutine_NotStartedError(gen) (__Pyx__Coroutine_NotStartedError(gen), (PyObject*)NULL)
24708 static void __Pyx__Coroutine_NotStartedError(CYTHON_UNUSED PyObject *gen) {
24709     const char *msg;
24710     if ((0)) {
24711         #ifdef __Pyx_Coroutine_USED
24712     } else if (__Pyx_Coroutine_Check(gen)) {
24713         msg = "can't send non-None value to a just-started coroutine";
24714     #endif
24715     #ifdef __Pyx_AsyncGen_USED
24716     } else if (__Pyx_AsyncGen_CheckExact(gen)) {
24717         msg = "can't send non-None value to a just-started async generator";
24718     #endif
24719     } else {
24720         msg = "can't send non-None value to a just-started generator";
24721     }
24722     PyErr_SetString(PyExc_TypeError, msg);
24723 }
24724 #define __Pyx_Coroutine_AlreadyTerminatedError(gen, value, closing)
(__Pyx__Coroutine_AlreadyTerminatedError(gen, value, closing), (PyObject*)NULL)
24725 static void __Pyx__Coroutine_AlreadyTerminatedError(CYTHON_UNUSED PyObject *gen, PyObject *value,
CYTHON_UNUSED int closing) {
24726     #ifdef __Pyx_Coroutine_USED
24727     if (!closing && __Pyx_Coroutine_Check(gen)) {
24728         PyErr_SetString(PyExc_RuntimeError, "cannot reuse already awaited coroutine");
24729     } else
24730     #endif
24731     if (value) {
24732         #ifdef __Pyx_AsyncGen_USED
24733         if (__Pyx_AsyncGen_CheckExact(gen))
24734             PyErr_SetNone(__Pyx_PyExc_StopAsyncIteration);
24735         else
24736         #endif
24737         PyErr_SetNone(PyExc_StopIteration);
24738     }
24739 }
24740 static
24741 PyObject *__Pyx_Coroutine_SendEx(__pyx_CoroutineObject *self, PyObject *value, int closing) {
24742     __Pyx_PyThreadState_declare
24743     PyThreadState *tstate;
24744     __Pyx_ExcInfoStruct *exc_state;
24745     PyObject *retval;

```

```

24746     assert(!self->is_running);
24747     if (unlikely(self->resume_label == 0)) {
24748         if (unlikely(value && value != Py_None)) {
24749             return __Pyx_Coroutine_NotStartedError((PyObject*)self);
24750         }
24751     }
24752     if (unlikely(self->resume_label == -1)) {
24753         return __Pyx_Coroutine_AlreadyTerminatedError((PyObject*)self, value, closing);
24754     }
24755     #if CYTHON_FAST_THREAD_STATE
24756     __Pyx_PyThreadState_assign
24757     tstate = __pyx_tstate;
24758     #else
24759     tstate = __Pyx_PyThreadState_Current;
24760     #endif
24761     exc_state = &self->gi_exc_state;
24762     if (exc_state->exc_type) {
24763         #if CYTHON_COMPILING_IN_PYPY || CYTHON_COMPILING_IN_PYSTON
24764         #else
24765         if (exc_state->exc_traceback) {
24766             PyTracebackObject *tb = (PyTracebackObject *) exc_state->exc_traceback;
24767             PyFrameObject *f = tb->tb_frame;
24768             assert(f->f_back == NULL);
24769             #if PY_VERSION_HEX >= 0x030B00A1
24770             f->f_back = PyThreadState_GetFrame(tstate);
24771             #else
24772             Py_XINCREF(tstate->frame);
24773             f->f_back = tstate->frame;
24774             #endif
24775         }
24776         #endif
24777     }
24778     #if CYTHON_USE_EXC_INFO_STACK
24779     exc_state->previous_item = tstate->exc_info;
24780     tstate->exc_info = exc_state;
24781     #else
24782     if (exc_state->exc_type) {
24783         __Pyx_ExceptionSwap(&exc_state->exc_type, &exc_state->exc_value, &exc_state->exc_traceback);
24784     } else {
24785         __Pyx_Coroutine_ExceptionClear(exc_state);
24786         __Pyx_ExceptionSave(&exc_state->exc_type, &exc_state->exc_value, &exc_state->exc_traceback);
24787     }
24788     #endif
24789     self->is_running = 1;
24790     retval = self->body((PyObject *) self, tstate, value);
24791     self->is_running = 0;
24792     #if CYTHON_USE_EXC_INFO_STACK
24793     exc_state = &self->gi_exc_state;
24794     tstate->exc_info = exc_state->previous_item;
24795     exc_state->previous_item = NULL;
24796     __Pyx_Coroutine_ResetFrameBackpointer(exc_state);
24797     #endif
24798     return retval;
24799 }
24800 static CYTHON_INLINE void __Pyx_Coroutine_ResetFrameBackpointer(__Pyx_ExcInfoStruct *exc_state) {
24801     PyObject *exc_tb = exc_state->exc_traceback;
24802     if (likely(exc_tb)) {
24803         #if CYTHON_COMPILING_IN_PYPY || CYTHON_COMPILING_IN_PYSTON
24804         #else
24805         PyTracebackObject *tb = (PyTracebackObject *) exc_tb;
24806         PyFrameObject *f = tb->tb_frame;
24807         Py_CLEAR(f->f_back);
24808         #endif
24809     }
24810 }
24811 static CYTHON_INLINE
24812 PyObject * __Pyx_Coroutine_MethodReturn(CYTHON_UNUSED PyObject* gen, PyObject *retval) {
24813     if (unlikely(!retval)) {
24814         __Pyx_PyThreadState_declare
24815         __Pyx_PyThreadState_assign
24816         if (!__Pyx_PyErr_Occurred()) {
24817             PyObject *exc = PyExc_StopIteration;
24818             #ifdef __Pyx_AsyncGen_USED
24819             if (__Pyx_AsyncGen_CheckExact(gen))
24820                 exc = __Pyx_PyExc_StopAsyncIteration;
24821             #endif
24822             __Pyx_PyErr_SetNone(exc);
24823         }
24824     }
24825     return retval;
24826 }
24827 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03030000 && (defined(__linux__) ||
PY_VERSION_HEX >= 0x030600B3)
24828 static CYTHON_INLINE
24829 PyObject * __Pyx_PyGen_Send(PyGenObject *gen, PyObject *arg) {
24830     #if PY_VERSION_HEX <= 0x030A00A1
24831     return _PyGen_Send(gen, arg);

```

```

24832 #else
24833     PyObject *result;
24834     if (PyIter_Send((PyObject*)gen, arg ? arg : Py_None, &result) == PYGEN_RETURN) {
24835         if (PyAsyncGen_CheckExact(gen)) {
24836             assert(result == Py_None);
24837             PyErr_SetNone(PyExc_StopAsyncIteration);
24838         }
24839         else if (result == Py_None) {
24840             PyErr_SetNone(PyExc_StopIteration);
24841         }
24842         else {
24843             _PyGen_SetStopIterationValue(result);
24844         }
24845         Py_CLEAR(result);
24846     }
24847     return result;
24848 #endif
24849 }
24850 #endif
24851 static CYTHON_INLINE
24852 PyObject *__Pyx_Coroutine_FinishDelegation(__pyx_CoroutineObject *gen) {
24853     PyObject *ret;
24854     PyObject *val = NULL;
24855     __Pyx_Coroutine_Undelegate(gen);
24856     __Pyx_PyGen__FetchStopIterationValue(__Pyx_PyThreadState_Current, &val);
24857     ret = __Pyx_Coroutine_SendEx(gen, val, 0);
24858     Py_XDECREF(val);
24859     return ret;
24860 }
24861 static PyObject *__Pyx_Coroutine_Send(PyObject *self, PyObject *value) {
24862     PyObject *retval;
24863     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject*) self;
24864     PyObject *yf = gen->yieldfrom;
24865     if (unlikely(gen->is_running))
24866         return __Pyx_Coroutine_AlreadyRunningError(gen);
24867     if (yf) {
24868         PyObject *ret;
24869         gen->is_running = 1;
24870         #ifdef __Pyx_Generator_USED
24871         if (__Pyx_Generator_CheckExact(yf)) {
24872             ret = __Pyx_Coroutine_Send(yf, value);
24873         } else
24874         #endif
24875         #ifdef __Pyx_Coroutine_USED
24876         if (__Pyx_Coroutine_Check(yf)) {
24877             ret = __Pyx_Coroutine_Send(yf, value);
24878         } else
24879         #endif
24880         #ifdef __Pyx_AsyncGen_USED
24881         if (__pyx_PyAsyncGenASend_CheckExact(yf)) {
24882             ret = __Pyx_async_gen_asend_send(yf, value);
24883         } else
24884         #endif
24885         #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03030000 && (defined(__linux__) ||
PY_VERSION_HEX >= 0x030600B3)
24886         if (PyGen_CheckExact(yf)) {
24887             ret = __Pyx_PyGen_Send((PyGenObject*)yf, value == Py_None ? NULL : value);
24888         } else
24889         #endif
24890         #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03050000 && defined(PyCoro_CheckExact)
&& (defined(__linux__) || PY_VERSION_HEX >= 0x030600B3)
24891         if (PyCoro_CheckExact(yf)) {
24892             ret = __Pyx_PyGen_Send((PyGenObject*)yf, value == Py_None ? NULL : value);
24893         } else
24894         #endif
24895         {
24896             if (value == Py_None)
24897                 ret = Py_TYPE(yf)->tp_iternext(yf);
24898             else
24899                 ret = __Pyx_PyObject_CallMethod1(yf, __pyx_n_s_send, value);
24900         }
24901         gen->is_running = 0;
24902         if (likely(ret)) {
24903             return ret;
24904         }
24905         retval = __Pyx_Coroutine_FinishDelegation(gen);
24906     } else {
24907         retval = __Pyx_Coroutine_SendEx(gen, value, 0);
24908     }
24909     return __Pyx_Coroutine_MethodReturn(self, retval);
24910 }
24911 static int __Pyx_Coroutine_CloseIter(__pyx_CoroutineObject *gen, PyObject *yf) {
24912     PyObject *retval = NULL;
24913     int err = 0;
24914     #ifdef __Pyx_Generator_USED
24915     if (__Pyx_Generator_CheckExact(yf)) {
24916         retval = __Pyx_Coroutine_Close(yf);

```

```

24917         if (!retval)
24918             return -1;
24919     } else
24920 #endif
24921 #ifdef __Pyx_Coroutine_USED
24922     if (__Pyx_Coroutine_Check(yf)) {
24923         retval = __Pyx_Coroutine_Close(yf);
24924         if (!retval)
24925             return -1;
24926     } else
24927     if (__Pyx_CoroutineAwait_CheckExact(yf)) {
24928         retval = __Pyx_CoroutineAwait_Close((__pyx_CoroutineAwaitObject*)yf, NULL);
24929         if (!retval)
24930             return -1;
24931     } else
24932 #endif
24933 #ifdef __Pyx_AsyncGen_USED
24934     if (__pyx_PyAsyncGenASend_CheckExact(yf)) {
24935         retval = __Pyx_async_gen_asend_close(yf, NULL);
24936     } else
24937     if (__pyx_PyAsyncGenAThrow_CheckExact(yf)) {
24938         retval = __Pyx_async_gen_athrow_close(yf, NULL);
24939     } else
24940 #endif
24941     {
24942         PyObject *meth;
24943         gen->is_running = 1;
24944         meth = __Pyx_PyObject_GetAttrStr(yf, __pyx_n_s_close);
24945         if (unlikely(!meth)) {
24946             if (!PyErr_ExceptionMatches(PyExc_AttributeError)) {
24947                 PyErr_WriteUnraisable(yf);
24948             }
24949             PyErr_Clear();
24950         } else {
24951             retval = PyObject_CallFunction(meth, NULL);
24952             Py_DECREF(meth);
24953             if (!retval)
24954                 err = -1;
24955         }
24956         gen->is_running = 0;
24957     }
24958     Py_XDECREF(retval);
24959     return err;
24960 }
24961 static PyObject *__Pyx_Generator_Next(PyObject *self) {
24962     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject*) self;
24963     PyObject *yf = gen->yieldfrom;
24964     if (unlikely(gen->is_running))
24965         return __Pyx_Coroutine_AlreadyRunningError(gen);
24966     if (yf) {
24967         PyObject *ret;
24968         gen->is_running = 1;
24969         #ifdef __Pyx_Generator_USED
24970         if (__Pyx_Generator_CheckExact(yf)) {
24971             ret = __Pyx_Generator_Next(yf);
24972         } else
24973         #endif
24974         #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03030000 && (defined(__linux__) ||
PY_VERSION_HEX >= 0x030600B3)
24975         if (PyGen_CheckExact(yf)) {
24976             ret = __Pyx_PyGen_Send((PyGenObject*)yf, NULL);
24977         } else
24978         #endif
24979         #ifdef __Pyx_Coroutine_USED
24980         if (__Pyx_Coroutine_Check(yf)) {
24981             ret = __Pyx_Coroutine_Send(yf, Py_None);
24982         } else
24983         #endif
24984         ret = Py_TYPE(yf)->tp_iternext(yf);
24985         gen->is_running = 0;
24986         if (likely(ret)) {
24987             return ret;
24988         }
24989         return __Pyx_Coroutine_FinishDelegation(gen);
24990     }
24991     return __Pyx_Coroutine_SendEx(gen, Py_None, 0);
24992 }
24993 static PyObject *__Pyx_Coroutine_Close_Method(PyObject *self, CYTHON_UNUSED PyObject *arg) {
24994     return __Pyx_Coroutine_Close(self);
24995 }
24996 static PyObject *__Pyx_Coroutine_Close(PyObject *self) {
24997     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
24998     PyObject *retval, *raised_exception;
24999     PyObject *yf = gen->yieldfrom;
25000     int err = 0;
25001     if (unlikely(gen->is_running))
25002         return __Pyx_Coroutine_AlreadyRunningError(gen);

```

```

25003     if (yf) {
25004         Py_INCREF(yf);
25005         err = __Pyx_Coroutine_CloseIter(gen, yf);
25006         __Pyx_Coroutine_Undelegate(gen);
25007         Py_DECREF(yf);
25008     }
25009     if (err == 0)
25010         PyErr_SetNone(PyExc_GeneratorExit);
25011     retval = __Pyx_Coroutine_SendEx(gen, NULL, 1);
25012     if (unlikely(retval)) {
25013         const char *msg;
25014         Py_DECREF(retval);
25015         if ((0)) {
25016             #ifdef __Pyx_Coroutine_USED
25017             } else if (__Pyx_Coroutine_Check(self)) {
25018                 msg = "coroutine ignored GeneratorExit";
25019             #endif
25020             #ifdef __Pyx_AsyncGen_USED
25021             } else if (__Pyx_AsyncGen_CheckExact(self)) {
25022             #if PY_VERSION_HEX < 0x03060000
25023                 msg = "async generator ignored GeneratorExit - might require Python 3.6+ finalisation (PEP
525)";
25024             #else
25025                 msg = "async generator ignored GeneratorExit";
25026             #endif
25027             #endif
25028             } else {
25029                 msg = "generator ignored GeneratorExit";
25030             }
25031             PyErr_SetString(PyExc_RuntimeError, msg);
25032             return NULL;
25033         }
25034         raised_exception = PyErr_Occurred();
25035         if (likely(!raised_exception || __Pyx_PyErr_GivenExceptionMatches2(raised_exception,
PyExc_GeneratorExit, PyExc_StopIteration))) {
25036             if (raised_exception) PyErr_Clear();
25037             Py_INCREF(Py_None);
25038             return Py_None;
25039         }
25040         return NULL;
25041     }
25042     static PyObject *__Pyx__Coroutine_Throw(PyObject *self, PyObject *typ, PyObject *val, PyObject *tb,
25043   PyObject *args, int close_on_genexit) {
25044         __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25045         PyObject *yf = gen->yieldfrom;
25046         if (unlikely(gen->is_running))
25047             return __Pyx_Coroutine_AlreadyRunningError(gen);
25048         if (yf) {
25049             PyObject *ret;
25050             Py_INCREF(yf);
25051             if (__Pyx_PyErr_GivenExceptionMatches(typ, PyExc_GeneratorExit) && close_on_genexit) {
25052                 int err = __Pyx_Coroutine_CloseIter(gen, yf);
25053                 Py_DECREF(yf);
25054                 __Pyx_Coroutine_Undelegate(gen);
25055                 if (err < 0)
25056                     return __Pyx_Coroutine_MethodReturn(self, __Pyx_Coroutine_SendEx(gen, NULL, 0));
25057                 goto throw_here;
25058             }
25059             gen->is_running = 1;
25060             if (0)
25061                 #ifdef __Pyx_Generator_USED
25062                 || __Pyx_Generator_CheckExact(yf)
25063                 #endif
25064                 #ifdef __Pyx_Coroutine_USED
25065                 || __Pyx_Coroutine_Check(yf)
25066                 #endif
25067             ) {
25068                 ret = __Pyx__Coroutine_Throw(yf, typ, val, tb, args, close_on_genexit);
25069                 #ifdef __Pyx_Coroutine_USED
25070             } else if (__Pyx_CoroutineAwait_CheckExact(yf)) {
25071                 ret = __Pyx_Coroutine_Throw((__pyx_CoroutineAwaitObject*)yf)->coroutine, typ, val, tb,
args, close_on_genexit);
25072             #endif
25073             } else {
25074                 PyObject *meth = __Pyx_PyObject_GetAttrStr(yf, __pyx_n_s_throw);
25075                 if (unlikely(!meth)) {
25076                     Py_DECREF(yf);
25077                     if (!PyErr_ExceptionMatches(PyExc_AttributeError)) {
25078                         gen->is_running = 0;
25079                         return NULL;
25080                     }
25081                     PyErr_Clear();
25082                     __Pyx_Coroutine_Undelegate(gen);
25083                     gen->is_running = 0;
25084                     goto throw_here;
25085                 }
25086                 if (likely(args)) {

```

```

25087         ret = PyObject_CallObject(meth, args);
25088     } else {
25089         ret = PyObject_CallFunctionObjArgs(meth, typ, val, tb, NULL);
25090     }
25091     Py_DECREF(meth);
25092 }
25093 gen->is_running = 0;
25094 Py_DECREF(yf);
25095 if (!ret) {
25096     ret = __Pyx_Coroutine_FinishDelegation(gen);
25097 }
25098 return __Pyx_Coroutine_MethodReturn(self, ret);
25099 }
25100 throw_here:
25101     __Pyx_Raise(typ, val, tb, NULL);
25102     return __Pyx_Coroutine_MethodReturn(self, __Pyx_Coroutine_SendEx(gen, NULL, 0));
25103 }
25104 static PyObject *__Pyx_Coroutine_Throw(PyObject *self, PyObject *args) {
25105     PyObject *typ;
25106     PyObject *val = NULL;
25107     PyObject *tb = NULL;
25108     if (!PyArg_UnpackTuple(args, (char *)"throw", 1, 3, &typ, &val, &tb))
25109         return NULL;
25110     return __Pyx_Coroutine_Throw(self, typ, val, tb, args, 1);
25111 }
25112 static CYTHON_INLINE int __Pyx_Coroutine_traverse_excstate(__Pyx_ExcInfoStruct *exc_state, visitproc
visit, void *arg) {
25113     Py_VISIT(exc_state->exc_type);
25114     Py_VISIT(exc_state->exc_value);
25115     Py_VISIT(exc_state->exc_traceback);
25116     return 0;
25117 }
25118 static int __Pyx_Coroutine_traverse(__pyx_CoroutineObject *gen, visitproc visit, void *arg) {
25119     Py_VISIT(gen->closure);
25120     Py_VISIT(gen->classobj);
25121     Py_VISIT(gen->yieldfrom);
25122     return __Pyx_Coroutine_traverse_excstate(&gen->gi_exc_state, visit, arg);
25123 }
25124 static int __Pyx_Coroutine_clear(PyObject *self) {
25125     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25126     Py_CLEAR(gen->closure);
25127     Py_CLEAR(gen->classobj);
25128     Py_CLEAR(gen->yieldfrom);
25129     __Pyx_Coroutine_ExceptionClear(&gen->gi_exc_state);
25130 #ifdef __Pyx_AsyncGen_USED
25131     if (__Pyx_AsyncGen_CheckExact(self)) {
25132         Py_CLEAR(((__pyx_PyAsyncGenObject*)gen)->ag_finalizer);
25133     }
25134 #endif
25135     Py_CLEAR(gen->gi_code);
25136     Py_CLEAR(gen->gi_frame);
25137     Py_CLEAR(gen->gi_name);
25138     Py_CLEAR(gen->gi_qualname);
25139     Py_CLEAR(gen->gi_modulename);
25140     return 0;
25141 }
25142 static void __Pyx_Coroutine_dealloc(PyObject *self) {
25143     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25144     PyObject_GC_UnTrack(gen);
25145     if (gen->gi_weakreflist != NULL)
25146         PyObject_ClearWeakRefs(self);
25147     if (gen->resume_label >= 0) {
25148         PyObject_GC_Track(self);
25149 #if PY_VERSION_HEX >= 0x030400a1 && CYTHON_USE_TP_FINALIZE
25150         if (PyObject_CallFinalizerFromDealloc(self))
25151 #else
25152         Py_TYPE(gen)->tp_del(self);
25153         if (Py_REFCNT(self) > 0)
25154 #endif
25155         {
25156             return;
25157         }
25158         PyObject_GC_UnTrack(self);
25159     }
25160 #ifdef __Pyx_AsyncGen_USED
25161     if (__Pyx_AsyncGen_CheckExact(self)) {
25162         /* We have to handle this case for asynchronous generators
25163            right here, because this code has to be between UNTRACK
25164            and GC_Del. */
25165         Py_CLEAR(((__pyx_PyAsyncGenObject*)self)->ag_finalizer);
25166     }
25167 #endif
25168     __Pyx_Coroutine_clear(self);
25169     PyObject_GC_Del(gen);
25170 }
25171 static void __Pyx_Coroutine_del(PyObject *self) {
25172     PyObject *error_type, *error_value, *error_traceback;

```



```

25173     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25174     __Pyx_PyThreadState_declare
25175     if (gen->resume_label < 0) {
25176         return;
25177     }
25178     #if !CYTHON_USE_TP_FINALIZE
25179     assert(self->ob_refcnt == 0);
25180     __Pyx_SET_REFCNT(self, 1);
25181     #endif
25182     __Pyx_PyThreadState_assign
25183     __Pyx_ErrFetch(&error_type, &error_value, &error_traceback);
25184     #ifndef __Pyx_AsyncGen_USED
25185     if (__Pyx_AsyncGen_CheckExact(self)) {
25186         __pyx_PyAsyncGenObject *agen = (__pyx_PyAsyncGenObject*)self;
25187         PyObject *finalizer = agen->ag_finalizer;
25188         if (finalizer && !agen->ag_closed) {
25189             PyObject *res = __Pyx_PyObject_CallOneArg(finalizer, self);
25190             if (unlikely(!res)) {
25191                 PyErr_WriteUnraisable(self);
25192             } else {
25193                 Py_DECREF(res);
25194             }
25195             __Pyx_ErrRestore(error_type, error_value, error_traceback);
25196             return;
25197         }
25198     }
25199     #endif
25200     if (unlikely(gen->resume_label == 0 && !error_value)) {
25201         #ifndef __Pyx_Coroutine_USED
25202         #ifndef __Pyx_Generator_USED
25203             if (!__Pyx_Generator_CheckExact(self))
25204             #endif
25205             {
25206                 PyObject_GC_UnTrack(self);
25207                 #if PY_MAJOR_VERSION >= 3 || defined(PyErr_WarnFormat)
25208                 if (unlikely(PyErr_WarnFormat(PyExc_RuntimeWarning, 1, "coroutine '%.50s' was never awaited",
25209                     gen->gi_qualname) < 0))
25209                     PyErr_WriteUnraisable(self);
25210                 #else
25211                 {PyObject *msg;
25212                 char *cmsg;
25213                 #if CYTHON_COMPILING_IN_PYPY
25214                 msg = NULL;
25215                 cmsg = (char*) "coroutine was never awaited";
25216                 #else
25217                 char *cname;
25218                 PyObject *qualname;
25219                 qualname = gen->gi_qualname;
25220                 cname = PyString_AS_STRING(qualname);
25221                 msg = PyString_FromFormat("coroutine '%.50s' was never awaited", cname);
25222                 if (unlikely(!msg)) {
25223                     PyErr_Clear();
25224                     cmsg = (char*) "coroutine was never awaited";
25225                 } else {
25226                     cmsg = PyString_AS_STRING(msg);
25227                 }
25228                 #endif
25229                 if (unlikely(PyErr_WarnEx(PyExc_RuntimeWarning, cmsg, 1) < 0))
25230                     PyErr_WriteUnraisable(self);
25231                 Py_XDECREF(msg);
25232             }
25233             #endif
25234             PyObject_GC_Track(self);
25235         }
25236     } else {
25237         PyObject *res = __Pyx_Coroutine_Close(self);
25238         if (unlikely(!res)) {
25239             if (PyErr_Occurred())
25240                 PyErr_WriteUnraisable(self);
25241         } else {
25242             Py_DECREF(res);
25243         }
25244     }
25245     __Pyx_ErrRestore(error_type, error_value, error_traceback);
25246     #if !CYTHON_USE_TP_FINALIZE
25247     assert(Py_REFCNT(self) > 0);
25248     if (--self->ob_refcnt == 0) {
25249         return;
25250     }
25251     {
25252         Py_ssize_t refcnt = Py_REFCNT(self);
25253         __Pyx_NewReference(self);
25254         __Pyx_SET_REFCNT(self, refcnt);
25255     }
25256     #if CYTHON_COMPILING_IN_CPYTHON
25257     assert(PyType_IS_GC(Py_TYPE(self)) &&
25258         __Pyx_AS_GC(self)->gc.gc_refs != __PyGC_REFS_UNTRACKED);

```

```

25259     _Py_DEC_REFTOTAL;
25260 #endif
25261 #ifdef COUNT_ALLOCS
25262     --Py_TYPE(self)->tp_frees;
25263     --Py_TYPE(self)->tp_allocs;
25264 #endif
25265 #endif
25266 }
25267 static PyObject *
25268 __Pyx_Coroutine_get_name(__pyx_CoroutineObject *self, CYTHON_UNUSED void *context)
25269 {
25270     PyObject *name = self->gi_name;
25271     if (unlikely(!name)) name = Py_None;
25272     Py_INCREF(name);
25273     return name;
25274 }
25275 static int
25276 __Pyx_Coroutine_set_name(__pyx_CoroutineObject *self, PyObject *value, CYTHON_UNUSED void *context)
25277 {
25278     PyObject *tmp;
25279     #if PY_MAJOR_VERSION >= 3
25280     if (unlikely(value == NULL || !PyUnicode_Check(value)))
25281     #else
25282     if (unlikely(value == NULL || !PyString_Check(value)))
25283     #endif
25284     {
25285         PyErr_SetString(PyExc_TypeError,
25286             "__name__ must be set to a string object");
25287         return -1;
25288     }
25289     tmp = self->gi_name;
25290     Py_INCREF(value);
25291     self->gi_name = value;
25292     Py_XDECREF(tmp);
25293     return 0;
25294 }
25295 static PyObject *
25296 __Pyx_Coroutine_get_qualname(__pyx_CoroutineObject *self, CYTHON_UNUSED void *context)
25297 {
25298     PyObject *name = self->gi_qualname;
25299     if (unlikely(!name)) name = Py_None;
25300     Py_INCREF(name);
25301     return name;
25302 }
25303 static int
25304 __Pyx_Coroutine_set_qualname(__pyx_CoroutineObject *self, PyObject *value, CYTHON_UNUSED void
25305 *context)
25306 {
25307     PyObject *tmp;
25308     #if PY_MAJOR_VERSION >= 3
25309     if (unlikely(value == NULL || !PyUnicode_Check(value)))
25310     #else
25311     if (unlikely(value == NULL || !PyString_Check(value)))
25312     #endif
25313     {
25314         PyErr_SetString(PyExc_TypeError,
25315             "__qualname__ must be set to a string object");
25316         return -1;
25317     }
25318     tmp = self->gi_qualname;
25319     Py_INCREF(value);
25320     self->gi_qualname = value;
25321     Py_XDECREF(tmp);
25322     return 0;
25323 }
25324 static PyObject *
25325 __Pyx_Coroutine_get_frame(__pyx_CoroutineObject *self, CYTHON_UNUSED void *context)
25326 {
25327     PyObject *frame = self->gi_frame;
25328     if (!frame) {
25329         if (unlikely(!self->gi_code)) {
25330             Py_RETURN_NONE;
25331         }
25332         frame = (PyObject *) PyFrame_New(
25333             PyThreadState_Get(), /*PyThreadState *tstate,*/
25334             (PyCodeObject*) self->gi_code, /*PyCodeObject *code,*/
25335             __pyx_d, /*PyObject *globals,*/
25336             0 /*PyObject *locals*/
25337         );
25338         if (unlikely(!frame))
25339             return NULL;
25340         self->gi_frame = frame;
25341     }
25342     Py_INCREF(frame);
25343     return frame;
25344 }
25345 static __pyx_CoroutineObject * __Pyx__Coroutine_New(

```

```

25345         PyTypeObject* type, __pyx_coroutine_body_t body, PyObject *code, PyObject *closure,
25346         PyObject *name, PyObject *qualname, PyObject *module_name) {
25347     __pyx_CoroutineObject *gen = PyObject_GC_New(__pyx_CoroutineObject, type);
25348     if (unlikely(!gen))
25349         return NULL;
25350     return __Pyx_Coroutine_NewInit(gen, body, code, closure, name, qualname, module_name);
25351 }
25352 static __pyx_CoroutineObject *__Pyx__Coroutine_NewInit(
25353     __pyx_CoroutineObject *gen, __pyx_coroutine_body_t body, PyObject *code, PyObject
25354     *closure,
25355     PyObject *name, PyObject *qualname, PyObject *module_name) {
25356     gen->body = body;
25357     gen->closure = closure;
25358     Py_XINCREF(closure);
25359     gen->is_running = 0;
25360     gen->resume_label = 0;
25361     gen->classobj = NULL;
25362     gen->yieldfrom = NULL;
25363     gen->gi_exc_state.exc_type = NULL;
25364     gen->gi_exc_state.exc_value = NULL;
25365     gen->gi_exc_state.exc_traceback = NULL;
25366     #if CYTHON_USE_EXC_INFO_STACK
25367     gen->gi_exc_state.previous_item = NULL;
25368     #endif
25369     gen->gi_weakreflist = NULL;
25370     Py_XINCREF(qualname);
25371     gen->gi_qualname = qualname;
25372     Py_XINCREF(name);
25373     gen->gi_name = name;
25374     Py_XINCREF(module_name);
25375     gen->gi_module_name = module_name;
25376     Py_XINCREF(code);
25377     gen->gi_code = code;
25378     gen->gi_frame = NULL;
25379     PyObject_GC_Track(gen);
25380     return gen;
25381 }
25382 /* PatchModuleWithCoroutine */
25383 static PyObject* __Pyx_Coroutine_patch_module(PyObject* module, const char* py_code) {
25384     #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
25385     int result;
25386     PyObject *globals, *result_obj;
25387     globals = PyDict_New(); if (unlikely(!globals)) goto ignore;
25388     result = PyDict_SetItemString(globals, "_cython_coroutine_type",
25389     #ifdef __Pyx_Coroutine_USED
25390         (PyObject*)__pyx_CoroutineType);
25391     #else
25392         Py_None);
25393     #endif
25394     if (unlikely(result < 0)) goto ignore;
25395     result = PyDict_SetItemString(globals, "_cython_generator_type",
25396     #ifdef __Pyx_Generator_USED
25397         (PyObject*)__pyx_GeneratorType);
25398     #else
25399         Py_None);
25400     #endif
25401     if (unlikely(result < 0)) goto ignore;
25402     if (unlikely(PyDict_SetItemString(globals, "_module", module) < 0)) goto ignore;
25403     if (unlikely(PyDict_SetItemString(globals, "__builtins__", __pyx_b) < 0)) goto ignore;
25404     result_obj = PyRun_String(py_code, Py_file_input, globals, globals);
25405     if (unlikely(!result_obj)) goto ignore;
25406     Py_DECREF(result_obj);
25407     Py_DECREF(globals);
25408     return module;
25409 ignore:
25410     Py_XDECREF(globals);
25411     PyErr_WriteUnraisable(module);
25412     if (unlikely(PyErr_WarnEx(PyExc_RuntimeWarning, "Cython module failed to patch module with custom
25413     type", 1) < 0)) {
25414         Py_DECREF(module);
25415         module = NULL;
25416     }
25417     #else
25418     py_code++;
25419     #endif
25420     return module;
25421 }
25422 /* PatchGeneratorABC */
25423 #ifndef CYTHON_REGISTER_ABCS
25424 #define CYTHON_REGISTER_ABCS 1
25425 #endif
25426 #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
25427 static PyObject* __Pyx_patch_abc_module(PyObject *module);
25428 static PyObject* __Pyx_patch_abc_module(PyObject *module) {
25429     module = __Pyx_Coroutine_patch_module(

```

```

25430     module, ""
25431 "if _cython_generator_type is not None:\n"
25432 "    try: Generator = _module.Generator\n"
25433 "    except AttributeError: pass\n"
25434 "    else: Generator.register(_cython_generator_type)\n"
25435 "if _cython_coroutine_type is not None:\n"
25436 "    try: Coroutine = _module.Coroutine\n"
25437 "    except AttributeError: pass\n"
25438 "    else: Coroutine.register(_cython_coroutine_type)\n"
25439 );
25440 return module;
25441 }
25442 #endif
25443 static int __Pyx_patch_abc(void) {
25444 #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
25445     static int abc_patched = 0;
25446     if (CYTHON_REGISTER_ABCS && !abc_patched) {
25447         PyObject *module;
25448         module = PyImport_ImportModule((PY_MAJOR_VERSION >= 3) ? "collections.abc" : "collections");
25449         if (!module) {
25450             PyErr_WriteUnraisable(NULL);
25451             if (unlikely(PyErr_WarnEx(PyExc_RuntimeWarning,
25452                                     ((PY_MAJOR_VERSION >= 3) ?
25453                                      "Cython module failed to register with collections.abc module" :
25454                                      "Cython module failed to register with collections module"), 1) < 0)) {
25455                 return -1;
25456             }
25457         } else {
25458             module = __Pyx_patch_abc_module(module);
25459             abc_patched = 1;
25460             if (unlikely(!module))
25461                 return -1;
25462             Py_DECREF(module);
25463         }
25464         module = PyImport_ImportModule("backports_abc");
25465         if (module) {
25466             module = __Pyx_patch_abc_module(module);
25467             Py_XDECREF(module);
25468         }
25469         if (!module) {
25470             PyErr_Clear();
25471         }
25472     }
25473 #else
25474     if ((0)) __Pyx_Coroutine_patch_module(NULL, NULL);
25475 #endif
25476     return 0;
25477 }
25478
25479 /* Generator */
25480 static PyMethodDef __pyx_Generator_methods[] = {
25481     {"send", (PyCFunction) __Pyx_Coroutine_Send, METH_O,
25482      (char*) PyDoc_STR("send(arg) -> send 'arg' into generator,\nreturn next yielded value or raise\nStopIteration.")},
25483     {"throw", (PyCFunction) __Pyx_Coroutine_Throw, METH_VARARGS,
25484      (char*) PyDoc_STR("throw(typ[,val[,tb]]) -> raise exception in generator,\nreturn next yielded\nvalue or raise StopIteration.")},
25485     {"close", (PyCFunction) __Pyx_Coroutine_Close_Method, METH_NOARGS,
25486      (char*) PyDoc_STR("close() -> raise GeneratorExit inside generator.")},
25487     {0, 0, 0, 0}
25488 };
25489 static PyMemberDef __pyx_Generator_memberlist[] = {
25490     {(char *) "gi_running", T_BOOL, offsetof(__pyx_CoroutineObject, is_running), READONLY, NULL},
25491     {(char *) "gi_yieldfrom", T_OBJECT, offsetof(__pyx_CoroutineObject, yieldfrom), READONLY,
25492      (char*) PyDoc_STR("object being iterated by 'yield from', or None")},
25493     {(char *) "gi_code", T_OBJECT, offsetof(__pyx_CoroutineObject, gi_code), READONLY, NULL},
25494     {0, 0, 0, 0, 0}
25495 };
25496 static PyGetSetDef __pyx_Generator_getsets[] = {
25497     {(char *) "__name__", (getter) __Pyx_Coroutine_get_name, (setter) __Pyx_Coroutine_set_name,
25498      (char*) PyDoc_STR("name of the generator"), 0},
25499     {(char *) "__qualname__", (getter) __Pyx_Coroutine_get_qualname,
25500      (setter) __Pyx_Coroutine_set_qualname,
25501      (char*) PyDoc_STR("qualified name of the generator"), 0},
25502     {(char *) "gi_frame", (getter) __Pyx_Coroutine_get_frame, NULL,
25503      (char*) PyDoc_STR("Frame of the generator"), 0},
25504     {0, 0, 0, 0, 0}
25505 };
25506 static PyTypeObject __pyx_GeneratorType_type = {
25507     PyVarObject_HEAD_INIT(0, 0)
25508     "generator",
25509     sizeof(__pyx_CoroutineObject),
25510     0,
25511     (destructor) __Pyx_Coroutine_dealloc,
25512     0,
25513     0,

```

```

25514     0,
25515     0,
25516     0,
25517     0,
25518     0,
25519     0,
25520     0,
25521     0,
25522     0,
25523     0,
25524     0,
25525     Py_TPFLAGS_DEFAULT | Py_TPFLAGS_HAVE_GC | Py_TPFLAGS_HAVE_FINALIZE,
25526     0,
25527     (traverseproc) __Pyx_Coroutine_traverse,
25528     0,
25529     0,
25530     offsetof(__pyx_CoroutineObject, gi_weakreflist),
25531     0,
25532     (iternextfunc) __Pyx_Generator_Next,
25533     __pyx_Generator_methods,
25534     __pyx_Generator_memberlist,
25535     __pyx_Generator_getsets,
25536     0,
25537     0,
25538     0,
25539     0,
25540     0,
25541     0,
25542     0,
25543     0,
25544     0,
25545     0,
25546     0,
25547     0,
25548     0,
25549     0,
25550     0,
25551     #if CYTHON_USE_TP_FINALIZE
25552         0,
25553     #else
25554         __Pyx_Coroutine_del,
25555     #endif
25556     0,
25557     #if CYTHON_USE_TP_FINALIZE
25558         __Pyx_Coroutine_del,
25559     #elif PY_VERSION_HEX >= 0x030400a1
25560         0,
25561     #endif
25562     #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM >= 0x07030800)
25563         0,
25564     #endif
25565     #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
25566         0,
25567     #endif
25568     #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
25569         0,
25570     #endif
25571 };
25572 static int __pyx_Generator_init(void) {
25573     __pyx_GeneratorType_type.tp_getattro = __Pyx_PyObject_GenericGetAttrNoDict;
25574     __pyx_GeneratorType_type.tp_iter = PyObject_SelfIter;
25575     __pyx_GeneratorType = __Pyx_FetchCommonType(&__pyx_GeneratorType_type);
25576     if (unlikely(!__pyx_GeneratorType)) {
25577         return -1;
25578     }
25579     return 0;
25580 }
25581
25582 /* CheckBinaryVersion */
25583 static int __Pyx_check_binary_version(void) {
25584     char ctversion[4], rtversion[4];
25585     PyOS_snprintf(ctversion, 4, "%d.%d", PY_MAJOR_VERSION, PY_MINOR_VERSION);
25586     PyOS_snprintf(rtversion, 4, "%s", Py_GetVersion());
25587     if (ctversion[0] != rtversion[0] || ctversion[2] != rtversion[2]) {
25588         char message[200];
25589         PyOS_snprintf(message, sizeof(message),
25590             "compiletime version %s of module '%.100s' "
25591             "does not match runtime version %s",
25592             ctversion, __Pyx_MODULE_NAME, rtversion);
25593         return PyErr_WarnEx(NULL, message, 1);
25594     }
25595     return 0;
25596 }
25597
25598 /* InitStrings */
25599 static int __Pyx_InitStrings(__Pyx_StringTabEntry *t) {
25600     while (t->p) {

```

```

25601     #if PY_MAJOR_VERSION < 3
25602     if (t->is_unicode) {
25603         *t->p = PyUnicode_DecodeUTF8(t->s, t->n - 1, NULL);
25604     } else if (t->intern) {
25605         *t->p = PyString_InternFromString(t->s);
25606     } else {
25607         *t->p = PyString_FromStringAndSize(t->s, t->n - 1);
25608     }
25609     #else
25610     if (t->is_unicode | t->is_str) {
25611         if (t->intern) {
25612             *t->p = PyUnicode_InternFromString(t->s);
25613         } else if (t->encoding) {
25614             *t->p = PyUnicode_Decode(t->s, t->n - 1, t->encoding, NULL);
25615         } else {
25616             *t->p = PyUnicode_FromStringAndSize(t->s, t->n - 1);
25617         }
25618     } else {
25619         *t->p = PyBytes_FromStringAndSize(t->s, t->n - 1);
25620     }
25621     #endif
25622     if (!t->p)
25623         return -1;
25624     if (PyObject_Hash(*t->p) == -1)
25625         return -1;
25626     ++t;
25627 }
25628 return 0;
25629 }
25630
25631 static CYTHON_INLINE PyObject* __Pyx_PyUnicode_FromString(const char* c_str) {
25632     return __Pyx_PyUnicode_FromStringAndSize(c_str, (Py_ssize_t)strlen(c_str));
25633 }
25634 static CYTHON_INLINE const char* __Pyx_PyObject_AsString(PyObject* o) {
25635     Py_ssize_t ignore;
25636     return __Pyx_PyObject_AsStringAndSize(o, &ignore);
25637 }
25638 #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII || __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT
25639 #if !CYTHON_PEP393_ENABLED
25640 static const char* __Pyx_PyUnicode_AsStringAndSize(PyObject* o, Py_ssize_t *length) {
25641     char* defenc_c;
25642     PyObject* defenc = _PyUnicode_AsDefaultEncodedString(o, NULL);
25643     if (!defenc) return NULL;
25644     defenc_c = PyBytes_AS_STRING(defenc);
25645     #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
25646     {
25647         char* end = defenc_c + PyBytes_GET_SIZE(defenc);
25648         char* c;
25649         for (c = defenc_c; c < end; c++) {
25650             if ((unsigned char) (*c) >= 128) {
25651                 PyUnicode_AsASCIIString(o);
25652                 return NULL;
25653             }
25654         }
25655     }
25656     #endif
25657     *length = PyBytes_GET_SIZE(defenc);
25658     return defenc_c;
25659 }
25660 #else
25661 static CYTHON_INLINE const char* __Pyx_PyUnicode_AsStringAndSize(PyObject* o, Py_ssize_t *length) {
25662     if (unlikely(__Pyx_PyUnicode_READY(o) == -1)) return NULL;
25663     #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
25664     if (likely(PyUnicode_IS_ASCII(o))) {
25665         *length = PyUnicode_GET_LENGTH(o);
25666         return PyUnicode_AsUTF8(o);
25667     } else {
25668         PyUnicode_AsASCIIString(o);
25669         return NULL;
25670     }
25671     #else
25672     return PyUnicode_AsUTF8AndSize(o, length);
25673     #endif
25674 }
25675 #endif
25676 #endif
25677 static CYTHON_INLINE const char* __Pyx_PyObject_AsStringAndSize(PyObject* o, Py_ssize_t *length) {
25678     #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII || __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT
25679     if (
25680     #if PY_MAJOR_VERSION < 3 && __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
25681         __Pyx_sys_getdefaultencoding_not_ascii &&
25682     #endif
25683         PyUnicode_Check(o)) {
25684         return __Pyx_PyUnicode_AsStringAndSize(o, length);
25685     } else
25686     #endif
25687     #if (!CYTHON_COMPILING_IN_PYPY) || (defined(PyByteArray_AS_STRING) && defined(PyByteArray_GET_SIZE))

```

```

25688     if (PyByteArray_Check(o)) {
25689         *length = PyByteArray_GET_SIZE(o);
25690         return PyByteArray_AS_STRING(o);
25691     } else
25692 #endif
25693     {
25694         char* result;
25695         int r = PyBytes_AsStringAndSize(o, &result, length);
25696         if (unlikely(r < 0)) {
25697             return NULL;
25698         } else {
25699             return result;
25700         }
25701     }
25702 }
25703 static CYTHON_INLINE int __Pyx_PyObject_IsTrue(PyObject* x) {
25704     int is_true = x == Py_True;
25705     if (is_true | (x == Py_False) | (x == Py_None)) return is_true;
25706     else return PyObject_IsTrue(x);
25707 }
25708 static CYTHON_INLINE int __Pyx_PyObject_IsTrueAndDecref(PyObject* x) {
25709     int retval;
25710     if (unlikely(!x)) return -1;
25711     retval = __Pyx_PyObject_IsTrue(x);
25712     Py_DECREF(x);
25713     return retval;
25714 }
25715 static PyObject* __Pyx_PyNumber_IntOrLongWrongResultType(PyObject* result, const char* type_name) {
25716 #if PY_MAJOR_VERSION >= 3
25717     if (PyLong_Check(result)) {
25718         if (PyErr_WarnFormat(PyExc_DeprecationWarning, 1,
25719             "__int__ returned non-int (type %.200s). "
25720             "The ability to return an instance of a strict subclass of int "
25721             "is deprecated, and may be removed in a future version of Python.",
25722             Py_TYPE(result)->tp_name)) {
25723             Py_DECREF(result);
25724             return NULL;
25725         }
25726         return result;
25727     }
25728 #endif
25729     PyErr_Format(PyExc_TypeError,
25730         "__%.4s__ returned non-%.4s (type %.200s)",
25731         type_name, type_name, Py_TYPE(result)->tp_name);
25732     Py_DECREF(result);
25733     return NULL;
25734 }
25735 static CYTHON_INLINE PyObject* __Pyx_PyNumber_IntOrLong(PyObject* x) {
25736 #if CYTHON_USE_TYPE_SLOTS
25737     PyNumberMethods *m;
25738 #endif
25739     const char *name = NULL;
25740     PyObject *res = NULL;
25741 #if PY_MAJOR_VERSION < 3
25742     if (likely(PyInt_Check(x) || PyLong_Check(x)))
25743 #else
25744     if (likely(PyLong_Check(x)))
25745 #endif
25746     return __Pyx_NewRef(x);
25747 #if CYTHON_USE_TYPE_SLOTS
25748     m = Py_TYPE(x)->tp_as_number;
25749     #if PY_MAJOR_VERSION < 3
25750     if (m && m->nb_int) {
25751         name = "int";
25752         res = m->nb_int(x);
25753     }
25754     else if (m && m->nb_long) {
25755         name = "long";
25756         res = m->nb_long(x);
25757     }
25758     #else
25759     if (likely(m && m->nb_int)) {
25760         name = "int";
25761         res = m->nb_int(x);
25762     }
25763     #endif
25764 #else
25765     if (!PyBytes_CheckExact(x) && !PyUnicode_CheckExact(x)) {
25766         res = PyNumber_Int(x);
25767     }
25768 #endif
25769     if (likely(res)) {
25770 #if PY_MAJOR_VERSION < 3
25771         if (unlikely(!PyInt_Check(res) && !PyLong_Check(res))) {
25772             #else
25773             if (unlikely(!PyLong_CheckExact(res))) {
25774 #endif

```

```

25775         return __Pyx_PyNumber_IntOrLongWrongResultType(res, name);
25776     }
25777 }
25778 else if (!PyErr_Occurred()) {
25779     PyErr_SetString(PyExc_TypeError,
25780         "an integer is required");
25781 }
25782 return res;
25783 }
25784 static CYTHON_INLINE Py_ssize_t __Pyx_PyIndex_AsSsize_t(PyObject* b) {
25785     Py_ssize_t ival;
25786     PyObject *x;
25787 #if PY_MAJOR_VERSION < 3
25788     if (likely(PyInt_CheckExact(b))) {
25789         if (sizeof(Py_ssize_t) >= sizeof(long))
25790             return PyInt_AS_LONG(b);
25791         else
25792             return PyInt_AsSsize_t(b);
25793     }
25794 #endif
25795     if (likely(PyLong_CheckExact(b))) {
25796         #if CYTHON_USE_PYLONG_INTERNALS
25797             const digit* digits = ((PyLongObject*)b)->ob_digit;
25798             const Py_ssize_t size = Py_SIZE(b);
25799             if (likely(__Pyx_sst_abs(size) <= 1)) {
25800                 ival = likely(size) ? digits[0] : 0;
25801                 if (size == -1) ival = -ival;
25802                 return ival;
25803             } else {
25804                 switch (size) {
25805                     case 2:
25806                         if (8 * sizeof(Py_ssize_t) > 2 * PyLong_SHIFT) {
25807                             return (Py_ssize_t) (((size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]);
25808                         }
25809                         break;
25810                     case -2:
25811                         if (8 * sizeof(Py_ssize_t) > 2 * PyLong_SHIFT) {
25812                             return -(Py_ssize_t) (((size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]);
25813                         }
25814                         break;
25815                     case 3:
25816                         if (8 * sizeof(Py_ssize_t) > 3 * PyLong_SHIFT) {
25817                             return (Py_ssize_t) (((size_t)digits[2]) << PyLong_SHIFT) | (size_t)digits[1]) <<
PyLong_SHIFT) | (size_t)digits[0]);
25818                         }
25819                         break;
25820                     case -3:
25821                         if (8 * sizeof(Py_ssize_t) > 3 * PyLong_SHIFT) {
25822                             return -(Py_ssize_t) (((size_t)digits[2]) << PyLong_SHIFT) | (size_t)digits[1]) <<
PyLong_SHIFT) | (size_t)digits[0]);
25823                         }
25824                         break;
25825                     case 4:
25826                         if (8 * sizeof(Py_ssize_t) > 4 * PyLong_SHIFT) {
25827                             return (Py_ssize_t) (((size_t)digits[3]) << PyLong_SHIFT) | (size_t)digits[2]) <<
PyLong_SHIFT) | (size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]);
25828                         }
25829                         break;
25830                     case -4:
25831                         if (8 * sizeof(Py_ssize_t) > 4 * PyLong_SHIFT) {
25832                             return -(Py_ssize_t) (((size_t)digits[3]) << PyLong_SHIFT) | (size_t)digits[2]) <<
PyLong_SHIFT) | (size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]);
25833                         }
25834                         break;
25835                 }
25836             }
25837         #endif
25838         return PyLong_AsSsize_t(b);
25839     }
25840     x = PyNumber_Index(b);
25841     if (!x) return -1;
25842     ival = PyInt_AsSsize_t(x);
25843     Py_DECREF(x);
25844     return ival;
25845 }
25846 static CYTHON_INLINE Py_hash_t __Pyx_PyIndex_AsHash_t(PyObject* o) {
25847     if (sizeof(Py_hash_t) == sizeof(Py_ssize_t)) {
25848         return (Py_hash_t) __Pyx_PyIndex_AsSsize_t(o);
25849     } #if PY_MAJOR_VERSION < 3
25850     } else if (likely(PyInt_CheckExact(o))) {
25851         return PyInt_AS_LONG(o);
25852     } #endif
25853     } else {
25854         Py_ssize_t ival;
25855         PyObject *x;
25856         x = PyNumber_Index(o);
25857         if (!x) return -1;

```



```

25858     ival = PyInt_AsLong(x);
25859     Py_DECREF(x);
25860     return ival;
25861 }
25862 }
25863 static CYTHON_INLINE PyObject * __Pyx_PyBool_FromLong(long b) {
25864     return b ? __Pyx_NewRef(Py_True) : __Pyx_NewRef(Py_False);
25865 }
25866 static CYTHON_INLINE PyObject * __Pyx_PyInt_FromSize_t(size_t ival) {
25867     return PyInt_FromSize_t(ival);
25868 }
25869
25870
25871 #endif /* Py_PYTHON_H */

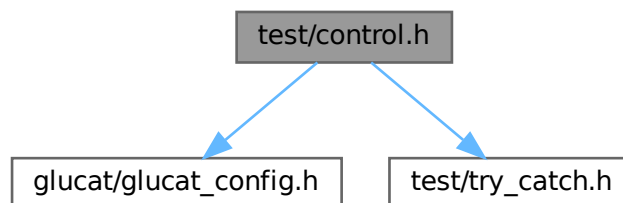
```

## 7.65 test/control.h File Reference

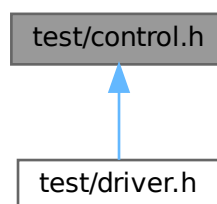
```
#include "glucat/glucat_config.h"
```

```
#include "test/try_catch.h"
```

Include dependency graph for control.h:



This graph shows which files directly or indirectly include this file:



### Classes

- class [glucat::control\\_t](#)

*Parameters to control tests.*

## Namespaces

- namespace `glucat`

## 7.66 control.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_CONTROL_H
00002 #define _GLUCAT_CONTROL_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     control.h : Define and set parameters to control tests
00006
00007     begin                : 2010-04-21
00008     copyright            : (C) 2010-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
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00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033 #include "glucat/glucat_config.h"
00034 #include "test/try_catch.h"
00035
00036 namespace glucat
00037 {
00038     class control_t
00039     {
00040     private:
00041         bool m_valid;
00042         bool valid() const
00043         { return m_valid; }
00044
00045         bool m_catch_exceptions;
00046         bool catch_exceptions() const
00047         { return m_catch_exceptions; }
00048
00049         static bool m_verbose_output;
00050
00051         control_t(int argc, char ** argv);
00052         // Enforce singleton
00053         // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00054         control_t() = default;
00055         ~control_t() = default;
00056         control_t(const control_t&) = delete;
00057         control_t& operator= (const control_t&) = delete;
00058
00059         friend class friend_for_private_destructor;
00060     public:
00061         static const control_t& control(int argc, char ** argv)
00062         { static const control_t c(argc, argv); return c; }
00063
00064         int call(intfn f) const;
00065         int call(intintfn f, int arg) const;
00066
00067         static bool verbose()
00068         { return m_verbose_output; }
00069     };
00070
00071     bool control_t::m_verbose_output = false;
00072
00073     control_t::

```

```

00089 control_t(int argc, char ** argv)
00090 : m_valid(true), m_catch_exceptions(true)
00091 {
00092     bool print_help = false;
00093     const std::string& arg_0_str = argv[0];
00094     const std::string program_name = arg_0_str.substr(arg_0_str.find_last_of('/')+1);
00095     for (int arg_ndx = 1; arg_ndx < argc; ++arg_ndx)
00096     {
00097         const std::string& arg_str = argv[arg_ndx];
00098         bool valid = false;
00099         if (arg_str.substr(0,2) == "--")
00100         {
00101             valid = true;
00102             const std::string& arg_name = arg_str.substr(2);
00103             if (arg_name == "help")
00104             {
00105                 this->m_valid = false;
00106                 print_help = true;
00107             }
00108             else if (arg_name == "verbose")
00109                 this->m_verbose_output = true;
00110             else if (arg_name == "no-catch")
00111                 this->m_catch_exceptions = false;
00112             else
00113                 valid = false;
00114         }
00115         if (!valid)
00116         {
00117             std::cout << "Invalid argument: " << arg_str << std::endl;
00118             this->m_valid = false;
00119             print_help = true;
00120         }
00121     }
00122     if (print_help)
00123     {
00124         std::cout << program_name << " for " << GLUCAT_PACKAGE_NAME << " version " << GLUCAT_VERSION << ":" <<
std::endl;
00125         std::cout << "Usage: " << program_name << " [option ...]" << std::endl;
00126         std::cout << "Options:" << std::endl;
00127         std::cout << "  --help      : Print this summary." << std::endl;
00128         std::cout << "  --no-catch  : Do not catch exceptions." << std::endl;
00129         std::cout << "  --verbose   : Produce more detailed test output." << std::endl;
00130     }
00131 }
00132
00133 inline
00134 int
00135 control_t::
00136 call(intfn f) const
00137 {
00138     {
00139         if (valid())
00140             return (catch_exceptions())
? try_catch(f)
: (*f)();
00141         else
00142             return 1;
00143     }
00144 }
00145
00146 inline
00147 int
00148 control_t::
00149 call(intintfn f, int arg) const
00150 {
00151     {
00152         if (valid())
00153             return (catch_exceptions())
? try_catch(f, arg)
: (*f)(arg);
00154         else
00155             return 1;
00156     }
00157 }
00158 }
00159 }
00160 }
00161 #endif // _GLUCAT_CONTROL_H

```

## 7.67 test/driver.h File Reference

```

#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include "test/tuning.h"
#include "test/try_catch.h"

```

```
#include "test/control.h"
#include <stdio>
Include dependency graph for driver.h:
```



## 7.68 driver.h

[Go to the documentation of this file.](#)

```
00001 #ifndef GLUCAT_TEST_DRIVER_H
00002 #define GLUCAT_TEST_DRIVER_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   driver.h : Header for example and timing test driver
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright             : (C) 2001-2021 by Paul C. Leopardi
00009   ****
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   ****
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030   ****
00031   See also Arvind Raja's original header comments in glucat.h
00032   ****/
00033
00034 #include "glucat/glucat.h"
00035 #include "glucat/glucat_imp.h"
00036 #include "test/tuning.h"
00037 #include "test/try_catch.h"
00038 #include "test/control.h"
00039 #include <stdio>
00040
00041 #endif // GLUCAT_TEST_DRIVER_H
```

## 7.69 test/timing.h File Reference

### Namespaces

- namespace [glucat](#)
- namespace [glucat::timing](#)

### Functions

- static double [glucat::timing::elapsed](#) (clock\_t cpu\_time)  
*Elapsed time in milliseconds.*

## Variables

- const double `glucat::timing::MS_PER_SEC` = 1000.0  
*Timing constant: milliseconds per second.*
- const double `glucat::timing::MS_PER_CLOCK` = `MS_PER_SEC` / `double(CLOCKS_PER_SEC)`  
*Timing constant: milliseconds per clock.*
- const int `glucat::timing::EXTRA_TRIALS` = 2  
*Timing constant: trial expansion factor.*

## 7.70 timing.h

[Go to the documentation of this file.](#)

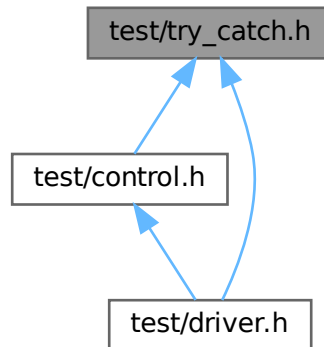
```

00001 #ifndef GLUCAT_TEST_TIMING_H
00002 #define GLUCAT_TEST_TIMING_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     timing.h : Common definitions for timing tests
00006     -----
00007     begin                : Tue 2012-03-27
00008     copyright            : (C) 2012 by Paul C. Leopardi
00009 *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
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00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 namespace glucat
00035 {
00036     namespace timing
00037     {
00038         const double MS_PER_SEC = 1000.0;
00039
00040         const double MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC);
00041
00042         const int EXTRA_TRIALS = 2;
00043
00044         inline
00045         static
00046         double
00047         elapsed(clock_t cpu_time)
00048         { return double(clock() - cpu_time) * MS_PER_CLOCK; }
00049     }
00050 }
00051 #endif // GLUCAT_TEST_TIMING_H

```

## 7.71 test/try\_catch.h File Reference

This graph shows which files directly or indirectly include this file:



### Namespaces

- namespace [glucat](#)

### Typedefs

- typedef int(\* [glucat::intfn](#)) ()  
*For exception catching: pointer to function returning int.*
- typedef int(\* [glucat::intintfn](#)) (int)  
*For exception catching: pointer to function of int returning int.*

### Functions

- int [glucat::try\\_catch](#) ([intfn](#) f)  
*Exception catching for functions returning int.*
- int [glucat::try\\_catch](#) ([intintfn](#) f, int arg)  
*Exception catching for functions of int returning int.*

## 7.72 try\_catch.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_TRY_CATCH_H
00002 #define _GLUCAT_TRY_CATCH_H
00003 /*****
00004      GluCat : Generic library of universal Clifford algebra templates
00005      try_catch.h : Catch exceptions
00006      -----
00007      begin                : Sun 2001-12-20
00008      copyright            : (C) 2001-2010 by Paul C. Leopardi
  
```

```

00009 *****
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031     See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 namespace glucat
00035 {
00037     typedef int (*intfn)();
00038
00040     typedef int (*intintfn)(int);
00041
00043     int try_catch(intfn f);
00044
00046     int try_catch(intintfn f, int arg);
00047
00049     int try_catch(intfn f)
00050     {
00051         int result = 0;
00052         try
00053         { result = (*f)(); }
00054         catch (const glucat_error& e)
00055         { e.print_error_msg(); }
00056         catch (const std::bad_alloc& e)
00057         { std::cerr << "bad_alloc" << std::endl; }
00058         catch (...)
00059         { std::cerr << "unexpected exception" << std::endl; }
00060         return result;
00061     }
00062
00064     int try_catch(intintfn f, int arg)
00065     {
00066         int result = 0;
00067         try
00068         { result = (*f)(arg); }
00069         catch (const glucat_error& e)
00070         { e.print_error_msg(); }
00071         catch (const std::bad_alloc& e)
00072         { std::cerr << "bad_alloc" << std::endl; }
00073         catch (...)
00074         { std::cerr << "unexpected exception" << std::endl; }
00075         return result;
00076     }
00077 }
00078 #endif // _GLUCAT_TRY_CATCH_H

```





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